

Chapter 2

Transmission Expansion Planning

A critical issue in the operation of electric energy systems is the capacity of the transmission lines that enable energy flows from generation nodes to demand nodes. In this chapter, we analyze the transmission expansion planning (TEP) problem, which allows a transmission planner to identify the optimal transmission reinforcements to be carried out with the aim of facilitating energy exchange among producers and consumers, e.g., by reducing generation or load-shedding costs. With this purpose, two models are described and analyzed: first, a deterministic model that solves the TEP problem based on a future demand forecast, and second, an adaptive robust optimization (ARO) model that takes into account the influence of different sources of uncertainty, such as future demand growth and the availability of generating units in the TEP problem. These two models are formulated using a static approach in which transmission expansion plans are made at a single point in time and for a future planning horizon.

2.1 Introduction

This chapter analyzes the transmission expansion planning (TEP) problem, which refers to the decision-making problem of determining the best transmission reinforcements to be made in an existing electric energy system. TEP is motivated, among other reasons, by the aging of the current infrastructure [20], expected demand growth, and the building of new renewable production facilities, usually located far away from demand centers. These issues make it essential to reinforce and to expand the existing transmission network in order to facilitate energy exchanges among producers and consumers as well as to guarantee supply–demand balance.

TEP is a critical issue in modern electric energy systems because transmission lines allow energy flows from generating to demand nodes. These demands must be supplied even in the worst situations, e.g., those corresponding to a peak load or the failure of a generating unit. Thus, transmission expansion plans should be decided in such a way that demands are efficiently supplied even if one of these situations

occurs. Note that the transmission expansion has twin objectives: on the one hand, facilitating economic trade, and on the other hand, engineering reliability.

The TEP problem is generally tackled under two different frameworks: centralized and competitive. In a centralized framework, an entity controls both generation and transmission facilities and is in charge of performing generation and transmission planning jointly. In a competitive environment, an independent and regulated entity is generally in charge of operating and expanding the transmission network with the aim of maximizing energy trade opportunities among producers and consumers. There is a third option that consists in considering the scope for a merchant investor that is profit-motivated in expanding the transmission system.

The first view, i.e., a central planner deciding both generation and transmission expansion, is analyzed in Chap. 4 of this book. Here, we adopt the second view, i.e., we consider that a single regulated entity decides the transmission expansion plans. The existence of this entity, known as the transmission system operator (TSO), is common in most European countries [14, 25]. In the US, TSOs have comparatively more limited attributes, and they are usually in charge of a specific region and therefore, are referred to as regional transmission organizations (RTOs) [19, 24].

The TSO (or RTO) decides the best way to reinforce and expand the existing transmission network with the goal of facilitating energy trade opportunities of the market players, e.g., by minimizing generation costs or by reducing load-shedding costs. Note that a joint economic and engineering objective is considered. By expanding the transmission network we reduce the generation and load-shedding costs (economic objective), but at the same time, we improve the reliability in the supply of demands (engineering objective).

The relevance of the TEP problem in electric energy systems has motivated significant research effort in this area over the past few decades. Pioneering work is due to Garver [12], who in 1970 proposed a linear programming problem that determines the transmission expansion plans based on the location of overloads. Since then, many relevant contributions have been published based on mathematical programming [1, 21, 23].

TEP is a complex decision-making problem since it generally involves a multiattribute objective, nonlinear constraints, and a nonconvex feasible region. As a result, different approaches have been proposed to deal with this complexity. These approaches are based on the application of decomposition techniques such as Benders decomposition [5, 18, 26, 27] or on the use of heuristics [5, 9, 22, 28].

An important observation is that the TEP problem is usually considered for a long-term planning horizon. When the TSO decides about the transmission expansion plan to be carried out, it should take into account the future demand growth, the availability of existing generating units, and the building of new generating facilities. This means that transmission expansion decisions are made within an uncertain environment, and such uncertainties must be properly represented in order to achieve informed expansion decisions. To do so, stochastic programming [11, 30] and robust optimization (RO) [7, 16, 29] have been used in a TEP context. On the one hand, stochastic programming is based on the generation of scenarios that describe the uncertain parameters [8]. However, this scenario generation usually requires knowing

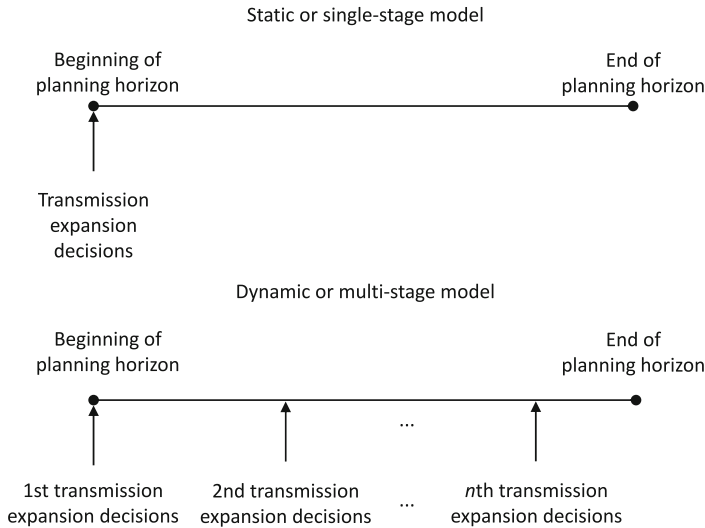


Fig. 2.1 Static and dynamic TEP models

the probability distribution functions of the uncertain parameters, which is generally a hard task. Moreover, a large enough number of scenarios must be generated to represent the uncertainty accurately, which increases the computational complexity of the problem. On the other hand, RO does not need scenarios to be generated but robust sets, which are generally simpler to obtain [2]. Additionally, RO models have a moderate size, which reduces the computational complexity compared with stochastic programming models. A general disadvantage of RO is that the results are usually too conservative. However, this is not a disadvantage for the TEP problem, in which a reliable supply of demands is required.

As previously mentioned, the TEP problem is generally considered for a long-term planning horizon, e.g., 30 years. In this sense, there are two expansion strategies. The first is to make the transmission expansion plans, i.e., to build the new transmission lines, at a single point in time (usually at the beginning of the planning horizon). The model that results from this strategy is known as a static or a single-stage model. The second one is to make the transmission expansion decisions at different points in time of the planning horizon. In this case, the model is known as a dynamic or a multistage model. This dynamic approach usually provides more accurate solutions since it allows the transmission planner to adapt to future changes in the system. However, it further increases the complexity of the TEP problem. Figure 2.1 illustrates the differences between these two expansion strategies. For the sake of simplicity, in this chapter we focus on a static approach. The use of dynamic models for making expansion decisions is described and analyzed in the following chapters of this book.

The remainder of this chapter is organized as follows. Section 2.2 provides a TEP-problem model considering a deterministic approach, in which transmission expansion decisions are made considering a future demand forecast. In this case,

the transmission expansion plan is determined so that the transmission network is capable of dealing with the worst expected demand realization in the future. The model in Sect. 2.2 is extended in Sect. 2.3 to consider the uncertainties faced by the TSO in carrying out the TEP-problem exercise. The model is formulated in this case using an adaptive robust optimization (ARO) approach. Both Sects. 2.2 and 2.3 include clarifying examples. Section 2.4 summarizes the chapter and discusses the main conclusions of the models and results reported in the chapter. Section 2.5 proposes some exercises to enable a deeper understanding of the models and concepts described in the chapter. Finally, Sect. 2.6 includes the GAMS code for one of the illustrative examples.

2.2 Deterministic Approach

The transmission infrastructure is a critical point in electric energy systems. Demands should be supplied even in the worst possible situations, e.g., during a peak demand period or during the failure of an important generating unit. Thus, transmission expansion plans should take into account such situations. For the sake of simplicity, we consider as the worst case a single-load scenario that corresponds to the maximum load demand expected in the planning horizon for which the TEP analysis is carried out. This assumption is usually made in the technical literature [1, 5, 18, 31].

In order to formulate the TEP problem, it is necessary to use binary variables to model whether a prospective transmission line is built. In systems of hundreds or thousands of nodes, there is a large number of transmission expansion options that need to be considered. This requires the use of a very large number of binary variables, which increases the complexity of the problem. To avoid formulating a very complex problem, we consider a static approach in which TEP decisions are made now for a future long-term planning horizon, e.g., 20 years. This assumption is typical in TEP problems because it allows us to formulate a comparatively easier problem to solve. However, note that under a deterministic assumption, it is also generally possible to formulate the TEP problem considering a dynamic approach.

The problem is formulated from the perspective of a TSO that aims at facilitating energy trade opportunities among producers and consumers. Therefore, the objective function of the TSO's problem is the minimization of generation and load-shedding costs. Since the TSO is responsible for building transmission lines, we also include their construction costs in the objective function.

The following sections provide the formulation of the TEP problem under a deterministic approach. This problem can be formulated as a mixed-integer nonlinear programming (MINLP) problem since it includes the products of continuous and binary variables. These kinds of problems are generally hard to solve, and the convergence of the MINLP problem to the optimum is not guaranteed [6]. However, it is possible to formulate an exact equivalent mixed-integer linear programming (MILP) problem, which allows us to solve the TEP problem by applying conventional branch-and-cut solvers [6, 15].

2.2.1 Notation

The main notation used in this chapter is provided below for quick reference. Other symbols are defined as needed throughout the chapter.

Indices

- d Demands.
- g Generating units.
- ℓ Transmission lines.
- n Nodes.
- ν Iterations.

Sets

- $r(\ell)$ Receiving-end node of transmission line ℓ .
- $s(\ell)$ Sending-end node of transmission line ℓ .
- Ω_n^D Demands located at node n .
- Ω_n^E Generating units located at node n .
- Ω^{L+} Prospective transmission lines.

Parameters

- B_ℓ Susceptance of transmission line ℓ [S].
- C_d^{LS} Load-shedding cost of demand d [\$/MWh].
- C_g^E Production cost of generating unit g [\$/MWh].
- F_ℓ^{\max} Capacity of transmission line ℓ [MW].
- \tilde{I}_ℓ^L Annualized investment cost of prospective transmission line ℓ [\$/MW].
- $\tilde{I}^{L, \max}$ Annualized investment budget for building prospective transmission lines [\$/].
- $P_d^{D, \max}$ Load of demand d [MW].
- $\underline{P}_d^{D, \max}$ Lower bound of the load of demand d [MW].
- $\overline{P}_d^{D, \max}$ Upper bound of the load of demand d [MW].
- $P_g^{E, \max}$ Production capacity of generating unit g [MW].
- $\overline{P}_g^{E, \max}$ Upper bound of the production capacity of generating unit g [MW].
- Γ^D Uncertainty budget for load demand.
- Γ^G Uncertainty budget for production capacity.

Binary Variables

- x_ℓ^L Binary variable that is equal to 1 if prospective transmission line ℓ is built and 0 otherwise.

Continuous Variables

- p_g^E Power produced by generating unit g [MW].
- p_ℓ^L Power flow through transmission line ℓ [MW].
- p_d^{LS} Load shed by demand d [MW].

- η Auxiliary variable to reconstruct objective function of the ARO problem gradually [\\$].
- θ_n Voltage angle at node n [rad].

2.2.2 MINLP Model Formulation

The deterministic TEP problem can be formulated as the following MINLP problem:

$$\min_{\Delta} \sum_{\ell \in \Omega^{L+}} \tilde{I}_{\ell}^L x_{\ell}^L + \sigma \left[\sum_g C_g^E p_g^E + \sum_d C_d^{LS} p_d^{LS} \right] \quad (2.1a)$$

subject to

$$\sum_{\ell \in \Omega^{L+}} \tilde{I}_{\ell}^L x_{\ell}^L \leq \tilde{I}^{L, \max} \quad (2.1b)$$

$$x_{\ell}^L = \{0, 1\} \quad \forall \ell \in \Omega^{L+} \quad (2.1c)$$

$$\sum_{g \in \Omega_n^E} p_g^E - \sum_{\ell | s(\ell)=n} p_{\ell}^L + \sum_{\ell | r(\ell)=n} p_{\ell}^L = \sum_{d \in \Omega_n^D} (P_d^{D^{\max}} - p_d^{LS}) \quad \forall n \quad (2.1d)$$

$$p_l^L = B_{\ell} (\theta_{s(\ell)} - \theta_{r(\ell)}) \quad \forall \ell \in \Omega^{L+} \quad (2.1e)$$

$$p_l^L = x_{\ell}^L B_{\ell} (\theta_{s(\ell)} - \theta_{r(\ell)}) \quad \forall \ell \in \Omega^{L+} \quad (2.1f)$$

$$-F_{\ell}^{\max} \leq p_{\ell}^L \leq F_{\ell}^{\max} \quad \forall \ell \quad (2.1g)$$

$$0 \leq p_g^E \leq P_g^{E^{\max}} \quad \forall g \quad (2.1h)$$

$$0 \leq p_d^{LS} \leq P_d^{D^{\max}} \quad \forall d \quad (2.1i)$$

$$-\pi \leq \theta_n \leq \pi \quad \forall n \quad (2.1j)$$

$$\theta_n = 0 \quad n: \text{ref.}, \quad (2.1k)$$

where variables in set $\Delta = \{x_{\ell}^L, p_{\ell}^L, p_g^E, p_d^{LS}, \theta_n\}$ are the optimization variables of problem (2.1).

