# Transformations of Digraphs Viewed as Intersection Digraphs

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## 1 Introduction

A great deal of research has been done in the area of transformations on graphs and digraphs, found in connection with work done in groups on graphs.

The best known and most thoroughly studied among these transformations has been the line graph, that was officially introduced as such by Whitney [40] in 1932, and by 1970 has been completely characterized by Krausz [28], van Rooij and Wilf [34] and Beineke [3]. The middle graph, was introduced independently by Chikkodimath and Sampathkumar [10], and Hamada and Yoshimura [20]. Middle graphs have been characterized in several ways by Akiyama et al. [1]. The total graph, was introduced in 1967 and studied by Behzad [2].

For over half a century transformations on digraphs, introduced as analogues of the corresponding transformations on graphs, have also received a great deal of attention. We refer to the line, total, and middle digraph, which have been introduced in 1960 by Harary and Norman [24], in 1964 by Chartrand and Stewart [9], and in 1981 (1977 in her Ph.D. thesis) by Zamfirescu [42], respectively. Characterizations have been given by Heuchenne [25] for the line digraph, by Zamfirescu [42] for the middle digraph, and by Skowronska et al. [36] for the total digraph. In addition, a lot of research has been done studying these transformations in various contexts [1–44].

Using intersections of sets belonging to a family of sets, in order to define the edge connections in a graph is so natural that it arose independently in a number of areas in connection with both pure and applied mathematics, and has been studied for over 7 decades. Let $U$ be a set, and $\mathcal{F} = \{F_i\}_i$ a finite family of non-empty subsets of $U$. The intersection graph $\Omega(\mathcal{F})$ is the graph with the vertex set $\mathcal{F}$ in which $\{F_i, F_j\}$ is an edge if and only if the intersection of the sets $F_i$ and $F_j$ is non-empty. At the same

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time, if $G = \Omega(\mathcal{F})$ then $\mathcal{F}$ is called a *set representation* of the graph $G$. As far as we know, the first person to formulate this definition in such a broad fashion, without restricting the nature of either the set $U$ or of the family $\mathcal{F}$ appears to have been Marczewski [30] in 1945. He also established that every graph is the intersection graph of some family of subsets of a finite set.

A lot of research has been done on various concepts that represent certain types of intersection graphs. Among these is the interval graph, $\Omega(\mathcal{F})$, where $U = \Re$, the real line, and each set $F_i$ in $\mathcal{F}$ is an interval; certain interval graphs with various sorts of restrictions, such as unit-interval graphs, and multiple interval graphs; $n$-dimensional interval graph; circular-arc graph, etc. The monograph written by Mc Kee and Mc Morris [31] on Intersection Graph Theory is an excellent resource, as well as a good reference for most notations used in this paper. For other ones, not defined here, please use Harary's Graph Theory [22].

On the other hand, the study of similar concepts for digraphs has just started. Beineke and Zamfirescu [4] and Sen et al. [35] introduced and studied in different contexts a natural analogue of the intersection graph model for digraphs. Beineke and Zamfirescu [4] made for the first time a connection between these new intersection digraphs and transformations on digraphs.

**Definitions**

A digraph $D = (V, A)$ has $V$ as vertex set, and $A$ as arc set. We may also use the notations $V(D)$ and $A(D)$, to denote $V$ and $A$, respectively. Note that, unless otherwise specified, from now on $D$ may have loops but no multiple arcs, $D$ is weakly connected, and has at least two points.

Let's consider a family of ordered pairs of subsets of a set $U$, and to each ordered pair let's assign a vertex $v \in V$. Let $S_v$ (source set) be the first set in the ordered pair assigned to $v$, and $T_v$ (terminal set) be the second one. The *intersection digraph* of this family of ordered pairs of sets, $\mathcal{F} = \{(S_v, T_v)\}_{v \in V}$, is the digraph $D$ that has $V$ as vertex set and $uv \in A$ iff $S_u \cap T_v \neq \emptyset$.

