Stochastic Dynamic Programming Solution of a Risk-Adjusted Disaster Preparedness and Relief Distribution Problem

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Abstract This chapter proposes a multistage stochastic optimization framework that dynamically updates the purchasing and distribution decisions of emergency commodities in the aftermath of an earthquake. Furthermore, the models consider the risk of exceeding the budget levels at any stage through chance constraints, which are then converted to Conditional Value-at-Risk constraints. Compared to the previous papers, our framework provides the flexibility of adjusting the level of conservativeness to the users by changing risk related parameters. Under some conditions, the resulting linear programming problems are solved through the Stochastic Dual Dynamic Programming algorithm. The preliminary numerical results are encouraging.

1 Introduction

This chapter proposes a dynamic and stochastic methodology to generate a risk-averse disaster preparedness and logistics plan that can mitigate demand and road capacity uncertainties. More specifically, we apply multistage stochastic optimization for dynamically purchasing and distributing emergency commodities with time dependent demands and road capacities. Several authors have dealt with problems similar to ours, but [2, 4] are the most related papers. In many cases, our approach can give less conservative solutions than [2], which considers a robust dynamic optimization framework. Furthermore, our approach gives more conservative solutions than [4], which considers a risk-neutral dynamic stochastic optimization framework with a finite number of scenarios.

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The structure of the chapter is as follows. In Sect. 2, we introduce multistage stochastic programming models that take risk into account. Section 3 presents the novelty in our application of the risk-averse Stochastic Dual Dynamic Programming (SDDP) algorithm, and Sect. 3.1 presents some preliminary numerical results. Finally, Sect. 4 summarizes the chapter and presents a few future research directions.

2 Risk-Adjusted Multistage Stochastic Programming Model

We formulate the problem through a risk-adjusted, T-stage stochastic programming model, where the decisions at the first-stage belong to the preparedness phase, and the decisions at later stages belong to the response phase of a disaster. The risk adjustments are achieved by adding probabilistic constraints to the risk-neutral formulation at stages $t = 1, \ldots, T-1$. A risk-neutral formulation and solution of this problem is given in [1].

We make the following two assumptions for the random vector $\boldsymbol{\xi}_t$ whose components are the demands and the road capacities: i—The distribution P_t of $\boldsymbol{\xi}_t$ is known, and this P_t is supported on a set $\Xi_t \subset \mathbb{R}^{d_t}$; ii—The random process $\left\{\boldsymbol{\xi}_t\right\}_{t=2}^{T}$ is stage-wise independent.

We formulate the T-stage problem through the following dynamic programming equations. At stage $t = 1$, the problem is

$$\begin{array}{lll} \text{Min} & \sum\limits_{i\in I}\left[\sum\limits_{l\in L} f_{il}y_{il} + \sum\limits_{k\in K} q_1^k r_{1i}^k\right] + \mathbb{E}\left[Q_2\left(\mathbf{x}_1, \boldsymbol{\xi}_2\right)\right] & \\ \text{s.t} & \sum\limits_{k\in K} b^k r_{1i}^k \leq \sum\limits_{l\in L} M_l y_{il} & \forall i \in I \\ & \sum\limits_{l\in L} y_{il} \leq 1 & \forall i \in I \\ & \text{Prob}\left\{Q_2\left(\mathbf{x}_1, \boldsymbol{\xi}_2\right) \leq \eta_2\right\} \geq 1 - \alpha_2 & \\ & y_{il} \in \{0, 1\}, r_{1i}^k \geq 0, \forall i \in I, l \in L, k \in K & \end{array} \tag{1}$$

where I, L, and K are the set of potential nodes to open storage facilities, the set of size categories of the facilities, and the set of commodity types, respectively, f_{il} is the fixed cost of opening a facility of size l in location i, q_t^k is the unit acquisition cost of commodity k at stage t, b^k is the unit space requirement for commodity k, M_l is the overall capacity of a facility of size l, r_{ti}^k is the amount of commodity k purchased at stage t in location i, y_{il} is the location i and the size l of a facility, η_t and α_t are the known budget limit and the significance level at stage t, respectively, and $\mathbf{x}_1$ is the vector with the components y_{il}'s and r_{1i}^k's. Furthermore, in (1), the first set of constraints limits the capacity of a facility, the second set of constraints restricts the number of facilities per node, and the chance constraint ensures that the second-stage cost-to-go function $Q_2\left(\mathbf{x}_1, \boldsymbol{\xi}_2\right)$ does not exceed the budget limit η_2 with high probability.