The aim of the TSO is to facilitate energy trading and at the same time, to minimize the costs incurred in building new transmission lines. On the other hand, the TSO is constrained by the requirements for maintaining grid reliability.

Therefore, objective function (2.1a) comprises the three terms below:

1. $\sum_{\ell \in \Omega^{L+}} I_{\ell}^L x_{\ell}^L$ is the annualized cost of building new transmission lines.
2. $\sum_g C_g^E p_g^E$ is the operating cost of generating units.
3. $\sum_d C_d^{LS} p_d^{LS}$ is the load-shedding cost.

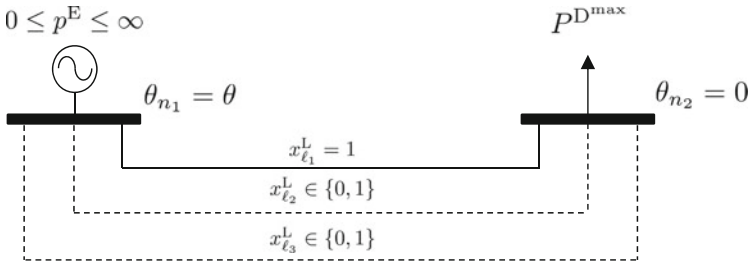


Fig. 2.2 Illustrative Example 2.1: two-node system (one stage)

The terms in 2 and 3 above are multiplied by the factor σ to make them comparable with investment costs. Since we consider that \tilde{I}_ℓ^L are annualized investment costs, then σ is equal to 8760 h, i.e., the total number of hours in a year.

Objective function (2.1a) is constrained by Eqs. (2.1b)–(2.1k). Constraint (2.1b) ensures that the cost of building new transmission lines is below the available budget when it exists. Constraints (2.1c) define binary variables x_ℓ^L that indicate whether a prospective line is built ($x_\ell^L = 1$) or not ($x_\ell^L = 0$). Constraints (2.1d) impose the generation–demand balance at each node of the system. Equations (2.1e) and (2.1f) define the power flows through existing and prospective transmission lines, respectively, which are limited by the corresponding capacity limits by constraints (2.1g). Note that subscripts $s(\ell)$ and $r(\ell)$ denote the sending-end and receiving-end nodes of transmission line ℓ , respectively. Constraints (2.1h) and (2.1i) impose bounds for the power produced by generating units and the unserved demand, respectively. Finally, constraints (2.1j) and (2.1k) impose bounds on voltage angles and fix to zero the voltage angle at the reference node, respectively.

The network is represented using a dc model without losses. This assumption is usually made in the technical literature and consists in considering that voltage magnitudes are approximately constant in the system and that voltage angle differences are small enough between two connected nodes [13]. This allows us to formulate the power-flow Eqs. (2.1e)–(2.1f) using linear expressions.

Illustrative Example 2.1 *Deterministic TEP: Static solution*

The deterministic TEP model (2.1) is applied to the two-node system depicted in Fig. 2.2. There is a generating unit located at node 1. Its production cost is C , and it has a very large capacity (for the sake of simplicity, we consider that its capacity is infinite). There is also a demand located at node 2 with a peak load equal to $P^{D^{max}}$ and a very large load-shedding cost (also considered infinite, i.e., load shedding is not possible).

Nodes 1 and 2 are connected through a transmission line ℓ_1 of capacity equal to F^{max} and susceptance equal to 1 p.u. It is possible to build two additional transmission lines (ℓ_2 and ℓ_3) between these nodes with the same characteristics as the existing one and an annualized investment cost equal to I with an unlimited budget. Node 2 is the reference node.

Considering the above data, model (2.1) results in the following optimization problem:

$$\min_{x_{\ell_2}^L, x_{\ell_3}^L, p^E, \theta} \quad Ix_{\ell_2}^L + Ix_{\ell_3}^L + \sigma Cp^E$$

subject to

$$\begin{aligned} x_{\ell_2}^L, x_{\ell_3}^L &= \{0, 1\} \\ p^E - p_{\ell_1}^L - p_{\ell_2}^L - p_{\ell_3}^L &= 0 \\ p_{\ell_1}^L + p_{\ell_2}^L + p_{\ell_3}^L &= P^{D^{\max}} \\ p_{\ell_1}^L &= \theta \\ p_{\ell_2}^L &= x_{\ell_2}^L \theta \\ p_{\ell_3}^L &= x_{\ell_3}^L \theta \\ -F^{\max} &\leq p_{\ell_1}^L \leq F^{\max} \\ -F^{\max} &\leq p_{\ell_2}^L \leq F^{\max} \\ -F^{\max} &\leq p_{\ell_3}^L \leq F^{\max} \\ 0 &\leq p^E \leq \infty \\ -\pi &\leq \theta \leq \pi. \end{aligned}$$

Note that if $0 \leq P^{D^{\max}} \leq F^{\max}$, the capacity of the existing transmission line allows the power to flow from the generating unit at node 1 to the demand at node 2. Therefore, we do not need to build any additional transmission lines. However, for values of $P^{D^{\max}} > F^{\max}$, the capacity of the existing transmission line ℓ_1 is not enough. If $F^{\max} < P^{D^{\max}} \leq 2F^{\max}$, then it is necessary to build one additional transmission line between nodes 1 and 2, while if $2F^{\max} < P^{D^{\max}} \leq 3F^{\max}$, then we need to build two additional transmission lines. Note that for $P^{D^{\max}} > 3F^{\max}$, the problem is infeasible, and additional expansion options must be considered.

That is, the optimal solution of the TEP problem for this illustrative example depends on the value of $P^{D^{\max}}$: we build zero, one, or two additional transmission lines depending on the value of the expected largest demand in the planning horizon. \square

Illustrative Example 2.2 *Deterministic TEP: Dynamic solution*

In Illustrative Example 2.1, we solve the TEP problem considering a static approach, in which transmission expansion plans are decided and made now for a future planning horizon. For illustration purposes, let us now consider that this planning horizon is divided into two time periods, t_1 and t_2 . The expected largest demands in these two time periods are different and equal to $P_{t_1}^{D\max}$ and $P_{t_2}^{D\max}$, respectively. Instead of considering a static approach, we consider that the transmission planner can build additional lines at the beginning of each of the two considered time periods, i.e., we consider a dynamic approach. Figure 2.3 illustrates the TEP problem in this case. Note that the remaining data are obtained from Illustrative Example 2.1.

Considering the above data, the TEP problem considering a dynamic approach results in the following optimization problem:

$$\begin{aligned} \min_{x_{\ell_2 t_1}^L, x_{\ell_3 t_1}^L, x_{\ell_2 t_2}^L, x_{\ell_3 t_2}^L, p_{t_1}^E, p_{t_2}^E, \theta_{t_1}, \theta_{t_2}} \quad & 2Ix_{\ell_2 t_1}^L + 2Ix_{\ell_3 t_1}^L + Ix_{\ell_2 t_2}^L + Ix_{\ell_3 t_2}^L \\ & + \sigma C (p_{t_1}^E + p_{t_2}^E) \end{aligned}$$

subject to

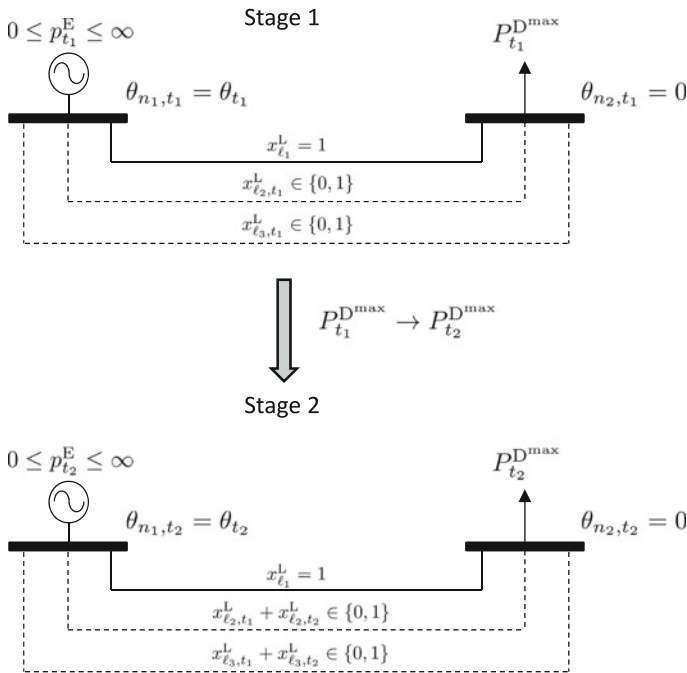


Fig. 2.3 Illustrative Example 2.2: two-node system (two stages)

$$\begin{aligned}
& x_{\ell_2 t_1}^L, x_{\ell_3 t_1}^L, x_{\ell_2 t_2}^L, x_{\ell_3 t_2}^L = \{0, 1\} \\
& x_{\ell_2 t_1}^L + x_{\ell_2 t_2}^L \leq 1 \\
& x_{\ell_3 t_1}^L + x_{\ell_3 t_2}^L \leq 1 \\
& p_{t_1}^E - p_{\ell_1 t_1}^L - p_{\ell_2 t_1}^L - p_{\ell_3 t_1}^L = 0 \\
& p_{\ell_1 t_1}^L + p_{\ell_2 t_1}^L + p_{\ell_3 t_1}^L = P_{t_1}^{D^{\max}} \\
& p_{\ell_1 t_1}^L = \theta_{t_1} \\
& p_{\ell_2 t_1}^L = x_{\ell_2 t_1}^L \theta_{t_1} \\
& p_{\ell_3 t_1}^L = x_{\ell_3 t_1}^L \theta \\
& -F^{\max} \leq p_{\ell_1 t_1}^L \leq F^{\max} \\
& -F^{\max} \leq p_{\ell_2 t_1}^L \leq F^{\max} \\
& -F^{\max} \leq p_{\ell_3 t_1}^L \leq F^{\max} \\
& 0 \leq p_{t_1}^E \leq \infty \\
& -\pi \leq \theta_{t_1} \leq \pi \\
& p_{t_2}^E - p_{\ell_1 t_2}^L - p_{\ell_2 t_2}^L - p_{\ell_3 t_2}^L = 0 \\
& p_{\ell_1 t_2}^L + p_{\ell_2 t_2}^L + p_{\ell_3 t_2}^L = P_{t_2}^{D^{\max}} \\
& p_{\ell_1 t_2}^L = \theta_{t_2} \\
& p_{\ell_2 t_2}^L = (x_{\ell_2 t_1}^L + x_{\ell_2 t_2}^L) \theta_{t_2} \\
& p_{\ell_3 t_2}^L = (x_{\ell_3 t_1}^L + x_{\ell_3 t_2}^L) \theta_{t_2} \\
& -F^{\max} \leq p_{\ell_1 t_2}^L \leq F^{\max} \\
& -F^{\max} \leq p_{\ell_2 t_2}^L \leq F^{\max} \\
& -F^{\max} \leq p_{\ell_3 t_2}^L \leq F^{\max} \\
& 0 \leq p_{t_2}^E \leq \infty \\
& -\pi \leq \theta_{t_2} \leq \pi.
\end{aligned}$$

The main differences between this model and that used in Illustrative Example 2.1 are as follows:

1. The subscripts t_1 and t_2 are used to denote the values of variables and parameters at time periods 1 and 2, respectively.
2. Prospective transmission lines built in the first time period remain built in the second time period.
3. Prospective transmission lines can be built only once.
4. The investment costs of transmission lines built in the first time period are twice the investment costs of transmission lines built in the second one since they are used (and thus amortized) twice.
5. Constraints are considered for both time periods.

