Preface

This book is based on a one-semester single variable advanced calculus course that I have been teaching at San Diego State University for many years. Mathematics departments in many schools offer such a course. The aim is a rigorous discussion of the concepts and theorems that are dealt with informally in the first two semesters of a beginning calculus course. As such, students are expected to gain a deeper understanding of the fundamental concepts of calculus, such as limits, continuity, the derivative and the Riemann integral. Success in this course is expected to prepare them for more advanced courses in real and complex analysis.

The first semester of advanced calculus can be followed by a rigorous course in multivariable calculus and an introductory real analysis course that treats the Lebesgue integral and metric spaces, with special emphasis on Banach and Hilbert spaces. I believe that each course requires a separate text.

Chapter 1 begins with a quick review of the properties of the set of real numbers as an ordered field. The concept of the limit of a sequence and the relevant rules are discussed rigorously. The completeness of the field of real numbers is introduced as the existence of the limit of a Cauchy sequence. I believe that this is better than the introduction of the notion of completeness via the existence of the least upper bound of a subset of real numbers that is bounded above. After all, students have been dealing with Cauchy sequences in the form of decimal approximations all along. An added advantage is the fact that the notion of completeness as the existence of the limit of a Cauchy sequence appears time and again within the general framework of a metric space that may not have an order relation, as in the cases of the field of complex numbers, Banach or Hilbert spaces. The least upper bound principle and the special nature of the convergence or divergence of a monotone sequence are also treated in Chapter 1. The notion of an infinite limit is discussed carefully since the convenient symbol ∞ can be misunderstood and mistreated by the student.

Chapter 2 discusses the continuity and limits of functions. I have chosen to limit the discussion to functions defined on intervals. I believe that the point set topology of more general sets belongs to a more advanced real analysis course. Many students encounter serious difficulties in the transition from informal to rigorous calculus anyway.

I emphasize the ε-δ definitions. Such emphasis is essential for the appreciation of the difference between mere pointwise continuity and uniform continuity on a set. One of the highlights of the chapter is the Intermediate Value Theorem that has bearing on the definitions of basic inverse functions that figure prominently in beginning calculus.

Chapter 3 takes up the derivative. The emphasis is on the nature of the error in local linear approximations to functions. That renders a rigorous proof of the celebrated chain rule quite straightforward. The student is also prepared for the generalization of the concept of the derivative to functions of more than a single real variable, even to functions between normed vector spaces. I have included a detailed discussion of convexity as a nice application of the Mean Value Theorem. Rigorous proofs of various versions of L'Hôpital's rule are not neglected.

Chapter 4 is on the Riemann integral. Integrability criteria in terms of upper and lower sums and the oscillation of a function are discussed. The two approaches complement each other. I establish the link with the usual introduction of the integral via arbitrary Riemann sums in a beginning calculus course, unlike some popular advanced calculus texts that neglect to mention that connection as if a new type of integral is being discussed. I have included a detailed discussion of improper integrals, including the comparison and Dirichlet tests. Some of the most important improper integrals that are encountered in practice require such tests. I emphasize Cauchy-type criteria for the convergence of an improper integral.

Chapter 5 is a review of a series of real numbers. I chose to provide details since this topic challenges students in a beginning calculus course where rigorous proofs are not provided. I emphasize Cauchy-type criteria for the convergence of series.

Chapter 6 discusses the convergence of sequences and series of real-valued functions on intervals. The distinction between mere pointwise and uniform convergence is emphasized, with ample examples. The nice behavior of sequences and series of functions with respect to integration and differentiation are not valid unless certain uniform convergence conditions are satisfied. The analyticity of functions defined via power series follows smoothly once the appropriate foundation involving uniform convergence is established. The chapter is concluded with the definition of familiar special functions via power series.

This book is an undergraduate text and not a monograph on a special topic. My writing has been inspired and influenced by a variety of authors over many years since my initial encounter with analysis as a student. I have been fortunate to have had teachers such as Stefan Warschawski, Errett Bishop, and William F. Lucas. The following is a short list of books that are relevant to the way I treated the topics that are included in this book (excellent classical texts in this classical subject):

1. Introduction to Calculus and Analysis, Vol. 1, by Richard Courant and Fritz John, Springer, 1998
2. Theory and Application of Infinite Series, by Konrad Knopp, Dover, 1990
3. The Elements of Real Analysis, Second Edition, by Robert G. Bartle, Wiley, 1976

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