Towards Cyclic Scheduling of Grid-Like Structure Networks

Robert Wójcik, Grzegorz Bocewicz and Zbigniew Banaszak

Abstract The paper treats about a grid-like topology of different means of transport acting in mesh-like streets network in which several modes interact each other via common shared hubs (i.e. stops, interchange stations, cross-platforms, etc.) as to provide mass customized passenger services, tailored to each travel destination. In that context, a grid-like layout of various transport modes such as tram, bus, train, subway where passenger flows are treated as multimodal processes can be seen as a real-life example of the considered case. The goal is to provide a declarative model representation enabling to state a constraint satisfaction problem aimed at multimodal transportation processes scheduling aimed at dedicated flows of passengers service. The main objective is to provide conditions guaranteeing the right match-up of local cyclic acting tram/bus/metro line schedules to a given passenger flow itineraries.

Keywords Grid structure · Cyclic scheduling · Multimodal processes · Declarative modeling · Constraint satisfaction problem

R. Wójcik (✉)
Department of Computer Engineering, Faculty of Electronics,
Wrocław University of Technology, Wroclaw, Poland
e-mail: robert.wojcik@pwr.wroc.pl

G. Bocewicz
Department of Computer Science and Management,
Koszalin University of Technology, Koszalin, Poland
e-mail: bocewicz@ie.tu.koszalin.pl

Z. Banaszak
Department of Business Informatics, Warsaw University of Technology,
Wroclaw, Poland
e-mail: Z.Banaszak@wz.pw.edu.pl

L. Borzemski et al. (eds.), *Information Systems Architecture and Technology: Proceedings of 36th International Conference on Information Systems Architecture and Technology – ISAT 2015 – Part I*,
Advances in Intelligent Systems and Computing 429, DOI 10.1007/978-3-319-28555-9_2

1 Introduction

Numerous road network patterns deployed in cities rang from the tightly structured mesh-like network with perpendicular roads in a regular raster pattern to the hierarchical network with sprawling secondary and tertiary roads feeding into arterial roads in a branch like system [4, 6]. A grid-like topology of different means of transport providing circular service and operating in such mesh-like streets network consists of different bus/tram/subway lines interacting each other via distinguished subsets of common shared hubs as to provide a variety of mass customized passenger services and guarantee no one area of city network gets preferential service treatment over another [8]. Multimodal processes [2, 3] executed in grid-like network of transport modes (GNTM), see Fig. 1 providing

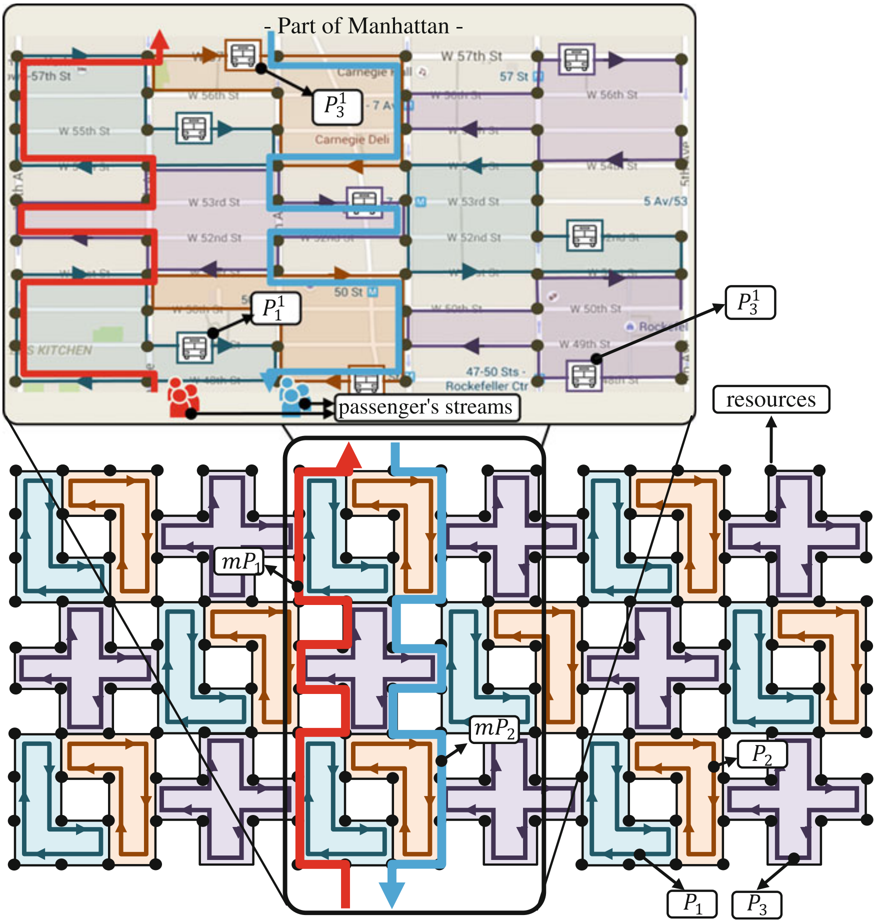

Fig. 1 An example of grid-like structure network

connection from origin to destination, can be seen as passengers and/or goods flows transferred between different modes to reach their destination [9]. The throughput of passengers and/or freight depends on geometrical and operational characteristics of GNTM. In that context the solutions of the layout designs exposing the grid-like structures are frequently observed [4].

The problems arising in these kind of networks concern multimodal routing of freight flows and supporting them multimodal transportation processes (MTP) scheduling. Since the transportation processes executed by particular lines are usually cyclic, hence the multimodal processes supported by them have also periodic character. That means, the periodicity of MTP depends on periodicity of local processes executed in GNTM.

Many models and methods aimed at cyclic scheduling have been considered so far [7]. Among them, the mathematical programming approach [1, 13], max-plus algebra [10], constraint programming [2], Petri nets [12] frameworks belong to the more frequently used. Most of them are oriented at finding of a minimal cycle or maximal throughput while assuming deadlock-free processes flow. However, the approaches trying to estimate the cycle time from cyclic processes structure and the synchronization mechanism employed (i.e. mutual exclusion instances) while taking into account deadlock phenomena are quite unique. In that context our main contribution is to propose a new modeling framework enabling to evaluate the cyclic steady state of a given GNTM encompassing the behavior typical for passenger transportation services (see Fig. 1a) in the mesh-like streets network.