In [4, 35], it was shown that every digraph is the intersection digraph of ordered pairs of subsets of some set $U$. In [43, 44], it was shown that the line, middle, total, and subdivision digraph of a digraph $D$ can all be generated as intersection digraphs of ordered pairs of subsets of a universal set of symbols $U$, which contains only vertices and arcs of the digraph $D$, the digraph to be transformed.

This type of intersection digraph representation, using only elements of the transformed digraph, could make possible a unique computer treatment of all these transformations of the same digraph.

Let the *intersection number*, $i\#(D)$, of a digraph $D$ be the minimum size of a set $U$, such that $D$ is the intersection digraph of ordered pairs of subsets of $U$. We are raising here the problem of expressing the intersection number of a transformed digraph as a function of the size of the vertex set or arc set of the original digraph that was transformed, and we solve this problem for most transformation digraphs we mentioned here.

The transformations on digraphs we consider in this paper are all based on the concept of *directed adjacency*, which throughout this paper will simply be called adjacency. This *adjacency* can be between two points ($x$ is adjacent to $y$, iff $xy$ is an arc), two arcs ($\alpha$ is adjacent to $\beta$, iff the ending point of the first arc is the starting point of the second, e.g. $\alpha = xy$ and $\beta = yz$), and one of each ($x$ is adjacent to any arc $\alpha = xz$ having $x$ as starting point, and any arc $\alpha = xz$ is adjacent to its ending point, in this case $z$). Furthermore, $x$ is called a *source* (sink), iff there are no points adjacent to (from) $x$, and $x$ is called a *carrier* iff it is adjacent both to exactly one other vertex, and from exactly one other vertex.

The transformations of the digraph $D$ express adjacencies within $D$ in various ways: The line digraph reflects the adjacencies among the arcs in $D$, the original digraph. The total digraph reflects the adjacencies between all elements of the original digraph: between vertices, between arcs, and between vertices and arcs (meaning one of each). The middle digraph reflects the adjacencies in $D$ between arcs, between vertices and arcs, but not the adjacencies between vertices. The well-known subdivision digraph reflects only the adjacencies in $D$, that exist between vertices and arcs.

Next we will define these 4 transformations for a digraph $D = (V, A)$, and mention theorems given in [44], that generate these transformations as intersection digraphs, using $U = V \cup A$, which means that the universal set $U$, of the intersection digraph consists only of elements of $D$.

The *line digraph*, denoted $\mathcal{L}(D)$, of the digraph $D$ has as vertex set $A$, the arc set of $D$, and there is an arc in $\mathcal{L}(D)$ from one vertex $\widehat{uv}$ [NB: $\widehat{uv}$ will denote the vertex in $\mathcal{L}(D)$, that represents the arc $uv$ in $D$] to another vertex $\widehat{wz}$ iff $v \equiv w$, i.e. the adjacency of the arcs in $D$ is preserved for the corresponding vertices in $\mathcal{L}(D)$.

The *total digraph*, denoted $\mathcal{T}(D)$, of the digraph $D$ has as vertex set $V \cup A$, and two such elements are connected by an arc in $\mathcal{T}(D)$ iff the corresponding elements in $D$ are adjacent in $D$.

The *middle digraph*, denoted $\mathcal{M}(D)$, of the digraph $D$, has as vertex set $V \cup A$, and two such vertices in $\mathcal{M}(D)$ are connected by an arc in $\mathcal{M}(D)$ iff they are not both vertices in $D$, and the corresponding elements in $D$ are adjacent in $D$.

The *subdivision digraph*, denoted $\mathcal{S}(D)$, of the digraph D, has as vertex set $V \cup A$, and two such elements are connected by an arc in $\mathcal{S}(D)$ iff one of them is an arc and the other one a vertex of $D$, and they are adjacent in $D$. This is equivalent to the more common definition of a subdivision digraph, which defines it is as the digraph we obtain from $D$ by attaching one extra point on each arc of $D$ and thus subdivide each arc into two new arcs in $\mathcal{S}(D)$.

