The Art of Anticipatory Decision Making

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Abstract This paper presents the recent advances of the theory of anticipatory networks and its applications in future-oriented decision-making. Anticipatory networks generalize earlier models of consequence anticipation in multicriteria decision problem solving. This theory is based on the assumption that the decision maker takes into account the anticipated outcomes of future decision problems linked in a prescribed manner by the causal relations with the present problem. Thus arises a multigraph of decision problems linked causally (the first relation) and representing one or more additional anticipation relations. Such multigraphs will be termed anticipatory networks. We will also present the notion of a superanticipatory system, which is an anticipatory system that contains a future model of at least one anticipatory system besides itself. It will be shown that non-trivial anticipatory networks are superanticipatory systems. Finally, we will discuss several real-life applications of anticipatory networks, including an application to establish efficient collaboration of human and robot teams.

Keywords Anticipatory networks · Superanticipatory systems · Multicriteria decision making · Anticipatory collaboration · Preference modelling

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1 Introduction

This paper presents the theory of anticipatory networks, which generalizes the ideas related to anticipatory models of consequences in multicriteria optimization problems presented in [12, 13, 18]. It is assumed that when making a decision, the decision maker takes into account the anticipated outcomes of each future decision problem linked by the causal relations with the present problem. In a network of linked decision problems the causal relations are defined between time-ordered nodes. The future scenarios of the causal consequences of each decision are modelled by multiple edges starting from an appropriate node. The network is supplemented by one or more relations of anticipation, or anticipatory feedback, which describes a situation where decision makers take into account the anticipated results of some future optimization problems while making their choice. They then use the causal dependences of future constraints and preferences on the choice just made to influence future outcomes in such a way that they fulfill the conditions contained in the definition of the anticipatory feedback relations.

Both types of relations as well as forecasts and scenarios regarding the future model parameters form an information model called an anticipatory network [18]. In Sect. 2 we will show the basic properties of anticipatory networks as well as a method for computing them.

Following [12] and [18], in Sect. 3 we will present an application of anticipatory networks to select compromise solutions to multicriteria planning problems with the additional preferences provided in the form of anticipatory trees and general networks. We propose a more general notion of preference structure as compared to [16] and [18] that allows us to separate the preferences included in the anticipatory network from those used in present-time decision making. The study of properties of the anticipatory networks led us to introduce the notion of superanticipatory systems in Sect. 4. By definition, an *anticipatory system* in the Rosen sense [11] makes its decisions based on a future model of itself and of the outer environment. A *superanticipatory system* S is a system that is anticipatory and contains a future model of at least one other anticipatory system whose outcomes may influence the current decisions of S by a so called anticipatory feedback relation. This notion is idempotent, i.e. the inclusion of other superanticipatory systems into the model of the future does not yield an extended class of systems, but we can classify them according to a grade that counts the number of nested anticipations. We will observe that most anticipatory networks can be regarded as superanticipatory systems if we assume that future decisions can be based on similar anticipatory principles as the present decision. The class of superanticipatory systems has been introduced in [17] and [15].

The above mentioned theory arose from the need to create an alternative approach to selecting solutions to multicriteria optimization problems, where the estimation of an unknown utility function is replaced by a direct multi-stage model of future consequences of the decision made [12]. The anticipatory behavior of decision makers corresponds to the above definition of anticipatory systems proposed by Rosen [11] and developed further by other researchers [3, 8, 10].

A bibliographic survey of these ideas can be found in [8]. The ability to create a model of the future (of the outer environment and of itself), which characterizes an anticipatory system, is also the prerequisite for an anticipatory network, where nodes model anticipatory systems that can influence each other according to causal order. In this paper, anticipatory networks are restricted to model decisions made in so-called networks of optimizers, where each node models an optimization problem [18]. Similarly to anticipatory networks of optimizers, one can construct networks with nodes modelling Nash equilibria, set choice problems, rankings, random or irrational decision makers, or hybrid networks containing nodes of all types [16]. It should be pointed out that most anticipatory networks, and all those considered in this paper, are model-based, so that their nodes correspond to weak anticipatory systems in the Dubois sense [3].

The networks of anticipatory agents can be constructed by applying future decision problem forecasts and scenarios of anticipated consequences. The latter can be provided by foresight projects. In the final Sect. 5 we will discuss further extensions and applications of anticipatory networks that may be useful e.g. to build holistic future models or to establish efficient collaboration of heterogeneous teams consisting of robots and humans.

2 Anticipatory Networks as Generic Causal Models

The idea behind introducing anticipatory networks as models of consequences was formulated in [12, 18]. The basic principle is to use forecasts and foresight scenarios to estimate the parameters of future decision-making agents and to build a network of them. The anticipated future consequences of a decision made are modelled as changes in constraints and/or preference structures of future decision problems. The nature of these changes is assumed known to the present-time decision maker(s). It may result from model-based forecasts or foresight as well. Then, the anticipated outcomes of future decision-making problems that—of course—depend on constraints and preference structures, serve as a source of additional information that can be used to solve the current problem. In addition, future decision-making agents may use the same principle to make their decisions and this must be taken into account at the preceding decision stages.

