

Rapid Optimal Lag Order Detection and Parameter Estimation of Standard Long Memory Time Series

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Abstract Objective of this paper is to highlight the rapid assessment (in a few minutes) of fractionally differenced standard long memory time series in terms of parameter estimation and optimal lag order assessment. Initially, theoretical aspects of standard fractionally differenced processes with long memory and related state space modelling will be discussed. An efficient mechanism based on theory to estimate parameters and detect optimal lag order in minimizing processing speed and turnaround time is introduced subsequently. The methodology is extended using an available result in literature to present rapid results of an optimal fractionally differenced standard long memory model. Finally, the technique is applied to a couple of real data applications illustrating its feasibility and importance.

Keywords ARFIMA process · Long memory · Fractional difference · Spectral density · Stationarity

1 Introduction

Background Information and Literature Review:

The stochastic analysis of time series began with the introduction of Autoregressive Moving Average (ARMA) model by [34], its popularization by [6] and subsequent developments of a number of path breaking research endeavours. In particular, in the early 1980s the introduction of long memory processes became an extensive practice among time series specialists and econometricians. In their papers [18, 23] proposed the class of fractionally integrated autoregressive moving average (ARFIMA or FARIMA) processes, extending the traditional autoregressive integrated moving average (ARIMA) series with a fractional degree of differencing. A hyperbolic decay of the autocorrelation function (acf), partial autocorrelation function (pacf) and an unbounded spectral density peak at or near the origin are two special characteristics of the ARFIMA family in contrast to exponential decay of the acf and a

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bounded spectrum at the origin in the traditional ARMA family. In addition to the mle (maximum likelihood estimation) approach, [16] have considered the estimation of parameters of ARFIMA using the frequency domain approach. References [11, 30, 31] have considered the estimation of ARFIMA parameters using the smoothed periodogram technique. Additional expositions presented in [1, 4, 5, 7, 10, 17, 29, 32] and references therein provide a comprehensive discussion about long memory series estimation. In yet another development, fractionally differenced long memory model parameters were estimated using maximum likelihood and least squares with their convergence rates, limiting distribution and strong consistency by [35]. Another interesting parameter assessment study employing state space modelling of ARFIMA series could be found in [19, 27].

An optimal lag order for the parent model of the ARFIMA series known as the Gegenbauer autoregressive moving average (GARMA) process driven by Gaussian white noise was established using state space modelling in [13] through the validation of the model mean square error (MSE) by the predictive accuracy. The technique was extended to a different dimension in [14] through the replacement of Gaussian white noise by heteroskedastic errors employing a Gegenbauer process highlighted in [12].

Unfortunately the fractionally differenced long memory model parameter assessment often has a high turnaround time or a very slow processing speed depending on the series length, lag order and the number of replications in delivering Monte Carlo evidence. In such a context a rapid mechanism to estimate model parameters and optimal lag order of even the simplest fractionally differenced standard long memory model in the form of an ARFIMA series is seemingly absent in the current literature. In lieu of it this paper addresses the void by presenting a mechanism that revolves around recent advancements in information technology in slashing the turnaround time and processing speed as an original contribution.

In summary, consideration of this paper will be given to a certain class of long memory ARFIMA processes generated by Gaussian white noise. In following it the paper will comprise of Sect. 2 providing preliminaries of fractionally differenced ARFIMA processes with long memory. ARFIMA processes and the use of truncated state space representations and Kalman Filter (KF) in estimating its long memory version will be considered in Sects. 3 and 4. The state space methodology will follow the work of [2, 3, 8, 9, 13, 15, 19, 21, 22, 27, 28]. This will be followed by Sect. 4 illustrating simulation results. Corroborating real data applications will be presented in Sects. 5 and 6 will comprise of concluding remarks.

2 Preliminaries

Certain preliminary definitions and concepts that are useful in comprehending the material in the subsequent sections of this paper are introduced next for clarity and completeness.

Definition 1.1

A stochastic process is a family of random variables $\{X_t\}$, indexed by a parameter t , where t belongs to some index set \mathcal{T} .

