

## Chapter 2

# Types of Projection. The Center of Projection

The object projection into the image plane can be carried out by straight lines or curves (rays) [1–15].

The projection carried out by straight lines (rays) is called *a rectilinear projection*. The projection carried out by curves (rays) is called *a curvilinear projection*.

*The center of projection* is the origin or source of the stream of projecting rays. Depending on the distance between the center of projection and the image plane, the distinction should be made between central (conical) projections and parallel (cylindrical) projections.

### 2.1 Central (conical) Projection

*Central (conic) projection* is a projection from the center, located at the finite distance from a plane of projection (Fig. 2.1). An array of rectilinear projecting rays forms a conic surface.

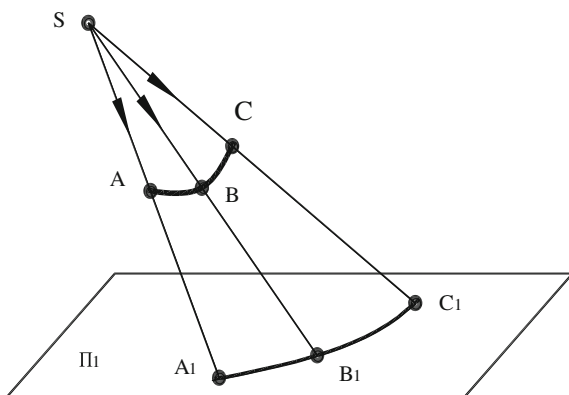
Examples of central projection are the shadow of an object from an electric bulb, a photo of a product, and an image of an object on the retina of the human eye.

### 2.2 Parallel (cylindrical) Projection

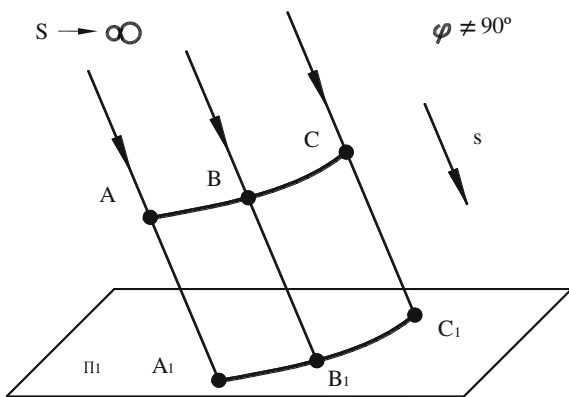
*Parallel (cylindrical) projection* is a projection from the center, located at a infinite distance from a plane of projection (Fig. 2.2). An array of rectilinear projecting rays forms a cylindrical surface as all rays are parallel.

An example of parallel projection is the shadow of an object from the rays of the Sun.

**Fig. 2.1** The central (*conic*) projection



**Fig. 2.2** Parallel (*cylindrical*) oblique-angled projection



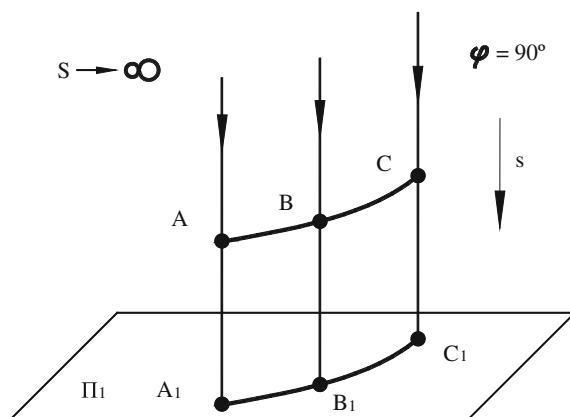
Depending on the size of the angle  $\varphi$  of an inclination of the projecting rays to the plane of projection, parallel (cylindrical) projection is divided into *oblique-angled* projection ( $\varphi \neq 90^\circ$ , Fig. 2.2) and *rectangular (orthogonal)* projecting ( $\varphi = 90^\circ$ , Fig. 2.3).

## 2.3 Properties of the Central (conic) Projection

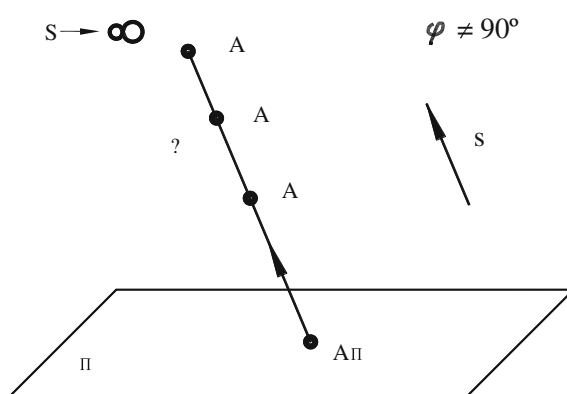
Some important properties of the central (conic) projection are:

1. A projection of a point is a point (Fig. 2.1).
2. A point in the space, with the given direction of projection, determines the location of the single projection of the point on the plane of projection (Figs. 2.2 and 2.3).
3. A projection of a point does not determine the location of the point in space (Fig. 2.4).