Constructive algorithms for computing the solutions to the current multicriteria decision making problem taking into account the above anticipatory preference information feedback may be applied if we know that:

- All agents whose decisions are modelled in the network are rational, i.e. they make their decisions complying with their preference structures.
- An agent can assess whether the outcomes of some or all future decision problems causally dependent on the present one are more or less desired. This dependence is described as relations (usually multifunctions) between the

decisions to be made now and the constraints and/or preference structures of future problems.

- The above assessments are transformed into decision rules for the current solution choice problem, which affect the outcomes of future problems in such a way that they comply with the agent's assessments. The decision rules so derived form an additional preference structure for the decision problem just considered.
- There exists a relevance hierarchy in the network; usually the more distant in the future an agent is, the less relevant the choice of solution. However, this rule is not a paradigm.

Anticipatory networks which contain only decision-making agents solving optimization problems are termed *optimizer networks.* According to [18], an *optimizer O* is a multivalued function that assigns to a set of feasible decisions U and to the preference structure P a subset of the set of optimal decisions $O_F(U, P) \subset U$ that is selected according to P and to a fixed set of optimization criteria F with values in an ordered space E. Throughout this paper we will assume that the optimization problems solved by the optimizers have the form

$$(F\colon U \to E) \to min(\theta), \tag{1}$$

where E is a vector space with a partial order $\leq_\theta$ defined by a convex cone θ, i.e. iff

$$x \leq_\theta y \Leftrightarrow y - x \in \theta \text{ for each } x, y \in E.$$

The solution to (1) is the set of nondominated points defined as

$$\Pi(U, F, \theta) := \{u \in U\colon\ [\forall v \in U\colon\ F(v) \leq_\theta F(u) \Rightarrow v = u]\}.$$

Thus the criteria F and the ordering cone θ characterize a given optimizer uniquely. Most frequently, the decision maker's aim is to select and apply just one nondominated solution to (1). Thus the role of the preference structure P that occurs in the definition of an optimizer is to restrict the set of nondominated points in the solution process. Without a loss of generality we can assume that P is defined explicitly by pointing out for each $u \in U$ which elements of U dominate u. These are termed *dominating sets* and form a *domination structure* [2] which models the way the decision maker takes into account additional information about preferences when making the decision. Therefore P can be defined as a family of subsets of U in the following way

$$P := \{\pi(u) \subset U\colon\ u \in \pi(u) \textit{ and } [\textit{if } v \in \pi(u) \textit{ and } w \in \pi(v) \textit{ then } w \in \pi(u)]\}_{u \in U},$$

i.e. for each $u \in U$ $\pi(u)$ is the set of elements preferred to u.

As in the case of orders defined by convex cones, the element $u \in U$ is *nondominated with respect to P* iff $\pi(u) \cap U = \{u\}$, which means that no other element of U is preferred to u. The set of nondominated points with respect to P will be denoted by $\Pi(U, F, P)$. If F and P are fixed or if F is an identity on U we will write just $\Pi(U)$.

In a common case, where the preference structure P is defined by a convex cone ζ,

$$\pi(u) := \pi(u,\zeta) = \{v \in U:\ F(v) \leq_{\zeta} F(u)\} \tag{2}$$

and $\Pi(U, F, P) = \Pi(U, F, \zeta)$. Conversely, in problem (1) $\Pi(U, F, \theta) = \Pi(U, F, P_\theta)$ with P_θ defined by (2). Now we can formulate the following:

Definition 1 The mapping $O(U, F, \theta, P)$, a multifunction of U, θ, P, as well as of F with values in the family of all subsets of U is termed a *free multicriteria optimizer* if for all U, F, θ, P $O(U, F, \theta, P) \subset \Pi(U, F, \theta)$ and the following implication holds

$$\Pi(U,F,\theta) \cap \Pi(U,F,P) \neq \varnothing \Rightarrow [O(U,F,\theta,P) \neq \varnothing \wedge O(U,F,\theta,P) \subset \Pi(U,F,\theta) \cap \Pi(U,F,P)].$$

If the latter condition is satisfied, but the Pareto optimality of $O(U, F, \theta, P)$ with respect to the problem (1) cannot be taken for granted, however either $O(U, F, \theta, P) \subset \Pi(U, F, \theta)$ or $O(U, F, \theta, P) \subset \Pi(U, F, P)$ then O will be termed simply a *free optimizer*.

If, beyond the criteria F, the ordering θ, and the preference structure P, an optimizer O takes into account an additional decision making rule R, such as a heuristics, numerical approximation, or a random choice rule from U then the set of solutions returned by this optimizer need not be contained in $\Pi(U, F, \theta)$. However, if $X := O(U, F, \theta, P)$ approximates in certain sense $\Pi(U, F, \theta)$, e.g. in terms of the Hausdorff distance, then O will be termed an *approximate multicriteria optimizer*. Since in most real-life optimization problems only approximate solutions are available, for the sake of brevity, whenever no ambiguity arises, approximate multicriteria optimizers will be referred to as multicriteria optimizers.

