

# Preface

This book is focused on the theory of linear operators on non-Archimedean Banach spaces. It is to some extent a sequel of the authors' recent work on linear operators on non-Archimedean Banach spaces as well as their spectral theory. There are not many books exclusively dealing with operator theory on non-Archimedean Banach spaces or their variants, and the authors wish to add to the scarce literature on the subject. A minimum necessary background material has been gathered which will allow a relatively friendly access to the book.

Beginning graduate students who wish to enter the field of non-Archimedean functional analysis should benefit from the material covered, but an expert reader might also find some of the results interesting enough to be sources of inspiration. Prerequisites for the book are the basic courses in classical real and complex analysis and some knowledge of basic functional analysis. Further, knowledge of basic algebra (groups, rings, fields, vector spaces) and some familiarity with  $p$ -adic numbers, such as Gouvêa's introductory book  *$p$ -adic numbers: An Introduction* (second edition, Springer, 2003), will be a huge plus. The student would gain a long way knowing the first four chapters of the book *Local Fields* by Cassels (1986, London Mathematical Society, Student Texts 3).

After reading this book, the reader might benefit a great deal if he/she moves on forward with deeper material like the book *Spectral Theory and Analytic Geometry over non-Archimedean Fields* by Berkovich which is geared toward a systematic treatment of the spectral theory for Banach algebras (respectively, Rigid analytic geometry) or the books *Non-Archimedean Functional Analysis* by Schneider and *Non-Archimedean Functional Analysis* by van Rooij. Those readers with more arithmetical inclinations will profit from the book  *$p$ -adic Analysis: A Short Course on Recent Work* by Koblitz.

The topics treated in the book range from a basic introduction to non-Archimedean valued fields, free Banach spaces, and (possibly unbounded) linear operators in the non-Archimedean setting to the spectral theory for some classes of linear operators and their perturbations. Although some parts of the material are taken from the book *Non-Archimedean Linear Operators and Applications* by Diagana, this book is more comprehensive as it covers many new topics. It

emphasizes the role of the theory of Fredholm operators which is used as an important tool. This approach in the study of the spectral theory of linear operators should play more roles in larger context than the ones covered in the book, and in this regard, the book is a good introduction to the spectral theory of linear operators in the non-Archimedean setting. Explicit descriptions of the spectra of some linear operators are worked out.

Chapter 1 is of a background nature. It covers non-Archimedean valued fields and contains many details and examples on non-Archimedean valuations, the topology induced by these valuations, and their extensions. Spherical completeness is defined and some related properties are proved and illustrated with examples. The Krull valuation is introduced.

Chapter 2 is also of a background nature and covers non-Archimedean Banach spaces. These spaces are complete normed vector spaces over a complete non-Archimedean valued field. Of special interest are the free Banach spaces, especially the  $p$ -adic Hilbert space, and they are studied in detail. A structure theorem for the  $p$ -adic Hilbert space is proved.

Chapter 3 is on the bounded linear operators. Various properties are stated and proved. Finite rank operators, completely continuous operators, and Fredholm operators are all discussed with a view toward the applications in spectral theory.

Chapter 4 introduces and studies properties of the Shnirel'man integral. Among other things, such an integral is used to construct the so-called Vishik spectral theorem.

Chapter 5 contains the determination of the spectrum of a perturbation of a bounded diagonal operator by finite rank operators. The technique uses the theory of Fredholm operators.

Chapter 6 treats general unbounded operators, closed operators, and the spectrum of unbounded operators and the unbounded Fredholm operators.

Chapter 7 is devoted to the study of spectral theory for the perturbations of an unbounded operator by operators of finite rank or by completely continuous operators. Special emphasis is put on the computation of the essential spectrum of these perturbed unbounded linear operators.

This book is intended for graduate and postgraduate students, mathematicians, and nonmathematicians such as physicists and engineers who are interested in functional analysis in the non-Archimedean context. Further, it can be used as an introduction to the study of linear operators in general and to the study of spectral theory in other special cases.

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