Chapter 2
Two-Degree-of-Freedom PID Controllers Structures

As in most of the existing industrial process control applications, the desired value of the controlled variable, or set-point, normally remains constant (regulatory control or disturbance rejection operation) but needs to be changed (servo-control or set-point tracking operation) we are mainly interested in the two-degree-of-freedom (2DoF) implementation of the PID control algorithms. The extra parameter that the 2DoF control algorithm provides is used to improve their servo-control behavior while considering the regulatory control performance and the closed-loop control system robustness [1–5]. This 2DoF feature can be incorporated both into a PI or a PID control algorithm. Although all the controllers with a proportional integral (PI) control algorithm are implemented in the same way, have the same transfer function, this is not the case with commercial controllers with proportional integral derivative (PID) control algorithms.

In fact, usually, the control algorithm implementation is manufacturer dependent and not all of its variations are available in the same controller. Even more, the controllers manufacturers use different names for the same PID algorithm [6]. The diversity of the PID control algorithms is evident in [7]. In addition, it would be the case that a tuning rule of interest had been obtained using a control algorithm different from the one implemented in the controller to tune. In this case, controller parameters conversion is required that will also indicate if the pursued equivalent controller exists.

On that basis, the most widely used PID control algorithms are presented in this chapter by also providing conversion formulae that allows to convert the parameters of one algorithm from those obtained for another formulation. As it will be seen this conversion will not always be possible, showing some formulations are more general than others.

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2.1 Proportional Integral Derivative Control Algorithm

Consider the general controller block diagram depicted in Fig. 2.1. The output or *control effort* of a proportional (P) integral (I) and derivative (D) control algorithm is given, in general, by

$$U(t) = Action\,\{U_P(t) + U_I(t) + U_D(t) + U_b\}, \tag{2.1}$$

if $0\,\% \le U(t) \le 100\,\%$, and 0 or $100\,\%$, depending on the controller action if the controller output reaches one of its limits.

In (2.1) U_P is the proportional term or *proportional control action*, given by

$$U_P(t) = K_p E(t) = K_p[R(t) - Y(t)], \tag{2.2}$$

with a proportional gain K_p; U_I is the integral term or *integral control action*, given by

$$U_I(t) = K_i \int_0^t E(\xi)\mathrm{d}\xi = K_i \int_0^t [R(\xi) - Y(\xi)]\mathrm{d}\xi, \tag{2.3}$$

with an integral gain K_i; and U_D the derivative term or *derivative control action*, given by

$$U_D(t) = K_d \frac{\mathrm{d}E(t)}{\mathrm{d}t} = K_d \frac{\mathrm{d}[R(t) - Y(t)]}{\mathrm{d}t}, \tag{2.4}$$

with a derivative gain K_d.The controller output *bias* U_b is usually set to $50\,\%$. In (2.1)–(2.4) controller inputs $R(t)$ and $Y(t)$, and output $U(t)$ change in the range from 0 to $100\,\%$.

The controller *Action* sign, $+1$ (Reverse) or -1 (Direct), must be selected equal to the controlled process gain sign to preserve the negative feedback characteristic of the control loop.

In the following, we will assume that the controller Action has been selected correctly, that all the closed-loop control variables are within their corresponding 0–$100\,\%$ range, and that the control system is initially at a steady-state stable operating

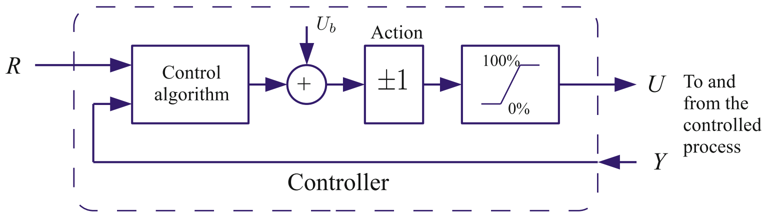

Fig. 2.1 Controller block diagram

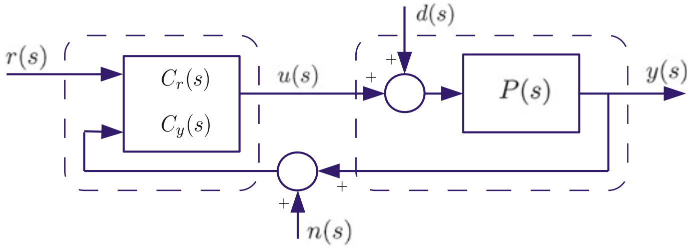

Fig. 2.2 Closed-loop control block diagram

point given by $\{R_o, Y_o, U_o\}$. Then, we only consider deviation variables $\{r, y, u\}$ around this operating point and then the controller output bias will not be included in following controllers equations.

