How I Saw, and How I See Fuzzy Sets

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Abstract This paper does not pretend a 'technical' presentation of a particular topic with an exhausting list of references; it just would like to contain some reflections of the author concerning how he sees, or better, he wishes, the future of current fuzzy logic that, in his view and at the risk of stagnation, cannot lie on any kind of 'logicism' but on 'scienticism'.

1 Introduction

It can be said that as it was originally introduced by Zadeh [16], fuzzy set theory mainly deals with two important linguistic phenomena, imprecision and non-random uncertainty, and that fuzzy sets can be applied, among others, to the study of dynamical systems whose behavior can be described by sets of imprecise linguistic rules, and to the random uncertainty associated to some linguistic statements [7, 13]. For instance, the theory of possibility can deal with non-random uncertainty, fuzzy control with dynamical systems, and fuzzy probability with random fuzzy events.

The ground of fuzzy set theory lies in the, historically not surprising, fact that predicates acting in a universe of discourse generate linguistic collectives in it; collectives [14], except when they degenerate in just a single classical set, are cloudy linguistic entities neither well known, nor easy to specify virtual or 'thought' entities whose appearances, or states, are just membership functions, fuzzy sets allowing to see their projections inside the fog of ordinary language. Hence, fuzzy sets can be seen as a starting point for the currently non existing scientific study of linguistic collectives. In sum and grossly speaking, fuzzy sets deal with ordinary language; they are mathematical entities contextually reflecting collectives, and modeled by their membership functions. They meant to pass from an old world of exact thinking

To Professor Peter Klement, with deep affection.

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represented by sets, to a new world of approximate thinking represented by them. The future of fuzzy sets can be seen around a new mathematical study of ordinary, or common sense, reasoning in which the central idea is, instead of 'deducing' from precise premises reflecting totally known information even if not fully describing something, to that of 'conjecturing' [9] from imprecise premises reflecting information partially known and also able to reach creative conclusions. That is, to increase the informative content of the premises or previous information; in sum, to be in touch with creativity.

Fuzzy sets have to do with both the representation of information, and to how new one can be obtained by just a 'previous thinking' as it is done always in searching for a new aspect of a problem, and that, latter on, should be either formalized, or checked against some reality to acquire the status of 'new' knowledge. Of course, in these processes of conjecturing those of deducing, abducing, and also lucubrating are included [11].

Without no doubt, it can also be said that the idea of fuzzy sets was born in the 'cultural' neighborhood of cybernetics, where analogical computers [1] were seriously taken into account. Fuzzy sets can be seen indeed as 'analogical entities' in contrast to the 'digital crisp sets' and, since most of the human knowledge is essentially analogical, it is not at all surprising that fuzzy sets can be suitable for representing, at least, expert knowledge. In fact, the first application of fuzzy sets to the control of machines, introduced in 1972 by the late Abe Mamdani [5], can be considered as a method for the management of imprecise expert knowledge, and who knows if, in a future, and provided analogical quantum computers [15] were actually constructed, fuzzy sets will not play some new role in their functioning. If from a philosophical and scientific point of view fuzzy sets are but measures, from a technological one they are just analogical tool constructs representing knowledge.

2 My First Steps into Fuzzy Logic

I entered into fuzzy logic by chance. It was through an interview, in a French newspaper, with the late professor Arnold Kaufmann in which he spoke on his then recently appeared book 'Ensembles flous'. The subject interested me since I was doing my research work on the probabilistic metrics introduced by Karl Menger, and knew his paper entitled 'Ensembles flous', a new concept that he translated into English by hazy sets. I bought Kaufmann's book, read a good part of it, and to some extent I was actually disappointed; my first glance at fuzzy sets make me to believe that they were just a simple generalization of sets. Nevertheless, since some of the examples in the book called upon my attention and made me curious, I decided to read the 1965 paper where fuzzy sets were originally introduced, whose title is 'Fuzzy Sets' and was written by Zadeh [16]. Before reading this paper, and as a consequence of both my mathematical formation, and the reading of Menger's paper, I was unable to see fuzzy sets unlinked with probability; indeed, hazy sets represent something like the probability of an element belonging to a set.

But the reading of Zadeh's paper suddenly changed my view. The subject was towards representing the multitude of imprecise predicates of which language is full; it was, for me, the first mathematical model for taking into account the imprecision that, permeating language, affects ordinary reasoning with concepts that are not definable like those managed in mathematics by 'if and only if' conditions, but only describable from their use in several contexts as they appear in dictionaries. It was for me a new land to be explored, and I was captivated as I see the possibility of building up mathematical models of common reasoning. I decide to start with such an exploration! At the end, in that time I was unhappy with the 'bourbakism' of which mathematics was full in Spain, and I was also worried by the giving up that, from time ago, logicians kept on ordinary reasoning.

