Preface

Admittedly, as useful a matter as the motion of fluid and related sciences has always been an object of thought. Yet until this day neither our knowledge of pure mathematics nor our command of the mathematical principles of nature have a successful treatment.

–Daniel Bernoulli

Incompressible Navier–Stokes equations describe the dynamic motion (flow) of incompressible fluid, the unknowns being the velocity and pressure as functions of location (space) and time variables. To solve those equations would mean to predict the behavior of the fluid under knowledge of its initial and boundary states. These equations are one of the most important models of mathematical physics. Although they have been a subject of vivid research for more than 150 years, there are still many open problems due to the nature of nonlinearity present in the equations. The nonlinear convective term present in the equations leads to phenomena such as eddy flows and turbulence. In particular the question of solution regularity for three-dimensional problem was appointed by Clay Mathematics Institute as one of the Millennium Problems, that is, the key problems in modern mathematics. This is, on one hand, due to the fact that the problem remains challenging and fascinating for mathematicians and, on the other hand, that the applications of the Navier–Stokes equations range from aerodynamics (drag and lift forces), through design of watercrafts and hydroelectric power plants, to the medical applications of the models of flow of blood in vessels.

This book is aimed at a broad audience of people interested in the Navier–Stokes equations, from students to engineers and mathematicians involved in the research on the subject of these equations.

It originated in part from a series of lectures of the first author given over the past 15 years at the Faculty of Mathematics, Informatics and Mechanics of the University of Warsaw; at summer schools at UNICAMP, Campinas, Brasil; and at Université Jean Monnet, Saint-Etienne, France. The lectures were based on the leading books on the then young theory of infinite dimensional dynamical systems, focused on mathematical physics, in particular, on Temam [220]; Chepyzhov and Vishik [61]; Doering and Gibbon [88]; Foiaş, Manley, Rosa, and Temam [99]; and Robinson [197].

The lectures at the Mathematics Faculty of the University of Warsaw were also attended by students and PhD students from the Faculty of Physics and Faculty of Geophysics, and it became clear that a routine mathematical lecture had to be extended to include additional aspects of hydrodynamics. Some students asked for "more physics and motivation" and "more real applications"; others were mainly interested in the mathematics of the Navier–Stokes equations, and yet others would like to see the Navier–Stokes equations in a more general context of evolution equations and to learn the theory of infinite dimensional dynamical systems on the research level. These several aspects of hydrodynamics well suited the tastes and interests of the lecturer, and also the second author was welcomed to join the project of the book at a later stage.

In consequence, the audience of the book is threesome:

Group I: Mathematicians, physicists, and engineers who want to learn about the Navier–Stokes equations and mathematical modeling of fluids

Group II: University teachers who may teach a graduate or PhD course on fluid mechanics basing on this book or higher-level students who start research on the Navier–Stokes equations

Group III: Researchers interested in the exchange of current knowledge on dynamical systems approach to the Navier–Stokes equations

Although, in principle, all these three groups can find interest in all chapters of the book, Chaps. 2–7 are primarily targeted at Group I, Chaps. 3, 4, 7, 8, 11, and 12 aimed mainly at Group II, and Chaps. 7–16 for Group III.

For a reader with reasonable background on calculus, functional analysis, and theory of weak solutions for PDEs, the whole book should be understandable.

The book was planned to be a monograph which could also be used as a textbook to teach a course on fluid mechanics or the Navier–Stokes equations. Typical courses could be "Navier–Stokes equations", "partial differential equations", "fluid mechanics", "infinite dimensional dynamics systems." To this end many chapters of this book include exercises. Moreover, we did not restrain ourselves to include a number of figures to liven the text and make it more intuitive and less formal. We believe that the figures will be helpful. Special care was undertaken to keep the individual chapters self-contained as far as possible to allow the reader to read the book linearly (in linear portions). That demanded several small repetitions here and there.

To understand the first chapters of this book, just the basic knowledge on calculus, that can be learned from any calculus textbook, should be enough.

The book is planned to be self-contained, but, to understand its last chapters, some knowledge from a textbook like "Partial Differential Equations" by L.C. Evans (which contains all necessary knowledge on functional analysis and PDEs) would be helpful. Each chapter contains an introduction that explains in simple words the nature of presented results and a section on bibliographical notes that will place it in the context of past and current research.

Several people greatly contributed, knowingly or not knowingly, to the creation of the book. Our thanks go to our colleagues and collaborators: Guy Bayada, Mahdi Boukrouche, Thomas Caraballo, José Langa, Pepe Real, James Robinson etc.

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