A linear control system is based on a linearized process model description that relates deviation variables from its operating point values. On that basis, the linearized closed-loop control system for variable deviations $r(s)$, $y(s)$, $u(s)$, and $d(s)$ is reduced as depicted in Fig. 2.2, where $P(s)$ is the transfer function of the controlled process model and $C_r(s)$ and $C_y(s)$ the controller aspects applied to the set-point and the feedback signal, respectively. The possible measurement noise $n(s)$ has been also included.

2.2 Two-Degree-of-Freedom (2DoF) PID Control Algorithms

The most widely used proportional integral derivative or PID control algorithms are briefly described below. Each formulation is provided by a specific notation for its parameters in order to distinguish the corresponding implementations when proceeding later on to provide the conversion equations from one algorithm to the other.

2DoF Standard PID

The "textbook" 2DoF proportional integral derivative control algorithm is the Standard PID whose output is given by the following [8–10]:

$$u(t) = K_p \left\{ e_p(t) + \frac{1}{T_i} \int_0^t e_i(\xi)\mathrm{d}\xi + T_d \frac{\mathrm{d}e_d(t)}{\mathrm{d}t} \right\}, \tag{2.5}$$

or

$$u(s) = K_p \left\{ e_p(s) + \frac{1}{T_i s} e_i(s) + \frac{T_d s}{\alpha T_d s + 1} e_d(s) \right\}, \tag{2.6}$$

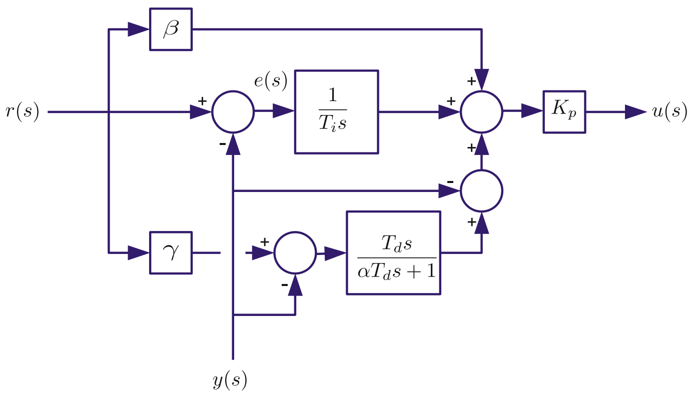

Fig. 2.3 Two-degree-of-freedom Standard PID controller

with

$$e_p(s) = \beta r(s) - y'(s), \tag{2.7}$$

$$e_i(s) = r(s) - y'(s), \tag{2.8}$$

$$e_d(s) = \gamma r(s) - y'(s), \tag{2.9}$$

$$y'(s) = y(s) + n(s), \tag{2.10}$$

where K_p is the *controller gain*, T_i the *integral time constant*, T_d the *derivative time constant*, β and γ the *set-point weights*, and α the *derivative filter constant*. The 2DoF PID block diagram is depicted in Fig. 2.3.

To avoid an extreme instantaneous change at the controller output signal when a set-point step change occurs normally γ is set to zero [11, 12]. In this case (2.6) reduces to

$$u(s) = K_p \left\{ \beta r(s) - y'(s) + \frac{1}{T_i s}\left[r(s) - y'(s)\right] - \left(\frac{T_d s}{\alpha T_d s + 1}\right) y'(s) \right\}, \tag{2.11}$$

that will be denoted as PID_2. In addition, in the following it is assumed that the measurement noise is filtered, then $y'(s) \approx y(s)$.

The controller output (2.11) may be rearranged, for analysis purposes, as follows:

$$u(s) = K_p \left(\beta + \frac{1}{T_i s}\right) r(s) - K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\alpha T_d s + 1}\right) y(s), \tag{2.12}$$

where the $C_r(s)$ and $C_y(s)$ controller aspects read as

$$C_r(s) = K_p \left(\beta + \frac{1}{T_i s}\right), \tag{2.13}$$

$$C_y(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\alpha T_d s + 1}\right), \tag{2.14}$$

being the controller parameters $\theta_c = \{K_p, T_i, T_d, \alpha, \beta, \gamma = 0\}$. Although the Standard form is the classical implementation of the PID control algorithm, the following forms are also found in the control literature [10, 12, 13].