**Fig. 2.3** Parallel  
(cylindrical) rectangular  
(orthogonal) projection



**Fig. 2.4** Ambiguity of the  
solution to an inverse problem  
of geometry for a point

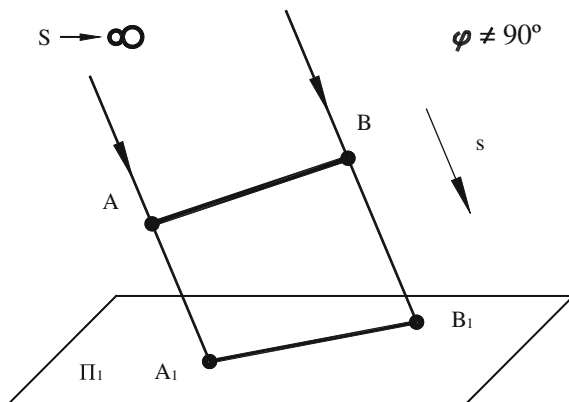


4. A projection of a straight line, which does not coincide with a direction of projection, is a straight line (Fig. 2.5).
5. A projection of a straight line, which coincides with a direction of projection, is a point (Fig. 2.6).

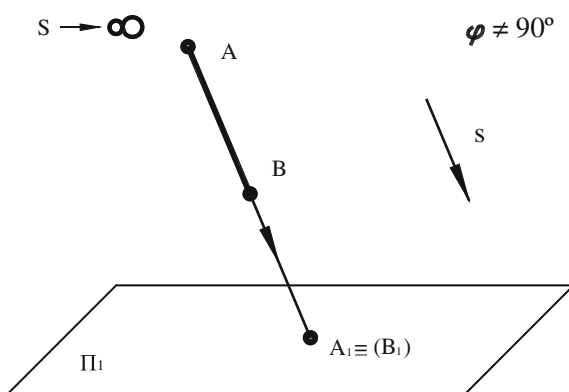
Central (conic) projecting is a special case of parallel (cylindrical) projecting. Therefore, the properties 1–5 of the central (conic) projection are also valid for parallel (cylindrical) projection.

Nevertheless, there are a number of properties, which are characteristic for parallel (cylindrical) oblique-angled projection, that differ for central (conic) projection in several respects.

**Fig. 2.5** Straight line  $AB$  does not coincide with the direction of projection  $s$



**Fig. 2.6** Straight line  $AB$  coincides with the direction of projection  $s$



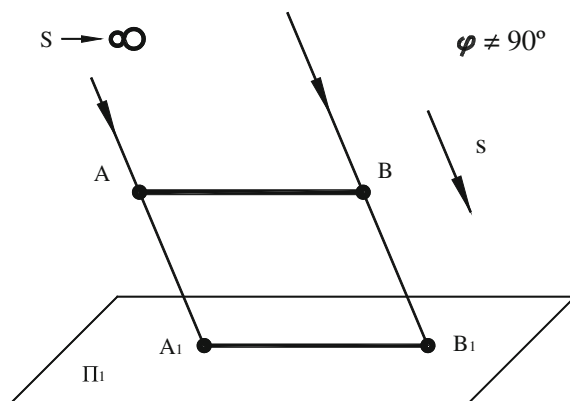
## 2.4 Properties of Parallel (cylindrical) Oblique-Angled Projection

6. A projection of a straight line segment or of a plane figure, which is parallel to a projection plane, appears in equal length or as a congruent figure, respectively (Fig. 2.7).
7. A projection of a point, dividing a spatial line segment in certain proportion, divides a projection of that segment in the same proportion (Fig. 2.8).
8. Projections of parallel straight lines are parallel (Fig. 2.9).