As in Illustrative Example 2.1, the optimal transmission expansion plan depends on the value of $P_{t_1}^{D^{\max}}$ and $P_{t_2}^{D^{\max}}$:

1. In the first time period:
 - If $0 \leq P_{t_1}^{D^{\max}} \leq F^{\max}$, then we build no prospective transmission line.
 - If $F^{\max} < P_{t_1}^{D^{\max}} \leq 2F^{\max}$, then we build one additional transmission line.
 - If $2F^{\max} < P_{t_1}^{D^{\max}} \leq 3F^{\max}$, then we build two additional transmission lines.
2. In the second time period, the expansion plan depends on $P_{t_2}^{D^{\max}}$ and also on the expansion plan considered in the first time period. Assuming that $P_{t_2}^{D^{\max}} \geq P_{t_1}^{D^{\max}}$ (which is generally true since the demand growth in a system is usually positive), the transmission expansion plan in the second time period is as follows:
 - If no prospective transmission line is built in the first time period, then we build no additional transmission line if $0 \leq P_{t_2}^{D^{\max}} \leq F^{\max}$, we build one additional transmission line if $F^{\max} < P_{t_2}^{D^{\max}} \leq 2F^{\max}$, and we build two additional transmission lines if $2F^{\max} < P_{t_2}^{D^{\max}} \leq 3F^{\max}$.
 - If one prospective transmission line is built in the first time period, then we build no additional transmission line if $F^{\max} < P_{t_2}^{D^{\max}} \leq 2F^{\max}$, and we build one additional transmission line if $2F^{\max} < P_{t_2}^{D^{\max}} \leq 3F^{\max}$.
 - If two prospective transmission lines are built in the first time period, then no additional transmission line is built.
3. If $P_{t_1}^{D^{\max}} > 3F^{\max}$ or $P_{t_2}^{D^{\max}} > 3F^{\max}$, then the problem is infeasible, and additional expansion options should be considered.

Note that Illustrative Example 2.2 is comparatively more complex than Illustrative Example 2.1 since the number of binary variables and constraints is approximately twice. However, considering a dynamic approach as in Illustrative Example 2.2, we can adapt to future changes in the system, and therefore, its solution is usually better than that obtained considering a static approach. \square

Illustrative Examples 2.1 and 2.2 are very simple, and their solutions are trivial. However, as the number of nodes of the system under study and the number of prospective transmission lines increase, the TEP problem becomes complex, and its solution is no longer trivial. Note also that problem (2.1) includes binary variables, as well as products of binary and continuous variables in constraints (2.1f), i.e., problem (2.1) is a MINLP problem. These problems are usually hard to solve. Nevertheless, it is possible to remove the nonlinearities, as explained in the following section.

2.2.3 Linearization of Products of Binary and Continuous Variables

MINLP model (2.1) provided in the previous section is nonlinear, due to the products of binary (x_ℓ) and continuous (θ_n) variables in constraints (2.1f). However, it is possible to replace these nonlinear constraints by the following sets of exact equivalent mixed-integer linear constraints:

$$-x_\ell^L F_\ell^{\max} \leq p_\ell^L \leq x_\ell^L F_\ell^{\max} \quad \forall \ell \quad (2.2a)$$

$$-(1 - x_\ell^L) M \leq p_\ell^L - B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) \leq (1 - x_\ell^L) M \quad \forall \ell, \quad (2.2b)$$

where M is a large enough positive constant [5, 31].

The working of Eqs. (2.2) is explained below.

On the one hand, let us consider that prospective transmission line ℓ is built, i.e., binary variable x_ℓ^L is equal to 1. In such a case, Eqs. (2.2) impose that $-F_\ell^{\max} \leq p_\ell^L \leq F_\ell^{\max}$ and $p_\ell^L - B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) = 0$. Note that these equations are equivalent to constraints (2.1f) and (2.1g) when $x_\ell^L = 1$.

On the other hand, let us consider that prospective transmission line ℓ is not built, i.e., binary variable x_ℓ^L is equal to 0. In such a case, Eqs. (2.2) impose that $p_\ell^L = 0$ and $-M \leq p_\ell^L - B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) \leq M$. First, we impose that the power flow through this transmission line is null. Second, we consider large enough bounds on the difference between the voltage angles at two nodes that are not connected through the disjunctive parameter M . These equations are equivalent to constraints (2.1f) and (2.1g) when $x_\ell^L = 0$. The interested reader is referred to [5, 31], which provide a discussion on how to select the value of parameter M effectively.

2.2.4 MILP Model Formulation

Using the linearization procedure described in the previous section, it is possible to reformulate the TEP problem considering a deterministic static approach as in the MILP problem below:

$$\min_{\Delta} \quad \sum_{\ell \in \Omega^{L+}} \tilde{I}_\ell^L x_\ell^L + \sigma \left[\sum_g C_g^E p_g^E + \sum_d C_d^{LS} p_d^{LS} \right] \quad (2.3a)$$

subject to

$$\sum_{\ell \in \Omega^{L+}} \tilde{I}_\ell^L x_\ell^L \leq \tilde{I}^{L, \max} \quad (2.3b)$$

$$x_\ell^L = \{0, 1\} \quad \forall \ell \in \Omega^{L+} \quad (2.3c)$$

$$\sum_{g \in \Omega_n^G} p_g^G - \sum_{\ell | s(\ell)=n} p_\ell^L + \sum_{\ell | r(\ell)=n} p_\ell^L = \sum_{d \in \Omega_n^D} (P_d^{D^{\max}} - p_d^{LS}) \quad \forall n \quad (2.3d)$$

$$p_\ell^L = B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) \quad \forall \ell \setminus \ell \in \Omega^{L+} \quad (2.3e)$$

$$-F_\ell^{\max} \leq p_\ell^L \leq F_\ell^{\max} \quad \forall \ell \setminus \ell \in \Omega^{L+} \quad (2.3f)$$

$$-x_\ell^L F_\ell^{\max} \leq p_\ell^L \leq x_\ell^L F_\ell^{\max} \quad \forall \ell \in \Omega^{L+} \quad (2.3g)$$

$$-(1 - x_\ell^L) M \leq p_\ell^L - B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) \leq (1 - x_\ell^L) M \quad \forall \ell \in \Omega^{L+} \quad (2.3h)$$

$$0 \leq p_g^E \leq P_g^{G^{\max}} \quad \forall g \quad (2.3i)$$

$$0 \leq p_d^{LS} \leq P_d^{D^{\max}} \quad \forall d \quad (2.3j)$$

$$-\pi \leq \theta_n \leq \pi \quad \forall n \quad (2.3k)$$

$$\theta_n = 0 \quad n: \text{ref.} \quad (2.3l)$$

Illustrative Example 2.3 *Deterministic TEP: Six-node system*

The deterministic TEP model (2.3) is applied to the six-node system depicted in Fig. 2.4. This system comprises six nodes, five generating units, four demands, and three transmission lines. The system is divided in two zones: region A (nodes 1–3) and region B (nodes 4–6), which are initially not connected. Note also that node six is initially isolated, and thus the demand at this node can be supplied only by generating unit g_5 .

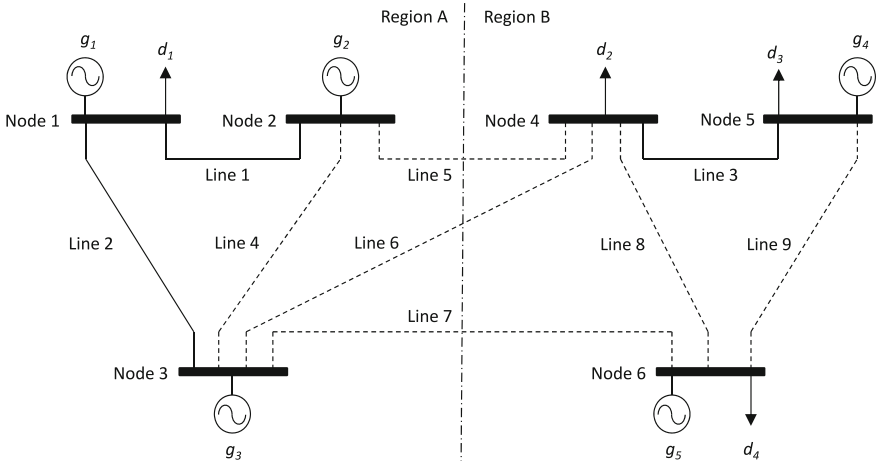


Fig. 2.4 Illustrative Example 2.3: six-node system

Table 2.1 Illustrative Example 2.3: data for generating units

Generating unit	Node	$P_g^{E^{\max}}$ [MW]	C_g^E [\$/MWh]
g_1	n_1	300	18
g_2	n_2	250	25
g_3	n_3	400	16
g_4	n_5	300	32
g_5	n_6	150	35

Table 2.2 Illustrative Example 2.3: data for demands

Demand	Node	$P_d^{D^{\max}}$ [MW]	C_d^{LS} [\$/MWh]
d_1	n_1	200	40
d_2	n_4	150	52
d_3	n_5	100	55
d_4	n_6	200	65

Table 2.3 Illustrative Example 2.3: data for existing transmission lines

Line	From node	To node	B_ℓ [S]	F_ℓ^{\max} [MW]
ℓ_1	n_1	n_2	500	150
ℓ_2	n_1	n_3	500	150
ℓ_3	n_4	n_5	500	150

Table 2.1 provides data for the generating units. The second column identifies the node allocation, while the third and fourth columns provide the capacity and production cost of each generating unit, respectively.

Table 2.2 provides data for the demands. The second column identifies the node allocation, while the third and fourth columns provide the maximum load demand and the load-shedding cost of each demand, respectively. Note that the optimal expansion plan is obtained based on a future demand forecast. Thus, these demands represent the worst realization of the demand at each node in the future, which corresponds to the largest expected demand at each node in the considered planning horizon.

Table 2.3 provides data for the existing transmission lines. The second and third columns identify the sending-end and receiving-end nodes, respectively, while the fourth and fifth columns provide the susceptance and capacity of each existing transmission line, respectively.

We consider that it is possible to build six different transmission lines, whose data are provided in Table 2.4. The second and third columns identify the sending-end and receiving-end nodes, respectively, while the fourth and fifth columns provide the susceptance and capacity of each prospective transmission line, respectively. The sixth column gives the annualized investment cost. The annualized investment budget is considered equal to \$3,000,000, which limits the number and type of prospective lines to be built.

Table 2.4 Illustrative Example 2.3: data for prospective transmission lines

Line	From node	To node	B_ℓ [S]	F_ℓ^{\max} [MW]	\tilde{I}_ℓ^L [\$]
ℓ_4	n_2	n_3	500	150	700,000
ℓ_5	n_2	n_4	500	200	1,400,000
ℓ_6	n_3	n_4	500	200	1,800,000
ℓ_7	n_3	n_6	500	200	1,600,000
ℓ_8	n_4	n_6	500	150	800,000
ℓ_9	n_5	n_6	500	150	700,000

Investment costs and investment budget are provided as annualized values. Therefore, factor σ is equal to 8760 (i.e., the number of hours in a year) to make the annualized costs and the load-shedding/generating costs comparable.

Finally, the reference node is node 1, the base power is 1 MW, and the base voltage is 1 kV.

The above data are used to solve the TEP problem (2.3). The optimal solution consists in building prospective lines ℓ_5 and ℓ_7 , i.e., lines connecting nodes 2–4 and 3–6, respectively. Note that the system is divided into two regions, A and B, which are originally independent. While most of the generation capacity is located in region A, most of the demand is located in region B. Moreover, the cheapest generating units are located in region A. This means that without building any prospective line, demands in region B are supplied by expensive generating units, and load-shedding occurs, resulting in load-shedding costs. Therefore, it is optimal to build prospective lines ℓ_5 and ℓ_7 , which connect regions A and B, and thus part of the demand in region B can be supplied by the cheap generating units in region A, which contributes to reducing generation and load-shedding costs. \square

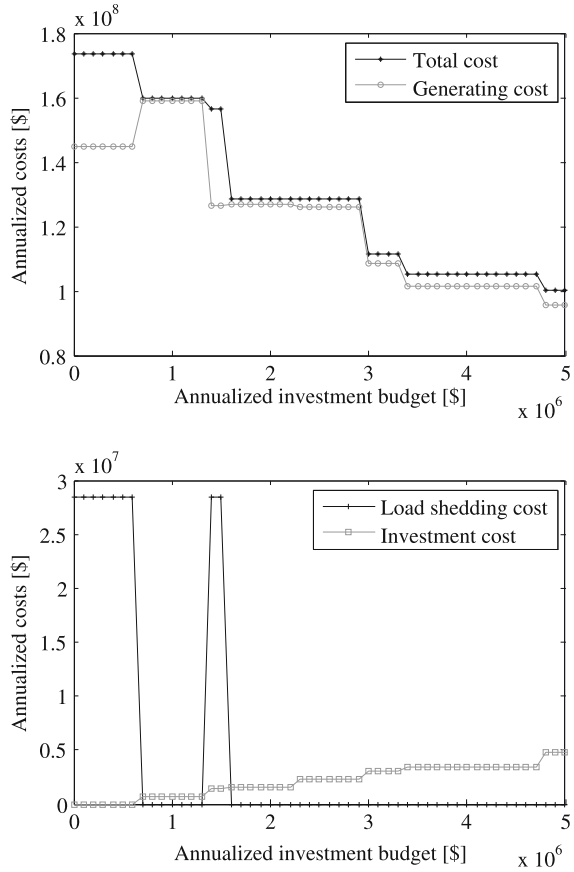
Illustrative Example 2.4 *Deterministic TEP: Impact of investment budget*

The expansion decisions obtained in Illustrative Example 2.3 are conditioned by the available investment budget. Note that building transmission lines ℓ_5 and ℓ_7 requires an annualized investment cost of \$3,000,000, i.e., the considered investment budget. A larger investment budget may allow the TSO, i.e., the planner, to reduce further the generation and load-shedding costs, as analyzed next.