Observe that in a free multicriteria optimizer O with $P := P_\zeta$, where $\zeta \subset E$ is a convex cone, from the basic properties of domination structures it follows that for all $\theta \subset \zeta$,

$$\begin{aligned} O(U,F,\theta,P) &\subset \{u \in U:\ [\forall v \in U:\ F(v) \leq_\theta F(u) \Rightarrow v = u]\} \cap \{u \in U:\ [\forall v \in U:\ F(v) \leq_\zeta F(u) \Rightarrow v = u]\} \\ &= \Pi(U,F,\zeta). \end{aligned}$$

In the above case the preference structure represented by the cone ζ may result from an iterative process of gradually restricting the set of nondominated points to (1). This technique is referred to as the *contracting cone method* [4] since the dual cones to an increasing sequence of ordering cones $\theta \subset \zeta_1 \subset \zeta_2 \ldots \subset \zeta$ contract as do the sets $\Pi(U, F, \theta)$, $\Pi(U, F, \zeta_1)$, …, $\Pi(U, F, \zeta)$. Here, we refer to this methodology to show its similarity to the anticipatory network technique described in the

Anticipatory Decision-Making Problem (ADMP, cf. Sect. 3). Indeed, it can be seen [18] that the more anticipatory feedbacks taken into account in an anticipatory network with the starting node, the more opportunities exist to confine the choice in problem (1) to a smaller subset of the set $\Pi(U, F, \theta, P)$. If for all $u, v \in U$

$$F(v) \leq_\theta F(u) \Rightarrow v \in \pi(u)$$

then we will say that P conforms to the criteria F and order θ; in short P is *conforming*. Observe that this is the case if $P := P_\zeta$ and $\theta \subset \zeta$. If P is conforming then to select an $X \subset \Pi(U, F, \theta)$ the action of the optimizer can be stretched on the whole set U, without computing $\Pi(U, F, \theta)$, otherwise it must be restricted to $\Pi(U, F, \theta)$ yielding a bi-level optimization problem. Let us note that the computation or even an approximation of $\Pi(U, F, \theta)$ can be a hard task, so the conforming P are sought in the first order of importance when solving (1).

If in a free multicriteria optimizer we fix the preference structure P and the ordering cone θ then the resulting mapping $O_{P,\theta}$ will be termed a *multicriteria selection rule* if for all U, and a given class Φ of E-valued functions for each $F \in \Phi$ it selects a non-empty subset of the nondominated subset $\Pi(U, F, \theta)$ of U with respect to θ, i.e. if

$$O_{P,\theta}(U,F) := \Pi(U,F,\theta,P) \subset \Pi(U,F,\theta) := \{u \in U: [\forall v \in U:\ F(v) \leq_\theta F(u) \Rightarrow v = u]\} \quad (3)$$

It is easy to see that if P is conforming then $O_{P,\theta}$ is a multicriteria selection rule. A multicriteria selection rule is termed *proper* iff $\Pi(U, F, \theta, P)$ contains a single point. For instance, if P is defined by (2) and $\pi(u) := \{v \in U: vTx \leq 0\}$ for a certain $x \in \theta^*$, where θ^* is the dual cone to a convex, pointed θ (i.e. such that $\theta \cap (-\theta) = \{0\}$) with $int(\theta) \neq$, and Φ is such that $F(U)$ is closed, bounded and strictly convex then for all U and F $\Pi(U, F, \theta, P) = \{u_0\}$ and $u_0 \in \Pi(U, F, \theta)$. Thus the selection rule $O_{P,\theta}$ is proper.

As already mentioned, besides their optimizing capabilities, optimizers may form networks with several new properties compared to the theory of single or sequential decision problems. In particular, in feed-forward networks of optimizers constraints and preference structures in some optimizers are causally linked to the solutions of other problems and may depend on their preference structures. Thus, in a network of optimizers, the parameters of the actual instances of optimization problems to be solved vary as the results of solving other problems in the network.

Definition 2 If $O_1 := X_1(U_1, F_1, \theta_1, P_1)$ and $O_2 := X_2(U_2, F_2, \theta_2, P_2)$ are free multicriteria optimizers then a constraint influence relation r between O_1 and O_2 is defined as

$$O_1 r\, O_2 \Leftrightarrow \exists \varphi:\ X_1 \to 2^{U_2}:\ X_2 = \varphi(X_1). \quad (4)$$

Acyclic r are termed causal constraint influence relations—in short, causal relations.

Causal relations are represented by a (causal) *network of optimizers*. Definition 2 models the situation where the decision maker anticipating a decision output at a future optimizer can react by creating certain decision alternatives or forbidding them. This is described by influencing the constraints by multifunctions φ depending on the outputs from the preceding problems. As in [18] and [16], from this point on the term *causal network* will refer to the graph of a causal constraint influence relation. To complete the definition of anticipatory networks, we will define the anticipatory feedback relation.

Definition 3 Suppose that G is a causal network consisting of free optimizers and that an optimizer O_i in G precedes another optimizer, O_j, in the causal order r. Then the *anticipatory feedback* between O_j and O_i in G is information concerning the model-based anticipated output from O_j, which serves as an input influencing the choice of decision at optimizer O_i. Such a relation will be denoted by $f_{j,i}$.

By the above definition, the existence of an anticipatory information feedback between the optimizers O_n and O_m means that both condition below apply:

- the decision maker at O_m is able to anticipate the decisions to be made at O_n,
- the results of this anticipation are to be taken into account when selecting the decision at O_m.

The anticipatory feedback relation does not need to be transitive. As in the case of causal relations, there may also exist multiple types of anticipatory information feedback in a network, each related to the different way the anticipated future optimization results are considered at optimizer O_m. The multigraph of r (cf. (4)) and one or more anticipatory feedbacks define an anticipatory network of optimizers:

Definition 4 A causal network of optimizers Ω with the starting node O_0 and at least one anticipatory feedback relation linking O_0 or an optimizer O_n causally dependent on O_0 with another node in the network will be termed an *anticipatory network* (of optimizers). Ω is termed *proper* iff $n=0$.