2 Grid-Like Structure Networks

The GNTM shown in Fig. 1 can be seen as a network of bus/trams lines providing periodic service along cyclic routes and can be modeled in terms of a System of Concurrently executed Cyclic Processes (SCCP) as shown in Fig. 2a, b [2, 5]. The structure of considered SCCP consists of patterns indexed by (i) and composed of following shapes "⌊", "⌈", "+" encompassing three kinds of **local cyclic processes**, viz. P_1, P_2,P_3, (distinguished in Fig. 2 by ${}^{(i)}P_1$, ${}^{(i)}P_2$, ${}^{(i)}P_3$, respectively). The processes follow the **routes** composed of transportation sectors and workstations (distinguished in Fig. 2b by the **set of resources** $R = \{R_1, \ldots, R_c, \ldots, R_{18}\}$, R_c—the cth resource). The local cyclic processes P_i contain the **streams** P_i^k (the kth stream of the ith local process P_i is denoted as P_i^k): $P_i = \{P_i^1, \ldots, P_i^k, \ldots P_i^{ls(n)}\}$. In the considered case all processes contain only the unique streams: $P_1 = \{P_1^1\}, P_2 = \{P_2^1\}, P_3 = \{P_3^1\}$. Apart from local processes, we consider two **multimodal processes** (i.e. processes executed along the routes consisting parts of the routes of local processes): mP_1, mP_2.

For example, the transportation route depicted by the red line corresponds to the multimodal process mP_1 supported by vehicles (busses and etc.), which in turn encompass local transportation streams P_3^1 and P_1^1. This means that the

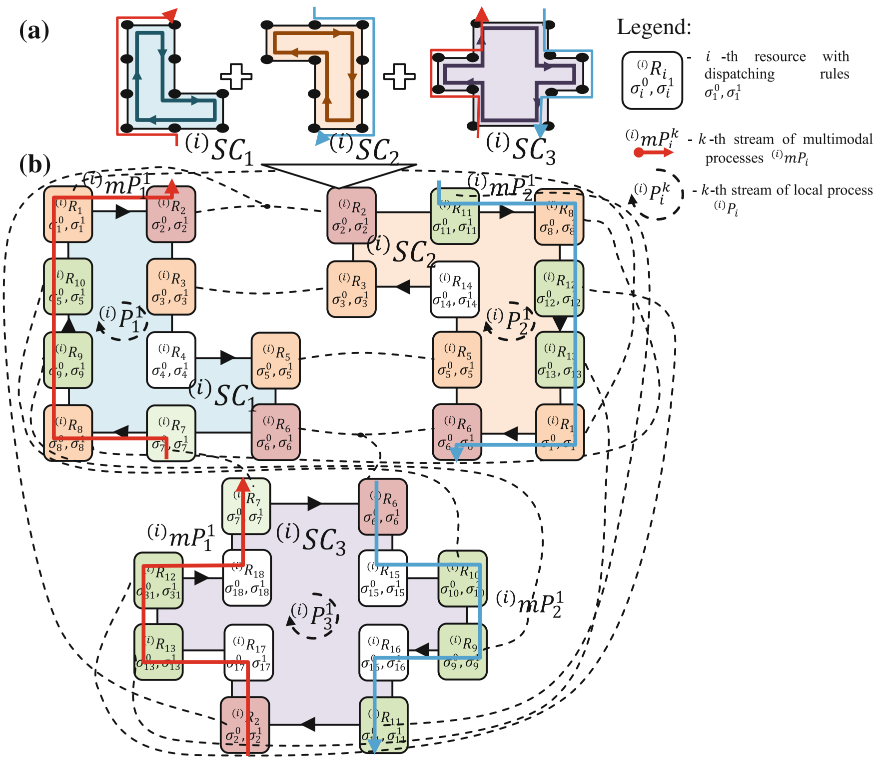

Fig. 2 The pattern composed by substructures: ${}^{(i)}SC_1$, ${}^{(i)}SC_2$,${}^{(i)}SC_3$ distinguished in Fig. 1 (**a**), and represented in terms of SCCP concept (**b**)

transportation routes specifying how a multimodal process is executed can be considered as composed of parts of the routes of local cyclic processes. Similar as in the case of local processes, each multimodal process consists of one stream: $mP_i = \{mP_i^1\}, i = 1, 2$.

Processes can interact with each other through shared resources, i.e. the transportation sectors. The routes p_i^k of the local processes P_i^k are as follows (see Fig. 2): $p_1^1 = (R_1, R_2, \ldots, R_{10})$, $p_2^1 = (R_{11}, R_8, R_{12}, R_{13}, R_1, R_6, R_5, R_{14}, R_3, R_2)$, $p_3^1 = (R_6, R_{15}, R_{10}, R_9, R_{16}, R_{11}, R_2, R_{17}, R_{13}, R_{12}, R_{18}, R_7)$.

Similarly the streams of cyclic multimodal processes: mP_1, mP_2, follow the routes: $mp_1^1 = ((R_7, R_8, R_9, R_{10}, R_1, R_2)) \cap ((R_2, R_{17}, R_{13}, R_{12}, R_{18}, R_7)) = (R_7, R_8, R_9, R_{10}, R_1, R_2, R_{17}, R_{13}, R_{12}, R_{18}, R_7)$, $mp_2^2 = (R_{11}, R_8, R_{12}, R_{13}, R_1, R_6) \cap (R_6, R_{15}, R_{10}, R_9, R_{16}, R_{11}) = (R_{11}, R_8, R_{12}, R_{13}, R_1, R_6, R_{15}, R_{10}, R_9, R_{16}, R_{11})$, where: $(R_7, \ldots, R_2) \cap (R_2, \ldots, R_7)/(R_{11}, \ldots, R_6) \cap (R_6, \ldots, R_{11})$—subsequences of routes $p_1^1, p_3^1/p_2^1, p_3^1$, defining the transportation sections of mp_1^1/mp_2^1, $u \cap v$—concatenation of sequences: $u = (u_1, \ldots, u_a)$, $v = (v_1, \ldots, v_b)$, $u \cap v = (u_1, \ldots, u_a, v_2, \ldots, v_b)$.