The lower plot of Fig. 2.5 depicts the investment and load-shedding costs, while the upper plot of Fig. 2.5 depicts the generation and the total, i.e., the value of objective function (2.3a), costs for different values of the investment budget. On the other hand, the expansion decisions for different investment budgets are provided in Table 2.5.

For values of the annualized investment budget lower than \$700,000, it is impossible to build any transmission lines since the cheapest transmission line has an annualized cost of \$700,000. In such a case, demands in region B are supplied by expensive generating units, and/or load-shedding occurs with the corresponding load-shedding cost, as can be observed in the lower plot of Fig. 2.5. As the investment

Fig. 2.5 Illustrative Example 2.4: costs versus investment budget



budget increases, it becomes possible to build some of the prospective transmission lines. Moreover, building some of the prospective transmission lines is most appropriate since this contributes to reducing the total costs by reducing the generation and/or the load-shedding costs. Note that for some values of the investment budget, the cost of load-shedding increases, and the generation cost decreases, i.e., load-shedding substitutes generation, and vice versa. Note also that the cost of building new transmission lines is significantly lower than the generation costs. Finally, we observe that there is no incremental progression in the building of new transmission lines as the investment budget increases, i.e., the optimal solution of the TEP problem does not consist in building additional transmission lines as the investment budget increases, but in considering different transmission expansion plans. \square

Table 2.5 Illustrative Example 2.4: expansion decisions for different investment budgets

Annualized investment budget [M\$]	Transmission lines built ^a
[0, 0.7)	–
[0.7, 1.4)	ℓ_9
[1.4, 1.6)	ℓ_5
[1.6, 2.3)	ℓ_7
[2.3, 3.0)	ℓ_4 and ℓ_7
[3.0, 3.4)	ℓ_5 and ℓ_7
[3.4, 4.8)	ℓ_6 and ℓ_7
[4, ∞)	ℓ_5 , ℓ_6 and ℓ_7

^a ℓ_4 : 2–3, ℓ_5 : 2–4, ℓ_6 : 3–4, ℓ_7 : 3–6, ℓ_8 : 4–6, ℓ_9 : 5–6

Table 2.6 Illustrative Example 2.5: expansion decisions for different values of the total demand in the system

Total demand in the system [MW]	Transmission lines built ^a
100	ℓ_5 and ℓ_9
200	ℓ_7 and ℓ_9
More than 300	ℓ_5 and ℓ_7

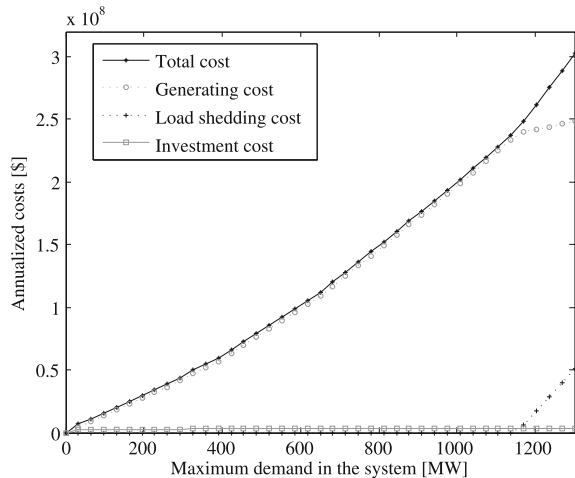
^a ℓ_4 : 2–3, ℓ_5 : 2–4, ℓ_6 : 3–4, ℓ_7 : 3–6, ℓ_8 : 4–6, ℓ_9 : 5–6

Illustrative Example 2.5 *Deterministic TEP: Impact of demand*

Illustrative Example 2.3 considers a deterministic approach in which expansion decisions are obtained for a given value of the demand. The problem is solved for the worst case, i.e., the largest expected demand in the considered planning horizon. However, expansion decisions are conditioned by this demand level, as can be observed in Table 2.6, which provides the transmission expansion decisions for different values of the demand in the system and a fixed annualized investment budget of \$3,000,000.

We observe that no matter what the demand in the system is, it is optimal to connect regions A and B using some of the prospective transmission lines. Moreover, initially isolated node 6 is always connected to the system since the generating unit and demand at this node are the most expensive (in terms of generation and load-shedding costs, respectively). However, note that there is no incremental progression in the expansion decisions.

Fig. 2.6 Illustrative Example 2.5: costs versus total demand



On the other hand, as the future demand forecast increases, expansion decisions change, as do the investment, load-shedding, and generation costs as well. These costs for different values of the total demand in the system are depicted in Fig. 2.6. □

2.3 Robust Approach

In the previous section, we described a deterministic model in which the TEP is parameterized based on the largest future demand forecast. We assume that we know this demand as well as the remaining data of the system, i.e., given all the information, it is possible to find the optimal transmission expansion plan that minimizes objective function (2.3a). However, the future is uncertain. It is difficult to forecast the maximum demand in the future if the planning horizon is long enough. Moreover, there are other sources of uncertainty that could contribute to the so-called *worst case*, e.g., availability of generating units, availability of transmission lines, or newly built generating units. Thus, it is necessary to consider an appropriate approach that takes into account the influence of these uncertainties on decision-making in the TEP problem. This is analyzed in this section.

As explained in the introductory chapter, there are generally two ways of handling this uncertainty. The first is to use stochastic programming, which requires generating scenarios of uncertain parameters whose probability distribution functions we need [8], which are generally not available. Additionally, stochastic programming usually leads to computationally complex problems. Therefore, we use in this chapter the second option, which is based on RO [2]. RO allows us to represent the uncertain parameters through robust sets, which are generally easier to obtain than probability distribution functions. Moreover, RO models are comparatively less complex than stochastic programming models.

2.3.1 Adaptive Robust Optimization Formulation

For the sake of simplicity, we assume that the uncertain parameters in the TEP problem (2.3) are (i) demand–load levels and (ii) available generating resources, i.e., we assume that uncertainty affects only parameters $P_d^{\text{D}^{\max}}$, $\forall d$, and $P_g^{\text{E}^{\max}}$, $\forall g$, respectively. Given this, we aim to determine the optimal transmission expansion plan, i.e., the optimal values of variables x_ℓ^{L} , $\forall \ell$, which optimizes objective function (2.3a), but anticipating the worst possible realizations of the uncertain parameters. To do so, we formulate an ARO problem, whose main characteristics are summarized below:

1. The optimal transmission expansion plan is sought by minimizing objective function (2.3a).
2. This optimal transmission expansion plan is sought by anticipating that once transmission expansion decisions are made, the worst uncertainty case will occur, i.e., assuming a given transmission expansion plan, uncertain parameters will take the values that maximize objective function (2.3a).
3. The worst-case realization of uncertain parameters is considered by anticipating that once this worst case is realized, the system adapts to it. That is, assuming that the transmission expansion decisions and uncertain parameters are fixed, we select the optimal values of the remaining variables (i.e., the operating decision variables) that minimize objective function (2.3a).

Note that the above decision sequence is consistent with reality. First, the transmission planner (the TSO) decides the transmission expansion plan to be implemented. Then, a *worst case* occurs, e.g., an unexpected peak demand in the system and/or the failure of some generating units. Finally, the system operator decides the best actions in order to minimize the operating costs.

The hierarchical structure described above can be represented using the three-level optimization problem below:

$$\begin{aligned}
 \min_{x_\ell^{\text{L}}} \quad & \sum_{\ell \in \Omega^{\text{L}+}} \tilde{I}_\ell^{\text{L}} x_\ell^{\text{L}} \\
 & + \max_{P_d^{\text{D}^{\max}}, P_g^{\text{E}^{\max}} \in \mathcal{Z}} \min_{\Delta \setminus x_\ell^{\text{L}} \in \Omega(x_\ell^{\text{L}}, P_d^{\text{D}^{\max}}, P_g^{\text{E}^{\max}})} \sigma \left[\sum_g C_g^{\text{E}} p_g^{\text{G}} + \sum_d C_d^{\text{LS}} p_d^{\text{LS}} \right]
 \end{aligned} \tag{2.4a}$$

subject to

$$\sum_{\ell \in \Omega^{\text{L}+}} I_\ell^{\text{L}} x_\ell^{\text{L}} \leq I^{\text{L}, \max} \tag{2.4b}$$

$$x_\ell^{\text{L}} \in \{0, 1\} \quad \forall \ell \in \Omega^{\text{L}+} \tag{2.4c}$$

In problem (2.4) we include set \mathcal{E} , which defines the uncertainty set and set $\Omega(x_\ell^L, P_d^{\text{Dmax}}, P_g^{\text{Emax}})$, which ensures the feasibility of the operating decision variables given the expansion decisions and the realizations of the uncertain parameters. Further details of these two sets are provided in the sections below.

2.3.2 Definition of Uncertainty Sets

In order to represent the uncertainty that appears in the TEP problem effectively, it is necessary to have an accurate definition of the uncertainty set \mathcal{E} . To do so, we consider a polyhedral uncertainty set, such as the one used in [2, 3, 29]. This uncertainty set is characterized by the following equations:

$$P_g^{\text{Emax}} \in [0, \bar{P}_g^{\text{Emax}}] \quad \forall g \quad (2.5a)$$

$$\frac{\sum_g (\bar{P}_g^{\text{Emax}} - P_g^{\text{Emax}})}{\sum_g \bar{P}_g^{\text{Emax}}} \leq \Gamma^G \quad (2.5b)$$

$$P_d^{\text{Dmax}} \in [\underline{P}_d^{\text{Dmax}}, \bar{P}_d^{\text{Dmax}}] \quad \forall d \quad (2.5c)$$

$$\frac{\sum_d (P_d^{\text{Dmax}} - \underline{P}_d^{\text{Dmax}})}{\sum_d (\bar{P}_d^{\text{Dmax}} - \underline{P}_d^{\text{Dmax}})} \leq \Gamma^D. \quad (2.5d)$$

Constraints (2.5a) and (2.5c) impose upper and lower bounds for P_g^{Emax} and P_d^{Dmax} , respectively. We consider that the lower bound of the available generating capacity is zero to represent the uncertainty in the availability of a generating unit or the uncertainty in building new generating units. On the other hand, constraints (2.5b) and (2.5d) limit the variability of uncertain variables P_g^{Emax} and P_d^{Dmax} , respectively, through the so-called uncertainty budgets Γ^G and Γ^D , as explained below.

Uncertainty budget Γ^G can take values between 0 and 1. If Γ^G is chosen equal to 0, then $P_g^{\text{Emax}} = \bar{P}_g^{\text{Emax}}$, $\forall g$, i.e., uncertainty in the available capacity of generating units is not considered. On the other hand, if Γ^G is chosen equal to 1, then P_g^{Emax} , $\forall g$, can take any value within the interval $[0, \bar{P}_g^{\text{Emax}}]$. This can be seen as the case of maximum uncertainty. Similarly, Γ^D can also take values between 0 and 1. If Γ^D is chosen equal to 0, then $P_d^{\text{Dmax}} = \underline{P}_d^{\text{Dmax}}$, $\forall d$, i.e., uncertainty in demand is not considered. However, if Γ^D is chosen equal to 1, then P_d^{Dmax} , $\forall d$, can take any value within the interval $[\underline{P}_d^{\text{Dmax}}, \bar{P}_d^{\text{Dmax}}]$. By choosing different values of Γ^G and Γ^D , we can analyze the impact of different levels of uncertainty on transmission expansion decisions.