2DoF Parallel PID

The parallel or "independent gains" PID control algorithm is

$$u(s) = \left(\beta_p K_p + \frac{K_i}{s}\right) r(s) - \left(K_p + \frac{K_i}{s} + \frac{K_d s}{\alpha_p K_d s + 1}\right) y(s), \tag{2.15}$$

where the $C_r(s)$ and $C_y(s)$ controller aspects read as

$$C_r(s) = \left(\beta_p K_p + \frac{K_i}{s}\right), \tag{2.16}$$

$$C_y(s) = \left(K_p + \frac{K_i}{s} + \frac{K_d s}{\alpha_p K_d s + 1}\right), \tag{2.17}$$

with parameters $\theta_{cp} = \{K_p, K_i, K_d, \alpha_p, \beta_p, \gamma_p = 0\}$. K_p is the proportional gain, K_i the integral gain, and K_d the derivative gain.

2DoF Series or "Industrial" PID

The 2DoF version of the series "interacting" implementation of the PID algorithm is

$$u(s) = K_p' \left(\beta' + \frac{1}{T_i' s}\right) r(s) - K_p' \left(1 + \frac{1}{T_i' s}\right) \left(\frac{T_d' s + 1}{\alpha' T_d' s + 1}\right) y(s), \tag{2.18}$$

where the $C_r(s)$ and $C_y(s)$ controller aspects read as

$$C_r(s) = K_p' \left(\beta' + \frac{1}{T_i' s}\right), \tag{2.19}$$

$$C_y(s) = K_p' \left(1 + \frac{1}{T_i' s}\right) \left(\frac{T_d' s + 1}{\alpha' T_d' s + 1}\right), \tag{2.20}$$

with parameters $\theta_c' = \{K_p', T_i', T_d', \alpha', \beta', \gamma' = 0\}$.

2DoF Ideal PID with Filter

A commonly used PID implementation in Internal Model Control (IMC)-based controller design is given by the following:

$$u(s) = K_p^* \left(\beta^* + \frac{1}{T_i^* s}\right) r(s) - K_p^* \left(1 + \frac{1}{T_i^* s} + T_d^* s\right) \left(\frac{1}{T_f s + 1}\right) y(s), \tag{2.21}$$

where the $C_r(s)$ and $C_y(s)$ controller aspects read as

$$C_r(s) = K_p^* \left(\beta^* + \frac{1}{T_i^* s} \right), \tag{2.22}$$

$$C_y(s) = K_p^* \left(1 + \frac{1}{T_i^* s} + T_d^* s \right) \left(\frac{1}{T_f s + 1} \right), \tag{2.23}$$

with parameters $\theta_c^* = \left\{ K_p^*, T_i^*, T_d^*, T_f, \beta^*, \gamma^* = 0 \right\}$. T_f is the controller intput filter time constant.

2.3 PID Control Algorithms Conversion Relations

As it can be observed from the presented PID forms, whereas the reference controller aspect takes the same form in all formulations, it is the feedback part the one that prevents a direct translation of the controller parameters from one formulation to another. This is important because some of the existing tuning rules have been conceived for a specific PID formulation. As an example, the derivations of the celebrated SIMC tuning [14] are with the Series or Industrial formulation in mind, whereas much of the other proposals are based on the Standard one. Due to the possibility that the control PID algorithm of the controller to tune be different to the one considered by the tuning rule to use it is necessary to have conversion relations to obtain "equivalent" parameters between two or more of them [15]. In what follows, we present conversion formulae to get the controller parameters for one specific PID formulation starting from the parameters got for another different one.

Conversion from a 2DoF Parallel PID to a Standard PID

A PID_2 controller (2.11) equivalent to the 2DoF Parallel PID (2.15) is found using the following relations:

$$K_p = K_p, \tag{2.24}$$

$$T_i = \frac{K_p}{K_i}, \tag{2.25}$$

$$T_d = \frac{K_d}{K_p}, \tag{2.26}$$

$$\alpha = \alpha_p K_p, \tag{2.27}$$

$$\beta = \beta_p, \tag{2.28}$$

$$\gamma = \gamma_p = 0. \tag{2.29}$$

There is a direct relation between the Standard and Parallel PID algorithms then this last one will not be further considered.