A resource conflict (caused by the application of the mutual exclusion protocol) is resolved with the aid of a priority dispatching rule [2, 3], which determines the order in which streams access shared resources. For instance, in the case of the resource R_2 (for substructures ${}^{(i)}SC_1$, ${}^{(i)}SC_2$, ${}^{(i)}SC_3$—see Fig. 2b), the priority dispatching rule: $\sigma_2^0 = (P_1^1, P_2^1, P_3^1)$, determines the order in which streams of local processes can access the shared resource R_2. That means, stream P_1^1 is allowed to access first, then the stream P_2^1 and next stream P_3^1, and then once again P_1^1, and so on. The SCCP shown in Fig. 2 is specified by the set of dispatching rules: $\Theta = \{\Theta^0, \Theta^1\}$, where: $\Theta^0 = \{\sigma_1^0, \ldots, \sigma_c^0, \ldots, \sigma_{18}^0\}, (\Theta^1 = \{\sigma_1^1, \ldots, \sigma_c^1, \ldots, \sigma_{18}^1\})$, $(\Theta^1 = \{\sigma_1^1, \ldots, \sigma_c^1, \ldots, \sigma_{18}^1\})$—set of rules determining the orders of local (multimodal) processes.

In general, the following notation is used:

- a sequence $p_i^k = (p_{i,1}^k, \ldots, p_{i,j}^k, \ldots, p_{i,lr(i)}^k)$ specifies **the route of the stream of the local process P_i^k** (the kth stream of the ith local process P_i). Its components $p_{i,j}^k \in R$ define the resources used in the execution of operations, In the rest of the paper, **the jth operation executed on the resource $p_{i,j}^k$ in the stream P_i^k** will be denoted by $o_{i,j}^k$; $lr(i)$ is the length of the cyclic process route.
- $x_{i,j}^k(l) \in \mathbb{N}$—the moment of operation beginning $o_{i,j}^k$ in the lth cycle,
- $t_i^k = (t_{i,1}^k, t_{i,2}^k, \ldots, t_{i,j}^k, \ldots, t_{i,lr(i)}^k)$ specifies **the operation times of local processes**, where $t_{i,j}^k$ denotes the time of execution of operation $o_{i,j}^k$.
- $mp_i^k = \left(mpr_{i_1}^{q_1}(a_{i_1}, b_{i_1}) \cap \ldots \cap mpr_{i_y}^{q_y}(a_{i_y}, b_{i_y})\right)$ specifies **the route of the stream mP_i^k from the multimodal process mP_i** (the kth stream of the ith multimodal process mP_i), where: $mpr_i^q(a, b)$ is the subsequence of the route p_i^q containing elements from $p_{i,a}^q$ to $p_{i,b}^q$. In the rest of the paper, **the jth operation executed in the stream mP_i^k** will be denoted by $mo_{i,j}^k$,
- $mx_{i,j}^k(l) \in \mathbb{N}$—the moment of operation beginning $mo_{i,j}^k$ in the lth cycle.
- $mt_i^k = \left(mt_{i,1}^k, mt_{i,2}^k, \ldots, m_{i,j}^k, \ldots, mt_{i,ldm(i)}^k\right)$ specifies **the operation times of multimodal processes**, where $mt_{i,j}^k$ denotes the time of execution of operation $mo_{i,j}^k$,
- $\Theta = \{\Theta^0, \Theta^1\}$ is the set of **priority dispatching rules**, $\Theta^i = \{\sigma_1^i, \ldots, \sigma_c^i, \ldots, \sigma_m^i\}$ is the set of priority dispatching rules for local ($i = 0$)/multimodal ($i = 1$) processes where: σ_c^i are sequence which determine the order in which the processes can be executed on the resource R_c.

Using the above notation, a SCCP can be defined as a tuple:

$$SC = ((R, SL), SM), \tag{1}$$

where $R = \{R_1, \ldots, R_c, \ldots, R_m\}$—the set of resources, $SL = (U, T, \Theta^0)$—the structure of local processes of SCCP, i.e.: U—the set of routes of local process, T—the set of sequences of operation times in local processes, $\Theta^0 = \{\sigma_1^0, \ldots, \sigma_c^0, \ldots, \sigma_m^0\}$—the set of priority dispatching rules for local processes. $SM = (M, mT, \Theta^1)$—the structure of multimodal processes of SCCP, i.e.: M—the set of routes of a multimodal process, mT—the set of sequences of operation times in multimodal processes, Θ^1—the set of priority dispatching rules for multimodal processes.

The behavior of the structure of SCCP (1) will be characterized by the schedule (2):

$$X' = ((X, \alpha), (mX, m\alpha)), \tag{2}$$

where $X = \left\{x_{1,1}^1, \ldots, x_{i,j}^k, \ldots, x_{n,lr(n)}^{ls(n)}\right\}$—a set of the moments of local processes operations beginning in $l = 0$ of the cycle, $x_{i,j}^k$—determines the value: $x_{i,j}^k(l) = x_{i,j}^k + \alpha$, α—periodicity of local processes executions, $mX = \left\{mx_{1,1}^1, \ldots, mx_{i,j}^k, \ldots, mx_{w,lm(w)}^{lsm(w)}\right\}$—a set of moments of operations of multimodal processes beginning in $l = 0$ of cycle, $mx_{i,j}^k$—determines the value $mx_{i,j}^k(l) = mx_{i,j}^k + m\alpha \cdot l$, $m\alpha$—periodicity of multimodal processes executions.