In [18], the anticipatory information feedback in causal networks of optimizers was applied to selecting a solution to an optimization problem modelled by the starting element in an anticipatory optimizer network G. Specifically, while making the decision, the decision maker takes into account the following information contained in G:

- forecasts concerning the parameters of future decision problems represented by the decision sets U, criteria F, and the ordering structure of the criteria values θ,
- the anticipation concerning the behavior of future decision makers acting at optimizers, represented by the preference structures P,

- the forecasted causal dependence relations r linking the parameters of optimizers in the network,
- the anticipatory relations pointing out which future outcomes are relevant when making decisions at specified nodes and the anticipatory feedback conditions.

We will now present a few key definitions that refer to solving multicriteria decision problems using an anticipatory network of optimizers as a source of additional preference information.

Definition 5 An anticipatory network (of optimizers) is said to be *solvable* if the process of considering all anticipatory information feedbacks results in selecting a non-empty solution set at the starting problem.

Definition 6 A causal graph of optimizers G that can be embedded in a straight line will be called a *chain* of optimizers. If it contains at least one anticipatory feedback $f_{i,0}$ then G will be termed an *anticipatory chain* (*of optimizers*)

The causal constraint influence relations $\varphi(j)$ (cf. Definition 2) are defined as $\varphi(j) := Y_j{}^\circ F_i$, where the multifunctions $Y_j{:}F_i(U_i) \rightarrow\rightarrow U_j$ model the dependence of the scope of decisions available at O_j on the optimization outcomes of the problem O_i. Following [12], the total restriction of the decision scope at O_j generated by Y_j is denoted by R_j, i.e.

$$R_j{:} = Y_i(F_i(U_i)).$$

The resulting restriction of the set of nondominated outcomes at O_j is denoted by S_j. In an anticipatory chain, as exemplified in Fig. 1, there is only one predecessor of each optimizer O_i, for i > 0, so in the above formula i can be replaced by $j - 1$. By definition, the causal relation represented by $\varphi_{i,j}$ is *non restrictive* iff $S_j = \Pi(U_j, F_j, \theta_j)$. We will say that $\varphi_{i,j}$ *complies with* O_j iff $S_j \subset \Pi(U_j, F_j, \theta_j)$.

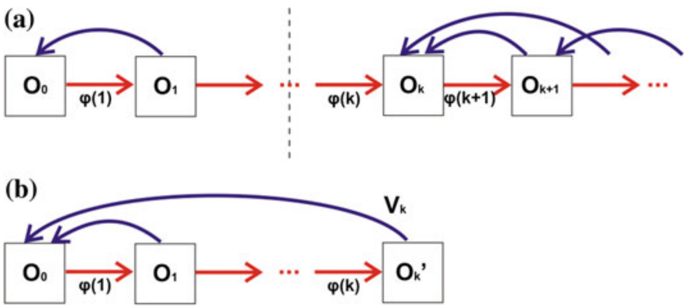

Fig. 1 Two examples of a chain of optimizers with anticipatory feedbacks: in figure **a** the anticipatory feedbacks linking O_k with O_{k+1}, and with further future optimizers have no influence on the decisions made at the starting node O_0. They are taking into account exclusively the decisions made at O_1, while the latter do not rely on any future considerations. The figure **b** depicts a case of anticipatively connected chain of optimizers. The temporal order complies in both cases with the causal relations defined by multifunctions $\varphi(j) := Y_j{}^\circ F_{j-1}$

Example 1 Figure 1 presents two examples of anticipatory chains of optimizers with different configurations of anticipatory feedbacks. In the case (a) above there are, in fact, two separate decision making problems, where a decision made at the optimizer $O_0 = (U_0, F_0, IR_+^{n(0)})$ is selected taking into account the anticipated outcomes at $O_1 = (U_1, F_1, IR_+^{n(1)})$ and $O_k = (U_k, F_k, IR_+^{n(k)})$ takes into account the outcomes of O_{k+1} and other future optimizers, but the decision made at O_0 is in no way related to the decisions made at the future decision nodes beyond O_1. Although O_1 influences future choices at $O_2, O_3, \ldots, O_k$, this optimizer does not select its decision in any way that may facilitate or hinder the achievement of any specific future goal. The solution of the problem $O_1 = (U_1, F_1, IR_+^{n(1)})$ is accomplished based on the local preference relation P_1 only. Thus the analysis of decisions in the anticipatory chain (a) may be decomposed into the analysis of the chains $O_0 \to O_1$ and $O_k \to O_{k+1} \to \cdots$. The case (b) presents an anticipatory chain equivalent to the decision situation considered in [12, 13], where the decision made at O_0 takes into account the outcomes of all subsequent problems, but there are no decision feedbacks between future optimizers.

Although the anticipatory chains constitute the simplest class of anticipatory networks, they are capable of describing a variety of sequential decision problems. A more advanced model is needed when an optimizer O_k influences two or more its immediate successors, say O_{k1} and O_{k2}, and the decision maker of O_k, or of any of its predecessors, is interested in the decision outcomes of both, O_{k1} and O_{k2}, or in the outcomes of two arbitrary optimizers that one following in the causal order O_{k1}, the other O_{k2}. If no optimizer is influenced by more than one immediate predecessor then the causal graph is a tree and we can formulate the following definition.