In terms of stochastic processes the concept of *stationarity* plays an important role in many applications.

Definition 1.2

A stochastic process $\{X_t; t \in \mathcal{T}\}$ is said to be strictly stationary if the probability distribution of the process is invariant under translation of the index, i.e., the joint probability distributions of $(X_{t_1}, \dots, X_{t_n})$ is identical to that of $(X_{t_1+k}, \dots, X_{t_n+k})$, for all $n \in \mathcal{Z}^+$ (Set of positive integers), $(t_1, \dots, t_n) \in \mathcal{T}$, $k \in \mathcal{Z}$ (set of integers). i.e.,

$$F(x_1, \dots, x_n; t_1, \dots, t_n) = F(x_1, \dots, x_n; t_1+k, \dots, t_n+k), \quad (1)$$

Definition 1.3

A stochastic process $\{X_t\}$ is said to be a Gaussian process if and only if the probability distribution associated with any set of time points is multivariate normal.

In particular, if the multivariate moments $E(X_{t_1}^{s_1} \dots X_{t_n}^{s_n})$ depend only on the time differences, the process is called stationary up to order s , when $s \leq s_1 + \dots + s_n$.

Note that, the second order stationarity is obtained by setting $s = 2$ and this weak stationarity asserts that the mean μ is a constant (i.e., independent of t) and the covariance function $\gamma_{t\tau}$ is dependent only on the time difference. That is,

$$E(X_t) = \mu, \text{ for all } t$$

and

$$\text{Cov}(X_t, X_\tau) = E[(X_t - \mu)(X_\tau - \mu)] = \gamma_{|t-\tau|}, \text{ for all } t, \tau.$$

Time difference $k = |t - \tau|$ is called the *lag*. The corresponding autocovariance function is denoted by γ_k .

Definition 1.4

The acf (autocorrelation function) of a stationary process $\{X_t\}$ is a function whose value at lag k is

$$\rho_k = \gamma_k / \gamma_0 = \text{Corr}(X_t, X_{t+k}), \text{ for all } t, k \in \mathcal{Z}, \quad (2)$$

Definition 1.5

The pacf (partial autocorrelation function) at lag k of a stationary process $\{X_t\}$ is the additional correlation between X_t and X_{t+k} when the linear dependence of X_{t+1} through to X_{t+k-1} is removed.

Definition 1.6

A time series is a set of observations on X_t , each being recorded at a specific time t , where $t \in (0, \infty)$.

Let $\{X_t\}$ be a stationary time series with autocovariance at lag k , $\gamma_k = \text{Cov}(X_t, X_{t+k})$, acf (autocorrelation at lag k) $\rho_k = \text{Corr}(X_t, X_{t+k})$ and the normalized spectrum or spectral density function (sdf), $f(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \rho_k e^{-i\omega k}$; $-\pi < \omega < \pi$, where ω is the Fourier frequency. There are two main types of time series uniquely identified

by the behaviour of ρ_k and $f(\omega)$. They are classified as *memory* types and their basic definitions in advance time series analysis are given as follows:

Short Memory: A stationary time series $\{X_t\}$ is *short memory* if $\sum |\rho_k| < \infty$. Then ρ_k decays exponentially depicting properties of $\rho_k \sim a^k$ for some $|a| < 1$ resulting in a finite spectrum at $\omega = 0$ or $\lim_{\omega \rightarrow 0} f(\omega)$ existing and being bounded.

A stationary ARMA process is, therefore, short memory.

Long Memory: A stationary time series $\{X_t\}$ is a standard *long memory* case if $\rho_k \sim k^{-d}$, $d > 0$ for large k and $\sum |\rho_k| = \infty$. Then ρ_k decays hyperbolically resulting in $\lim_{\omega \rightarrow 0} f(\omega)$ being unbounded at $\omega = 0$.

In her paper [20] provides a number of alternative characteristics of long memory processes. Interestingly, [33] introduced a characteristic-based clustering method to capture the characteristic of long-range dependence (self-similarity).