Although the only references in the first Zadeh's paper on fuzzy sets are the purely mathematical books by Birkhoff, Halmos, and Kleene, and since from very young I kept a deep interest in Bertrand Russell's philosophical writings, I remembered the Russell's paper 'On Vagueness' and believed that there should be some links between fuzzy sets and vagueness. This idea conducted me to the 1972 paper by Aldo De Luca and Settimo Termini, where they established the then new concept of a 'fuzzy entropy', and I thought it is nothing else than a measure of the vagueness or, by duality of its classicality, booleanity, or crispness, the linguistic label of a fuzzy set presents. This idea make me to think that 'fuzziness' is just a restriction of the vagueness of a predicate whenever it can be represented by a fuzzy set, and this was for me a challenging philosophical idea that, many years ahead, conducted me to see that fuzzy sets are nothing else than measures of the meaning of predicates. My first papers on fuzzy sets dealt, between 1976 and 1978, with trying to find functionally expressible mathematical formulas able to represent fuzzy entropies, but different and more general than the unique logarithmic fuzzy entropy shown by De Luca and Termini in his paper. In addition, I also tried to relate them with the Sugeno's fuzzy integral since, in the meantime, I was acquainted with Michio Sugeno in Toulouse, and with his 1974 Ph.D. Thesis.

Early after these worries, I began to be interested in the subject of fuzzy connectives and fuzzy inference. For the first I was essentially motivated by the fact that fuzzy connectives can't show the same properties in all contexts, and that distributivity is, for instance, a very constraining and crisp property. What conducted me in such direction were the papers by Bellman and Giertz [2], and that on negation by Lowen [4]. My dedication to Probabilistic Metric Spaces, that bring me to know Bert Schweizer and Able Sklar after meeting Karl Menger in Chicago, introduced me to solve Functional Equations, and I got the idea of characterizing the (continuous) strong negations by just solving an easy functional equation. Since I was familiar with Schweizer and Sklar's t-norms (a restriction by adding associativity to those introduced by Menger [6]), I can introduce in fuzzy logic these ordered semi-groups in the unit interval. Finally, the fact that as examples of his Compositional Rule of Inference, Zadeh did show some that were non-preserving the classical Modus Ponens when the input is just the antecedent of the rule, I tried to study this 'Modus' in fuzzy logic by formulating it as Hardegree did in Orthomodular lattices [3].

To end this section, that corresponds with the time in which I met for the first time professor Peter Klement and in Barcelona, let me remember again that, as another consequence of Menger's work, but due to the idea of mathematically modeling the breaking of synonymous chains, I introduced in fuzzy logic the T-indistinguishabilities, or T-equivalences, allowing to relating such problem with that of Poincaré concerning the physical continuum. I would like to say that Menger's trace in fuzzy logic or, at least, in my contributions to it, is certainly of some relevance.

3 Zadeh's Fuzzy Sets Are but Measures of Meaning

For a lot of time after 1965, the mathematical nature of fuzzy sets in relation with the meaning of their linguistic label, was not clearly explained. They were simply viewed as membership functions generalizing the characteristic function of crisp sets and, supposedly, representing its meaning in the universe of discourse but without counting with a meaning's operational description [8]. If philosophers largely debated on the meaning of 'meaning', they never attended the representation of meaning, and it lacked a scientific study that today can be considered started with the work of Zadeh, and in a form close to the Wittgenstein of the 'Philosophical Investigations', when he states that almost always 'the meaning of a word is its use in language'. How can even if not defined, the use or management in language of a linguistic label be mathematically described?

If P is a linguistic label, or predicate, acting in a universe of discourse X through the elemental statements 'x is P', for a suitably management of P the two binary relations in X, that empirically come from linguistic perception, from its use,