Consider SCCP following the grid-like structure shown in Fig. 1 and created as a result of multiple composition of the structure shown in Fig. 2. Formally, the grid-like structure is defined as SC (1) structure, that can be decomposed into the set of isomorphic substructures: $SC^* = \{SC_1, \ldots, SC_i, \ldots, SC_{lc}\}$ and following:

(a) each substructure $SC_i \in SC^*$ of the structure SC is defined analogically as (1):

$$SC_i = ((Rp_i, SLp_i), SMp_i). \tag{3}$$

where $Rp_i \subset R$—the set of resources of sub-structure SC_i, SLp_i—level of local processes of substructure SC_i, including: local processes $Pp_i \subset P$ with corresponding route sequences $Up_i \subset U$; the operation times $Tp_i \subset T$; the set of dispatching rules Θ_i^0. SMp_i—level of multimodal processes of substructure SC_i, including fragments of multimodal processes $mP_j(a, b)$ (fragment of the process mP_j related with executing the operation from a, $a + 1$, …, b) forming the set mPp_i.

(b) $SC^* = \{SC_1, \ldots, SC_i, \ldots, SC_{lc}\}$ is a set of substructures of the structure SC if [3]: $\bigcup_{i=1}^{lc} Rp_i = R$—substructures include all resources of the structure SC, $\bigcup_{i=1}^{lc} Pp_i = P$; $\prod_{i=1}^{lc} Pp_i = \varnothing$ and $\bigcup_{i=1}^{lc} Up_i = U$; $\prod_{i=1}^{lc} Up_i = \varnothing$—substructures use all the local processes and one process occurs in exactly one structure, $\prod_{i=1}^{lc} mPp_i = \varnothing$—elements of mPp_i occurs in exactly one substructure.

(c) Two sub-structures SC_a, $SC_b \in SC^*$ are called isomorphic if:

- each resource $R_a \in Rp_a$ of substructure SC_a is corresponding to exactly one resource $R_b \in Rp_b$ of the structure SC_b: $R_b = f(R_a)$,
- each process P_a/mP_a (local as well as multimodal) of the substructure SC_a is corresponding to exactly one process P_b/mP_b of the structure SC_b: $P_b = f(P_a)$,
- routes p_b/mp_b and p_a/mp_a of the corresponding processes are sequences consisting of corresponding resources,
- each operation $o^h_{a,j}\Big/mo^h_{a,j}$ executed within the substructure SC_a is corresponding to exactly one operation $o^h_{b,j}\Big/mo^h_{b,j}$ executed within the substructure $SC_b : o^h_{a,j} = f\left(o^h_{b,j}\right)/mo^h_{a,j} = f\left(mo^h_{b,j}\right)$;
- dispatching rules σ^l_a/σ^l_b of the corresponding resources are sequences consisting of elements $s^l_{a,d}/s^l_{b,d}$ indicating the streams of corresponding processes.

The structure shown in Fig. 1 consists of three types of isomorphic substructures presented in Fig. 2 and denoted as ${}^{(i)}SC_1$, ${}^{(i)}SC_2$, ${}^{(i)}SC_3$. Each of them includes one local processes ${}^{(i)}P_1$, ${}^{(i)}P_2$, ${}^{(i)}P_3$ respectively and one (${}^{(i)}SC_1$, ${}^{(i)}SC_2$) or two (${}^{(i)}SC_3$.) fragments of multimodal processes.

3 Problem Formulation

Given a grid-like structure SC (1), where the dispatching rules Θ are unknown. An answer is sought to the question whether there are such values Θ that can guarantee that the cyclic behavior represented by the schedule X' (2) will be attainable in the structure SC (1).

The grid-like structure SC (1) can be decomposed into a set of isomorphic substructures $SC^* = \{SC_1, \ldots, SC_i, \ldots, SC_{lc}\}$. Therefore, the selection of dispatching rules Θ can be carried out independently for each substructure. If, for every substructure SC_i, there is a subset of parameters Θ that guarantee its cyclic behavior, then the considered problem should provide an answer to the following question: **Does there exist a way of substructures *SC**, that can guarantee the cyclic work of the system *SC*?**

In order to answer this question the **operator of substructures composition** $\oplus$ is introduced as well as constraint assuming the result of composition of two substructures SC_a, SCb through mutually shared resources ($Rp_a \cap Rp_b \neq \varnothing$) results in $SC_a \oplus SC_b = SC_c$ where:

$$SC_c = ((Rp_c, SLp_c), SMp_c), \tag{4}$$

and $Rp_c = Rp_a \cup Rp_b$—the set of resources, and variables characterizing SLp_c are determined in the following way: $Pp_c = Pp_a \cup Pp_b$; $Up_c = Up_a \cup Up_b$; $Tp_c = Tp_a \cup Tp_b$, $\Theta_c^0 = \{\sigma_{k,c}^0 | k = 1 \ldots lk\}$, where:

$$\sigma_{k,c}^l = \begin{cases} \sigma_{k,a}^l & \text{for } R_k \in Rp_a \text{ and } R_k \notin Rp_b \\ \sigma_{k,b}^l & \text{for } R_k \in Rp_b \text{ and } R_k \notin Rp_a \, , \\ \vartheta(\sigma_{k,a}^l, \sigma_{k,b}^l) & \text{for } R_k \in Rp_a \text{ and } R_k \in Rp_b \end{cases} \tag{5}$$

$\vartheta\left(\sigma_{k,a}^0, \sigma_{k,b}^0\right)$—function determining the dispatching rules for the mutual resource R_k of the composed structures. SMp_c consist of mPp_c composed of parts of multimodal processes following the sets mPp_a and mPp_b [3].

4 Cyclic Scheduling

4.1 Determining Cyclic Steady Processes

Figure 2 shows in detail the substructures arrangement of the system from Fig. 1. There is three type of elementary isomorphic substructures ${}^{(i)}SC_1$, ${}^{(i)}SC_2$, ${}^{(i)}SC_3$ which are put together by means of integrating mutual resources. As Fig. 2 shows, for every substructure ${}^{(i)}SC_1$, ${}^{(i)}SC_2$,${}^{(i)}SC_3$ processes are implemented in the same manner: operations are performed along the same routes, the same dispatching rules are applied, etc. In this context the introduced operator of substructures composition ($\oplus$) SC can be shown as a multiple composition:

$$SC = \oplus_{i=1}^{lc}\left({}^{(i)}SC_1 \oplus^{(i)} SC_2 \oplus^{(i)} SC_3\right), \tag{6}$$