Definition 7 A causal graph of optimizers G that is a tree and contains at least two anticipatory feedbacks $f_{i,0}$ and $f_{j,0}$, each of them starting at optimizers that are not mutually causally conected, will be termed a *proper anticipatory tree* (*of optimizers*).

As a consequence, any anticipatory tree contains at least one optimizer that influences two or more its immediate successors. Such nodes in an anticipatory tree are termed *bifurcation optimizers*.

Along with chains, anticipatory trees are another special class of anticipatory networks that may be solved with dedicated algorithms. These are based on the decomposition of a tree into chains and on a subsequent analysis of them, starting from chains having a common bifurcation optimizer that is most distant in time [18]. Let us observe that if all anticipatory feedbacks in a tree were situated on one of its chains then the analysis of this tree could be reduced to just one chain. Anticipatory trees that possess this property are not proper.

Besides of bifurcation optimizers, the anticipatory trees contain a new phenomenon related to the anticipatory component of the multigraph, namely the *spurious anticipatory feedbacks*. These appear when the decision maker at an optimizer O_k would like to take into account the future outcomes of an optimizer O_m, but O_k and O_m are not causally connected, so there is no way to influence O_m to

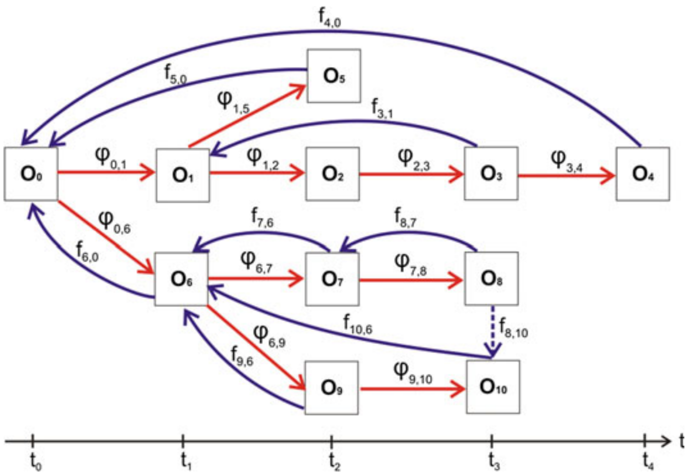

Fig. 2 An example of a tree of optimizers, with three bifurcation optimizers O_0, O_1 and O_6. Causal relations are defined by the multifunctions $\varphi_{i,j} := Y_i^\circ\ F_{i-1}$. Eight anticipatory feedback relations are denoted by $f_{k,m}$, where k is an index of a future node with the outcome taken into account by the optimizer O_m. There is also one spurious anticipatory feedback $f_{8,10}$

force or suggest a decision choice satisfying the condition defined in an anticipatory feedback.

An example of a tree of optimizers that contains a spurious anticipatory feedbacks is shown in Fig. 2.

3 Decision Making Problems in General Anticipatory Networks

In a non-trivial anticipatory network the following problem can be formulated:

Anticipatory Decision-Making Problem (ADMP, [17, 18]). For all chains of optimizers in an anticipatory network G with finite decision sets find the set of all admissible sequences of decisions $(u_0, \ldots, u_n)$ that minimize the function

$$g(u_0, \ldots, u_n) := \sum_{i \in J(0)} h(u_i, q(0, i)) w_{0,i} \tag{5}$$

and such that for all i, $1 \le i < n$, the truncated decision chain $(u_i, \ldots, u_n)$ minimizes

$$g(u_i, \ldots, u_n) := \sum_{j \in J(i)} h\left(u_j,\ q(i, j)\right) w_{i,j}, \tag{6}$$

where $J(i)$, $i = 0, 1, \ldots, n$, denote the sets indices of decision units in G, which are in the anticipatory feedback relations with O_i and $w_{i,j}$ are positive coefficients corresponding to the relevance of each anticipatory feedback relation between the optimizers O_i and O_j. The function h may be defined as

$$h\left(u_j, q(i,j)\right) := \left\| F_j\left(u_j\right) - q(i,j) \right\|, \tag{7}$$

where $q(i, j)$ are user-defined reference levels of criteria F_i, for $i=0,1,\ldots,n$.

From the formulation of the above decision-making problem it follows that the decision maker at O_0, while selecting the first element of an admissible decision sequence $u_0 \in U$ uses the anticipatory network G and the function g as an auxiliary preference structure to solve the problem (1). The key notion for the theory of anticipatory decision making can now be defined as follows:

Definition 8 A solution to the ADMP, a family of decision sequences $u_{0,m(0)}, \ldots, u_{N,m(N)}$ minimizing (5)–(7), will be called *anticipatory paths.*

Constructive solution algorithms for solving the ADMP take into account the information contained in an anticipatory network G. These have been proposed in [18] (Algorithms 1 and 2) for a class of anticipatory networks with discrete decision sets U_i, when the graph of causal relation r is either a chain or a tree. The anticipatory feedback conditions have been defined there as a requirement of O_i that the decisions at O_j, for j from a certain index set $J(i)$ such as O_i precedes O_j in causal order r are selected from the subsets $\{V_{ij}\}_{j \in J(i)}$, $V_{ij} \subset U_j$. Usually, this means that the values of criteria F_j admitted on V_{ij} are of special importance to the decision makers and can be defined as reference sets [14]. The general principles behind these algorithms are as follows:

- Decompose the anticipatory network into causal chains of optimizers linked by causal relations.
- Identify *elementary cycles* in each chain in the anticipatory network, i.e. cycles which do not contain other such cycles except themselves, consisting of causal relations along chains and anticipatory feedback relations.
- Solve the decision problem for each chain, by eliminating the elementary cycles.
- Use the logical conditions that defined the anticipatory requirements to bind the solution sequences to the common parts of the anticipatory chains.

