Chapter 2
Simple Electro-Magnetic Circuits

2.1 Introduction

The simplest component which utilizes electro-magnetic interaction is the coil. A coil is an energy storage component, which stores energy in magnetic form. Air-cored coils are frequently used (for example, in loudspeaker filters), but coils with a core of (possibly gapped-) magnetic material are more common, because of their increased inductance (or reduced size), which may come at the cost of reduced maximum field strength and increased non-linearity. In this chapter we will develop a generic model of a coil with linear and non-linear self-inductance. Furthermore, the effect of coil resistance is considered. The use of phasors is introduced in this chapter as a means to verify simulation of such circuits when connected to a sinusoidal source.

2.2 Linear Inductance

The physical representation of the coil considered here is given in Fig. 2.1. The figure shows a coil with n turns which is wrapped around a toroidally shaped non-gapped magnetic core with cross-sectional area A_m. The permeability of the material is given as μ and the average flux path length is equal to l_m. Analog to Eq. (1.6), the magnetic reluctance of the circuit is: $R_m = {}^{l_m}/_{A_m \mu}$ and the inductance is $L = {}^{n^2 \mu A_m}/_{l_m} = {}^{n^2}/_{R_m}$.

Electronic supplementary material The online version of this chapter (doi: 10.1007/978-3-319-29409-4_2) contains supplementary material, which is available to authorized users.

A. Veltman et al., *Fundamentals of Electrical Drives*, Power Systems,
DOI 10.1007/978-3-319-29409-4_2

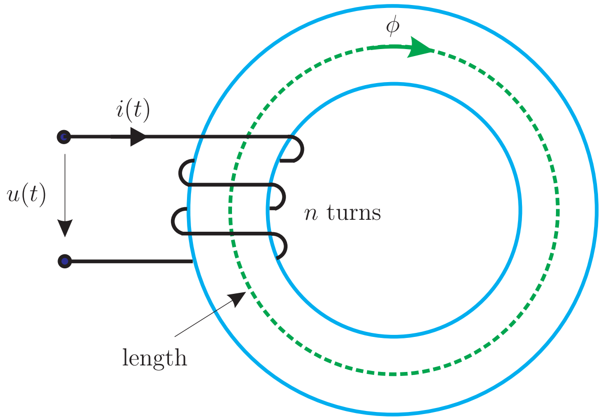


Fig. 2.1 Toroidal inductance

The relation between the magnetic flux and the current in the coil is described by the expression

$$\psi = Li \tag{2.1}$$

With Faraday's law

$$u = \frac{\mathrm{d}\psi}{\mathrm{d}t} \tag{2.2}$$

Equation (2.1) can be rewritten to the more familiar differential form of the coil's voltage terminal equation

$$u = L\frac{\mathrm{d}i}{\mathrm{d}t} \tag{2.3}$$

Equation (2.3) can be integrated on both sides and rewritten as the general equation

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} u(t)\mathrm{d}t \tag{2.4}$$

The whole integrated history of the inductor voltage is reflected by the inductor current, so Eq. (2.4) can be expressed in a more practical form, starting at $t = 0$ with initial condition $i(0)$, according to

$$i(t) = \frac{1}{L}\int_{0}^{t} u(t)\mathrm{d}t + i(0) \tag{2.5}$$

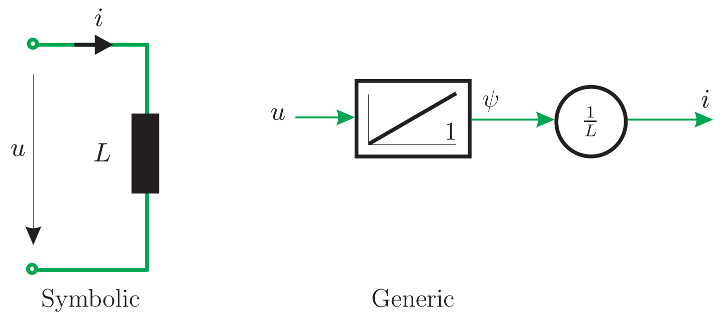


Fig. 2.2 Symbolic and generic model of a linear inductance

This integral form can be developed further

$$\Delta i = \frac{\Delta\psi}{L} \tag{2.6}$$

$$\underbrace{\psi(t) - \psi(0)}_{\Delta\psi} = \int_0^{t_0} u(t)\mathrm{d}t \tag{2.7}$$

introducing the concept of "incremental flux linkage" $\Delta\psi = \psi(t) - \psi(0)$. The equation basically states that a flux-linkage variation corresponds with a voltage-time integral (the so-called volt-second) when the resistance is zero.

A symbolic and generic model of the ideal coil is given in Fig. 2.2. With the model of Fig. 2.2, we will now simulate the time-response of a coil in reaction to a voltage pulse of magnitude $\hat{u}$ and duration T, starting at $t = t_0$, as displayed in Fig. 2.3. Integrating the supply voltage u over time gives the flux-linkage ψ in the coil, which linearly increases from 0 at $t = t_0$ to $\hat{u}T$ at $t = T$. The current is obtained by dividing the flux ψ by L.