- $x =_P y \Leftrightarrow x$ shows the property named P equally than y shows it $\Leftrightarrow x$ is equally P than y,
- $x \leq_P y \Leftrightarrow x$ is less P than y,

should be known [2, 14]. When both relations $\leq_P$ and $=_P$ do coincide, it is said that the use of P in X is *precise*, *rigid*, or *crisp*, $=_P$ is an equivalence, and X is partitioned in the equivalence classes in the quotient set $X/=$, $[x] = \{y \in X; y =_P x\}$. Instead and when $\leq_P \neq =_P$ that, provided it can be supposed $=_P = \leq_P \cap \leq_P^{-1}$, implies that it is not $\leq_P \subseteq \leq_P^{-1}$, it is said that the use of P in X is *imprecise, flexible,* or *fuzzy*. In any case, the graph $(X, \leq_P)$ represents the *qualitative*, or *primary*, meaning of P in X. In this way, the previously amorphous universe of discourse X, is softly structured thanks to the use of P in it. The simple and usual act of 'speaking' on a property recognizable in the elements of X, endows X with the arcs of this graph; an idea corresponding with the intuitive one that rational speech tries to introduce some kind of 'ordering' in the universe of discourse, also corresponding to the establishment of some necessary link between ordering and understanding. Nevertheless, the graph does not exhaust the 'full meaning' of P in X, and when it is $\leq_P = \emptyset$, it can be said that P is metaphysically used in X, that P is *metaphysical*, or *meaningless* in X [11]. Notice that it is thanks to the relation $\leq_P$ that can be seen the variability of the property named P along the elements of X; that it is $\leq_P \neq =_P$, is what permits to say that the use of P is imprecise in X.

If P is not metaphysically used in X, that is, if $\leq_P \neq \emptyset$, then a *measure* of the extent of P in X, is a mapping $\mu_P \colon X \to [0, 1]$, such that

(1) $x \leq_P y \Rightarrow \mu_P(x) \leq \mu_P(y)$
(2) z maximal for $\leq_P \Rightarrow \mu_P(z) = 1$
(3) z minimal for $\leq_P \Rightarrow \mu_P(z) = 0$.

Once the graph $(X, \leq_P)$ is known it can be said that P is *measurable* in X, and once a measure μ_P is known that it is *effectively measurable* in X [14].

The three former properties are not sufficient, in general, to specify a measure, but there is only a single one if the predicate is precise; to specify a measure either more information on the use of P, or to establish a reasonable hypothesis on it, is necessary.

In any case, each measure μ_P is the membership function of a fuzzy set in X labeled P. Fuzzy sets are defined by the measures of the extent up to which the elements in X are P show the property named P; shortly speaking it can be said that fuzzy sets are measures of meaning, like probabilities are measures of random uncertainty, and fuzzy entropies are measures of fuzziness. It should be noticed that each quantity $(X, \leq_P, \mu_P)$ represents a good enough knowledge on the meaning of P in X for its scientific consideration; it can be said that such quantities are the typically scientific *domestication* of meaning [10], and can offer a new perspective for studying both fuzzy sets and fuzzy logic.

It should be noticed that if the use of P in X is rigid, it is $x =_P y \Leftrightarrow x \leq_P y$ & $y \leq_P x \Rightarrow \mu_P(x) = \mu_P(y)$, and hence, μ_P is constant in the classes modulo P, the only values μ_P can take are 0 or 1, $\mu_P^{-1}(1)$ is the crisp subset specified by P in X, $\mu_P^{-1}(0)$ its classical complement, and one of them can be empty.

In praxis, a fuzzy set is designed by means of the information on its linguistic label that is available and that, most of the times, is not the full relation $\leq_P$, but a part of it; there are cases in which obtaining $\leq_P$ can be very difficult. Hence and very often, neither it is always $\leq_P$ completely known, nor it can be stated that the designed membership function μ_P^* is truly a measure, but some unknown approximation to it. Consequently, the designer cannot work with $\leq_P$ but only with the total order defined by x $\leq_{\mu_P^*} y \Leftrightarrow \mu_P^*(x) \leq \mu_P^*(y)$, called the *working meaning* of P in X. Provided μ_P^* were actually a measure or, at least, it can be supposed it verifies property (1), and since then, $x \leq_P y \Rightarrow \mu_P^*(x) \leq \mu_P^*(y) \Leftrightarrow x \leq_{\mu_P^*} y$, implies $\leq_P \subseteq \leq_{\mu_P^*}$, that is, the working meaning extends the qualitative meaning of P. The act of measuring P, modifies its qualitative meaning by adding more arcs to it [14].

Notice that since in most cases the relation $\leq_P$ has not a total, or linear, character, it cannot coincide with the linear orders $\leq_{\mu_P}$. When there is coincidence, it is said that the measure *perfectly reflects* the qualitative meaning of P. It is easy to proof that, provided $\leq_P$ is reflexive and transitive, then $=_P$ is an equivalence relation, and that the mapping $C : X \to X/ =_P$, assigning to each x the equivalence class $[x] = C(x)$, verifies $x \leq_P y \Leftrightarrow [x] \leq_P^* [y] \Leftrightarrow C(x) \leq_P^* C(y)$. Hence, not only C

perfectly reflects the qualitative meaning of P in X, but this idea opens the door for defining 'qualitative measures' of a predicate by taking, instead of the unit interval, some non-numerical posets with which more possibilities of perfectly reflecting the qualitative meaning could appear.