Thus it is possible to reduce the analysis of anticipatory trees to a recursive analysis of anticipatory chains in the tree. Moreover, a general network can be decomposed into trees or chains, which makes it possible to apply solution rules for chains iteratively, gradually eliminating solved trees and chains. However, the solution procedures for anticipatory trees cannot be directly adopted for the solution of the problems where there may exist units that are influenced causally by two or more predecessors without taking into account synchronization problems.

Such networks can model problems where multiple resources, provided as outcomes of a number of different and independent decision processes, determine the scope of a later-stage decision. For example, to optimize the decisions in a potential future joint venture created to develop a new product (so-called NPD problem), the outputs provided by the potential future partners of this joint venture should be considered. It can be shown that taking into account the possibility of

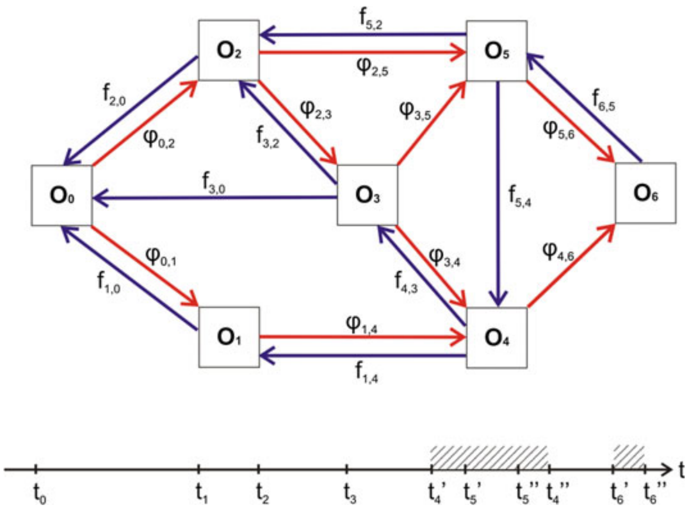

Fig. 3 A causal network of seven optimizers, where O_0, O_2, and O_3 are bifurcation optimizers, while O_4, O_5, and O_6 are each influenced by two predecessors. The shadowed areas between t_4' and t_4'' and between t_6' and t_6'' on the time axis denote the synchronization intervals for the simultaneous influence of O_1 and O_3 on the outcomes of O_4, and of O_4 and O_5 on O_6, respectively. The synchronisation interval for O_5, $[t_5', t_5'']$, is contained in $[t_4', t_4'']$. The anticipatory feedback $f_{5,4}$ between O_5 and O_4 is induced by the information flow from O_5 to O_4

creating future production alliances and representing such relations in an anticipatory network results in a competitive advantage over agents optimizing their own future outputs only. An example of a general anticipatory network is shown in Fig. 3.

To analyze general networked optimizers, it will need to be assumed that if an optimizer O_p is directly influenced by more than one predecessor then the aggregation rules are defined for each subset of influencing factors generated by the preceding optimizer (e.g. as an intersection or a union of the sets of feasible alternatives, each one imposed by a different preceding optimizer).

In addition, these rules must take into account the synchronization of influence that was not necessary in the case of anticipatory trees. Specifically, the simultaneous action of predecessors on O_p may be restricted to the prescribed time intervals. This is depicted above in Fig. 3, where t_i' and t_i'' denote the start and end of a synchronization time interval for the i-th optimizer.

In the most common situation, where the influence of preceding optimizers imposes a logical product of individual influences, the synchronization problem reduces to analysing the time intervals when the intersection of constraints resulting from multiple influencing multifunctions can still yield a feasible solution. However, in general, all combinations of logical conditions binding independent influences should be considered, including the situation where one agent's influence results in removing another agent's constraints. The analysis of such cases requires further studies, which, however, can be based on the solution scheme presented above and in [18].

The emergence of *induced anticipatory feedback* is a new phenomenon that could not occur in anticipatory chains of trees. In the example provided in Fig. 3 the anticipatory feedback $f_{5,4}$ between O_5 and O_4 is induced, but it is not spurious, as e.g. the spurious feedback $f_{8,10}$ in Fig. 2, although there is no causal relation between these optimizers. There are four necessary conditions for the existence of an induced anticipatory feedback. The first one is precedence in time of the decision made at the optimizer which is the target of the anticipatory feedback. The second is the existence of an information exchange between the optimizers that are source and target of induced anticipatory feedback. The third is the existence of at least one other optimizer that is influenced directly or indirectly by both, the source and the target optimizers of the induced anticipatory feedback, and the joint influence is restrictive, i.e. the influenced optimizer may select the decision from the intersection of sets provided as values of influencing multifunctions. The fourth condition is most specific and requires that the sets that define the anticipatory feedbacks at the source optimizer and commonly influenced optimizers were appropriately configured with respect to the values of influencing multifunctions. This is exemplified below.