Note: Processes in which the decay of ρ_k takes a shape in between exponential and hyperbolic arcs are known as *intermediate memory*. It implies the acf plot of such a process will be neither exponential nor hyperbolic but corresponding to a curve in between the shapes.

Remark: Partial autocorrelation function (pacf) of each memory type will provide corresponding shapes related to that of the acf.

2.1 Fractionally Differenced Long Memory Processes

When a long memory process is subject to the technique of fractional differencing it becomes a fractionally differenced long memory series. Due to its importance in time series econometrics as a method, fractional differencing becomes the next topic of interest.

2.1.1 Fractional Differencing

Suppose that $\{Y_t\}$ is a long memory stationary time series with an unbounded spectrum at the origin. It can be shown that the time series Y_t can be transformed to a short memory series X_t through a fractional filter of the form

$$X_t = (1 - B)^d Y_t, \quad d \in (0, 0.5),$$

where d could take any real fractional value within the open interval $(0, 0.5)$ such that the full ARFIMA(p,d,q) model becomes

$$\phi(B)(1 - B)^d Y_t = \theta(B)\varepsilon_t, \quad (3)$$

where $\phi(B)$ and $\theta(B)$ are stationary AR(p) and invertible MA(q) operators, B the backshift operator such that $\phi(B) = 1 - \phi_1 B - \dots - \phi_p B^p$, $\theta(B) = 1 + \theta_1 B + \dots + \theta_q B^q$ have zeros outside the unit circle and $\{\varepsilon_t\} \sim WN(0, \sigma^2)$.

See for example [18, 23] for details.

The ARFIMA process is a special case of the GARMA(p,d,q) model given by

$$\phi(B)(1 - 2uB + B^2)^d Y_t = \theta(B)\varepsilon_t, \quad (4)$$

in which the polynomial index $u = 1$ reduces the Gegenbauer expression $(1 - 2uB + B^2)$ to $(1 - B)^{2d}$ resulting in a standard long memory model with a polynomial power term $2d$.

The above fractional differencing is used in long memory time series modelling and analysis.

Note: A fractionally differenced stationary ARFIMA process is considered long memory if the memory parameter $d \in (0, 0.5)$.

Remark: For convenience a fractionally differenced ARFIMA(0,d,0) series is considered hereafter in the analysis of the paper.

3 State Space Representation of an ARFIMA Time Series

Consider the Wold representation of a Gaussian ARFIMA(0,d,0) process with $\varepsilon_t \sim \text{NID}(0, \sigma^2)$ given by

$$X_t = \psi(B)\varepsilon_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}, \quad (5)$$

where $\psi_0 = 1$, $\sum_{j=0}^{\infty} \psi_j^2 = \infty$ and each of the coefficients are defined by the equation $\psi_j = \frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)}$.

Now the m th order moving average approximation to (5) is obtained by truncating the right hand side at lag m , such that

$$X_{t,m} = \sum_{j=0}^m \psi_j \varepsilon_{t-j}, \quad (6)$$

where $\{X_{t,m}\}$ is a truncated ARFIMA process that will vary with the chosen truncation lag order m , which is fixed and finite.

3.1 State Space Representation of ARFIMA Model

In this approach, a dynamic time series is transformed into a suitable equivalent system comprising of two fundamental (*Measurement/Observation* and *State/Transient*) equations. As has been shown in the literature, this equivalent state space representation is not unique for time series. It will be similar to the state space representation of the ARFIMA series of [9]. Following this approach, (6) is equivalent to:

$$\begin{aligned} X_{t,m} &= Z\alpha_t + \varepsilon_t, \\ \alpha_{t+1} &= T\alpha_t + H\varepsilon_t, \end{aligned} \quad (7)$$

where $\alpha_{t+1} = [X(t+1|t), X(t+2|t), X(t+3|t), \dots, X(t+m|t)]'$ is the $m \times 1$ state vector with elements

$\alpha_{j,t+1} = E(X_{t+j,m} | \mathcal{F}_t)$, $\mathcal{F}_t = \{X_{t,m}, X_{t-1,m}, \dots\}$, and

$$Z = [1, 0, \dots, 0], \quad T = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 \\ \vdots & \dots & \dots & 0 & 1 \\ 0 & 0 & \dots & \dots & 0 \end{bmatrix}, \quad H = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_m \end{bmatrix},$$

Z, T, H are suitably chosen matrices with dimensions $1 \times m, m \times m$, and $m \times 1$.