2.3 Coil Resistance

In practical situations, the resistance of the coil wire can usually not be neglected. Wire resistance can simply be modeled as a resistor in series with the ideal coil. The modified symbolic model is shown in Fig. 2.4.

Figure 2.4 shows that the coil flux is no longer equal to the integrated supply voltage u. Instead, the variable u_L is introduced, which refers to the voltage across the "ideal" (zero resistance) inductance $u_L = {}^{\mathrm{d}\psi}/_{\mathrm{d}t}$. The terminal equation for this circuit is now given by expression (2.8).

$$u = iR + \frac{\mathrm{d}\psi}{\mathrm{d}t} \tag{2.8}$$

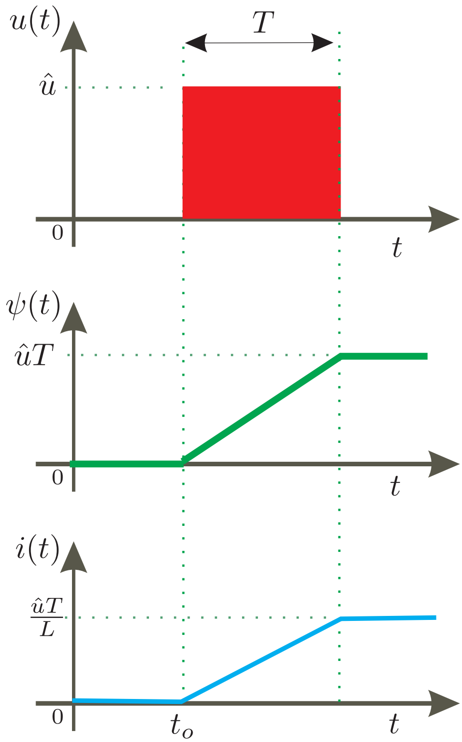


Fig. 2.3 Transient response of inductance

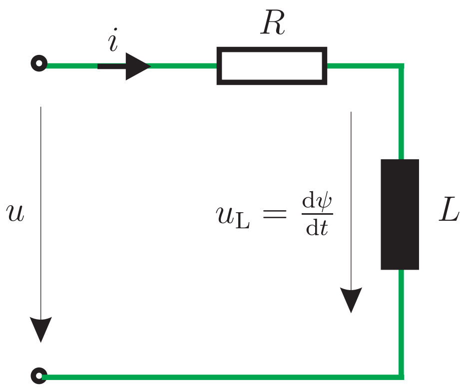


Fig. 2.4 Symbolic model of linear inductance with coil resistance

where R represents the coil resistance. The corresponding generic model of the lumped parameter "L, R" circuit is shown in Fig. 2.5. The generic model clearly shows how the inductor voltage u_{L} is decreased by the resistor voltage caused by the current through the coil.

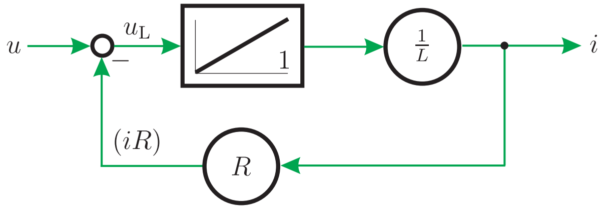
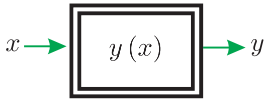


Fig. 2.5 Generic model of linear inductance with coil resistance

Fig. 2.6 Non-linear generic building block



2.4 Magnetic Saturation

As discussed in Chap. 1, the maximum magnetic flux density in magnetic materials is limited. Above the saturation flux density, the magnetic permeability μ drops and the material will increasingly behave like air, i.e., $\mu \to \mu_0$ when flux density is increased further. Since motors usually work at high flux density levels, with noticeable saturation, it is essential to incorporate saturation in our coil model.

The relationship between flux-linkage and current is in the magnetically linear case determined by the inductance, as shown in Fig. 1.14. In reality, the $\psi(i)$ relationship is only relatively linear over a limited region (in case the magnetic circuit contains "iron" (steel) core), as shown in Fig. 1.16. The generic model according to Fig. 2.5 needs to be revised in order to cope with the general case.

The generic building block for non-linear functions [7] is shown in Fig. 2.6. The double edged box indicates a non-linear module with input variable x and output variable y. The relationship between output and input is shown as $y(x)$ (y as a function of the input x). In some cases, a symbolic graph of the function that is implemented may also be shown on this building block.

The non-linear module has the coil flux ψ as input and the current i as output. Hence, the non-linear function of the module is described as $i(\psi)$, which expresses the current of the coil as a function of the coil flux. The terminal equation (2.8) remains unaffected by the introduction of saturation, only the gain module $1/L$ shown in Fig. 2.5 must be replaced by the non-linear module described above. The revised generic model of the coil is shown in Fig. 2.7.