In the Fig. 1 below we exemplify all these transformations for a digraph $D_0$, with the vertex set $V(D_0) = \{a, b, c, d, e, f, g\}$, where the two types of vertices and the three types of arcs of the transformed digraphs are marked in such a way that they intuitively show their provenience: The empty (bold) points represent the vertices of the original digraph $D_0$, (respectively those vertices corresponding to arcs in $D_0$), while the wavy (double) [plain] arcs in any transformed or original digraph represent the adjacencies that exist between vertices in the original digraph $D_0$ (represent the adjacencies that exist between arcs in $D_0$) [represent the adjacencies that exist between vertices and arcs and arcs and vertices in $D_0$].

## 2 Results

**Theorem A** [44]
$\mathcal{L}(D)$, the line digraph of $D = (V, A)$, is the intersection digraph of the family $\mathcal{F}$ of ordered pairs of subsets of the universal set $U = V$, defined by:
$\mathcal{F} = \{(S_{\widehat{uv}}, T_{\widehat{uv}})\}_{\widehat{uv} \in A(\mathcal{L}(D))}$, where $S_{\widehat{uv}} = \{v\}$, and $T_{\widehat{uv}} = \{u\}$.

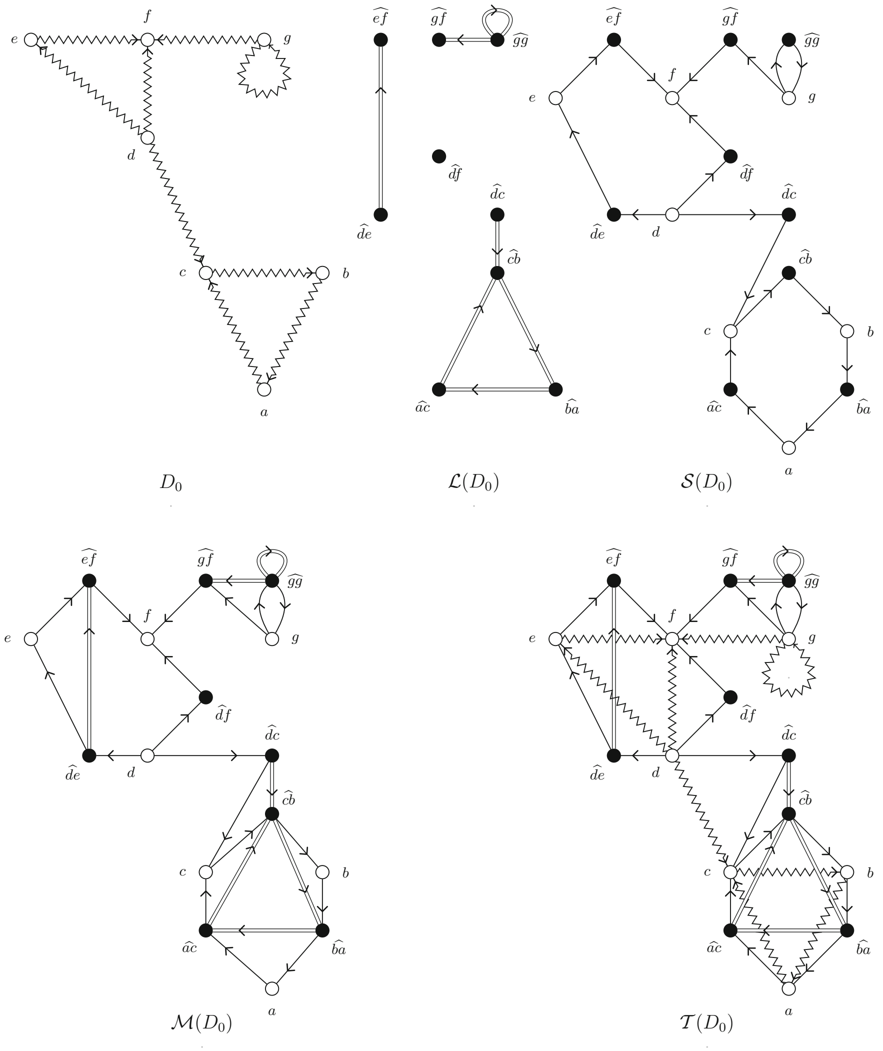


**Fig. 1** The Figure shows a digraph and its line, subdivision, middle, and total digraph, respectively

**Theorem B** [44]

$\mathcal{T}(D)$, the total digraph of $D = (V, A)$, is the intersection digraph of the family $\mathcal{F} = \{(S_\varepsilon, T_\varepsilon)\}_{\varepsilon \in U}$, of ordered pairs of subsets of the universal set $U = V \cup A$, defined by:

$S_\varepsilon = \{\varepsilon\} \bigcup \{\widehat{\varepsilon u} : \widehat{\varepsilon u} \in A\}$ and $T_\varepsilon = \{\widehat{u\varepsilon} : \widehat{u\varepsilon} \in A\}$, for all $\varepsilon \in V$,
$S_\varepsilon = S_{\widehat{uv}} = \{\widehat{uv}, v\}$ and $T_\varepsilon = T_{\widehat{uv}} = \{u\}$, for all $\varepsilon \in A$, $\varepsilon = \widehat{uv}$.

**Theorem C** [44]

$\mathcal{M}(D)$, the middle digraph of $D = (V, A)$, is the intersection digraph of the family $\mathcal{F} = \{(S_\varepsilon, T_\varepsilon)\}_{\varepsilon \in U}$, of ordered pairs of subsets of the universal set $U = V \cup A$, defined by:

$S_\varepsilon = \{\varepsilon\}$ and $T_\varepsilon = \{\widehat{u\varepsilon} : \widehat{u\varepsilon} \in A\}$, for all $\varepsilon \in V$,
$S_\varepsilon = S_{\widehat{uv}} = \{\widehat{uv}, v\}$ and $T_\varepsilon = T_{\widehat{uv}} = \{u\}$, for all $\varepsilon \in A$, $\varepsilon = \widehat{uv}$.

**Theorem D** [44]

$\mathcal{S}(D)$, the subdivision digraph of $D = (V, A)$, is the intersection digraph of the family $\mathcal{F} = \{(S_\varepsilon, T_\varepsilon)\}_{\varepsilon \in U}$, of ordered pairs of subsets of the universal set $U = V \cup A$, defined by:

$S_\varepsilon = \{\varepsilon\}$ and $T_\varepsilon = \{\mu \in U : \varepsilon$ adjacent to $\mu$, and exactly one of $\varepsilon, \mu$ is an arc$\}$.
Next, we will aim at minimizing the number of symbols in the universal set, $U$.

The *intersection number*, i#(G), of an undirected graph $G$ is the minimum size of a set $U$, such that $G$ is the intersection graph of subsets of $U$. For the undirected case, Erdös et al. [12] showed that the intersection number of $G$ equals the minimum number of complete subgraphs needed to cover its edges. Sen et al. [35] proved an analogous result for digraphs. They defined the *generalized complete bipartite subdigraph* (abbreviated GBS) to be the subdigraph generated by vertex sets $X$, $Y$, the arcs of which are all $xy$ such that $x \in X$, and $y \in Y$. Note that $X$ and $Y$ need not be disjoint (this is how loops are covered) which justifies the "generalized" term. If $K$ is a GBS we shall call $X(K)$ and $Y(K)$ its $X$, respectively $Y$, sets. They gave the following:

**Theorem E** ([35]). The intersection number of a digraph equals the minimum number of GBSs required to cover its arcs.

We shall further give results that will express the intersection numbers of transformation digraphs of a digraph $D$, as functions of the numbers of vertices of $D$, that are sinks, sources or not sinks or not sources.