Conversion from a 2DoF Series PID to a Standard PID

It is possible to obtain a Standard 2DoF PID controller (2.11) equivalent to the 2DoF Series PID (2.18) using the following relations:

$$K_p = F_c' K_p', \tag{2.30}$$

$$T_i = F_c' T_i', \tag{2.31}$$

$$T_d = \frac{(1 - \alpha' F_c') T_d'}{F_c'}, \tag{2.32}$$

$$\alpha = \frac{F_c' \alpha'}{1 - \alpha' F_c'}, \quad \alpha' < 1 + \frac{T_i'}{T_d'}, \tag{2.33}$$

$$\beta = \frac{\beta'}{F_c'}, \tag{2.34}$$

$$\gamma = \gamma' = 0, \tag{2.35}$$

$$F_c' = 1 + \frac{(1 - \alpha') T_d'}{T_i'}. \tag{2.36}$$

where F_c' (2.36) is the PID_{2s} to PID_2 *conversion factor*. It takes into account the derivative filter constant α'.

The conversion constraint in (2.33) usually holds then we may say that there is a Standard PID equivalent to a Series one.

Conversion from a 2DoF Ideal PID with Filter to a Standard PID

A Standard 2DoF PID controller (2.11) equivalent to the Ideal PID with filter (2.21), denoted by PID_{2F}, can be obtained using the following relations:

$$K_p = F_c^* K_p^*, \tag{2.37}$$

$$T_i = F_c^* T_i^*, \tag{2.38}$$

$$T_d = \frac{T_d^*}{F_c^*} - T_f, \quad T_d^* > F_c^* T_f, \tag{2.39}$$

$$\alpha = \frac{F_c^* T_f}{T_d^* - F_c^* T_f}, \tag{2.40}$$

$$\beta = \frac{\beta^*}{F_c^*}, \tag{2.41}$$

$$\gamma = \gamma^* = 0, \tag{2.42}$$

$$F_c^* = 1 - \frac{T_f}{T_i^*}, \tag{2.43}$$

$$T_f < T_i^*, \quad \text{for PI } T_f = 0. \tag{2.44}$$

where F_c^* (2.44) is the PID_{2F} to PID_2 *conversion factor*.

In this case, an equivalent PID_2 controller cannot always be obtained as shown in (2.39) and (2.44).

Using the conversion factors presented above, exact equivalent feedback ($C_y(s)$) and set-point ($C_r(s)$) controllers transfer functions for a PID_2 (2.12) may be obtained for 2DoF PID controllers given by (2.15), (2.18), and (2.21).

Exact equivalent controllers guarantee to obtain the same control system performance and robustness in case a 2DoF PID controller is replaced with a PID controller with a different 2DoF algorithm.

Conversion from a 2DoF Standard PID to a Series PID

In the other direction a 2DoF Series PID controller equivalent to a 2DoF Standard one can be found using the following relations:

$$K_p' = F_c K_p, \tag{2.45}$$

$$T_i' = F_c T_i, \tag{2.46}$$

$$T_d' = \frac{(1+\alpha)T_d}{F_c}, \tag{2.47}$$

$$\alpha' = \frac{\alpha F_c}{1+\alpha}, \tag{2.48}$$

$$\beta' = \frac{\beta}{F_c}, \tag{2.49}$$

$$\gamma' = \gamma = 0, \tag{2.50}$$

$$F_c = 0.5\left[1 + \frac{\alpha T_d}{T_i} + \sqrt{1 - \frac{(4+2\alpha)T_d}{T_i} + \frac{\alpha^2 T_d^2}{T_i^2}}\right]. \tag{2.51}$$

Due to the constraint imposed by (2.51) there will not always exist a Series PID equivalent to a Standard PID. It will only exist if

$$\alpha^2\left(\frac{T_d}{T_i}\right)^2 - (4+2\alpha)\left(\frac{T_d}{T_i}\right) + 1 > 0. \tag{2.52}$$

If the PID_2 derivative filter constant is taken as $\alpha = 0.1$ there is a Series equivalent PID controller only if $T_i > 4.20\ T_d$. Figure 2.4 shows that this constrain increases as α increases.