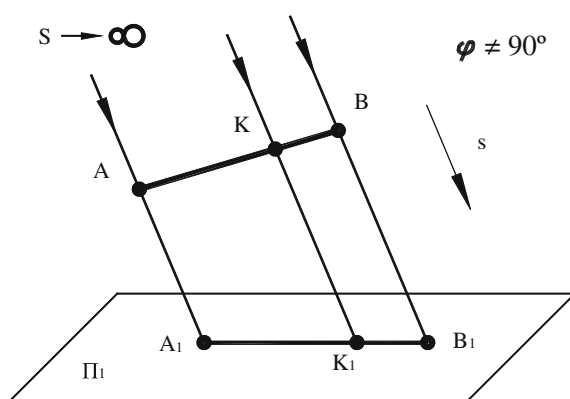
Parallel oblique projection is a generalization of parallel projection. In the former case, the angle between projecting rays and the planes of projection is not  $90^\circ$  ( $\varphi \neq 90^\circ$ ). Therefore the properties of parallel oblique projection also apply to parallel projection.

Rectangular (orthogonal) projection is a generalization of parallel projection. The angle between projecting rays and the planes of projection is  $90^\circ$  ( $\varphi = 90^\circ$ ).

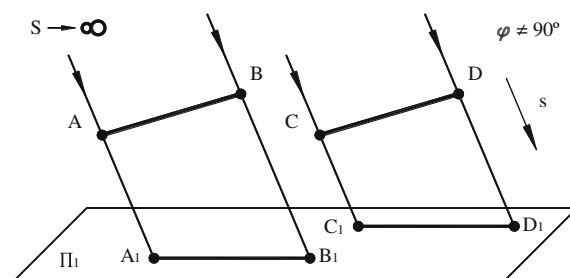
**Fig. 2.7** Projection  $A_1B_1$  of the segment  $AB$  is parallel to it and equal in length



**Fig. 2.8** Projection  $K_1$  of point  $K$  divides projection  $A_1B_1$  of segment  $AB$  in the same proportion, i.e., lengths  $AK:KB = A_1K_1:K_1B_1$



**Fig. 2.9** Parallel projections  $A_1B_1$  and  $C_1D_1$  of the parallel straight lines  $AB$  and  $CD$



Therefore the properties of parallel rectangular (orthogonal) projection also apply to parallel projection.

Consider some important properties, which are characteristic only for parallel rectangular (orthogonal) projection, that differ for central (conic) projection in certain respects.

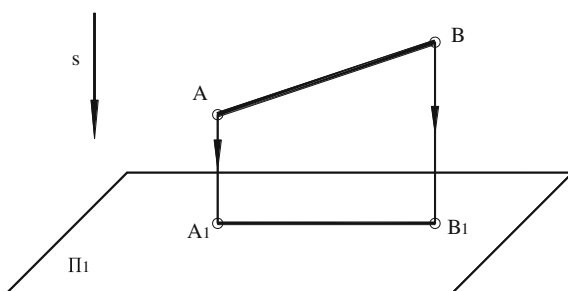
## 2.5 Properties of Parallel Rectangular (orthogonal) Projection

9. In orthogonal projection, the length of a projection of the segment, which is not parallel to the plane of projection, is less than the true (actual) length of that segment (Fig. 2.10).
10. If a plane figure coincides with a projection direction, its projection is a straight line segment (Fig. 2.11).

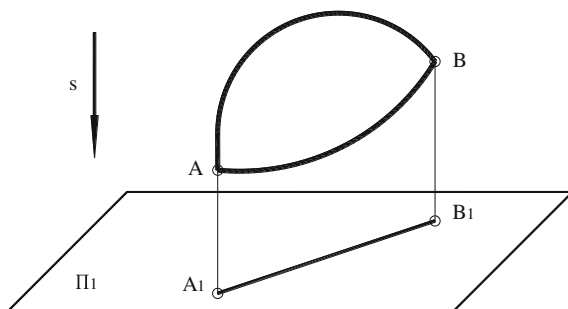
Orthogonal projection has 28 principal properties.

The structural scheme of the types of projection is presented in Fig. 2.12.

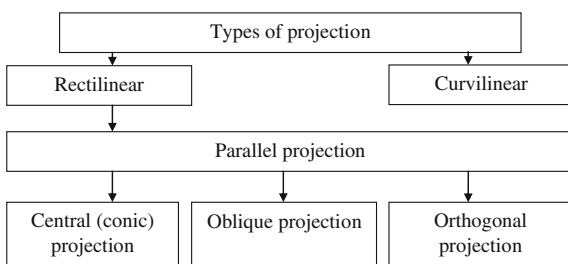
**Fig. 2.10** Projection  $A_1B_1$  of segment  $AB$  is less than the true length of this segment



**Fig. 2.11** Projection  $A_1B_1$  of an *orthogonal plane* figure is a segment



**Fig. 2.12** Types of projection



## References

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