2.5 Use of Phasors for Analyzing Linear Circuits

The implementation of generic circuits (such as those discussed in this chapter) in PLECS allows us to study models for a range of conditions. The use of a sinusoidal excitation waveform is of most interest given their use in electrical machines and

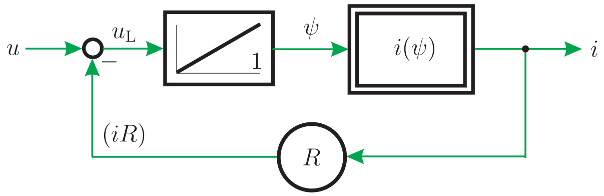


Fig. 2.7 Generic model of general inductance model with coil resistance

actuators. However, there must be a way to perform "sanity checks" on the results given by simulations. Analysis by way of phasors provides us with a tool to look at the *ac* steady-state results of linear circuits. The underlying principle of this approach lies with the fact that a sinusoidal excitation function, for example, the applied voltage, will cause a sinusoidal output function of the same frequency, be it that the amplitude and phase (with respect to the excitation function) will be different. For example, in the symbolic circuit shown in Fig. 2.4, the excitation function will be defined as $u(t) = \hat{u}\sin(\omega t)$, where $\hat{u}$ and ω represent the peak amplitude and angular frequency (rad/s), respectively. Note that the latter is equal to $\omega = 2\pi f$, where f represents the frequency in Hz. The output variables are the flux-linkage $\psi(t)$ and current $i(t)$ waveforms. Both of these will also be sinusoidal, be it that their amplitude and phase differ from the input signal $u(t)$. In general, a sinusoidal function can be described by

$$x(t) = \hat{x}\sin(\omega t + \rho) \tag{2.9}$$

This function can also be written in complex notation as

$$x(t) = \Im\left\{\hat{x}e^{\mathrm{j}(\omega t+\rho)}\right\} \tag{2.10}$$

Equation (2.10) makes use of "Euler's rule" $e^{\mathrm{j}y} = \cos y + \mathrm{j}\sin y$. The imaginary part of this expression is defined as $\Im\left\{e^{\mathrm{j}y}\right\} = \sin y$. $\Im\{\}$ is the imaginary operator, which takes the imaginary part from a complex number. Note that the analysis would be identical with $x(t)$ in the form of a cosine function. In the latter case it would be more convenient to use the real component of $\hat{x}e^{\mathrm{j}(\omega t+\rho)}$, using the real operator $\Re\{\}$. Equation (2.10) can be rewritten to separate the time dependent component $e^{\mathrm{j}\omega t}$ namely:

$$x(t) = \Im\left\{\underbrace{\hat{x}e^{\mathrm{j}\rho}}_{\underline{x}}\, e^{\mathrm{j}\omega t}\right\} \tag{2.11}$$

The time independent component in Eq. (2.11) is known as a "*phasor*" and is generally identified by the notation $\underline{x}$. In general the phasor will have a real and imaginary component and can therefore be represented in a complex plane.

In many cases it is also convenient to use the time differential of $x(t)$ namely ${}^{\mathrm{d}x}/_{\mathrm{d}t}$. The time differential of the function $x(t) = \Im\left\{\underline{x}\, e^{\mathrm{j}\omega t}\right\}$ is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \Im\left\{\mathrm{j}\omega\underline{x}\, e^{\mathrm{j}\omega t}\right\} \tag{2.12}$$

which implies that the differential of the phasor $\underline{x}$ is calculated by simply multiplying $\underline{x}$ with $\mathrm{j}\omega$.

2.5.1 Application of Phasors to a Linear Inductance with Resistance Network

As a first example of the use of phasors, we will analyze a coil with linear inductance and non-zero wire resistance, as shown in Fig. 2.4. We need to calculate the steady-state flux-linkage and current waveforms of the circuit. The differential equation set for this system is

$$u = iR + \frac{\mathrm{d}\psi}{\mathrm{d}t} \tag{2.13a}$$

$$\psi = Li \tag{2.13b}$$

The flux-linkage differential equation is found by substitution of Eq. (2.13b) into (2.13a) which gives

$$u = \frac{R}{L}\psi + \frac{\mathrm{d}\psi}{\mathrm{d}t} \tag{2.14}$$

The applied voltage will be $u = \hat{u}\sin\omega t$, hence the phasor representation of the input signal according to (2.11) is: $\underline{u} = \hat{u}$.

The flux-linkage will also be a sinusoidal function, albeit with different amplitude and phase: $\psi = \hat{\psi}\,\sin\left(\omega t + \rho_{\psi}\right)$. The parameters $\hat{\psi}\, and\ \rho_{\psi}$ are the unknowns at this stage. In phasor representation, the flux time function can be written as $\psi = \Im\left\{\underline{\psi}\, e^{\mathrm{j}\omega t}\right\}$ where $\underline{\psi} = \hat{\psi}\, e^{\mathrm{j}\rho_{\psi}}$.