We will study the case of the line digraph separately, as it has an additional property: it satisfies the *Heuchenne Condition*, abbreviated here as H condition.

We say that a digraph fulfills the *H condition* iff for every four of its vertices, call them $u$, $v$, $w$ and $z$, not necessarily distinct, the existence of the arcs $uv$, $wv$, $wz$ implies the existence of the arc $uz$.

**Theorem F** ([25]). A digraph is a line digraph iff the H condition is fulfilled.

Let D satisfy the H condition, and let $\mathcal{C} = \{K_\sigma\}_{1 \le \sigma \le i\#(D)}$ be a set of minimum size, of GBSs that cover all arcs in $D$.

In $D$, let $uv$ be an arc in some $K_\sigma \in \mathcal{C}$. It is easy to see that, given the H condition, all arcs adjacent from $u$, and all arcs adjacent to $v$ in $D$, must also belong to $K_\sigma$ since $\mathcal{C}$ is of minimum size. We can now define an equivalence relation $R$ on the arc set

$A(D)$ by stating that two arcs are related iff one of the following is fulfilled: (a) they have the same starting point; (b) they have the same ending point; (c) there is an arc in $A(D)$ from the starting point of one arc to the ending point of the other. It is easy to see that the set of GBSs induced by the equivalence classes generated by $R$ is of minimum size, and we proved the following lemma.

**Lemma 1** *If H condition holds then $\mathcal{C}$ is uniquely determined in D.*

Next, we can see that, if we apply Lemma 1 to $\mathcal{L}(D)$, which by Theorem F fulfills condition H, then each GBS, $K_\sigma \in C$, induced in $\mathcal{L}(D)$ by the relation $R$ defined above, corresponds to exactly one vertex in $D$. That vertex in $D$ is 1) adjacent from all arcs of $D$ that correspond to the vertices in $X(K_\sigma)$, which means that it is not a source, and 2) adjacent to all arcs of $D$ that correspond to the vertices in $Y(K_\sigma)$, which means that it is not a sink. Since $K_\sigma$ contains at least one arc, that vertex in $D$ must be neither a source nor a sink. This proves the next lemma and theorem.

**Lemma 2** *There is a one-to-one correspondence between the set $\mathcal{C}$ of GBSs and the set of all vertices in D, that are neither sources nor sinks.*

**Theorem 1** *$i\#(\mathcal{L}(D))$, the intersection number of $\mathcal{L}(D)$ equals the number of vertices of D that are neither sources nor sinks.*

Let's consider now the subdivision digraph, $\mathcal{S}(D)$, of the digraph $D$. It is easy to see that, since in $\mathcal{S}(D)$ in every semipath (NB: walk in the graph without following the directions of the arcs, see [22]), every second vertex is a carrier, $\mathcal{S}(D)$ satisfies the H condition. From Lemma 1 we know that $\mathcal{S}(D)$ has a unique minimum set of GBSs that cover all its arcs, and each such GBS is induced by the arc set of one of the equivalence classes generated by the equivalence relation $R$, defined for Lemma 1. In fact, the point (c) in the definition of $R$ cannot occur in $\mathcal{S}(D)$, and thus each GBS in $\mathcal{S}(D)$ is a star (see [22]), which (a) has a source as the center, and any remaining vertex is a sink, or (b) has a sink as the center, and any remaining vertex is a source. We can attach these GBSs to only those vertices in $\mathcal{S}(D)$, that correspond to vertices in $D$. To each source (sink) will correspond exactly one GBS, consisting in a star with $n$ arms, where $n$ is the out-degree (in-degree) of the source (sink) in $D$. To each of the other vertices, we will attach exactly two GBSs, one for the in-coming arcs, and the other for the out-coming arcs. We thus proved:

**Lemma 3** *$i\#(\mathcal{S}(D))$ equals the number of vertices of D that are not sources, added to the number of vertices of D that are not sinks.*

From now on, let's consider that $D$ contains no loops.