2.3.3 Feasibility of Operating Decision Variables

Given the expansion decision variables, x_ℓ^L , and the realizations of the uncertain parameters, $P_d^{\text{D}^{\max}}$ and $P_g^{\text{E}^{\max}}$, we define set $\Omega(x_\ell^L, P_d^{\text{D}^{\max}}, P_g^{\text{E}^{\max}})$ to ensure the feasibility of the operating decision variables $\Delta \setminus x_\ell^L$ as follows:

$$\begin{aligned} \Omega(x_\ell^L, P_d^{\text{D}^{\max}}, P_g^{\text{E}^{\max}}) = \{ \Delta \setminus x_\ell^L : \\ \sum_{g \in \Omega_n^{\text{E}}} p_g^{\text{E}} - \sum_{\ell | s(\ell)=n} p_\ell^L + \sum_{\ell | r(\ell)=n} p_\ell^L \\ = \sum_{d \in \Omega_n^{\text{D}}} (P_d^{\text{D}^{\max}} - p_d^{\text{LS}}) : \lambda_n \quad \forall n \end{aligned} \quad (2.6a)$$

$$p_\ell^L = B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) : \phi_\ell^L \quad \forall \ell \setminus \ell \in \Omega^{\text{L}+} \quad (2.6b)$$

$$p_\ell^L = x_\ell^L B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) : \phi_\ell^{\text{L}+} \quad \forall \ell \in \Omega^{\text{L}+} \quad (2.6c)$$

$$-F_\ell^{\max} \leq p_\ell^L \leq F_\ell^{\max} : \phi_\ell^{\text{L},\min}, \phi_\ell^{\text{L},\max} \quad \forall \ell \quad (2.6d)$$

$$0 \leq p_g^{\text{E}} \leq P_g^{\text{E}^{\max}} : \phi_g^{\text{E},\min}, \phi_g^{\text{E},\max} \quad \forall g \quad (2.6e)$$

$$0 \leq p_d^{\text{LS}} \leq P_d^{\text{D}^{\max}} : \phi_d^{\text{D},\min}, \phi_d^{\text{D},\max} \quad \forall d \quad (2.6f)$$

$$-\pi \leq \theta_n \leq \pi : \phi_n^{\text{N},\min}, \phi_n^{\text{N},\max} \quad \forall n \quad (2.6g)$$

$$\theta_n = 0 : \chi^{\text{ref}} \quad n: \text{ref.} \} . \quad (2.6h)$$

Note that the dual variable associated to each constraint is provided following a colon.

Equations (2.6a) ensures the generation–demand balance at each node of the system. Equations (2.6b) and (2.6c) define the power flows through existing and prospective transmission lines, respectively, which are bounded by the corresponding capacity limits by Eqs. (2.6d). Equations (2.6e) and (2.6f) impose bounds on the power of generating units and on the load-shedding, respectively. Finally, Eqs. (2.6g) and (2.6h) define bounds on voltage angles and fix to zero the voltage angle at the reference node, respectively.

2.3.4 Detailed Formulation

Given the definitions of the uncertainty sets and the operating feasibility region provided in the previous sections, the TEP problem considering an ARO approach can be formulated using the following model:

$$\begin{aligned}
\min_{x_\ell^L} \quad & \max_{p_g^{E^{\max}}, p_d^{D^{\max}}} \quad \min_{p_g^E, p_d^D, p_\ell^L, \theta_n} \quad \sum_{\ell \in \Omega^{L+}} \tilde{I}_\ell^L x_\ell^L \\
& + \sigma \left[\sum_g C_g^E p_g^E + \sum_d C_d^{LS} p_d^{LS} \right]
\end{aligned} \tag{2.7a}$$

subject to

$$\sum_{g \in \Omega_n^E} p_g^E - \sum_{\ell | s(\ell)=n} p_\ell^L + \sum_{\ell | r(\ell)=n} p_\ell^L = \sum_{d \in \Omega_n^D} (p_d^{D^{\max}} - p_d^{LS}) \quad \forall n \tag{2.7b}$$

$$p_\ell^L = B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) \quad \forall \ell \in \Omega^{L+} \tag{2.7c}$$

$$p_\ell^L = x_\ell^L B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) \quad \forall \ell \in \Omega^{L+} \tag{2.7d}$$

$$-F_\ell^{\max} \leq p_\ell^L \leq F_\ell^{\max} \quad \forall \ell \tag{2.7e}$$

$$0 \leq p_g^E \leq P_g^{E^{\max}} \quad \forall g \tag{2.7f}$$

$$0 \leq p_d^{LS} \leq P_d^{D^{\max}} \quad \forall d \tag{2.7g}$$

$$-\pi \leq \theta_n \leq \pi \quad \forall n \tag{2.7h}$$

$$\theta_n = 0 \quad n: \text{ref.} \tag{2.7i}$$

subject to

$$P_g^{E^{\max}} \in [0, \bar{P}_g^{E^{\max}}] \quad \forall g \tag{2.7j}$$

$$\frac{\sum_g (\bar{P}_g^{E^{\max}} - P_g^{E^{\max}})}{\sum_g \bar{P}_g^{E^{\max}}} \leq \Gamma^G \tag{2.7k}$$

$$P_d^{D^{\max}} \in [\underline{P}_d^{D^{\max}}, \bar{P}_d^{D^{\max}}] \quad \forall d \tag{2.7l}$$

$$\frac{\sum_d (P_d^{D^{\max}} - \underline{P}_d^{D^{\max}})}{\sum_d (\bar{P}_d^{D^{\max}} - \underline{P}_d^{D^{\max}})} \leq \Gamma^D \tag{2.7m}$$

subject to

$$\sum_{\ell \in \Omega^{L+}} I_\ell^L x_\ell^L \leq I^{L, \max} \tag{2.7n}$$

$$x_\ell^L = \{0, 1\} \quad \forall \ell \in \Omega^{L+}. \tag{2.7o}$$

Note that the above problem (2.7) has a three-level structure whose three optimization problems include as optimization variables the expansion decision variables x_ℓ^L , the worst realizations of uncertain parameters $P_g^{E^{\max}}, P_d^{D^{\max}}$, and the operating decision variables $p_g^E, p_d^D, p_\ell^L, \theta_n$, respectively. Constraints (2.7b)–(2.7i) define the feasibility of the operating decision variables. Constraints (2.7j)–(2.7m) define the uncertainty sets. Finally, constraints (2.7n) and (2.7o) define the investment budget and the binary variables that represent which transmission lines are built, respectively.

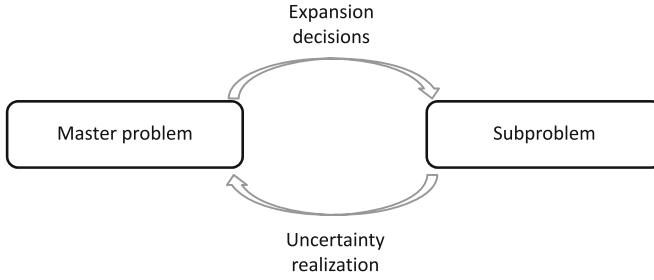


Fig. 2.7 Adaptive robust optimization approach: interactions between master problem and subproblem

2.3.5 Solution Procedure

The ARO (2.7) is difficult to solve since its multilevel structure renders an NP-hard problem. In order to solve this kind of problem, several algorithms are available in the technical literature based on extended versions of Benders decomposition [4, 16, 17] and on constraint-and-column generation methods [17, 29, 32]. In Benders-based methods, dual information from the so-called subproblem is used to build the objective function of the so-called master problem gradually. On the other hand, the constraint-and-column generation methods use cutting-plane strategies based solely on primal cuts that involve only primal decision variables. These methods generally perform computationally better than Benders' methods.

The three-level optimization problem (2.7) is decomposed into a master problem and a subproblem that exchange information on primal decision variables and that are iteratively solved until convergence to an optimal solution is achieved. Figure 2.7 schematically represents the interactions between these two problems.

The sections below provide detailed formulations of the master problem and the subproblem.

2.3.5.1 Master Problem

Considering a Benders framework, the master problem associated with problem (2.7) is formulated below:

$$\min_{x_\ell^L, p_{g,v'}^G, p_{d,v'}^D, p_{\ell,v'}^L, \theta_{n,v'}, \eta} \sum_{\ell \in \Omega^{L+}} \tilde{I}_\ell^L x_\ell^L + \eta \quad (2.8a)$$

subject to

$$\sum_{\ell \in \Omega^{L+}} I_\ell^L x_\ell^L \leq I^{L,\max} \quad (2.8b)$$

$$x_\ell^L = \{0, 1\} \quad \forall \ell \in \Omega^{L+} \quad (2.8c)$$

$$\sum_{g \in \Omega_n^E} p_{g,v'}^E - \sum_{\ell | s(\ell)=n} p_{\ell,v'}^L + \sum_{\ell | r(\ell)=n} p_{\ell,v'}^L = \sum_{d \in \Omega_n^D} (P_{d,v'}^{D^{\max,*}} - p_{d,v'}^{LS}) \quad \forall n, \forall v' \leq v \quad (2.8d)$$

$$p_{\ell,v'}^L = B_\ell (\theta_{s(\ell),v'} - \theta_{r(\ell),v'}) \quad \forall \ell \setminus \ell \in \Omega^{L+}, \forall v' \leq v \quad (2.8e)$$

$$p_{\ell,v'}^L = x_\ell^L B_\ell (\theta_{s(\ell),v'} - \theta_{r(\ell),v'}) \quad \forall \ell \in \Omega^{L+}, \forall v' \leq v \quad (2.8f)$$

$$-F_\ell^{\max} \leq p_{\ell,v'}^L \leq F_\ell^{\max} \quad \forall \ell, \forall v' \leq v \quad (2.8g)$$

$$0 \leq p_{g,v'}^E \leq P_{g,v'}^{E^{\max,*}} \quad \forall g, \forall v' \leq v \quad (2.8h)$$

$$0 \leq p_{d,v'}^{LS} \leq P_{d,v'}^{D^{\max,*}} \quad \forall d, \forall v' \leq v \quad (2.8i)$$

$$-\pi \leq \theta_{n,v'} \leq \pi \quad \forall n, \forall v' \leq v \quad (2.8j)$$

$$\theta_{n,v'} = 0 \quad n: \text{ref.} \quad \forall v' \leq v \quad (2.8k)$$

$$\eta \geq \sigma \left[\sum_g C_g^G p_{g,v'}^G + \sum_d C_d^{LS} p_{d,v'}^{LS} \right] \quad \forall v' \leq v, \quad (2.8l)$$

where v is the iteration index and $v' = 1, \dots, v$.

The optimization variables of this master problem are the expansion decision variables x_ℓ^L , the operating decision variables $p_{g,v'}^E, p_{d,v'}^D, p_{\ell,v'}^L, \theta_{n,v'}$ (one per iteration of the algorithm), and auxiliary variable η , which is used to reconstruct objective function (2.7a) gradually. Uncertain parameters $P_g^{E^{\max,*}}$ and $P_d^{D^{\max,*}}$ are fixed to their optimal values obtained from the subproblem solution at each iteration and used as input data of the master problem.

The size of master problem (2.8) increases with the iteration counter v since a new set of constraints (2.8d)–(2.8l) is incorporated at each iteration of the algorithm. Note that if $v = 0$, then constraints (2.8d)–(2.8l) are not included in the master problem.

Master problem (2.8) is a MINLP problem since it includes binary variables x_ℓ^L and nonlinearities in constraints (2.8f). However, these nonlinear constraints can be replaced by equivalent mixed-integer linear Eqs. (2.2), as explained in Sect. 2.2.3. Therefore, the master problem is recast as an MILP problem that can be solved using conventional branch-and-cut solvers [6, 15].

2.3.5.2 Subproblem

The subproblem associated with (2.7) is given below:

$$\max_{P_d^{D^{\max}}, P_g^{E^{\max}} \in \Xi} \min_{\Delta \setminus x_\ell^L \in \Omega} (x_\ell^{L,*}, P_d^{D^{\max}}, P_g^{E^{\max}}) \quad \sigma \left[\sum_g C_g^E p_g^E + \sum_d C_d^{LS} p_d^{LS} \right], \quad (2.9a)$$

where the expansion decisions are considered to be fixed to $x_\ell^{L,*}$. These expansion decisions are obtained from the solution of master problem (2.8) at each iteration of the algorithm.