Rewriting Eq. (2.14) using these phasors, we obtain

$$\underline{u} = \frac{R}{L}\underline{\psi} + \mathrm{j}\omega\underline{\psi} \tag{2.15}$$

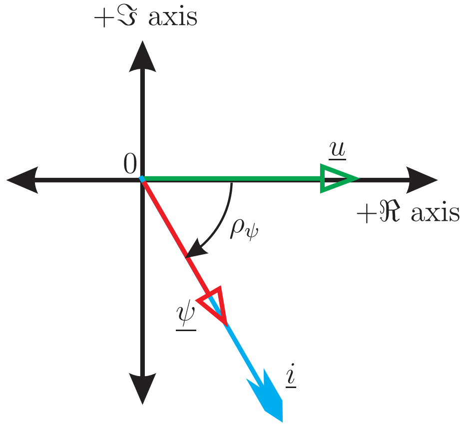


Fig. 2.8 Complex plane with phasors: $\underline{u}$, $\underline{\psi}$, $\underline{i}$

from which we can calculate the flux phasor by reordering, namely

$$\underline{\psi} = \frac{\underline{u}}{\left(\frac{R}{L} + \mathrm{j}\omega\right)} \tag{2.16}$$

The amplitude and phase angle of the flux phasor are now

$$\hat{\psi} = \frac{\hat{u}}{\sqrt{\left(\frac{R}{L}\right)^2 + \omega^2}} \tag{2.17a}$$

$$\rho_\psi = -\arctan\left(\frac{\omega L}{R}\right) \tag{2.17b}$$

and the corresponding current phasor is according to Eq. (2.13b): $\underline{i} = \underline{\psi}/L$.

The transformation of phasors back to corresponding time variable functions is carried out with the aid of Eq. (2.11). A graphical representation of the input and output phasors is given in the complex plane shown in Fig. 2.8.

2.6 Tutorials

2.6.1 Tutorial 1: Analysis of a Linear Inductance Model

In this chapter we analyzed a linear inductance and defined the symbolic and generic models as shown in Fig. 2.2. The aim of this tutorial is to build a PLECS model from this generic diagram. An example as to how this can be done is given in Fig. 2.9. Indicated in Fig. 2.9 is the inductance model in the form of an integrator and gain module. Also given are two “step” modules which, together with a “Sum” unit,

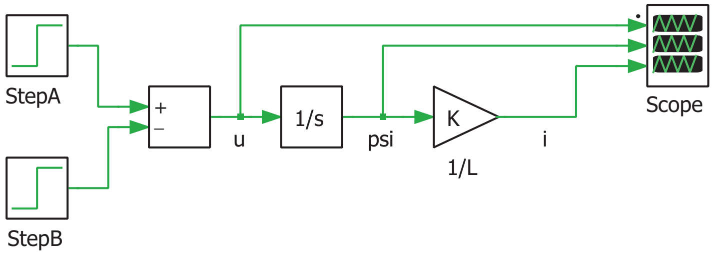


Fig. 2.9 PLECS model of linear inductance with excitation function

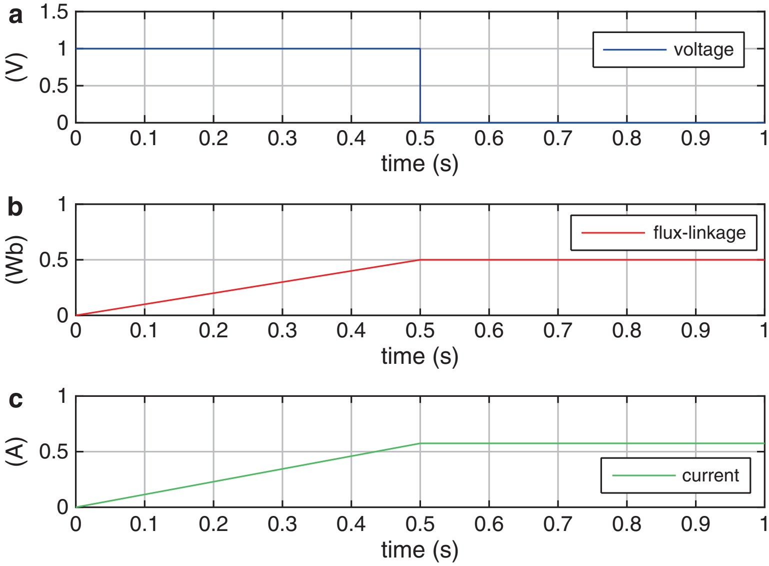


Fig. 2.10 PLECS results: ideal inductance simulation

generate a voltage pulse of magnitude 1 V. This pulse should start at $t = 0$ and end at $t = 0.5$ s. Build this circuit and also add a "Scope" module which allows you to display your data. In this exercise we look at the input voltage waveform, the flux-linkage, and current versus time functions. Once you have built the circuit you need to run this simulation. For this purpose you need to set the "stop time" (under Simulations/simulation parameters dialog window) to 1 s. The inductance value used in this case is $L = 0.87$ H, which should be set in the "Integrator" module dialog box. The results which should appear from your simulation after running this PLECS file are given in Fig. 2.10.