For later stages $t = 2, \ldots, T-1$ and for a realization $\boldsymbol{\xi}_t^s$ of $\boldsymbol{\xi}_t$, the cost-to-go functions $Q_t\left(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t^s\right)$ are given by

$$
\begin{array}{llll}
\text{Min} & \sum_{k\in K}\left[\sum_{i\in I} q_t^k r_{ti}^k + \sum_{(i',j')\in A} c_{ti'j'}^k m_{ti'j'}^k + \sum_{j\in J} p_t^k w_{tj}^k\right] + \mathbb{E}\left[Q_{t+1}\left(\mathbf{x}_t, \boldsymbol{\xi}_{t+1}\right)\right] & & \\
\text{s.t} & z_{ti}^k + \sum_{(i,j')\in A} m_{tij'}^k - \sum_{(j',i)\in A} m_{tj'i}^k & = r_{t-1,i}^k + z_{t-1,i}^k & \forall i \in I, k \in K \\
& \sum_{(i',j)\in A} m_{ti'j}^k - \sum_{(j,i')\in A} m_{tji'}^k + w_{tj}^k & = \nu_{tj}^{ks} & \forall j \in J, k \in K \\
& \sum_{k\in K} b^k \left(m_{ti'j'}^k + m_{tj'i'}^k\right) & \le \kappa_{ti'j'}^s & \forall \left(i', j'\right) \in A \\
& \sum_{k\in K} b^k \left(z_{ti}^k + r_{ti}^k\right) & \le \sum_{l\in L} M_l y_{il} & \forall i \in I \\
& \text{Prob}\left\{Q_{t+1}\left(\mathbf{x}_t, \boldsymbol{\xi}_{t+1}\right) \le \eta_{t+1}\right\} & \ge 1 - \alpha_{t+1} & \\
& r_{ti}^k, m_{ti'j'}^k, w_{tj}^k, z_{ti}^k \ge 0 \forall i \in I, j \in J, k \in K, (i', j') \in A & &
\end{array}
\tag{2}
$$

where J and A are the set of nodes that represent shelters and the set of arcs that represent roads in the network, respectively, $c_{ti'j'}^k$ is the unit transportation cost of commodity k through arc $\left(i', j'\right)$, p_t^k is the unit shortage cost of commodity k, $m_{ti'j'}^k$ is the amount of commodity k transported through arc $\left(i', j'\right)$, w_{tj}^k and z_{ti}^k are the shortage amount of commodity k in shelter j and the amount of commodity k stored in location i, respectively, ν_{tj}^{ks} and $\kappa_{ti'j'}^s$ are the demand for the commodity k in shelter j and the road capacity of arc $\left(i', j'\right)$ for a realization s, respectively, and $\mathbf{x}_t$ is the vector with components r_{ti}^k's and z_{ti}^k's; all values depend on stage t. Moreover, in (2), the first set of constraints represents the flow conservation with $z_{1,i}^k = 0 \forall i \in I, k \in K$, the second set of constraints is for the demand satisfaction, and the third set of constraints is for the road capacity. The stage T problem has the same three sets of constraints as in (2), but there are no more acquisition decisions and the remaining inventories are penalized through a unit holding cost h_T^k. Hence, the objective function at $t = T$ becomes

$$
\text{Min} \sum_{k\in K}\left[\sum_{i\in I} h_T^k z_{Ti}^k + \sum_{(i',j')\in A} c_{Ti'j'}^k m_{Ti'j'}^k + \sum_{j\in J} p_T^k w_{Tj}^k\right].
$$

It was suggested in [5] to replace the chance constraint by the $\mathsf{CV@R}_\alpha$-type constraint, where $\mathsf{CV@R}_\alpha$ is given by

$$
\mathsf{V@R}_\alpha\left[Q_t\left(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t\right)\right] + \alpha^{-1}\mathbb{E}\left[Q_t\left(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t\right) - \mathsf{V@R}_\alpha\left[Q_t\left(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t\right)\right]\right]_+ \tag{3}
$$

where the Value-at-Risk ($\mathsf{V@R}_\alpha$) in (3) is, by definition, the left-side $(1-\alpha)$-quantile of the distribution of $Q_t\left(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t\right)$, and

$$
\left[Q_t - \mathsf{V@R}_\alpha\left(Q_t\right)\right]_+ = \max\left\{Q_t - \mathsf{V@R}_\alpha\left(Q_t\right), 0\right\}.
$$

A problem with a CV@R-type constraint is that it can make the problem infeasible. Consequently, it could be convenient to move the CV@R-type constraint into the objective function; that is, we redefine the cost-to-go function as

$$V_{\lambda_t}\left[Q_t\left(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t\right)\right] := (1-\lambda_t)\,\mathbb{E}\left[Q_t\left(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t\right)\right] + \lambda_t \mathsf{CV@R}_{\alpha_t}\left[Q_t\left(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t\right)\right] \tag{4}$$

where $\lambda_t \in [0, 1]$ is a parameter that can be tuned for a tradeoff between minimizing on average and risk control.