Subproblem (2.9) is a bilevel problem that can be converted into an equivalent single-level problem as explained below. The lower-level problem in (2.9) is continuous and linear (and thus convex) in its decision variables $\Delta \setminus x_\ell^L \in \Omega \left(x_\ell^{L,*}, P_d^{\text{Dmax}}, P_g^{\text{Emax}} \right)$. Therefore, it can be replaced by its Karush–Kuhn–Tucker (KKT) conditions, which are necessary and sufficient conditions for optimality. These KKT conditions are included as constraints of the upper-level problem, rendering a single-level problem as follows:

$$\max_{\Delta_{\text{SUB}}} \quad \sigma \left[\sum_g C_g^{\text{E}} p_g^{\text{E}} + \sum_d C_d^{\text{LS}} p_d^{\text{LS}} \right] \quad (2.10\text{aa})$$

subject to

$$P_g^{\text{Emax}} \in \left[0, \overline{P}_g^{\text{Emax}} \right] \quad \forall g \quad (2.10\text{ab})$$

$$\frac{\sum_g \left(\overline{P}_g^{\text{Emax}} - P_g^{\text{Emax}} \right)}{\sum_g \overline{P}_g^{\text{Emax}}} \leq \Gamma^{\text{G}} \quad (2.10\text{ac})$$

$$P_d^{\text{Dmax}} \in \left[\underline{P}_d^{\text{Dmax}}, \overline{P}_d^{\text{Dmax}} \right] \quad \forall d \quad (2.10\text{ad})$$

$$\frac{\sum_d \left(\underline{P}_d^{\text{Dmax}} - P_d^{\text{Dmax}} \right)}{\sum_d \overline{P}_d^{\text{Dmax}} - \underline{P}_d^{\text{Dmax}}} \leq \Gamma^{\text{D}} \quad (2.10\text{ae})$$

$$\sum_{g \in \Omega_n^{\text{E}}} p_g^{\text{E}} - \sum_{\ell | s(\ell)=n} p_\ell^{\text{L}} + \sum_{\ell | r(\ell)=n} p_\ell^{\text{L}} = \sum_{d \in \Omega_n^{\text{D}}} (P_d^{\text{Dmax}} - p_d^{\text{LS}}) \quad \forall n \quad (2.10\text{af})$$

$$p_\ell^{\text{L}} = B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) \quad \forall \ell \setminus \ell \in \Omega^{\text{L}+} \quad (2.10\text{ag})$$

$$p_\ell^{\text{L}} = x_\ell^{L,*} B_\ell (\theta_{s(\ell)} - \theta_{r(\ell)}) \quad \forall \ell \in \Omega^{\text{L}+} \quad (2.10\text{ah})$$

$$-F_\ell^{\text{max}} \leq p_\ell^{\text{L}} \leq F_\ell^{\text{max}} \quad \forall \ell \quad (2.10\text{ai})$$

$$0 \leq p_g^{\text{G}} \leq P_g^{\text{Emax}} \quad \forall g \quad (2.10\text{aj})$$

$$0 \leq p_d^{\text{LS}} \leq P_d^{\text{Dmax}} \quad \forall d \quad (2.10\text{ak})$$

$$-\pi \leq \theta_n \leq \pi \quad \forall n \quad (2.10\text{al})$$

$$\theta_n = 0 \quad n: \text{ref.} \quad (2.10\text{am})$$

$$\sigma C_g^{\text{G}} - \lambda_{n(g)} + \phi_g^{\text{E,max}} - \phi_g^{\text{E,min}} = 0 \quad \forall g \quad (2.10\text{an})$$

$$\sigma C_d^{\text{D}} - \lambda_{n(d)} + \phi_d^{\text{D,max}} - \phi_d^{\text{D,min}} = 0 \quad \forall d \quad (2.10\text{ao})$$

$$\lambda_{s(\ell)} - \lambda_{r(\ell)} - \phi_\ell^{\text{L}} + \phi_\ell^{\text{L,max}} - \phi_\ell^{\text{L,min}} = 0 \quad \forall \ell \setminus \ell \in \Omega^{\text{L}+} \quad (2.10\text{ap})$$

$$\lambda_{s(\ell)} - \lambda_{r(\ell)} - \phi_\ell^{\text{L}+} + \phi_\ell^{\text{L,max}} - \phi_\ell^{\text{L,min}} = 0 \quad \forall \ell \in \Omega^{\text{L}+} \quad (2.10\text{aq})$$

$$\begin{aligned} & \sum_{\ell \setminus \ell \in \Omega^{\text{L}+} | s(\ell)=n} B_\ell \phi_\ell^{\text{L}} + \sum_{\ell \in \Omega^{\text{L}+} | s(\ell)=n} x_\ell^{L,*} B_\ell \phi_\ell^{\text{L}+} - \sum_{\ell \setminus \ell \in \Omega^{\text{L}+} | r(\ell)=n} B_\ell \phi_\ell^{\text{L}} \\ & - \sum_{\ell \in \Omega^{\text{L}+} | r(\ell)=n} x_\ell^{L,*} B_\ell \phi_\ell^{\text{L}+} + \phi_n^{\text{N,max}} - \phi_n^{\text{N,min}} = 0 \quad \forall n \setminus n: \text{ref.} \end{aligned} \quad (2.10\text{ar})$$

$$\begin{aligned} & \sum_{\ell \in \Omega^{L+} | s(\ell)=n} B_\ell \phi_\ell^L + \sum_{\ell \in \Omega^{L+} | s(\ell)=n} x_\ell^{L,*} B_\ell \phi_\ell^{L+} - \sum_{\ell \in \Omega^{L+} | r(\ell)=n} B_\ell \phi_\ell^L \\ & - \sum_{\ell \in \Omega^{L+} | r(\ell)=n} x_\ell^{L,*} B_\ell \phi_\ell^{L+} + \phi_n^{N,\max} - \phi_n^{N,\min} - \chi^{\text{ref}} = 0 \quad n: \text{ref.} \end{aligned} \quad (2.10\text{as})$$

$$0 \leq \phi_\ell^{L,\max} \perp F_\ell^{\max} - p_\ell^L \geq 0 \quad \forall \ell \quad (2.10\text{at})$$

$$0 \leq \phi_\ell^{L,\min} \perp p_\ell^L + F_\ell^{\max} \geq 0 \quad \forall \ell \quad (2.10\text{au})$$

$$0 \leq \phi_g^{E,\max} \perp P_g^{E,\max} - p_g^E \geq 0 \quad \forall g \quad (2.10\text{av})$$

$$0 \leq \phi_g^{E,\min} \perp p_g^E \geq 0 \quad \forall g \quad (2.10\text{aw})$$

$$0 \leq \phi_d^{D,\max} \perp P_d^{D,\max} - p_d^{\text{LS}} \geq 0 \quad \forall d \quad (2.10\text{ax})$$

$$0 \leq \phi_d^{D,\min} \perp p_d^{\text{LS}} \geq 0 \quad \forall d \quad (2.10\text{ay})$$

$$0 \leq \phi_n^{N,\max} \perp \pi - \theta_n \geq 0 \quad \forall n \quad (2.10\text{az})$$

$$0 \leq \phi_n^{N,\min} \perp \theta_n + \pi \geq 0 \quad \forall n, \quad (2.10\text{ba})$$

where variables in set $\Delta^{\text{SUB}} = \{P_d^{\text{D,max}}, P_g^{\text{E,max}}, p_g^{\text{E}}, p_d^{\text{D}}, p_\ell^{\text{L}}, \theta_n, \lambda_n, \phi_\ell^{\text{L}}, \phi_\ell^{\text{L,+}}, \phi_\ell^{\text{L,max}}, \phi_\ell^{\text{L,min}}, \phi_g^{\text{E,max}}, \phi_g^{\text{E,min}}, \phi_d^{\text{D,max}}, \phi_d^{\text{D,min}}, \phi_n^{\text{N,max}}, \phi_n^{\text{N,min}}, \chi^{\text{ref}}\}$ are the optimization variables of subproblem (2.10). These optimization variables are the worst realizations of the uncertain parameters $P_d^{\text{D,max}}$ and $P_g^{\text{E,max}}$, the operating decision variables $p_g^{\text{E}}, p_d^{\text{D}}, p_\ell^{\text{L}}$, and θ_n , as well as the dual variables $\lambda_n, \phi_\ell^{\text{L}}, \phi_\ell^{\text{L,max}}, \phi_\ell^{\text{L,min}}, \phi_g^{\text{E,max}}, \phi_g^{\text{E,min}}, \phi_d^{\text{D,max}}, \phi_d^{\text{D,min}}, \phi_n^{\text{N,max}}, \phi_n^{\text{N,min}}$, and χ^{ref} . Note that expansion decision variables $x_\ell^{L,*}$ are considered to be given parameters of subproblem (2.10) with values fixed to their optimal values obtained from the solution of the master problem (2.8) at the corresponding iteration.

Constraints (2.10ab)–(2.10ae) represent the uncertainty sets, constraints (2.10af)–(2.10am) are the primal constraints of the lower-level problem in (2.9), constraints (2.10an)–(2.10as) result from differentiating the Lagrangian of the lower-level problem in (2.9) with respect to lower-level variables, and constraints (2.10at)–(2.10ba) are the complementarity conditions.

Note that dual variables λ_n in constraints (2.10an)–(2.10ap) include different subscripts, namely $n(g), n(d), s(\ell), r(\ell)$, which indicate the node in which generating unit g is located, the node in which demand d is located, the sending-end node of transmission line ℓ , and the receiving-end node of transmission line ℓ , respectively.

Subproblem (2.10) is nonlinear, due to the complementarity constraints (2.10at)–(2.10ba). Complementarity constraints have the form $0 \leq a \perp b \geq 0$, which is equivalent to nonlinear constraints $a \geq 0, b \geq 0$, and $ab = 0$. However, these

constraints can be replaced by the following exact equivalent mixed-integer linear expressions, as explained in [10]:

$$a \geq 0 \quad (2.11a)$$

$$b \geq 0 \quad (2.11b)$$

$$a \leq Mu \quad (2.11c)$$

$$b \leq M(1 - u), \quad (2.11d)$$

where u is an auxiliary binary variable and M is a large enough positive constant.

The working of Eqs. (2.11) is explained below. The complementarity constraints impose that either a or b must be equal to zero. On one hand, if binary variable u is equal to 0, then $a = 0$ by Eqs. (2.11a) and (2.11c), and $0 \leq b \leq M$ by Eqs. (2.11b) and (2.11d). On the other hand, if binary variable u is equal to 1, then $b = 0$ by Eqs. (2.11b) and (2.11d) and $0 \leq a \leq M$ by Eqs. (2.11a) and (2.11c). Using Eqs. (2.11), we impose that either a or b is equal to zero and that the other one can take any value in a large enough interval defined by the disjunctive parameter M . Reference [10] discusses how to select the values of this parameter.

2.3.5.3 Algorithm

The master problem and the subproblem defined in the previous sections are solved iteratively. The optimal solution of the master problem at each iteration is used to solve the subproblem and vice versa. The iterative algorithm continues until convergence is attained. The detailed steps of this iterative algorithm are provided below:

- Step 1** Set lower (LB) and upper (UB) bounds to $-\infty$ and ∞ , respectively.
- Step 2** Set the iteration counter to $v = 0$.
- Step 3** Solve master problem (2.8). Obtain the optimal solution of variables $x_\ell^{L,*}$, $p_{g,v'}^{E,*}$, $p_{d,v'}^{D,*}$, $p_{\ell,v'}^{L,*}$, $\theta_{n,v'}^*$, and η^* . Note that if $v = 0$, then constraints (2.8d)–(2.8i) are not considered in the master problem.
- Step 4** Update the lower bound using Eq. (2.12) below:

$$LB = \sum_{\ell \in \Omega^{L+}} \tilde{I}_\ell^L x_\ell^{L,*} + \eta^*. \quad (2.12)$$

Note that the master problem is a relaxed version of the original problem in which variable η is used to reconstruct the original problem progressively at each iteration. Therefore, the value of the lower bound increases with the iteration counter as the master problem approximates the original problem.

- Step 5** Solve subproblem (2.10) by considering the optimal values of variables $x_\ell^{L,*}$ obtained in **Step 3** to be given parameters. Obtain the optimal solution of variables in the set $\Delta^{\text{SUB},*}$.
- Step 6** Update the upper bound using Eq. (2.13) below:

$$UB = \min \left\{ UB, \sum_{\ell \in \Omega^{L+}} \tilde{I}_{\ell}^L x_{\ell}^{L,*} + \sigma \left[\sum_g C_g^G p_g^{G,*} + \sum_d C_d^{LS} p_d^{LS,*} \right] \right\}. \quad (2.13)$$

Note that the subproblem is a more constrained version of the original problem since variables x_{ℓ}^L are fixed to given values. Thus, the upper bound decreases with the iteration counter as variables x_{ℓ}^L change and approximate their optimal values.

- Step 7** If $UB - LB$ is lower than a predefined tolerance ϵ , the algorithm terminates. The optimal solution is $x_{\ell}^{L,*}$. If not, then continue with the following step.
- Step 8** Update the iteration counter, $\nu \leftarrow \nu + 1$, and set $P_{d,\nu}^{D^{max,*}} = P_d^{D^{max,*}}$ and $P_{g,\nu}^{E^{max,*}} = P_g^{E^{max,*}}$, where $P_d^{D^{max,*}}$ and $P_g^{E^{max,*}}$ are the optimal values obtained from the solution of the subproblem in **Step 5**.
- Step 9** Continue with **Step 3**.
- For the sake of clarity, the algorithm flowchart is depicted in Fig. 2.8.