Vector α_t consists of stochastic elements that have evolved from the process. H consists of an m number of ψ_\bullet coefficients. The specification comprises of the initial state distribution, $\alpha_1 \sim N(a_1, P_1)$, where $a_1 = 0$ and P_1 is the Toeplitz matrix with elements $p_{hk} = \sum_j \psi_j \psi_{j+|h-k|}$, such that the state space configuration will be based on the Wold representation of (6).

Note: Similar results could be obtained using the corresponding $AR(m')$ approximation by truncating (5) such that $\pi(B) = (1 - B)^d \approx \sum_{j=0}^{m'} d_j \varepsilon_{t-j}$. See [9, 19] for a comparison of the two approximations in the fractionally integrated case.

The most popular algorithm utilized by state space modelling specialists for estimation and prediction is known as the KF and becomes the focal point of Sect. 3.2.

3.2 KF and Estimation Process

KF was introduced by [25, 26] to provide estimates of parameters in a state space model of a time series or a linear dynamic system disturbed by Gaussian white noise. Approximate maximum likelihood estimation and prediction of time series parameters can be executed by adopting a state space approach coupled with a set of recursions called the KF.

Due to the presence of stochastic elements in the system it uses a series of measurements observed over time containing random variations (noise) to return pseudo-innovations of the model in creating the pseudo log-likelihood and quasi profile likelihood functions. This gives the optimal quasi maximum likelihood estimates (QMLE's) of the model parameters as shown in Tables 1 and 2 in Sect. 4. Let $\{x_t, t = 1, \dots, n\}$ be a time series. The likelihood function of an approximating $MA(m)$ model could be evaluated using the KF set of recursions for $t = 1, \dots, n$:

Table 1 QMLE results due to the MA approximation

m	29	30	31	32	33	34	35
\hat{d}	0.2285	0.2288	0.2304	0.2317	0.2320	0.2376	0.2362
$\hat{\sigma}$	0.9664	0.9625	0.9622	0.9650	0.9649	0.9571	0.9660
Model-bias	-0.0051	-0.0087	-0.0074	-0.0033	-0.0031	-0.0053	0.0022

Table 2 QMLE results due to AR approximation

m	9	10	11	12	13
\hat{d}	0.2238	0.2223	0.2215	0.2222	0.2243
$\hat{\sigma}$	0.9638	0.9633	0.9643	0.9657	0.9662
Model-bias	-0.0124	-0.0144	-0.0142	-0.0121	-0.0095

$$\begin{aligned}
 v_t &= x_t - Z a_t, & f_t &= Z P_t Z', \\
 & & K_t &= (T P_t Z')/f_t, \\
 a_{t+1} &= T a_t + K_t v_t, & P_{t+1} &= T P_t T' + H H' - K_t K_t'/f_t,
 \end{aligned} \tag{8}$$

where K_t is the *Kalman gain* (which shows the effect of estimate of the previous state to the estimate of the current state), and f_t the prediction error variance

The KF returns the pseudo-innovations v_t , such that if the MA(m) approximation were the true model, $v_t \sim \text{NID}(0, \sigma^2 f_t)$, so that the quasi log-likelihood of (d, σ^2) is (apart from a constant term)

$$\ell(d, \sigma^2) = -\frac{1}{2} \left(n \ln \sigma^2 + \sum_{t=1}^n \ln f_t + \frac{1}{\sigma^2} \sum_{t=1}^n \frac{v_t^2}{f_t} \right). \tag{9}$$