where $\oplus_{i=1}^{lc}\left({}^{(i)}SC\right) =^{(1)} SC \oplus \cdots \oplus^{(i)} SC \oplus \cdots \oplus^{(lc)} SC$—means composition according to Eqs. (4) and (5) i.e. each substructure ${}^{(i)}SC$ is put together with the others by means of integrating the resources belonging to the same set of corresponding resources. For example, the structure ${}^{(i)}SC_1$ from Fig. 2 is put together with ${}^{(i)}SC_2$ by the resources ${}^{(i)}R_2$, ${}^{(i)}R_3$, ${}^{(i)}R_5$, ${}^{(i)}R_6$ and with ${}^{(i)}SC_3$ by the resources ${}^{(i)}R_7$, ${}^{(i)}R_6$, ${}^{(i)}R_9$, ${}^{(i)}R_{10}$. Due to the same manner of process execution, as well as the same manner of substructures composition, the cyclic schedule representing the behavior of the whole structure can be perceived as a composition of corresponding (isomorphic) schedules:

$$X' = \uplus_{i=1}^{lc}\left({}^{(i)}X'_1 \uplus {}^{(i)}X'_2 \uplus {}^{(i)}X'_3\right), \tag{7}$$

where ${}^{(i)}X'_Z$—the cyclic schedule of the substructure ${}^{(i)}SC_Z$:

$$ {}^{(i)}X'_Z = \left(\left({}^{(i)}X_Z, {}^{(i)}\alpha_Z\right), \left({}^{(i)}mX_Z, {}^{(i)}m\alpha_Z\right)\right), \tag{8}$$

${}^{(i)}X_Z/{}^{(i)}mX_Z$—set of the initiation moments of local/multimodal process operations of the substructure ${}^{(i)}SC_Z$; ${}^{(i)}\alpha_Z/{}^{(i)}m\alpha_Z$—periodicity of local/multimodal processes executions; $\uplus_{i=1}^{lc}\left({}^{(i)}X'\right) = {}^{(1)}X' \uplus \ldots \uplus {}^{(i)}X' \uplus \ldots \uplus {}^{(lc)}X'$—composition of schedules ${}^{(i)}X'$, ${}^{(a)}X' \uplus {}^{(b)}X'$—the schedule composition ${}^{(a)}X'$, ${}^{(b)}X'$:

$$ {}^{(a)}X' \uplus {}^{(b)}X' = \left(\left({}^{(a)}X \cup {}^{(b)}X, lcm\left({}^{(a)}\alpha, {}^{(b)}\alpha\right)\right), \left({}^{(a)}mX \cup {}^{(b)}mX, lcm\left({}^{(a)}m\alpha, {}^{(b)}m\alpha\right)\right) \tag{9}$$

In order to determine the schedule X' it is enough to know the schedule ${}^{(i)}X'_1 \uplus {}^{(i)}X'_2 \uplus {}^{(i)}X'_3$ of composition ${}^{(i)}SC_1 \oplus {}^{(i)}SC_2 \oplus {}^{(i)}SC_3$. However, to make the composition (7) possible, it is necessary to make sure that the operations executed according to ${}^{(i)}X'_Z$, do not lead to deadlocks.

In order to determine such parameters as dispatching rules ${}^{(i)}\Theta_Z$ of the substructures ${}^{(i)}SC_1$, ${}^{(i)}SC_2$, ${}^{(i)}SC_3$ (Fig. 2) that guarantee the attainability of the cyclic schedule ${}^{(i)}X'_Z$ within the structure, it is possible to apply the constraint satisfaction problem (10) [11]:

$$PS'_{REXi} = \left(\left(\left\{ {}^{(i)}X'_Z, {}^{(i)}\Theta_Z, {}^{(i)}\alpha'_z\right\}, \{D_X, D_\Theta, D_\alpha\}\right), \{C_L, C_M, C_D\}\right) \tag{10}$$

where ${}^{(i)}X'_Z$, ${}^{(i)}\Theta_Z$, ${}^{(i)}\alpha'_z$—decision variables, ${}^{(i)}X'_Z$—cyclic schedule (8) of substructure ${}^{(i)}SC_Z$, ${}^{(i)}\Theta_Z = \left\{ {}^{(i)}\Theta^0_z, {}^{(i)}\Theta^1_z\right\}$—the set of priority dispatching rules for ${}^{(i)}SC_Z$, ${}^{(i)}\alpha'_z = \left({}^{(i)}\alpha_z, {}^{(i)}m\alpha_z\right)$—periodicity of local/multimodal processes for ${}^{(i)}SC_Z$, D_X, D_Θ, D_α—domains determining admissible value of decision variables.

$\{C_L, C_M, C_D\}$—the set of constraints C_L and C_M describing SCCP behavior; C_L—constraints determining cyclic steady state of local processes, i.e. their cyclic schedule [2], C_M—constraints determining multimodal processes behavior [2], C_D—constraints that guarantee the smooth implementation of the stream on mutual resources.

The schedule ${}^{(i)}X'_Z$ that meets all the constraints from the given set $\{C_L, C_M, C_D\}$ is the solution sought for the problem (10). It means that it is possible to smoothly execute the operations occurring in ${}^{(i)}SC_Z$ as well as in neighboring substructures.

4.2 Principle of Match-up Structures Coupling

The constraints C_L, C_M guarantee that in the substructure ${}^{(i)}SC_Z$ from Fig. 2 the processes will be executed in a cyclic and deadlock-free manner [2]. These constraints, however, cannot ensure the lack of interferences between the operations of neighboring substructure streams with the substructure ${}^{(i)}SC_Z$. In order to avoid interferences of this kind, additional constraints C_D, are introduced, which describe the relationships between the process operations of the constituted structures. For that purpose the principle of match-up structures coupling is applied.

The idea of the principle of match-up structures coupling is to attain the cyclic schedule X'_c (that does not lead to any collisions between operations) in the substructure SC_c, gained as a result of the composition $SC_a \oplus SC_b$. The cyclic schedule is a composition of the schedules X'_a, X'_b: $X'_c = X'_a \uplus X'_b$ (7) if:

- the value of the periodicity of schedule X'_a is the total multiple of the periodicity of schedule X'_b: $m\alpha_a MOD\ m\alpha_b = 0$; and $\alpha_a MOD \alpha_b = 0$
- the operations of mutual resources $Rk = Rp_a \cap Rp_b = \left\{R_{k_1}, \ldots, R_{k_i}, \ldots, R_{k_q}\right\}$ are executed without mutual interferences.