Based on the example presented in Fig. 3, the phenomenon of induced feedback may be explained as follows: due to the assumed information exchange, the decision maker responsible for selecting the decision at O_4 knows both, the multifunction $\varphi_{5,6}$ that influences O_6 and the anticipatory feedback $f_{6,5}$. Similarly, the decision maker at O_5 knows the multifunction $\varphi_{4,6}$ and the actual choice made by O_4. Then O_4 can select such a decision $v \in \Pi(U_4)$ so that O_5 is forced to select a decision from the set $V_{5,4}$ that defines the anticipatory feedback $f_{5,4}$ to get a satisfactory solution of the problem solved by the commonly influenced optimizer O_6, specifically an element of $V_{6,5}$.

Of course, the more decision problems commonly influenced by the same pair of optimizers in a network, the more likely is the occurrence of induced anticipatory feedbacks. In addition, more complex configurations of induced feedbacks may arise if in an anticipatory network the same optimizer is causally influenced by three of more causally independent predecessors or when the information flow on the causal influences, anticipatory feedbacks, or preference structures is asymmetric.

The above presented phenomenon of induced anticipatory feedback, its extensions and relations to the topology and other properties of the network is a subject of an ongoing research that may potentially discover new applications of anticipatory networks as well as new problems to be investigated.

4 Anticipatory Networks as Superanticipatory Systems

Let us observe that in the above presented approach to solving anticipatory networks, we have assumed that anticipation is a universal principle governing the solution of optimization problems at all stages. In particular, future decision makers modelled at the starting decision node O_0 can in the same way take into account the network of their relative future optimizers when making their decisions. Thus, the future model of the decision maker at O_0 contains models of future agents including their respective future models. This has led us to introduce the notion of superanticipatory systems [15, 17] which directly generalize anticipatory systems in the Rosen sense [11] and weak anticipation in the Dubois sense [3]:

Definition 9 *A superanticipatory system* is an anticipatory system that contains at least one non-trivial model of another future anticipatory system.

Since a superanticipatory system is required to contain a model of *another* system, the above definition excludes the case where an anticipatory system models itself recursively. This is discussed later in this section.

By definition, this notion is idempotent, i.e. the inclusion of other superanticipatory systems in the model of the future of a superanticipatory system does not yield an extended class of systems since every superanticipatory system is also anticipatory.

Superanticipatory systems can be classified according to a grade that counts the number of nested superanticipations.

Definition 10 A superanticipatory system S *is of grade* n if it contains a model of a superanticipatory system of grade $n - 1$. An anticipatory system which does not contain any model of another anticipatory system is defined as superanticipatory of grade 0.

Let us note that the actual grade n of a superanticipatory system S depends on the accuracy of the model of other systems used by S. In addition, when constructing its model of the environment, S may underestimate the actual content of the other system models. Then, according to Definition 10, the grade of superanticipation of S should be regarded as a grade of the model, rather than the actual grade of the physical system.

It may be conjectured that if a superanticipatory system uses an empirical and rational modelling approach then it is more likely that the other systems will have models of a higher grade than S has estimated based on experiments. Thus the grade of the rational system S, when determined based on the information coming solely from the same system, can be regarded as the lower bound of an actual grade. Perfect knowledge of the grade can be attributed to a hypothetical ideal external observer only.

When referring to an anticipatory network, which is always a result of a certain modelling compromise, the following statement can be formulated

Theorem 1 Let $G = (O, r, f)$ be an anticipatory network, where O is the (finite) family of optimizers, r is the causal influence relation, and f is the anticipatory feedback relation. If G contains an anticipatory chain C such that there exist exactly

n optimizers in C, $\{O_{C,1}, \ldots, O_{C,n}\} \subset C = (O_1, \ldots, O_N, r, f)$, $N \geq n$, with the following property:

$$\forall i \in \{1, \ldots, n\} J_C(i) \neq \varnothing \text{ and } (\exists j \neq i\colon\ O_{C,j} r\, O_{C,i} \textit{ and } i \in J_C(j)), \tag{8}$$

where $J_C(i)$ is the set of indices of optimizers in G, which are in the anticipatory feedback relation with O_i. and no other chain in G has the property (8) with $m > n$ then G is a superanticipatory system of a grade of at least n.

The proof of the above Theorem 1 follows directly from the definitions of anticipatory networks (Definition 4) and superanticipatory systems (Definitions 9 and 10). Its first version appeared in [17].

It is easy to see that an anticipatory network containing a chain on n optimizers, each one linked with O_0 and with all its causal predecessors with an anticipatory feedback is an example of a superanticipatory system of grade n.

The notion of superanticipation is obviously related to the general recursive properties of anticipation. By definition, superanticipation makes sense only when the anticipation of the future is based on a predictive model. Problems to be solved that arise in a natural way are related to the accuracy of such models and to the grade of superanticipation. They are also related to the relation between *internal* (system) time, when the model is built and analyzed, and *external* real-life time, when the modelled objects evolve. A brief discussion of other recursive approaches related to anticipation such as recursion in Rosen's theory, Dubois' meta-anticipation, and information set models in multi-step games is given in [17].