The scale parameter σ^2 can be concentrated out of the likelihood function, so that

$$\hat{\sigma}^2 = \sum_t \frac{v_t^2}{f_t},$$

and the quasi profile log likelihood is

$$\ell_{\sigma^2}(d) = -\frac{1}{2} \left[n(\ln \hat{\sigma}^2 + 1) + \sum_{t=1}^n \ln f_t \right]. \tag{10}$$

The maximisation of (10) can be performed by a quasi-Newton algorithm, after a reparameterization which constrains d in the subset of $\mathcal{R}(0, 0.5)$. For convenience we use the following reparameterization: $\theta_1 = \exp(2d)/(1 + \exp(2d))$. Furthermore, a discussion about the KF formulation of the likelihood function can be found in [24]. The simulation results given next provides an interesting assessment.

4 Simulation Results

The Monte Carlo simulation experiment was based on the state space model of the previous section and was executed through the KF recursive algorithm, which is already established in the literature. To propel the speed of it Fast Fourier transforms (FFT) were utilized and to a certain degree it could also be classified as a hybrid model. It enabled to slash the processor speed and turnaround time illustrating the rapidity of optimal lag order detection and estimation in proposing a creative component. The programs were parallelized and run on high speed multiple servers with random access memory (RAM) capacities ranging from 24–1024 gigabytes using the MATLAB R2011b software version. QMLE's of d and σ^2 due to an approximate likelihood through a state space approach using MA and AR approximations for an ARFIMA model driven by Gaussian white noise are shown in Tables 1 and 2 utilizing fast FFT.

By using FFT to convert functions of time to functions of frequency, the above state space approach is illustrated by fitting a Gaussian ARFIMA process with model initial values of $d = 0.2$, $\sigma = 1$ and the results are as follows in terms of both the MA and AR approximations:

For this particular simulation the likelihood is monotonically increasing with m due to the use of a log likelihood function. Figure 1 displays the implied spectral density of $X_{t,m}$ corresponding to the above parameter estimates. For $m > 1$ they are characterised by a spectral peak around the frequency $\cos^{-1}0$ and are side lobes due to the truncation of the MA filter (A Fourier series oscillation overshoot near a discontinuity that does not die out with increasing frequency but approaches a finite limit known as *Gibbs phenomenon*). The autoregressive estimates do not suffer from the Gibbs phenomenon. It is illustrated in Fig. 2.

