Preface

This book was written expressly to serve as a textbook for a one- or two-semester introductory graduate course in the theory of measure and Lebesgue integration, usually designated in graduate programs as *real analysis.* In writing this book we have naturally been concerned with the level of preparation of the prospective reader. Such a reader who has mastered the majority of our earlier book *An Introduction to Analysis* (which is referred to in the text as [**I**]) will be more than adequately prepared to profitably read this book. However, at the other end of the spectrum, any beginning graduate student who has reached a certain level of mathematical maturity, which may be taken to mean the ability to follow and construct ε-δ arguments, should have no difficulty in mastering the material in this book. We have deliberately made this book as self-contained as possible.

In keeping with our pedagogical intent, we have provided in each chapter a copious supply of examples and a lengthy collection of problems. *Some of the problems that appear in these problem sets are stated simply as facts (see, for example the first sentence of Problem 1D). In this case, the reader is supposed to supply a proof of the stated fact.* Hints are provided for the more challenging problems. These problem sets constitute an integral part of the book and the reader should study them along with the text. Working problems is, of course, critically important in the study of mathematics because that is how mathematics is learned. In this textbook it is particularly important because many topics of interest are first introduced in the problem sets. Not infrequently, the solution of a problem depends on material in one or several previous problems, a fact that instructors should keep in mind when assigning problems to a class.

While, as noted, this book is intended to serve as a textbook for a course, it is our hope that the wealth of carefully chosen examples and problems will also make it useful to the reader who wishes to study *real analysis* individually. This certainly has proved to be the case for our earlier books [**I**] and [**II**].

One novel feature of our development of integration theory in this book is that the Lebesgue integral is defined axiomatically (see Chapter 3), and only after its usual properties have been revealed is it proved that there is a one-to-one correspondence between measures and Lebesgue integrals. To our knowledge, this approach is original with the authors and it was sketched in our earlier text [**II**]. Our exposition has the advantage that the Lebesgue integral is developed before, and independently of, any discussion of measure spaces. This order of doing things seemed to be quite successful in classroom use. In particular, preliminary versions of this book have been used successfully in graduate courses at The University of Michigan, Indiana University, and Texas A&M University. We take this opportunity to thank the many students in these courses who pointed out inaccuracies in the text and problem sets. Any remaining mistakes are entirely the authors' responsibility.

In writing this book no systematic effort has been made to attribute results or to assign historical priorities. The association of particular names to theorems serves primarily as a memory aid.

The notation and terminology used throughout the book are in essential agreement with those to be found in contemporary textbooks. In particular, the symbols $\mathbb{N}, \mathbb{N}_0, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$, and $\mathbb{C}$ always represent the systems of positive integers, nonnegative integers, integers, rational numbers, real numbers, and complex numbers, respectively. The closure of a set A in a topological space is denoted $\overline{A}$ or A^-. One basic convention valid throughout the book is that all vector spaces encountered are either real or complex, and the relevant field of scalars should be clear from the context.

The numbering system for examples, propositions, theorems, remarks, etc., is somewhat standard. For instance, Proposition 4.21 is the 21st fact stated in Chapter 4 and it is followed by Example 4.22. Also, Problem 3Z is the 26th problem at the end of Chapter 3 and is followed by Problem 3AA.

The reader who is interested in the historical development of measure and Lebesgue integration theory would do well to consult von Neumann [**vN**] and Halmos [**H**]. Excellent comprehensive reference books on this subject are Fremlin [**F**] and Bogachev [**B**]. Useful treatments of analytic sets are to be found in Lusin [**L**] and Kuratowski [**CK**] (historical) and Kechris [**AK**] (contemporary). For further information on functional analysis, one might consult Brown-Pearcy [**II**], Dunford-Schwartz [**DS**], and Rudin [**R**]. Similarly, for Fourier analysis, Zygmund [**Z**], Stein [**S**], and Muscalu-Schlag [**MS**] are excellent references.

Of course, of necessity, many topics in measure theory and integration are not dealt with in this book; for instance, vector measures and Haar measure.

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