Neither $\mathcal{M}(D)$ nor $\mathcal{T}(D)$ satisfies the H condition, generally.

We will show next that, in the case of both $\mathcal{M}(D)$ and $\mathcal{T}(D)$, although the covering of the arcs by a set of GBSs of minimum size may not be unique, their intersection numbers are equal to the intersection number of $\mathcal{S}(D)$. We will do this by extending the GBSs we formed for the $\mathcal{S}(D)$ to also cover all the arcs that are in $\mathcal{M}(D)$ or $\mathcal{T}(D)$ but not in $\mathcal{S}(D)$, by allocating each such arc, say $xy$, new to $\mathcal{S}(D)$, to the GBS that contains all arcs in $\mathcal{S}(D)$ adjacent to $y$. Similarly, we could allocate $xy$ to $x$, instead of to $y$, thus defining a generally different set of GBSs, that cover all arcs in $\mathcal{M}(D)$ or $\mathcal{T}(D)$.

In order to prove that this new set of GBSs is of minimum size it is enough to show that we cannot construct a GBS in $\mathcal{M}(D)$ or $\mathcal{T}(D)$ that contains two arcs $\alpha$ and $\beta$, that belong to two different GBSs in $\mathcal{S}(D)$. Any arc in $\mathcal{S}(D)$ joins a vertex that represents a vertex in $D$ with (i.e. to or from) a vertex that represents an arc in $D$. The latter must also be a carrier in $\mathcal{S}(D)$. If $\alpha$ and $\beta$ have a common endpoint, then this can only represent a vertex in $D$, as it is not a carrier. In this case they must be in the same GBS in $\mathcal{S}(D)$. If the starting point of $\alpha$ is the same point as the ending of $\beta$, then by the definition of the GBS, we would need to have a loop at that point, which is not allowed in $\mathcal{S}(D)$, even if $D$ had loops. If $\alpha$ and $\beta$ do not have a common endpoint, say $\alpha$ is the arc $xz$ and $\beta$ is the arc $yt$, with all endpoints distinct, then the GBS must also contain the arcs $xt$ and $yz$. Let's assume, without loss of generality, that $x$ and $t$ represent vertices, while $y$ and $z$ represent arcs in the original $D$, that we transformed. In addition, note that the arc in $D$ represented by $y$ must be adjacent to the arc in $D$ represented by $z$. A contradiction follows from the fact that $y$ and $z$ must both be carriers in $\mathcal{S}(D)$, and $D$ may not contain a loop. We therefore proved the following results.

**Lemma 4** *No GBS in $\mathcal{M}(D)$ or $\mathcal{T}(D)$ may contain two arcs that belong to two different GBSs in $\mathcal{S}(D)$.*

**Theorem 2** *If $D$ contains no loops, $i\#(\mathcal{T}(D)) = i\#(\mathcal{M}(D)) = i\#(\mathcal{S}(D))$, that is the intersection numbers of $\mathcal{T}(D)$, $\mathcal{M}(D)$ and $\mathcal{S}(D)$ are all equal to the number of vertices of $D$ that are not sources, added to the number of vertices of $D$ that are not sinks.*

We would like to note here, that Lemma 4 is no longer true when $D$ has loops, as the number of GBSs covering all arcs in $\mathcal{T}(D)$ might be reduced from the one covering $\mathcal{S}(D)$. For instance, the subgraph induced by the vertex set $\{g, \widehat{gg}, \widehat{gf}\}$ in $\mathcal{T}(D_0)$ in our Figure forms one GBS, while in $\mathcal{S}(D)$ and $\mathcal{M}(D)$ the same subgraph must be covered by two GBSs, due to the lack of the loop at the vertex $g$ in $\mathcal{S}(D)$ and $\mathcal{M}(D)$.

The problem of finding equivalent results for other transformations of digraphs, such as various power digraphs, remains also open.

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