4 On the Other Types of Fuzzy Sets

In those cases in which the measure does not perfectly reflect the qualitative meaning, and since in science is not at all rare to manage measures with complex values, it could be suitable to substitute the real interval [0, 1] by the complex one $\{a + bi; a, b \in [0, 1]\}$, the complex circle, endowed with the usual partial order $a_1 + b_1 i \leq a_2 + b_2 i \Leftrightarrow a_1 \leq a_2$ & $b_1 \leq b_1$, and with analogous properties [14] to the former (1), (2), and (3). This substitution cannot guarantee that a complex-valued measure will perfectly reflect the qualitative meaning, but just that it can offer more possibilities for it, since the working order will be not linear. This substitution that can be equivalently seen by taking an interval-valued measure, just changing a + bi by the interval $[a, b]$, and corresponding to a particular type of the so-called type-2 fuzzy sets reflecting that the value of the measure carries with the uncertainty coming from only being sure that it is in the interval $[a, b]$.

Analogously, and instead of the real or the complex unit intervals, it can be taken the set $[0, 1]^{[0,1]}$, of the fuzzy sets in the unit interval (type-2 fuzzy sets) that contains images isomorphic to both the unit interval and the complex unit interval, and for those cases in which the only that can be asserted is that the value of the measure is, for instance, either 'around 0.7', or 'high' [12]. In this form, all the types of fuzzy sets currently considered, are integrated thanks to the quantities, either numerical or functional, representing the meaning of its linguistic label.

The full meaning of a linguistic label P is not unique, but it is actually context-dependent and purpose-driven. Each quantity $(X, \leq_P, \mu_P)$, real, complex or fuzzy valued, is obtained through what the designer can know, in a given context, of the use, action or behavior of P in X, or through some reasonable hypothesis he could be able to make on such behavior. This last is the often considered case in most applications, in which the real-valued measure, the membership function, is supposed to be trapezoidal, or just triangular.

Once seen that the membership functions of fuzzy sets mean nothing else than a 'measure of the meaning' of its linguistic label, it can be remembered the famous words of Lord Kelvin shortened to 'If you cannot measure it, it is not science'. There are, notwithstanding and at least, two aspects introducing important differences between Lord Kelvin's times and ours. In the first place, it is the fact that if, lets it say, science is essentially concerned with matter and energy, fuzzy set theory is concerned with knowledge and information, and directly related with the so called Information Technologies. In a second place, in Lord Kelvin's science there were and are known systematic procedures and laboratory methods, to measure the basic parameters of the studied things, but now and for what concerns, for instance, the design of membership functions, the situation is different and more linked to some analogy with virtual objects, than with physically real objects. It is not the same to study the chemical composition of an organic product, or the movement of a star, than to study the meaning of a written piece, or the control of a machine whose behavior is known by the knowledge of the experts in their functioning and once linguistically described. Nevertheless, this is the kind of problems currently worrying Artificial Intelligence.

5 The Evoluation of Fuzzy Logic

Anyway, the evolution of fuzzy logic towards Zadeh's Computing with Words and Perceptions, CwW for short [17], is conducting towards the mathematical representation of statements larger and more complex than the more or less simple rules considered in fuzzy control [8]. This will mean to face with the necessity of considering different ways of expressing conditional statements, and the already known linguistic connectives 'and', 'or', 'not', etc., since there is not a universal form of expressing them in language, like it is in classical logic and set theory, but respectively represented in fuzzy logic by residuated implications, S-implications, conjunctive implications, t-norms, t-conorms, negation functions, etc. For all that there are a lot of mathematical models facilitating to fuzzy logic a remarkable armamentarium for the representation of statements, and for doing deductive inferences with them, but what is not yet clear enough are the linguistic subjects to which such armamentarium is applicable, and to which is not. For instance, fuzzy logic only considers functionally expressible connectives, but no suitable criteria are known for recognizing this hypothesis in concrete cases, and, analogously, the use of non strong but continuous negations is not yet spread into fuzzy logic applications to represent language. Even more, almost always the used connectives are min, max, prod, and 1-id; there exists a big separation between what is employed by practitioners of fuzzy logic, and what is kept in the theoretical armamentarium generated by mathematicians.