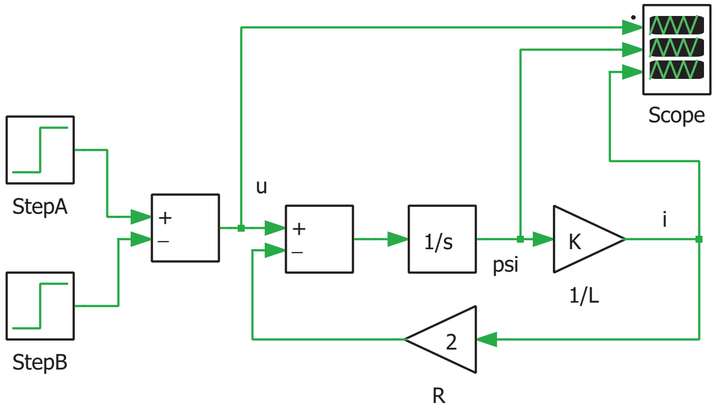


Fig. 2.11 PLECS model of linear inductance with resistance and excitation function

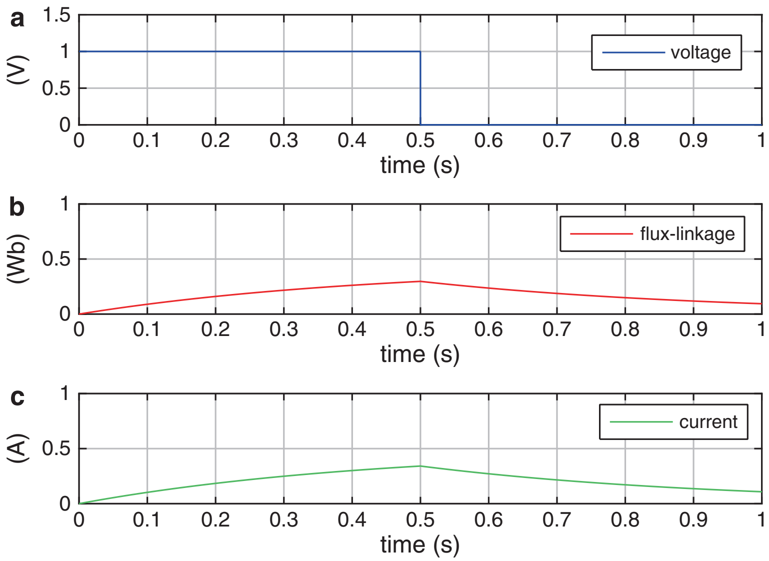


Fig. 2.12 PLECS results: inductance simulation, with coil resistance

The dynamic model as discussed above is to be extended to the generic model shown in Fig. 2.5. Add a coil resistance of $R = 2\,\Omega$ to the PLECS model given in Fig. 2.9. The new model should be of the form given in Fig. 2.11.

Run the simulation again, in which case the results should be of the form given in Fig. 2.12.

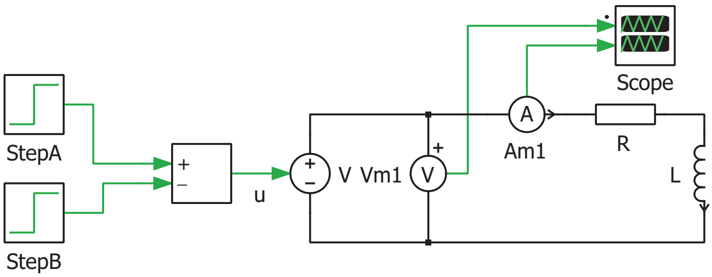


Fig. 2.13 PLECS "symbolic" model: linear inductance with coil resistance

2.6.2 Tutorial 2: Symbolic Model Analysis of a Linear Inductance Model

In this tutorial we will consider an alternative implementation of tutorial 1, based on the use of symbolic models (where possible) instead of "control" blocks as used in the previous case. Build a PLECS model of the symbolic model shown in Fig. 2.4 with the excitation and circuit parameters as discussed in tutorial 1. Note that "symbolic" modules in PLECS are known as "Electrical" blocks.

An example of a PLECS implementation is given in Fig. 2.13 on page 39. The "scope" module given in Fig. 2.13 displays the results of the simulation. The simulation results obtained with this simulation should match those given in Fig. 2.12 where it is noted that the flux plot is not shown in this case, given that it is not directly generated by a symbolic model. Furthermore, a "Voltmeter" (Vm1) and "Ammeter" (Am1) are used to measure the voltage and current, respectively.

2.6.3 Tutorial 3: Analysis of a Non-linear Inductance Model

In Sect. 2.4 we have discussed the implications of saturation effects on the flux-linkage/current characteristic. In this tutorial we aim to modify the simulation model discussed in the previous tutorial (see Fig. 2.11) by replacing the linear inductance component with a non-linear function module as shown in the generic model (see Fig. 2.7). In this case, the flux-linkage/current $\psi\,(i)$ relationship is taken to be of the form $\psi = \tanh\,(i)$ as shown in Fig. 2.14. Note that in this example the gradient of the flux-linkage/current curve becomes zero for currents in excess of ± 3 A. In reality, the gradient will be non-zero when saturation occurs.