Formally, the constraints that guarantee the lack of interferences while executing the process operations on mutual resources are defined in the following way:

Constraints for local process operations In order to guarantee the smooth process implementation on the resource $R_{k_i} \in Rk$ the extension of the conventional constraints of non-superimposition of time intervals is used. The two operations $o^h_{i,j}, o^s_{q,r}$ do not interfere (on the mutually shared resource R_{k_i}) if the operation $o^h_{i,j}$ begins (moment $x^h_{i,j}$) after the release (with the delay Δt) of the resource by the operation $o^s_{q,r}$ (moment x^s_{q,r^*} of the subsequent operation initiation) and releases the resource (moment x^h_{i,j^*} of the subsequent operation initiation) before the beginning of the next execution of the operation $o^s_{q,r}$ (moment $x^s_{q,r} + \alpha_b$). The collision-free execution of the local process operations is possible if the following constraint holds:

$$\begin{gathered}\left[\left(x^h_{i,j} \geq x^s_{q,r^*} + k'' \cdot \alpha_b + \Delta t\right) \wedge \left(x^h_{i,j^*} + k' \cdot \alpha_a + \Delta t \leq x^s_{q,r} + \alpha_b\right)\right] \\ \vee \left[\left(x^s_{q,r} \geq x^h_{i,j^*} + k' \cdot \alpha_a + \Delta t\right) \wedge \left(x^s_{q,r^*} + k'' \cdot \alpha_b + \Delta t \leq x^h_{i,j} + \alpha_a)\right]\right.\end{gathered} \tag{11}$$

where $j^* = (j+1)MODlr(i), \quad r^* = (r+1)MODlr(q)$,

$$k' = \begin{cases} 0 \text{ when } j+1 \leq lr(i) \\ 1 \text{ when } j+1 < lr(i) \end{cases}, \quad k'' = \begin{cases} 0 \text{ when } r+1 \leq lr(q) \\ 1 \text{ when } r+1 < lr(q) \end{cases}, \tag{12}$$

Constraints for multimodal processes In order to guarantee an interference-free implementation of the multimodal processes (when the condition of mutual exclusion is applied) the applied conditions are similar to those used for local processes. Two operations $mo^h_{i,j}, mo^s_{q,r}$ can be executed without any interferences on the mutually shared resource $R_{k_i} \in Rk$ if one operation is executed between the subsequent executions of the other. In this context, the collision-free execution of the multimodal process operations is possible if the following constraint is satisfied:

$$\left[\left(mx^h_{i,j} \geq mx^s_{q,r^*} + k'' \cdot m\alpha_b + \Delta t\right) \wedge \left(mx^h_{i,j^*} + k' \cdot m\alpha_a + \Delta t \leq mx^s_{q,r} + m\alpha_b\right)\right] \\ \vee \left[\left(mx^s_{q,r} \geq mx^h_{i,j^*} + k' \cdot m\alpha_a + \Delta t\right) \wedge \left(mx^s_{q,r^*} + k'' \cdot m\alpha_b + \Delta t \leq mx^h_{i,j} + m\alpha_a\right)\right] \quad (13)$$

where $j*$, r^*, $k^{'}$ and $k^{''}$ defined as in Eq. (11), $mx^h_{i,j^*}, mx^s_{q,r}$—initiation moments of the operations $mo^h_{i,j}, mo^s_{q,r}$ of substructures SC_a, SC_b, respectively;mx^h_{i,j^*}, mx^s_{q,r^*}—initiation moments of operations executed after $mo^h_{i,j}, mo^s_{q,r}$, respectively.

The constraints (11) and (13) must be satisfied so that the composition of two substructures $SC_c = SC_a \oplus SC_b$ of the known cyclic behaviors, is also characterized by the cyclic behavior X'_c. If these constraints are satisfied, the manner of executing operations on mutual resources Rk determines the form of dispatching rules $\sigma^0_{k,c}$ (5), and, to be more exact, the form of functions $\vartheta\left(\sigma^0_{k,a}, \sigma^0_{k,b}\right)$ and $\vartheta\left(\sigma^1_{k,a}, \sigma^1_{k,b}\right)$. The $\vartheta\left(\sigma^l_{k,a}, \sigma^l_{k,b}\right)$ is determined based on moments of operations executed on the R_k:

$$\vartheta\left(\sigma^l_{k,a}, \sigma^l_{k,b}\right) = \left(s^l_{k,1,c}, \ldots, s^l_{k,j,c}, \ldots, s^l_{k,lh_c,c}\right) \text{ when } x^l_{k,1,c} < \cdots < x^l_{k,lh_c,c}, \quad l \in \{0, 1\} \quad (14)$$

where $s^l_{k,j,c}$—jth element of the rule $\sigma^l_{k,c}$ determining the stream of the process of the lth behavior level initiating its operation on the resource R_k in the moment: $x^l_{k,j,c}; s^l_{k,j,c}$ is one of the elements of the rules $\sigma^l_{k,a}, \sigma^l_{k,b}; x^0_{k,j,c} \in X_a \cup X_b$.

In other words, there are such dispatching rules on mutual R_k as the sequence of operations resulting from the schedules X'_a, X'_b satisfying the constraints (11) and (13).