The expectation and the CV@R in (4) usually make the problem analytically untractable. A possible way to deal with this problem is to use Sample Average Approximation (SAA). That is, sample $\boldsymbol{\xi}_t$ from its distribution P_t to obtain $\mathscr{S}_t := \left\{\boldsymbol{\xi}_t^1, \ldots, \boldsymbol{\xi}_t^{N_t}\right\}$, where N_t is the sample size at stage t. Then, setting $\lambda_t = 0$ in (4) and for a fixed $\overline{\mathbf{x}}_{t-1}$, solve the stage t problem to obtain the N_t optimal values $Q_t\left(\overline{\mathbf{x}}_{t-1}, \boldsymbol{\xi}_t^1\right), \ldots, Q_t\left(\overline{\mathbf{x}}_{t-1}, \boldsymbol{\xi}_t^{N_t}\right)$. Let $Q_{t,(1)} < Q_{t,(2)} < \cdots < Q_{t,(\iota)} < \cdots < Q_{t,(N_t)}$ be the order statistics obtained from these optimal values, and ι be the smallest integer that satisfies $\iota \geq N_t\left(1-\alpha_t\right)$. This $Q_{t,(\iota)}$ is an estimate of $\mathsf{V@R}\left[Q_t\left(\overline{\mathbf{x}}_{t-1}, \boldsymbol{\xi}_t\right)\right]$ so that (4) is estimated through

$$\frac{(1-\lambda_t)}{N_t}\sum_{s=1}^{N_t} Q_t\left(\overline{\mathbf{x}}_{t-1}, \boldsymbol{\xi}_t^s\right) + \lambda_t Q_{t,(\iota)} + \frac{\lambda_t}{N_t\alpha_t}\sum_{s=1}^{N_t}\left[Q_t\left(\overline{\mathbf{x}}_{t-1}, \boldsymbol{\xi}_t^s\right) - Q_{t,(\iota)}\right]_+ .$$

3 Stochastic Dual Dynamic Programming Applications

The Stochastic Dual Dynamic Programming (SDDP) algorithm was introduced in [3], and the risk-averse SDDP algorithm was applied to an SAA problem in [6]. Furthermore, a detailed description of the risk-neutral SDDP algorithm applied to an SAA problem was given in [1]. We do not give further detail on the SDDP algorithm, but refer to the papers above.

The novelty in our application of the risk-averse SDDP follows from the following proposition.

Proposition 1 *For a realization $\boldsymbol{\xi}_t^s$ of $\boldsymbol{\xi}_t$ and at a given $\overline{\mathbf{x}}_{t-1}$, a subgradient $\mathbf{g}_t^s$ of $V_{\lambda_t}\left[Q_t\left(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t\right)\right]$ is computed through*

$$\mathbf{g}_t^s = \begin{cases} -\left(1-\lambda_t+\lambda_t\alpha_t^{-1}\right)\mathbf{B}_t^{sT}\boldsymbol{\pi}_t^s - \left(\lambda_t - \lambda_t\alpha_t^{-1}\right)\mathbf{B}_t^{(\iota)T}\boldsymbol{\pi}_t^{(\iota)} & \text{if } Q_t\left(\overline{x}_{t-1}, \xi_t^s\right) > Q_{t,(\iota)} \\ -(1-\lambda_t)\mathbf{B}_t^{sT}\boldsymbol{\pi}_t^s - \lambda_t\mathbf{B}_t^{(\iota)T}\boldsymbol{\pi}_t^{(\iota)} & \text{if } Q_t\left(\overline{x}_{t-1}, \xi_t^s\right) \leq Q_{t,(\iota)} \end{cases}$$

where $\boldsymbol{\pi}_t^s$ is the vector of dual variables corresponding to the first set of constraints for $t = 3, \ldots, T$, and to the first and the second set of constraints for $t = 2$, $\mathbf{B}_t^s$ is the *matrix whose entries are given by the coefficients of* $r^k_{t-1,i}$ *and* $z^k_{t-1,i}$ *for* $t = 3, \ldots, T$, *and by the coefficients of* $r^k_{t-1,i}$ *and* y_{il} *for* $t = 2$, *and* $\mathbf{B}^{(\iota)}_t$ *and* $\boldsymbol{\pi}^{(\iota)}_t$ *correspond to* $Q_{t,(\iota)}$.

Then, a subgradient $\hat{\mathbf{g}}_t$ of (4) is estimated through $\hat{\mathbf{g}}_t = \frac{1}{N_t} \sum_{s-1}^{N_t} \mathbf{g}^s_t$.