The coil resistance of the coil is increased to $R = 100\,\Omega$. An example of a Simulink implementation is given in Fig. 2.15. The block diagram clearly shows the presence of the non-linear module used to implement the function $i\,(\psi)$. The non-linear module has the form of a "look-up" table which requires two vectors to be entered. Upon opening the dialog box for this module, provide the following

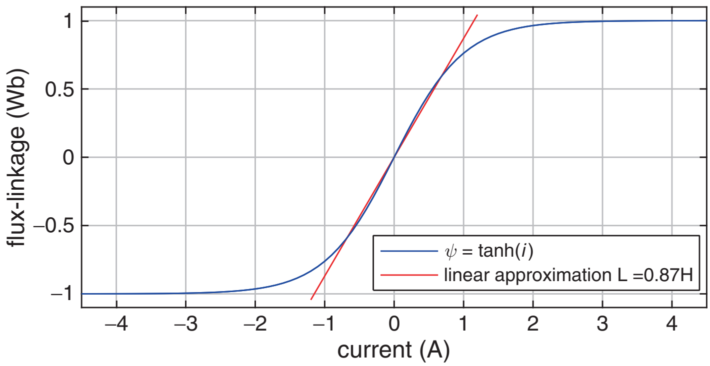


Fig. 2.14 Flux-linkage/current $\psi\,(i)$ relationship

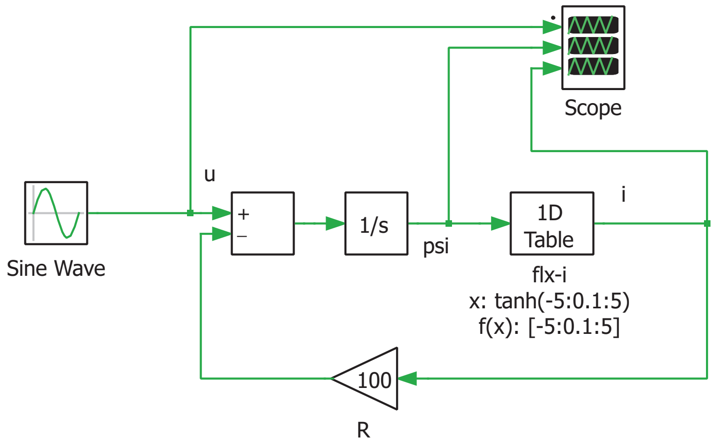


Fig. 2.15 PLECS model of non-linear inductance with sinusoidal excitation function

entries under: "vector of input values": set to `tanh([-5:0.1:5])` and "vector of output values": set to `[-5:0.1:5]`. Also given in Fig. 2.15 is a "sine wave" module, which in this case must generate the function $u = \hat{u}\cos\omega t$, where $\omega = 100\ \pi$ (rad/s) and $\hat{u}$ is initially set to $\hat{u} = 140\sqrt{2}$ V. Note that a cosine function is used. This means that in the "Sine Wave" dialog box (under "Phase") a phase angle entry is required, which must be set to $\pi/2$ (PLECS knows the meaning of "π" hence you can write this as "pi").

Once the new PLECS model has been completed, run this simulation for a time interval of 40 ms. For this purpose set the "stop time" (under Simulations/simulation parameters dialog window) to 40 ms. Save the results from the "Scope" module in the form of a "xxx.csv" file. An example of the results obtained with this simulation under the present conditions is given in Fig. 2.16. The results as given in Fig. 2.16 also include two "M-file" functions, which represent the results obtained via a phasor analysis to be discussed below.

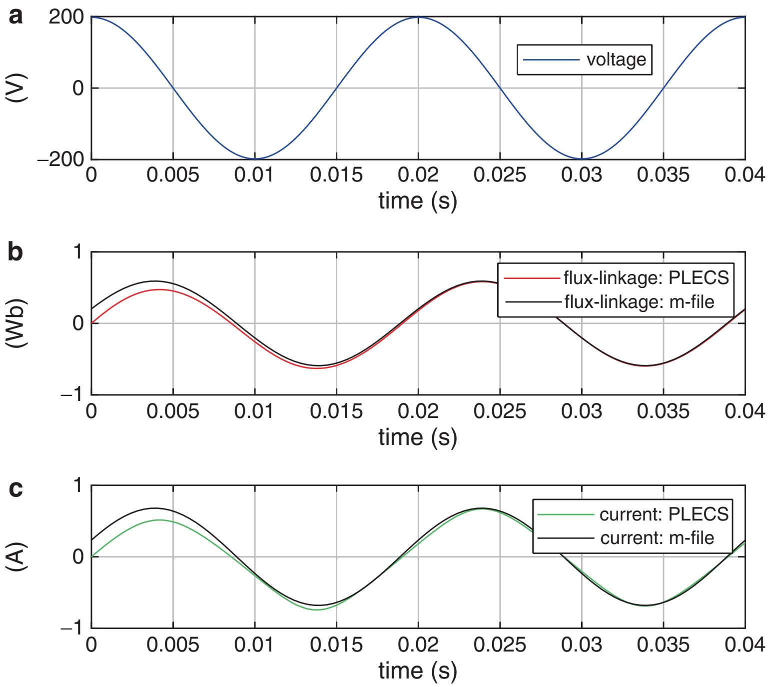


Fig. 2.16 PLECS/M-file results: inductance simulation, with coil resistance and non-linear $i(\psi)$ function

To obtain some idea as to whether or not the simulation results discussed in this tutorial are correct, we calculate the steady-state flux-linkage and current versus time functions by way of a phasor analysis. An observation of the current amplitude shows that, according to Fig. 2.14, operation is within the linear part of the current/flux-linkage curve. Assume a linear approximation of this function as shown in Fig. 2.14. This approximation corresponds to an inductance value of $L = 0.87\,\mathrm{H}$.