As can be seen in same figure quadratic inequality (2.52) can be approximated by the following straight line for $0 \leq \alpha \leq 1.0$:

$$\frac{T_i}{T_d} > 4.05 + 1.80\ \alpha. \tag{2.53}$$

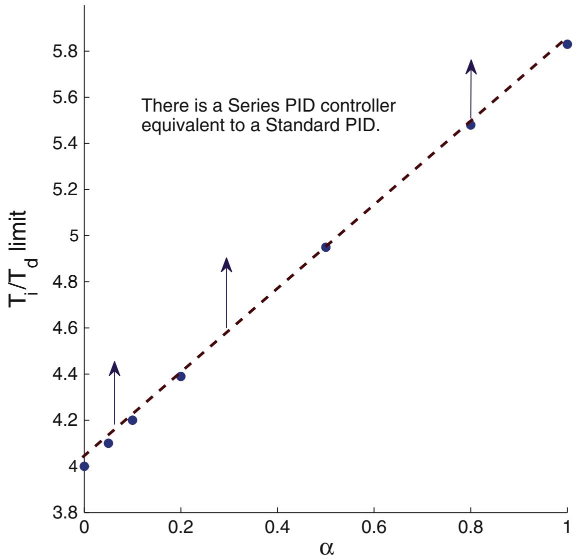

Fig. 2.4 T_i/T_d condition to obtain a Series PID equivalent to a Standard PID

Conversion from a 2DoF Standard PID to an Ideal PID with Filter

The PID_{2F} is a more general control algorithm and, as indicated above, not always an equivalent PID_2 controller may be obtained from the PID_{2F} but it is always possible to obtain a PID_{2F} control algorithm equivalent to the PID_2 using the following relations:

$$K_p^* = F_{cf} K_p, \tag{2.54}$$

$$T_i^* = F_{cf} T_i, \tag{2.55}$$

$$T_d^* = \left(\frac{1+\alpha}{F_{cf}} \right) T_d, \tag{2.56}$$

$$T_f = \alpha T_d, \tag{2.57}$$

$$\beta^* = \frac{\beta}{F_{cf}}, \tag{2.58}$$

$$\gamma^* = 0, \tag{2.59}$$

$$F_{cf} = 1 + \frac{\alpha T_d}{T_i}. \tag{2.60}$$

where F_{cf} (2.60) is the PID_2 to PID_{2F} *conversion factor*.

Considering the above we may say that in the 2DoF PID controllers parametric space $\theta_c' \subset \theta_c \subset \theta_c^*$. Controller parameters conversion equations show that the derivative filter constant $(\alpha, \alpha_p, \alpha')$ must be take into account to obtain an equivalent controller with a different control algorithm from a given one.

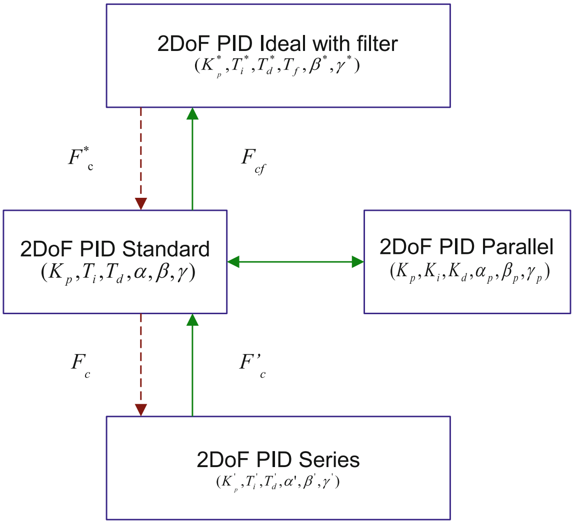

Fig. 2.5 2DoF PID controllers conversion

To summarize the above relations a 2DoF PID controllers conversion chart is shown in Fig. 2.5. The solid arrows indicate directions on where there are always equivalent controllers and the dashed arrows the directions on where there are constraints to obtain equivalent controllers. As can be seen in this chart the 2DoF Ideal PID with filter is the most general proportional integral derivative control algorithm.

2.4 PID Controller with Two Input Filters

The different signals that enter the PID controller are normally filtered in different ways before they enter the controller. However, as pointed out in [16], a proper choice of these filters can improve the performance of the feedback loop considerably. Therefore, it is important to keep these filters in mind during the design procedures. In order to include into the controller design the measurement noise filter and also to have more freedom for the servo-control design, the control algorithm may be aggregated with two input filters as depicted in Fig. 2.6 [16, 17]. These filters should be considered as an integral part of the design procedure.

The control algorithm is of independent gains (ideal parallel PID implementation) whose output signal is given by [8]:

$$u(s) = K_p \left[r'(s) - y'(s)\right] + \frac{K_i}{s} \left[r'(s) - y'(s)\right] - K_d s y'(s), \tag{2.61}$$

where K_p is the controller *proportional gain*, K_i the *integral gain*, and K_d the *derivative gain* ($\gamma = 0$).