Illustrative Example 2.6 Two-stage ARO TEP

We consider the six-node system analyzed in Illustrative Example 2.3. The technical data of generating units, demands, and both existing and prospective transmission

Fig. 2.8 TEP flowchart of the ARO algorithm

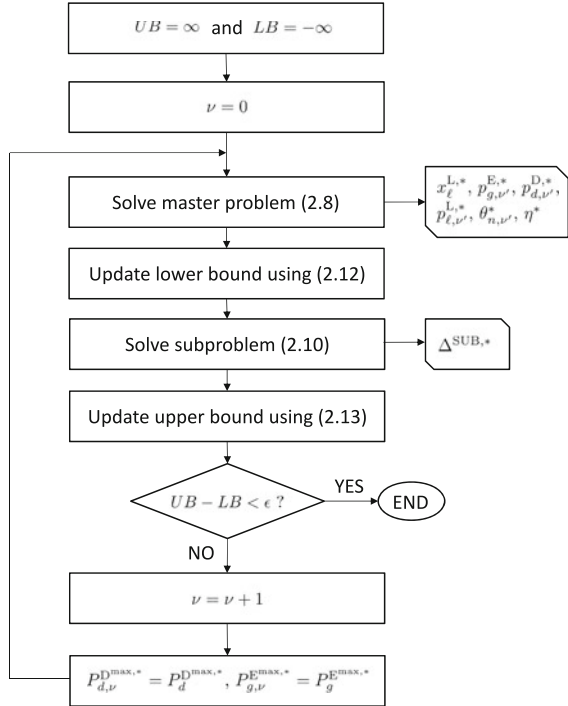


Table 2.7 Illustrative Example 2.6: data for uncertainty sets of demands

Demand	$\underline{P}_d^{\text{D}^{\text{max}}}$ [MW]	$\overline{P}_d^{\text{D}^{\text{max}}}$ [MW]
d_1	180	220
d_2	135	165
d_3	90	110
d_4	180	220

lines, as well as the available investment budget are obtained from Illustrative Example 2.3.

Here, we consider an ARO approach for the TEP problem, and therefore, we consider uncertainty in the available capacity of generating units and the maximum demand at each node of the system, as explained below:

1. We consider that $P_g^{\text{E}^{\text{max}}}$ is uncertain and can take values between 0 and the generating capacity values provided in the third column of Table 2.1.
2. We consider that $P_d^{\text{D}^{\text{max}}}$ is uncertain and can take values between $\underline{P}_d^{\text{D}^{\text{max}}}$ and $\overline{P}_d^{\text{D}^{\text{max}}}$, which are provided in the second and third columns of Table 2.7, respectively.

We consider that the uncertainty budgets for demands and generating units are $\Gamma^{\text{D}} = 0.5$ and $\Gamma^{\text{G}} = 0.2$, respectively. This means that the level of uncertainty is higher in the demand than in the capacity of generating units. For example, with the considered uncertainty budgets, up to 20% of the generating capacity may be unavailable.

With these data, we solve the TEP problem with an ARO approach using the algorithm described in Sect. 2.3.5.3. For the sake of clarity, the steps and results of each iteration of the algorithm are provided below:

- Step 1** We set the lower bound to $LB = -\infty$ and the upper bound to $UB = \infty$.
- Step 2** We set the iteration counter to $\nu = 0$.
- Step 3** We solve master problem (2.8). We obtain the optimal solution of variables $x_\ell^{\text{L},*} = 0$, $\ell = 4, \dots, 9$; and $\eta^* = -\infty$. Note that since $\nu = 0$, constraints (2.8d)–(2.8l) are not included in the master problem at this iteration.
- Step 4** We update the lower bound using Eq. (2.12), $LB = -\infty$.
- Step 5** We solve subproblem (2.10) by considering the optimal value of variables $x_\ell^{\text{L},*}$ obtained in **Step 3** as given parameters. We obtain the optimal solution of variables $P_g^{\text{E}^{\text{max},*}}$ and $P_d^{\text{D}^{\text{max},*}}$, which are provided in Table 2.8. The first and second columns respectively give the generating unit and the corresponding $P_g^{\text{E}^{\text{max},*}}$, while the third and fourth columns provide the demand and the corresponding $P_d^{\text{D}^{\text{max},*}}$, respectively.

Subproblem (2.10) corresponds to the worst realization (in terms of generation and load-shedding costs) of uncertain parameters once the expansion decision variables, i.e., variables $x_\ell^{\text{L},*}$, are fixed. As observed from the results provided in Table 2.8, the worst realization corresponds to the case in which a peak load occurs for demands

Table 2.8 Illustrative Example 2.6: solution of subproblem (2.10) for iteration $\nu = 0$

Generating unit	$P_g^{E_{\max,*}}$ [MW]	Demand	$P_d^{D_{\max,*}}$ [MW]
g_1	300	d_1	180
g_2	250	d_2	160
g_3	400	d_3	90
g_4	170	d_4	220
g_5	60		

d_2 and d_4 (located at nodes 4 and 6, respectively, both in Region B) and when the generation capacity of generating units g_4 and g_5 (located at nodes 5 and 6, respectively, both in Region B) is limited. If we look at the data for generating units and demands provided in Tables 2.1 and 2.2, then we observe that most of the demand is located in Region B. Moreover, Region B is initially not connected to Region A, and node 6 is isolated. Therefore, a critical situation would be the case of a peak demand in Region B (and mainly at node 6) and the failure of generating units in the same region. This is consistent with the results achieved for the subproblem.

In this step, we also obtain solutions for operating decision variables $p_g^{E,*}$, $p_d^{D,*}$, $p_\ell^{L,*}$, and θ_n^* . However, these variables are not used in the following steps of the algorithm.

Step 6 We update the upper bound using Eq. (2.13), $UB = 2.351 \times 10^8$.

Step 7 We compute $UB - LB = \infty$. Since this difference is not small enough, we continue with the following step.

Step 8 We update the iteration counter, $\nu = 1$, and set $P_{g,v_1}^{E_{\max,*}} = P_g^{E_{\max,*}}$ and $P_{d,v_1}^{D_{\max,*}} = P_d^{D_{\max,*}}$, where $P_g^{E_{\max,*}}$ and $P_d^{D_{\max,*}}$ are obtained from the solution of subproblem in **Step 5**.

Step 9 Continue with **Step 3**.

Step 3 We solve master problem (2.8). We obtain the optimal solution of variables $x_\ell^{L,*} = 1$, $\ell = 5, 7$; $x_\ell^{L,*} = 0$, $\ell = 4, 6, 8, 9$; and $\eta^* = 1.169 \times 10^8$.

Master problem (2.8) corresponds to the response of the TSO to the worst situation, i.e., given the worst realization of uncertain parameters obtained in subproblem in **Step 5** above, the TSO decides the optimal transmission expansion plans to minimize generation and load-shedding costs, as well as its investment costs.

Step 4 We update the lower bound using Eq. (2.12), $LB = 1.199 \cdot 10^8$.

Step 5 We solve subproblem (2.10) by considering the optimal value of variables $x_\ell^{L,*}$ obtained in **Step 3** as given parameters. We obtain the optimal solution of variables $P_g^{E_{\max,*}}$ and $P_d^{D_{\max,*}}$, which are provided in Table 2.9. The first and second columns respectively give the generating unit and the corresponding $P_g^{E_{\max,*}}$, while the third and fourth columns provide the demand and the corresponding $P_d^{D_{\max,*}}$, respectively.

Table 2.9 Illustrative Example 2.6: solution of subproblem (2.10) for iteration $\nu = 1$

Generating unit	$P_g^{E_{\max,*}}$ [MW]	Demand	$P_d^{D_{\max,*}}$ [MW]
g_1	300	d_1	195
g_2	250	d_2	165
g_3	400	d_3	110
g_4	20	d_4	180
g_5	150		

As previously explained, subproblem (2.10) corresponds to the worst realization (in terms of generation and load-shedding costs) of uncertain parameters once the expansion decision variables (i.e., $x_\ell^{L,*}$) are fixed. However, now we consider the updated values of expansion decision variables obtained in the master problem for iteration $\nu = 1$. As a consequence, the worst realization of uncertain parameters is different from that obtained in the previous iteration. For example, considering the updated expansion decisions, initially isolated node 6 is now connected to node 3. Therefore, the worst realization that in the previous iteration was a peak load and the failure of the generating unit at this node is not that harmful in this case.

Step 6 We update the upper bound using Eq. (2.13), $UB = 1.334 \times 10^8$.

Step 7 We compute $UB - LB = 0.135 \times 10^8$. Since this difference is not small enough, we continue with the following step.

Step 8 We update the iteration counter, $\nu = 2$, and set $P_{g,v_1}^{E_{\max,*}} = P_g^{E_{\max,*}}$ and $P_{d,v_1}^{D_{\max,*}} = P_d^{D_{\max,*}}$.

Step 9 Continue with **Step 3**.

Step 3 We solve master problem (2.8). We obtain the optimal solution of variables $x_\ell^{L,*} = 1$, $\ell = 5, 7$; $x_\ell^{L,*} = 0$, $\ell = 4, 6, 8, 9$; and $\eta^* = 1.304 \times 10^8$.

Step 4 We update the lower bound using Eq. (2.12), $LB = 1.334 \times 10^8$.

Step 5 We solve subproblem (2.10) by considering the optimal value of variables $x_\ell^{L,*}$ obtained in **Step 3** as given parameters. Since the expansion decisions are the same as those obtained for $\nu = 1$, the optimal solution of subproblem (2.10) corresponds with that provided in Table 2.9.

Step 6 We update the upper bound using Eq. (2.13), $UB = 1.334 \times 10^8$.

Step 7 We compute $UB - LB = 0$. This means that the algorithm has converged, and so it terminates.

The optimal solution of the TEP problem using an ARO approach that takes into account the uncertainty in demands and the availability of generating units consists in building transmission lines ℓ_5 and ℓ_7 , i.e., transmission lines connecting nodes 2 and 4 and nodes 2 and 6, respectively. \square

Table 2.10 Illustrative Example 2.7: investment decisions for different uncertainty budgets^a

		Γ^G				
		0	0.25	0.5	0.75	1
Γ^D	0	ℓ_5, ℓ_7	ℓ_5, ℓ_7	ℓ_4, ℓ_7, ℓ_9	ℓ_4, ℓ_5, ℓ_9	-
	0.25	ℓ_5, ℓ_7	ℓ_5, ℓ_7	ℓ_5, ℓ_7	ℓ_4, ℓ_7, ℓ_9	-
	0.5	ℓ_5, ℓ_7	ℓ_5, ℓ_7	ℓ_5, ℓ_7	ℓ_4, ℓ_7, ℓ_9	-
	0.75	ℓ_5, ℓ_7	ℓ_5, ℓ_7	ℓ_5, ℓ_7	ℓ_4, ℓ_7, ℓ_9	-
	1	ℓ_5, ℓ_7	ℓ_5, ℓ_7	ℓ_5, ℓ_7	ℓ_4, ℓ_7, ℓ_9	-

^a ℓ_4 : 2–3, ℓ_5 : 2–4, ℓ_6 : 3–4, ℓ_7 : 3–6, ℓ_8 : 4–6, ℓ_9 : 5–6

Illustrative Example 2.7 Two-stage ARO TEP: Impact of uncertainty budgets

In Illustrative Example 2.6 we consider that uncertainty budgets Γ^D and Γ^G are equal to 0.5 and 0.2, respectively. Now we analyze its influence on the expansion decisions. Table 2.10 provides the optimal transmission expansion decisions for different values of these uncertainty budgets.

We obtain different expansion decisions depending on the considered uncertainty budgets. These expansion decisions are mainly conditioned by the value of Γ^G , as explained below:

1. For low values of Γ^G , it is optimal to build ℓ_5 and ℓ_7 . These values of the uncertainty budget represent the case in which the uncertainty in the available capacity of generating units is not very high.
2. For values of Γ^G larger than or equal to 0.5, it becomes optimal to build ℓ_4 , ℓ_5 , ℓ_7 , or ℓ_9 , depending on the considered values of Γ^D . These values of Γ^G may represent the case of a system with high uncertainty in the available capacity of generating units, e.g., in a system in which building new generating units is highly uncertain.
3. The value of $\Gamma^G = 1$ represents the case in which $P_g^{G,\max}$ can take any value between 0 and $\bar{P}_g^{G,\max}$, i.e., all units may be unavailable. Therefore, this case is not realistic.

The optimal expansion plan if no uncertainty is considered (i.e., if $\Gamma^G = \Gamma^D = 0$) consists in building transmission lines ℓ_5 and ℓ_7 . This solution corresponds to solving the deterministic TEP problem (2.3) with $P_d^{\text{Dmax}} = \underline{P}_d^{\text{Dmax}}, \forall d$, and $P_g^{\text{Emax}} = \bar{P}_g^{\text{Emax}}, \forall g$. This expansion plan is significantly different from that obtained for a high level of uncertainty (e.g., if $\Gamma^G = 0.75$ and $\Gamma^D = 1$), which consists in building transmission lines ℓ_4 , ℓ_7 , and ℓ_9 . This highlights the importance of modeling the uncertainty when the TEP is decided. If the deterministic solution is implemented, and then one of the worst situations considered in the ARO model occurs, the system may experience significant costs. \square

2.4 Summary

This chapter analyzes the TEP problem. TEP consists in deciding on the optimal reinforcement of the transmission capacity of an existing electric energy network by appropriately selecting the type and number of transmission lines to be built. TEP is a relevant problem in electric energy systems that require a secure, economic, and reliable supply of demand.