The input function $u = \hat{u}\cos\omega t$ may also be written as

$$u(t) = \Re\left\{\underbrace{\hat{u}}_{\underline{u}}\, e^{\mathrm{j}(\omega t)}\right\} \tag{2.18}$$

where in this case the phasor $\underline{u} = \hat{u} = 140\sqrt{2}\,\mathrm{V}$.

The actual phasor analysis must be done in MATLAB which also allows you to use complex numbers directly. For example, you can specify a phasor `xp=3+j*5` (in MATLAB form) and a reactance `X=100*pi*L`, where $L = 0.87\,\mathrm{H}$.

Write an M-file which will calculate the current and flux phasors. In addition calculate and plot the instantaneous current and flux versus time waveforms and add the results from the PLECS simulation (generated in the form of a “xxx.csv” file).

An example of such an M-file is given at the end of this tutorial, which also shows the code required to plot the results from the PLECS model.

The results obtained after running this M-file are shown in Fig. 2.16 (in "black"), together with the earlier PLECS results. A comparison between the results obtained via the PLECS model and phasor analysis (see Fig. 2.16) shows that the waveforms merge towards the end of the simulation time. In the first part of the simulation the transient effects dominate, hence the discrepancy between the simulation results and those calculated via a (steady-state *ac*) phasor analysis.

2.6.3.1 M-File Code

```
%Tutorial 3, chapter 2
close all
L=0.87;                                %inductance value (H)
R=100;%resistance
dat = csvread('tut3ch2data.csv',1,0) % read in data from
    PLECS
close all
L=0.87;                                %inductance value (H)
R=100;                                 %resistance
subplot(3,1,1)
plot(dat(:,1),dat(:,2));               % voltage input
xlabel(' (a) time (s)')
ylabel('voltage (V)')
grid
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
subplot(3,1,2)
plot(dat(:,1),dat(:,3),'r');           % flux-linkage
xlabel(' (b) time (s)')
ylabel('\psi (Wb)')
grid
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
subplot(3,1,3)
plot(dat(:,1),dat(:,4),'g');           % current
xlabel(' (c) time (s)')
ylabel(' current  (A)')
grid
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%complex analysis
u_ph=140*sqrt(2);                      %voltage phasor
w=2*pi*50;                             %excitation frequency
                                          (rad/)
X=w*L;%reactance
i_ph=u_ph/(R+j*X);                     %current phasor
i_pk=abs(i_ph);                        %peak current value
i_rho=angle(i_ph);                     % angle current phasor
psi_ph=i_ph*L;%flux phasor
psi_pk=abs(psi_ph);                    %peak value flux
psi_rho=angle(psi_ph);                 %angle current phasor
%%%%%%%%%%%plot results
```

```
time=[0:40e-3/100:40e-3];
i_t=i_pk*cos(w*time+i_rho);          %current/time function
psi_t=psi_pk*cos(w*time+psi_rho);    %flux/time function
subplot(3,1,3)
hold on
plot(time,i_t,'k');                  %add result to plot 3
legend('PLECS','m-file')
subplot(3,1,2)
hold on
plot(time,psi_t,'k');                 %add result to plot 2
legend('PLECS','m-file')
```

2.6.4 Tutorial 4: PLECS Based Analysis of a Non-linear Inductance Model with Revised Excitation Condition

It is instructive to repeat the analysis given in tutorial 3 by changing the peak supply voltage to $\hat{u} = 240\sqrt{2}$ V in the PLECS model and M-file. An example of the results, which should appear after running your files, is given in Fig. 2.17.

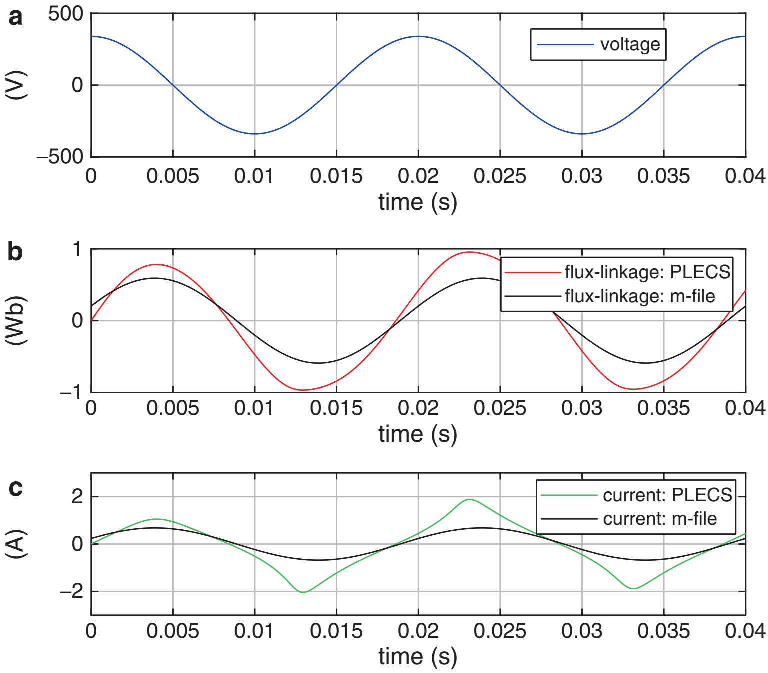


Fig. 2.17 PLECS/M-file results: induction simulation, with coil resistance, non-linear $i(\psi)$, and higher peak voltage

A comparison between the results obtained via the phasor analysis and PLECS simulation shows that the two are now decidedly different. The reason for the discrepancy is that the increased supply voltage level has increased the flux levels, which forces operation of the inductance into the non-linear regions of the flux-linkage/current curve. Note that the phasor analysis uses the same $L = 0.87\,\mathrm{H}$ inductance value. To prevent invalid conclusions, we must be aware that this *ac* phasor analysis tool is only usable for linear models.