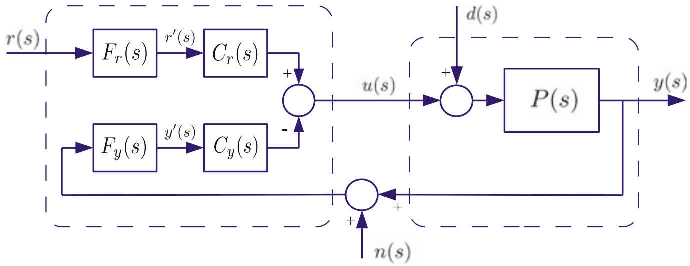

Fig. 2.6 Closed-loop control system of a controller with two input filters

The set-point r and feedback y signals are filtered before they enter the controller. Then r' and y' in (2.61) are given by

$$r'(s) = F_r(s)r(s), \;\; y'(s) = F_y(s)\,[y(s) + n(s)]\,. \tag{2.62}$$

Using (2.62) into (2.61), it is obtained that

$$u(s) = \left(K_p + \frac{K_i}{s}\right) F_r(s)r(s) - \left(K_p + \frac{K_i}{s} + K_d s\right) F_y(s)\,[y(s) + n(s)]\,. \tag{2.63}$$

In a compact form (2.63) is expressed as

$$u(s) = C_r(s)F_r(s)r(s) - C_y(s)F_y(s)\,[y(s) + n(s)]\,. \tag{2.64}$$

The *set-point filter* $F_r(s)$ is selected strictly proper and given by the transfer function

$$F_r(s) = \frac{\sigma T_r s + 1}{(T_r s + 1)^2}, \tag{2.65}$$

where T_r is its time constant and σ an adjustable parameter. Filter (2.65) avoids to have a step change in the controller output when a set-point step change is made.

The *feedback filter* ("noise filter") $F_y(s)$ is selected of first order for PI controllers, given by

$$F_y(s) = \frac{1}{D_{fy}(s)} = \frac{1}{T_f s + 1}, \tag{2.66}$$

with time constant T_f, and of second order for PID controllers, given by

$$F_y(s) = \frac{1}{D_{fy}(s)} = \frac{1}{T_f^2/2s^2 + T_f s + 1}, \tag{2.67}$$

to provide high-frequency roll-off (measurement noise attenuation) with either controllers.

Input filters transfer function gains are constrained to be equal, $\lim_{s\to 0} F_r(s) = \lim_{s\to 0} F_y(s)$, to ensure that in steady state the controller integral action operates on the error signal.

Considering F_r and F_y as part of the "controller" be designed the selectable parameters of the set-point controller are $\theta_{cr} = \left\{K_p, K_i, T_r, \sigma, \gamma = 0\right\}$, and corresponding to the feedback controller $\theta_{cy} = \left\{K_p, K_i, K_d, T_f\right\}$. Then, parameters of the controller as a whole are $\theta_c \doteq \theta_{cr} \bigcup \theta_{cy} = \left\{K_p, K_i, K_d, T_f, T_r, \sigma, \gamma = 0\right\}$.

The set-point and feedback signal filters combination with the PID control algorithm is denoted as PID_{2IF} controller. For tuning rules comparison, in addition to the quantitative performance and robustness indices and the responses shapes, the process control-oriented characteristics of the PID_{2IF} controllers must bring to the front.

With the PID_{2IF} controllers there is not any abrupt change at the controller output when a step change is made on the set-point. To mimic this characteristic with a 2DoF PID controller its proportional set-point weight β must be made zero. With this, the second degree of freedom is lost and the servo-control response delayed.

The other important characteristic of the PID_{2IF} controllers is their frequency response roll-off. It is normal that in process control applications the feedback signal be corrupted with high-frequency measurement noise. If this noise is not properly filtered it will generate high control signal variations resulting in a deterioration of the final control element. If a measurement noise filter is added to a Standard PID controller *after* its tuning the filter dynamics will affect the control system robustness and performance. Then, it is essential that both these characteristics be part of the controller design from the beginning.

Chapter Remarks

The (2DoF) PID algorithm implementations are presented as well as the conversion relations between their parameters.

From the presented PID algorithms the Ideal PID with filter is the more general one.

The aggregation of the deal PID control algorithm with two input filters allows to include two important industrial features: high-frequency roll-off and lack of a control effort abrupt change on a step set-point modification.

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