3.1 Numerical Results

We consider three consumable emergency commodity types, 10 potential locations for facilities, and 30 shelters in the two boroughs of Istanbul. The data for costs and volumes of commodities, the data for costs and capacities of facilities, and the population data are the same as in [1]. Furthermore, [7] estimated the total numbers of buildings that are prone to be damaged at various levels for an earthquake of magnitude 7.3 on the Richter scale; these data are also summarized in [1].

We model the random demand ν^k_{tj} for commodity k at shelter j and random capacity $\kappa_{ti'j'}$ for any arc (i', j') at stage t $(t = 2, \ldots, T)$ as follows:

$$\nu^k_{tj} = \delta^k_t \left(\varsigma_{t-1,j} + \varsigma_{t,j} \right) \forall j \in J \text{ and } \kappa_{ti'j'} = \eta * \tfrac{\tau(t)}{\omega(i',j')/\gamma_{ti'j'}} \forall (i', j') \in A$$

where δ^k_t is the amount of commodity k needed by a single individual during stage t, $\varsigma_{t-1,j}$ is the number of evacuees who were expected to arrive at shelter j by the end of stage $(t-1)$, and $\varsigma_{t,j}$ is the random additional number of evacuees who arrive at shelter j at stage t. Moreover, η is the capacity of a single vehicle, $\tau(t)$ is the length of stage t, $\omega(i', j')$ is the actual distance between nodes i' and j', and $\gamma_{ti'j'}$ is the random speed of the vehicle. Both $\varsigma_{t,j}$ and $\gamma_{ti'j'}$ are assumed to be normal; see [1].

We consider $T = 6$ stages, and concentrate on the first 72 h in the aftermath of an earthquake. The stopping criterion of the SDDP algorithm is the maximum number of iterations, which is 100. All computational experiments are conducted on a workstation with Windows 2008 Server, three Intel(R) Xeon(R) CPU E5-2670 CPUs of 2.60 GHz, and 4 GB RAM. The linear programming problems are solved by ILOG CPLEX Callable Library 12.2.

So far we have only experimented with risk-related parameters, namely λ and α. Values of λ closer to 1 and values of α closer to 0 make the 6-stage problems more risk-averse. In Fig. 1, for $\alpha = 1\,\%$ (on the left) the lower bounds on the 6-stage costs for $\lambda = 0.4$ and $\lambda = 0.5$ stabilize at almost the same value. For $\alpha = 5\,\%$ (on the right), however, the lower bound for the more risk-averse case ($\lambda = 0.5$) stabilizes at a value which is much lower than the lower bound of the less risk-averse case ($\lambda = 0.4$); this is due to the fact that for the $\lambda = 0.5$ case, facilities store more emergency commodities, and hence the shortage amounts and the penalty costs are much lower compared to the $\lambda = 0.4$ case.

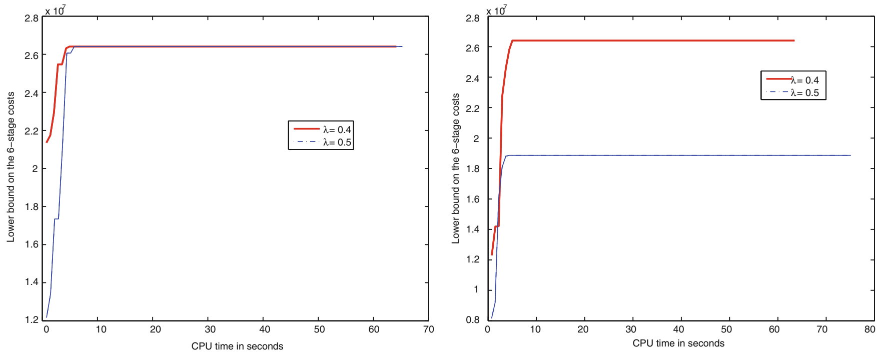

Fig. 1 Changes in the lower bound on the 6-stage costs for $\alpha = 1\,\%$ on the *left* and $\alpha = 5\,\%$ on the *right*

4 Conclusions

In this chapter, we formulate a short-term disaster management problem through a multistage stochastic programming model. The model takes the risk of exceeding the budget level at that stage into account through a chance constraint, which is then converted into a $\mathsf{CV@R}$-type constraint. Because the $\mathsf{CV@R}$-type constraint can make the problem infeasible, that constraint is further added to the objective function. Under some assumptions, the resulting problem is solved through the Stochastic Dual Dynamic Programming (SDDP) algorithm. The numerical results are very preliminary, but nevertheless encouraging; the model responds to the risk factors, namely λ and α, as it should. Furthermore, the solution time of a risk-adjusted problem is not worse than a risk-neutral one.

Future research should include the derivation of a stopping rule for the risk-adjusted SDDP. Moreover, more numerical experiments should be done concerning the risk factors and testing the model sensitivities to various cost parameters.

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