We adopt the perspective of a TSO that decides a transmission expansion plan with the aim of facilitating energy trading among producers and consumers, by reducing the generation and load-shedding costs. To do so, two different approaches are developed:

1. A deterministic approach in which optimal transmission plans are obtained by considering the largest expected demand in the planning horizon.
2. An ARO approach in which optimal transmission expansion plans are obtained by taking into account the uncertainty in the demand and the capacity of generating units.

Different illustrative examples are provided to show the working and applicability of the two models described. From these examples, as well as from the theoretical framework described in this chapter, we obtain the following conclusions:

1. Investment costs in transmission lines are comparatively lower than the generation and load-shedding costs. Therefore, it is possible to reduce generation and load-shedding costs by employing limited resources in building new transmission lines.
2. TEP is carried out within an uncertain environment. RO is a practical tool that allows us to represent uncertain parameters by robust sets at a reduced computational cost.
3. TEP is a computationally complex problem. Therefore, it requires implementing some simplifications, e.g., a static approach or a dc power flow, especially if the system under study is very large.
4. The TEP problem is analyzed in this chapter by considering that the generation capacity of the system under study is fixed. The extension of the TEP problem to considering the joint expansion of transmission and generation capacity is analyzed in Chap. 4.

2.5 End-of-Chapter Exercises

2.1 Why is TEP needed? Who decides about it? What is its main purpose?

2.2 Describe the advantages and disadvantages of using a static approach such as that used in this chapter (not a dynamic one) for the formulation of the TEP problem.

2.3 Determine the optimal transmission expansion plan in the modified Garver's system using the deterministic model (2.3). This system is depicted in Fig. 2.9, and

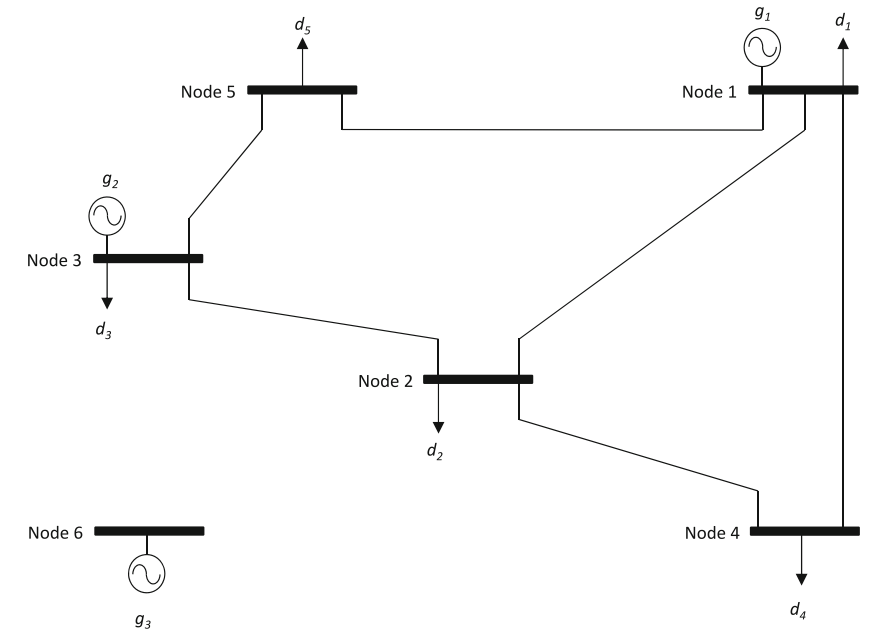


Fig. 2.9 Exercise 2.3: modified Garver’s system

Table 2.11 Exercise 2.3: data for generating units of the modified Garver’s system

Generating unit	Node	$P_g^{G^{\max}}$ [MW]	C_g^G [\$/MWh]
g_1	n_1	200	24
g_2	n_3	200	28
g_3	n_6	300	16

Table 2.12 Exercise 2.3: data for demands of the modified Garver’s system

Demand	Node	$P_d^{D^{\max}}$ [MW]	C_d^{LS} [\$/MWh]
d_1	n_1	110	49
d_2	n_2	132	51
d_3	n_3	88	80
d_4	n_4	132	65
d_5	n_5	88	39

its data are provided in Tables 2.11, 2.12, 2.13, and 2.14. Consider an annualized investment budget equal to \$12 million, a base power of 1 MW, and a base voltage of 1 kV.

Table 2.13 Exercise 2.3: data for existing transmission lines of the modified Garver's system

Line	From node	To node	B_ℓ [S]	F_ℓ^{\max} [MW]
ℓ_1	n_1	n_2	250	100
ℓ_2	n_1	n_4	133	80
ℓ_3	n_1	n_5	500	100
ℓ_4	n_2	n_3	500	100
ℓ_5	n_2	n_4	250	100
ℓ_6	n_3	n_5	250	100

Table 2.14 Exercise 2.3: data for prospective transmission lines of the modified Garver's system

Line	From node	To node	B_ℓ [S]	F_ℓ^{\max} [MW]	I_ℓ^L [\$]
ℓ_7	n_2	n_5	323	100	3,491,000
ℓ_8	n_2	n_6	333	100	3,379,000
ℓ_9	n_3	n_6	500	100	5,406,000
ℓ_{10}	n_4	n_6	333	100	3,379,000

2.4 Expand the deterministic TEP model (2.3) to consider a dynamic approach whereby expansion decisions can be made at different points in time. Then apply this dynamic problem to obtain the optimal TEP decisions in Illustrative Example 2.3, considering three time periods and assuming a constant demand growth of 5% at each time period.

2.5 Expand the deterministic TEP model (2.3) to consider losses through transmission lines, as done in [1].

2.6 Expand the deterministic TEP model (2.3) to consider the uncertainty in the maximum demand through a set of scenarios, i.e., obtain a stochastic programming model from the deterministic TEP model (2.3). Is this stochastic model more efficient than the ARO model provided in Sect. 2.3? Why or why not?

2.7 Write the equivalent mixed-integer linear expressions corresponding to complementarity constraints (2.10at)–(2.10ba) using the Fortuny–Amat transformation described by Eqs. (2.11).

2.8 Determine the optimal transmission expansion plan in the modified Garver's system using the ARO approach described in Sect. 2.3. This system is depicted in Fig. 2.9, and its data are provided in Tables 2.11, 2.13, 2.14, and 2.15. Consider an annualized investment budget equal to \$12 million and uncertainty budgets for demands and generating units equal to $\Gamma^D = 0.5$ and $\Gamma^G = 0.2$, respectively.

2.9 In the ARO approach described in Sect. 2.3, the so-called uncertainty budgets, Γ^G and Γ^D , are used to model the level of uncertainty in generating units and

Table 2.15 Exercise 2.8: data for uncertainty sets of demands of the modified Garver's system

Demand	\underline{P}_d^D [MW]	\overline{P}_d^D [MW]
d_1	100	120
d_2	120	144
d_3	80	96
d_4	120	144
d_4	80	96

demands, respectively. However, uncertainty in generating units and demands may be different in different regions of a system. It is possible to define uncertainty budgets for different regions, i.e., Γ_r^G and Γ_r^D , where r indicates the region of the system, as explained in [29]. Solve Illustrative Example 2.6, considering that the uncertainties in Regions A and B are different. Region A has significant uncertainty on the available capacity of generating units ($\Gamma_A^G = 0.5$), but no uncertainty on demand levels ($\Gamma_B^D = 0$), while Region B has no uncertainty on the available capacity of generating units ($\Gamma_B^G = 0$), but high uncertainty on demand levels ($\Gamma_B^D = 0.5$). How different are the results from those obtained in Illustrative Example 2.6? Why?

2.6 GAMS Code

A GAMS code for solving problem Illustrative Example 2.3 is provided below:

```

1  SETS
2  n                               /n1*n6 /
3  g                               /g1*g5 /
4  d                               /d1*d4 /
5  l                               /l1*l9 /
6  pros(l)                        /l4*l9 /
7  ex(l)                           /l1*l3 /
8  mapG(g,n)                       /g1.n1,g2.n2,g3.n3,g4.n5,g5.n6 /
9  mapD(d,n)                       /d1.n1,d2.n4,d3.n5,d4.n6 /
10 ref(n)                           /n1 /
11 mapSL(l,n)                      /l1.n1,l2.n1,l3.n4,l4.n2,l5.n2,l6.
    n3,l7.n3,l8.n4,l9.n5 /
12 mapRL(l,n)                      /l1.n2,l2.n3,l3.n5,l4.n3,l5.n4,l6.
    n4,l7.n6,l8.n6,l9.n6 /;

15 TABLE LDATA(L,*)
16          B      FLmax      IC
17  11          500      150      0
18  12          500      150      0
19  13          500      150      0
20  14          500      150      700000
21  15          500      150      1400000

```

```

22 16          500      200      1800000
23 17          500      200      1600000
24 18          500      150      800000
25 19          500      150      700000;

27 SCALAR ILmax
28 /3000000/;

30 TABLE DDATA(d,*)
31          PDmax      LScost
32 d1          200      40
33 d2          150      52
34 d3          100      55
35 d4          200      65;

37 TABLE GDATA(g,*)
38          PEmax      Gcost
39 g1          300      18
40 g2          250      25
41 g3          400      16
42 g4          300      32
43 g5          150      35;

45 SCALAR SIGMA
46 /8760/;

48 SCALAR M
49 /5000/;

51 VARIABLES
52 Z
53 PL(l)
54 THETA(n);

56 POSITIVE VARIABLES
57 PG(g)
58 PLS(d);

60 BINARY VARIABLES
61 x(l);

63 EQUATIONS EQ3A, EQ3B, EQ3D, EQ3E, EQ3Fa, EQ3Fb,
          EQ3Ga, EQ3Gb, EQ3Ha, EQ3Hb, EQ3I, EQ3J, EQ3Ka,
          EQ3Kb, EQ3L;

65 EQ3A..          Z=E=SUM(l$pros(l),LDATA(l,
          'IC')*x(l))+SIGMA*(SUM(g,GDATA(g,'Gcost')*PG(g)
          )+SUM(d,DDATA(d,'LSCOST')*PLS(d)));
66 EQ3B..          SUM(l$pros(l),LDATA(l,'IC'
          )*x(l))=L=ILmax;
67 *EQUATIONS 3C ARE DEFINITION OF BINARY VARIABLES

```

```

68 EQ3D(n) ..                                SUM(g$mapG(g,n),PG(G))-SUM
      (l$mapSL(l,n),PL(l))+SUM(l$mapRL(l,n),PL(l))=E=
      SUM(d$mapD(d,n),DDATA(d,'PDMAX')-PLS(d));
69 EQ3E(l)$EX(l) ..                          PL(l)=E=LDATA(l,'B')*(SUM(
      n$mapSL(l,n),THETA(n))-SUM(n$mapRL(l,n),THETA(n)
      ));
70 EQ3Fa(l)$EX(l) ..                        -LDATA(l,'FLmax')=L=PL(l);
71 EQ3Fb(l)$EX(l) ..                        PL(l)=L=LDATA(l,'FLmax');
72 EQ3Ga(l)$PROS(l) ..                      -x(l)*LDATA(l,'FLmax')=L=
      PL(l);
73 EQ3Gb(l)$PROS(l) ..                      PL(l)=L=x(l)*LDATA(l,'
      FLmax');
74 EQ3Ha(l)$PROS(l) ..                      -(1-x(l))*M=L=PL(l)-LDATA(
      l,'B')*(SUM(n$mapSL(l,n),THETA(n))-SUM(n$mapRL(
      l,n),THETA(n)));
75 EQ3Hb(l)$PROS(l) ..                      PL(l)-LDATA(l,'B')*(SUM(
      n$mapSL(l,n),THETA(n))-SUM(N$mapRL(l,n),THETA(n)
      ))=l=(1-x(l))*M;
76 EQ3I(g) ..                              PG(g)=L=GDATA(g,'PEmax');
77 EQ3J(d) ..                              PLS(d)=L=DDATA(d,'PDmax');
78 EQ3Ka(n) ..                              -3.14=L=THETA(n);
79 EQ3Kb(n) ..                              THETA(n)=L=3.14;
80 EQ3L(n)$REF(n) ..                        THETA(n)=L=0;

82 MODEL TEP_DET /ALL/;

84 SOLVE TEP_DET USING MIP MINIMIZING Z;

```

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