5 Computational Experiments

The evaluation of the cyclic behaviour (the existence of the schedule X') of the grid-like structure SC from Fig. 1 can be obtained as a result of evaluating the parameters of isomorphic substructures ${}^{(i)}SC_1$, ${}^{(i)}SC_2$,${}^{(i)}SC_3$ from Fig. 2 (it is assumed that the all the operation times are the same and equal to $t^k_{i,j} = 5$). The problem PS'_{REXi} (10) was formulated in which the constraints C_L, C_M determining the relationships between the behaviour and the structure are formulated according to [2]. In order to formulate the constraints C_D the principle of match-up structures coupling was applied. It is necessary that they guarantee a collision-free execution of stream operations ${}^{(i)}P^1_1, {}^{(i)}P^1_2$, (on the resources ${}^{(i)}R_1$, ${}^{(i)}R_3, {}^{(i)}R_5, {}^{(i)}R_8$), ${}^{(i)}P^1_1, {}^{(i)}P^1_3$, (on the ${}^{(i)}R_7, {}^{(i)}R_9$, ${}^{(i)}R_{10}$), ${}^{(i)}P^1_2, {}^{(i)}P^1_3$, (on the ${}^{(i)}R_{11}, {}^{(i)}R_{12}$, ${}^{(i)}R_{13}$) and ${}^{(i)}P^1_1, {}^{(i)}P^{1(i)}_2 P^1_3$, (on the ${}^{(i)}R_2, {}^{(i)}R_6$). The relations regarding to these constraints C_D were shown in Fig. 2 by dot dashed lines. The problem PS'_{REXi}, formulated in this manner, was implemented and solved in the constraint programming environment OzMozart (CPU Intel Core 2 Duo 3 GHz RAM 4 GB). The first acceptable solution was obtained in less than one second. The result of the problem solution are the cyclic schedule ${}^{(i)}X'$ and the dispatching rules ${}^{(i)}\Theta$ shown in Fig. 3 and Table 1. It shows that the operations executed on the mutual resources do not superimpose on each

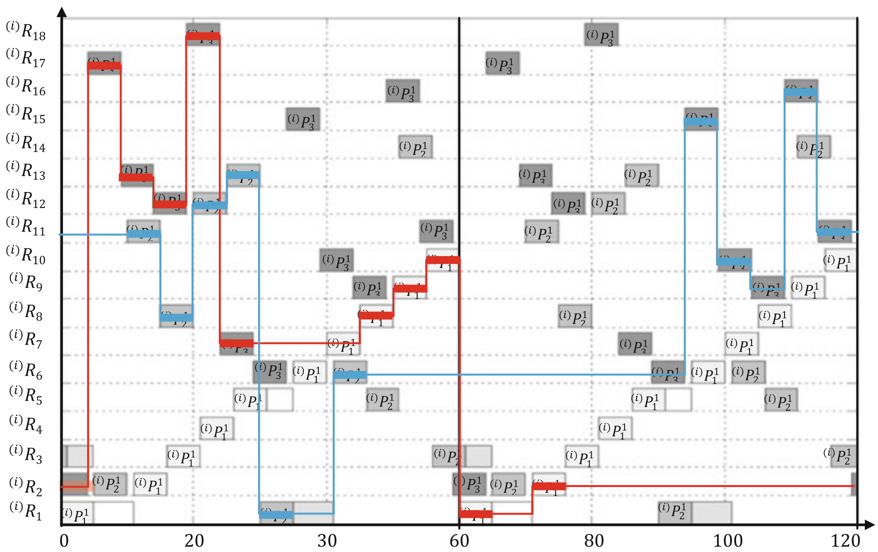

Fig. 3 The schedule of SCCP following ${}^{(i)}SC_1 \oplus^{(i)} SC_2 \oplus^{(i)} SC_3$ from Fig. 2b

Table 1 The dispatching rules of SCCP following ${}^{(i)}SC_1 \oplus^{(i)} SC_2 \oplus^{(i)} SC_3$ from Fig. 2b

Dispatching rule for local processes				Dispatching rule for multimodal processes		
${}^{(i)}\sigma^0_1$	$({}^{(i)}P^1_1, {}^{(i)}P^1_2)$	${}^{(i)}\sigma^0_6$	$({}^{(i)}P^1_1, {}^{(i)}P^1_2, {}^{(i)}P^1_3)$	${}^{(i)}\sigma^1_1$	${}^{(i)}\sigma^1_{12}$	$({}^{(i)}m^1P^1_1, {}^{(i)}m^1P^1_2)$
${}^{(i)}\sigma^0_2$	$({}^{(i)}P^1_1, {}^{(i)}P^1_3, {}^{(i)}P^1_2)$	${}^{(i)}\sigma^0_7$	$({}^{(i)}P^1_1, {}^{(i)}P^1_3)$	${}^{(i)}\sigma^1_8$	${}^{(i)}\sigma^1_{13}$	$({}^{(i)}m^1P^1_1, {}^{(i)}m^1P^1_2)$
${}^{(i)}\sigma^0_3$	$({}^{(i)}P^1_1, {}^{(i)}P^1_2)$	${}^{(i)}\sigma^0_8$	$({}^{(i)}P^1_2, {}^{(i)}P^1_1)$	${}^{(i)}\sigma^0_9$		$({}^{(i)}m^1P^1_1, {}^{(i)}m^1P^1_2)$
${}^{(i)}\sigma^0_5$	$({}^{(i)}P^1_1, {}^{(i)}P^1_2)$	${}^{(i)}\sigma^0_7$	$({}^{(i)}P^1_3, {}^{(i)}P^1_1)$	${}^{(i)}\sigma^0_{10}$		$({}^{(i)}m^1P^1_1, {}^{(i)}m^1P^1_2)$