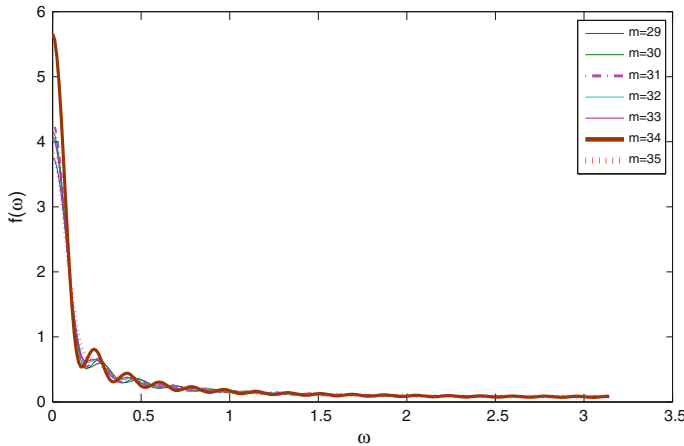


Fig. 1 Spectral density of an ARFIMA(0, d ,0) series using MA approximation

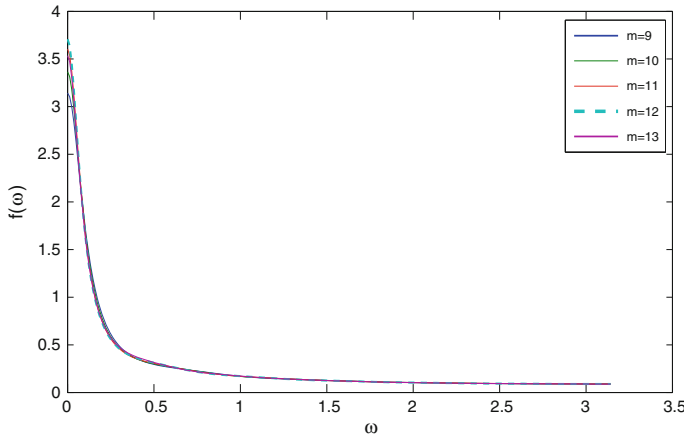


Fig. 2 Spectral density of an ARFIMA(0, d ,0) series using AR approximation

By recalling from Sect. 1 that the state space methodology presented in Sect. 3 was utilized by [13] for a GARMA(0, d ,0) series in concluding that the optimal lag order falls within [29, 35] for an MA and [9, 13] for an AR approximation. The result is applicable to the ARFIMA(0, d ,0) model considered in this paper, since it is a special case of the GARMA(0, d ,0) series (Refer: the explanation related to Eq. (2) in Sect. 2). By using the result and the minimum model-bias of estimators in Tables 1 and 2 it is clear that for the series generated through the MA approximation the **optimal lag order** is **30**, and for the series created through the AR approximation it is **10**. More importantly the processing or turnaround times taken to deliver the results were: **approximately 3 minutes** for the MA representation and **approximately 5 minutes** for the AR representation. It clearly proves the cost effectiveness of the process, since if run without the FFT's the processing time will vary between 45 min to 17 h depending on the utilized server, number of iterations and incorporated lags. The difference in processing turnaround times of the two estimation techniques (MA and AR) is due to the varying Monte Carlo error as explained in [13].

The simulation study of the MA approximation also revealed that the asymptotic variance of long memory parameter d was $\frac{\pi^2}{24n}$ independent of the series length (n) corroborating the result of [13]. These developments motivated the author to apply the methodology of this paper towards real applications involving Nile river outflow and Australia Consumer Price Index (ACPI) data, which depict characteristics of standard long memory (based on literature) and becomes the topic of the next section.

5 Empirical Evidence

Nile River outflow and ACPI data have been premier real data sets utilized by time series econometricians and statisticians over the years due to their close relationship with standard long memory. The chosen data sets have a significant impact in

Table 3 QMLE results for Nile River Data

Method	\hat{d}
MA-Approximation	0.291 (0.012)
AR-Approximation	0.278 (0.019)

Table 4 QMLE results for ACPI data

Method	\hat{d}
MA-Approximation	0.331 (0.009)
AR-Approximation	0.319 (0.015)

econometrics, since an assessment of the Nile river outflow is important to irrigation and agricultural production yields that affect the economies of many third world African nations, while an ACPI benchmark affects the economic stability of a developed first world country. The long memory feature becomes evident from the *sdf* plots of the datasets with infinite peaks close to the origin. Furthermore the *acf* and *pacf* plots depict long memory through hyperbolically decaying arcs. In lieu of it Nile River data from 1870 to 2011 and ACPI data from 3rd quarter, 1948 to 2nd quarter 2015 were considered and fitted to the hybrid ARFIMA(0, d ,0) state space model discussed in this paper. The datasets were downloaded from <https://datamarket.com> and <http://www.abs.gov.au> websites and the corresponding results are provided in Tables 3 and 4.

Note: The values in brackets right adjacent to the parameter estimates of d are the standard errors. From the estimate values it is evident that the long memory property is preserved since with both the approximations of the applications $0 < \hat{d} < 0.5$.

Concluding remarks are provided next based on the details and results of the preceding sections.

6 Concluding Remarks

The facts provided in Sects. 1–5 of this paper highlight and address various issues that are prevalent in the current literature. In terms of such issues one major void is the lack of a rapid estimation process for standard long memory models. In lieu of it a hybrid state space modelling technique based on the combined utilization of the KF and FFT is introduced as a novel contribution to the existing body of knowledge. Thereafter by employing the method and an existing result in the literature an optimal lag order is established for a standard long memory process by way of two distinct estimation techniques as a secondary contribution. It is established by ascertaining the smallest bias of each estimator within the optimal lag order interval. Finally, the developed methodology is applied to a real data sets in hydrology and economics in symbolizing the long memory attribute.

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