

## Preface

Students from the University of Toronto have participated in the annual Putnam Competition since its inception in 1938. Many of them have competed with distinction, but others have not fared so well. Because of the way in which the Putnam scripts are marked, it is necessary to make significant progress on a problem to earn one or two points.

In 2001, I started a mathematics competition for undergraduates at the University of Toronto to offer the experience of solving problems under a time constraint. My goal was to provide a paper that would be more widely accessible while maintaining some challenge and interest. A secondary goal was to produce a set of problems that might find a place in regular college and university courses. So, I have collected them here in the hope that teachers and students may find them useful.

The problems have many sources. Some are original; some are side results in research papers; some have been contributed; some have appeared in the *American Mathematical Monthly*, and some, of unknown provenance, have been lying in folders in my office. Each problem is graded out of 10, with opportunity for part marks.

So that students have some choice, there are eight to ten problems on each paper. I expected that students normally should attempt no more than five, and to enforce this, they were told that only their best five problems would be marked unless the score on these five was at least 30. Inevitably, there were students who correctly solved more than five.

The first two years of college mathematics should provide sufficient background. Students should have knowledge of secondary level algebra and geometry, elementary linear algebra, basic number theory, elementary combinatorics, techniques of single variable calculus, as well as fundamental results on limits, continuous functions, and convergence of sequences and series.

In Chap. 1, I present the questions as they were given. However, the solutions are in the following chapters, arranged according to their subject matter in chronological order. Where a problem can be classified in more

than one way, I have provided the statement of the problem in each relevant chapter, but the solution appears in only one chapter, indicated by a number in parentheses at the beginning of its statement in Chap. 1. At the beginning of some solutions, there is a name in square brackets of either a competitor or a colleague, to acknowledge the source of an approach to the problem. However, I take full responsibility for the actual presentation.

The book concludes with two appendices. The first establishes notation and conventions and lists definitions and results that might not be readily available to every student. The second lists the names of students who performed well in the competition.

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Toronto, Canada

Edward J. Barbeau

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Barbeau, E.J.

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