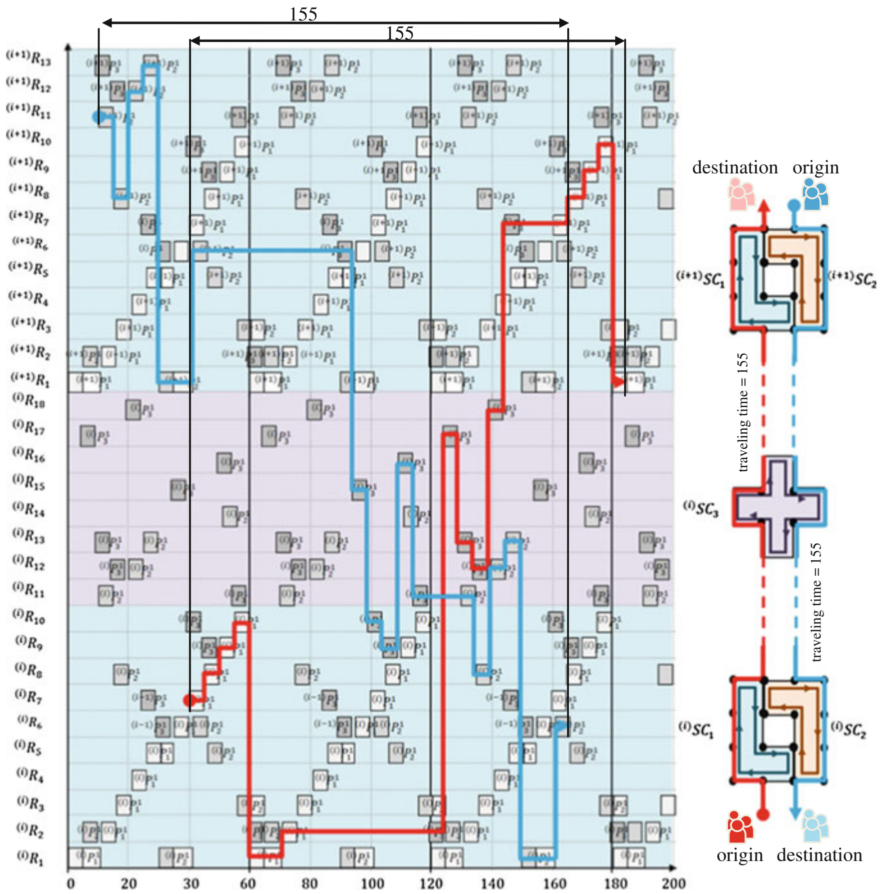

Fig. 4 The Gantt's chart of grid-like network from Fig. 1 including both local and multimodal processes

other. According to Eq. (7) the attained schedule is a component of the schedule X' that characterizes the behavior of the whole structure *SC*.

The schedule X' (7) being a multiple composition of the schedules ${}^{(i)}X_1'$, ${}^{(i)}X_2', {}^{(i)}X_3'$ is presented in Fig. 4. It is evident that the composition of these schedules of all the substructures of the structure *SC* does not lead to interferences in the execution of the operation. On the basis of the obtained schedules it is also possible to determine (according to Eq. (14)) the dispatching rules for all the resources of the structure *SC*; the rules are presented in Table 1.

Referring back to the layout presented in Fig. 1a, the obtained schedule should be treated as an illustration of vehicle movement (local processes) and the method of executing transportation routes (multimodal processes) in a network consisting of numerous fragments of the same type. It should be emphasized that the periodicity of local processes in the network of this kind amounts to $\alpha = 60$ u.t. (units time), and the journey times the passengers have to spent while traveling along the red or blue streams (processes ${}^{(i)}mP_1^1$, ${}^{(i)}mP_2^1$) are equal to 155 u.t.

6 Conclusions

A declarative modeling approach to MTP scheduling in GNTM networks environment is considered. Opposite to traditional approach a given network of local cyclic acting different modes of transportation services is assumed. In considered regular network, composed of elementary, structurally isomorphic subnetworks, the passengers pass their origin-destination itineraries along routes composed of local transportation means. The solution sought assumes that schedules of locally acting subnetworks composed of a set of assumed transportation lines will match-up the schedules of assumed set of passengers itinerary. The relevant sufficient conditions guaranteeing such a match-up exists were provided.

References

1. Abara, J.: Applying integer linear programming to the fleet assignment problem. Interfaces **19**, 4–20 (1989)
2. Bocewicz, G., Nielsen, I., Banaszak, Z.: Automated guided vehicles fleet match-up sched-uling with production flow constraints. Eng. Appl. Artif. Intell. **30**, 49–62 (2014)
3. Bocewicz, G., Banaszak, Z., Pawlewski, P.: Multimodal cyclic processes scheduling in fractal structure networks environment. In: Proceedings of the 19th World Congress: The International Federation of Automatic Control, Cape Town, 8939–8946 (2014)
4. Courtat, T.: Walk on city maps—mathematical and physical phenomenology of the city, a geometrical approach. Modeling and simulation. Université Paris-Diderot—Paris VII (2012). https://tel.archives-ouvertes.fr/tel-00714310
5. Krenczyk, D., Kalinowski, K., Grabowik, C.: Integration production planning and scheduling systems for determination of transitional phases in repetitive production. Hybrid Artif. Intell. Syst. LNCS **7209**, 274–283 (2012)
6. Kelly, G., McCabe, H.: A survey of procedural techniques for city generation. ITB J. **14**, 87–130 (2006)
7. Levner, E., Kats, V., Alcaide, D., Pablo, L., Cheng, T.C.E.: Complexity of cyclic scheduling problems: A state-of-the-art survey. Comput. Ind. Eng. **59**(2), 352–361 (2010)
8. Levinson, D., Huang, A.: A positive theory of network connectivity. Environ. Plan. **39**, 308–325 (2012)
9. Moudani, W., Mora-Camino, F.: A dynamic approach for aircraft assignment and maintenance scheduling by airlines. J. Air Transp. Manag. **6**, 233–237 (2000)
10. Polak, M., Majdzik, P., Banaszak, Z., Wójcik, R.: The performance evaluation tool for automated prototyping of concurrent cyclic processes. Fundamenta Informaticae **60**(1–4), 269–289 (2004)

11. Sitek, P., Wikarek, J.: A hybrid framework for the modelling and optimisation of decision problems in sustainable supply chain management. Int. J. Prod. Res., 1–18 (2015). doi:10.1080/00207543.2015.1005762
12. Song, J.-S., Lee, T.-E.: Petri net modeling and scheduling for cyclic job shops with blocking. Comput. Ind. Eng. **34**(2), 281–295 (1998)
13. Von Kampmeyer, T.: Cyclic scheduling problems. Ph.D. Dissertation, Mathematik/Informatik, Universität Osnabrück (2006)

Springer

http://www.springer.com/978-3-319-28553-5

Information Systems Architecture and Technology: Proceedings of 36th International Conference on Information Systems Architecture and Technology – ISAT 2015 – Part I

Borzemski, L.; Grzech, A.; Swiatek, J.; Wilimowska, Z. (Eds.)

2016, XIV, 239 p. 84 illus., 49 illus. in color., Softcover
ISBN: 978-3-319-28553-5