A recursive anticipation can be applied in n-stage games, when one player anticipates the behavior of the others, cf. e.g. [7]. From the point of view of player G_1, anticipation is defined here for $k(G_1)$ steps forward and includes the anticipatory models for the other players $G_2, \ldots, G_N$. Each player can also possess a model of themselves (G_1) and of some or all the remaining participants with an anticipation horizon of $k(G_i)$ moves, $i = 2, \ldots, N$. Player G_1 thus fulfills the definition of a superanticipatory system and the game can be represented as an evolving anticipatory network. However, when the future moves of the other players result from a deterministic algorithm rather than from a decision-making model, anticipation may be based on the knowledge of the (deterministic) function identified with the operation of that algorithm. This may happen when a human player plays a deterministic game with a computer, or when a machine-machine interaction is modelled. Such games could be modelled by the master-slave (or driver-response) structure of the coupled system analogous to the leader-follower relation in multi stage Stackelberg games [5, 9].

5 Conclusions and a Discussion of Future Research Directions

The theory of anticipatory systems emerged originally in order to explain behavioral phenomena in systems biology, yet it turned out soon that it may also explain the collaboration and conflict patterns of human, artificial, as well as hybrid autonomous systems. Despite the efforts of its founder, Rosen, and Rosen's successors, in original formulation it contained a notion of 'internal model' of itself and other systems' future that has been regarded as vague and hard to implement constructively. The hitherto attempts to create a constructive theory of 'forward systems' symmetric to delayed control systems yielded a formal description that provided a correct computational framework in few cases only. A breakthrough was possible due to the introduction of the notion of anticipatory feedback that makes possible to describe the interaction between the models of the future systems and present-time decision makers, and of extending the anticipatory modelling to nested systems, whereas future generations of anticipatory systems have the same right as the present ones to define anticipatory feedbacks. The ability to define an anticipatory feedback is restricted to the case where the agent linked by an anticipatory feedback can causally influence the target agent of this feedback.

This paper examined the principle ideas concerning anticipatory networks as a new tool to model the multicriteria decision problems and the basic methods for solving them. Their extension, so-called superanticipatory systems, was also presented. We have also shown some potential further extensions of the theory of anticipation and presented research-in-progress on these topics.

Anticipatory networks may be applied to model and solve a broad range of problems. Apart from the above-mentioned potential uses in foresight, roadmapping, and socio-econometric modelling, there are further potential fields of application, such as:

- Anticipatory modelling of sustainable development: the underlying assumption of the anticipatory network theory, namely that the present decision maker wants to ensure that future decision makers have the best possible opportunities to make satisfactory decisions corresponds to the 'future generation' paradigm of sustainability theory. These 'future generations' are modelled by other network nodes.
- Anticipatory planning based on results of foresight studies, such as development trends, scenarios, and relevance rankings of key technologies, strategic goals, etc. Such planning can use deterministic as well as stochastic planning techniques and include multi-step game models.
- Anticipatory coordination of robotic swarms and human-robot systems, where anticipation is coped with multi-stage cooperative and leader-follower game models.

Anticipatory networks can also contribute to solving the stochastic optimization problems related to portfolio selection [1], road traffic management [6, 20], and to implementing the knowledge contained in foresight scenarios in a clear, formal way. Further real-life applications are discussed in [18] and [19].

Specifically, the development of the theory outlined above has been motivated by the problem of modelling the process of finding feasible foresight scenarios based on the identification of future decision-making processes and on anticipating their outcomes. Such an anticipatory network was applied in a recent information technology foresight project to build a strategy for a Regional Creativity Support Center [19]. Scenarios, such as those defined and used in foresight and strategic planning [4], may depend on the choice of a decision in one of the networked optimization problems and can be external-event driven. When included in a causal network of optimizers, the anticipation of future decisions and alternative external events would allow us to generate alternative structures of optimizers in the network.

Anticipatory networks, those that contain solely optimizers as well as hybrid ones [15], extend the plethora of modelling tools that can be used to formulate and solve decision making problems taking into account new future-dependent preference structures. When regarded as a class of world models for robotic systems, anticipatory networks provide a flexible representation of the outer environment, while superanticipation allows us to model collective decision phenomena in autonomous robot swarms. Further studies on this class of models may also contribute to the general theory of causality and lead to discovering surprising links to theoretical biology, quantum physics and the causal fields theory, as well as to the mirror neuron research in neurosciences. The theory of anticipatory networks links the ideas of anticipatory systems and models with multicriteria decision making, game theory, and the algorithmics. The formal methods that will be used to further develop the anticipatory networks theory include multigraphs and hypergraphs, dynamic programming in partially ordered spaces, controlled discrete-event systems, and general causality theory.

The above links and methods will make the research on networked anticipation truly interdisciplinary and may provide intriguing ties to the hard consciousness problem in cognitive sciences and the nature of time as qualia in the philosophy of mind. The relations to the theory of cooperative systems, specifically anticipatory robots, predictive and anticipatory control, foresight and backcasting, as well as to other areas of applicable basic research will assure the existence of a variety of potential real-life applications.

Finally, le us note that the above presented progress in the theory of anticipation and causality has appeared as a parsimonious effect of a foresight project devoted to modelling the ICT and AI futures, namely as a methodology to filter out the irrational technological and economic scenarios and to perform the technological and strategic planning.

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