In sum, it seems that fuzzy logic is approaching the time in which it should face a turning point. The great subjects fuzzy logic deals with are linguistic imprecision and non-random uncertainty, not to say anything on the very important but scientifically almost pending subjects of ambiguity, the presence of multiple meanings, and common sense non-deductive reasoning [11] with imprecise, non-randomly uncertain, and ambiguous words.

The only way to properly afford it is, in the author's view, the transformation of fuzzy logic in a kind of 'physics' of imprecision, non-random uncertainty and ambiguity. That is, in a new experimental science that, based in Natural Language, can count with mathematical models able to give important parameters to be experimentally computed at each case once their can be found in the same study of language, and not by abstract mathematical thought considerations. What is needed is to transform the study of language from a logic one in a scientific one.

When fuzzy logic was initially developed in the past Century's seventies and eighties, almost the only back referents for its study were classical and multiple-valued logics, but now it should be centered in Natural Language. If current fuzzy logic already meant an important progress in the way asked by John von Neumann of introducing mathematical analysis in the study of those subjects without a just 'yes' or 'not' hypothesis for its validity, it can be the right moment to go a step ahead and turning towards the Artificial Intelligence's 'Gordian Knot' of trying to reach computers thinking like people usually do.

6 Conclusion

Up to some point, and although many papers of a mathematical character, even with some of them of a true mathematical quality, are being continuously published in the setting of 'theoretic fuzzy logic', its evolution seems to be actually stagnated because of some moving away of what is the essence of fuzzy logic. By one side, those papers remain practically unknown or, at least, not considered, for those who devote their efforts to the applications of fuzzy logic, and by the other the motivation of their authors is almost always purely abstract; in them, it rarely appears a 'real fuzzy problem' to which either their results could be applied to, or just it can be suggested by the paper's content. It seems as if in current fuzzy logic it were two streams, that of mathematicians and that of engineers, but fuzzy logic should be an integrated study of what is 'fuzzy' and, in principle, that practitioners ignore the obtained mathematical results, marks a limit in their capability of designing fuzzy systems. There is, perhaps, some kind of isolation between both types of researchers, the most relevant of the ones not mixed with the most relevant of the others. This and to some extent, goes against the cross-fertilization of both groups and can contribute to the closing of the first in their own mathematical interest. Anyway, and in the last years, I hopefully heard on mixed groups working in some specific projects. Notwithstanding, and as far as I know, such projects are on very specific topics not directly related with CwW.

In the author's view, and by looking at what is fuzzy and what is for its study, the great challenge for the best continuation of theoretic fuzzy logic lies in the problems that are in the back of the new Zadeh's 'Computing with Words and Perceptions', where the problems that were essential for the introduction of fuzzy sets could acquire all their relevance as clearly dealing with Natural Language's complex phrases, and with the non-deductive varieties of Commonsense Reasoning. Nevertheless, the natural and dynamic characters of both language and reasoning, seems to suggest that a new, and scientific, study of them cannot be completely afforded by only counting with the abstract reasoning reached through mathematical theorems that only can be successfully applied provided all what is being supposed for their proofs is actually verified in a concrete and actual situation. Something that is, usually, very difficult to check as, it happens, for instance, when trying to use an S-implication function for linguistic rules in which the representation of the negation of their antecedents is actually unknown.

It is in the thought of the author that the main subjects of ‘fuzzy logic’ or, by extension CwW, are both the representation and technical management of the imprecision and the uncertainty pervading natural language and commonsense reasoning in non-trivial statements. In some cases, for instance, the meaning of the components of a large statement is only captured after having captured the full meaning of the full statement, something different of what is done in logic where always it is done by departing from the meaning of the components.

To afford those subjects it seems recommendable to face them as they are, natural phenomena of which, and in addition, we have a scarce knowledge that, notwithstanding, should be increased by the only way it can be followed for any natural phenomena, namely, by experimenting in controlled forms as it is typical of science. It is with the conjunction of experimentation and mathematical modeling how measurable parameters can be obtained and deep conclusions attained. Science always needs to count with suitable frames for representing what it deals with, thanks to which some mathematical models could be established and that, at its turn, facilitates some numerical parameters necessary to going on with more experimentation. À la Popper, research is always an un-ended quest.

A new experimental science dealing with linguistic imprecision and uncertainty, both random and not random, seems to appear in the horizon and into the complex knitting of language. It is an enterprise that jointly with, and close to, fully knowing the brain’s functioning, could contribute to capture what is rationality by going far from old metaphysical ideas, and by means of the single way mankind has for acquiring safe knowledge, the scientific method.

Would young researchers in the XXI Century devote their efforts to such a challenging enterprise!

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