2.6.5 Tutorial 5: PLECS Based Electro-magnetic Circuit Example

This tutorial makes use of the magnetic model introduced previously (see Sect. 1.8.1) which is to be connected to a 100 V, 50 Hz sinusoidal voltage source. The coil resistance R of the coil is assumed to be $500\,\Omega$. Build a PLECS based model, which shown the magnetic structure and symbolic (electrical) circuit. Add a scope module to show: applied voltage, current, flux linked with the coil, and the coil MMF. Use the geometry parameters as defined in Sect. 1.8.1. The PLECS model according to Fig. 2.18 is an implementation of said problem. Readily observable are the "electrical" ("black" connections) and magnetic ("red" connections) components together with the meters used to measure voltage, current, and MMF. Furthermore, a meter dPhi is present, which measures the circuit flux differential ${}^{d\phi}/_{dt}$, hence a integrator must used to generate the circuit flux ϕ. The flux-linkage $\psi = n\phi$ is found by adding a gain module after the integrator with gain 1000, which is the number of turns of the coil. the simulation results by way of three "SCOPE" submodules. The results displays on the Scope module show the required variables for a time interval of 40 ms (Fig. 2.19).

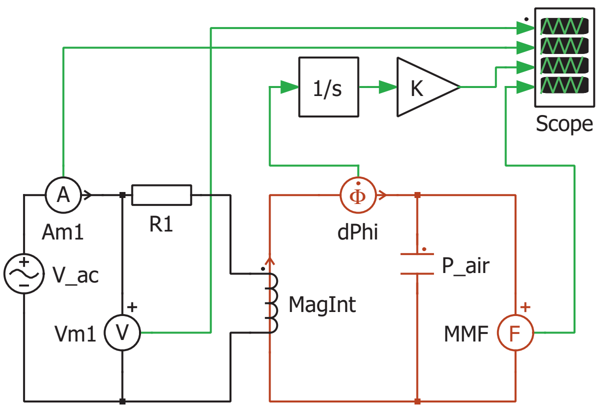


Fig. 2.18 PLECS simulation: electro-magnetic circuit example

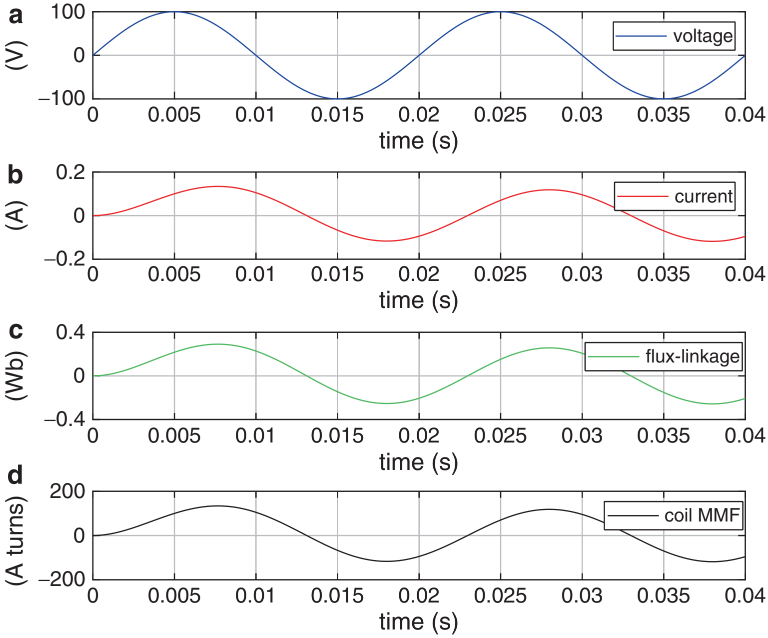


Fig. 2.19 Simulation results for electro-magnetic circuit example

It is instructive to briefly consider the results shown on the scope module:

- Current: this waveform lags the voltage waveform as expected because the coil has inductance and resistance.
- Flux linkage: this waveform is identical to the current waveform, but the magnitude is different. This is to be expected as the flux linkage is equal to $\psi = L i$, where L is the inductance which according to Sect. 1.8.1 was found to be 2.19 H.
- Coil MMF: this waveform is identical to the current waveform, but the magnitude is different. This is to be expected as the coil MMF is equal to MMF $= n\, i$, where n is the number of coil turns, set to 1000.

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Fundamentals of Electrical Drives
Veltman, A.; Pulle, D.W.J.; De Doncker, R.W.S.
2016, XXI, 341 p. 297 illus., 243 illus. in color.,
Hardcover
ISBN: 978-3-319-29408-7