

Preface to the Second Edition

Writing the second edition of this text was a wonderful opportunity for a more mature presentation than I was able to give 13 years ago. The intent still is to give a self-contained introduction to the theory of ordered sets and to its connections to other areas. I tried to shine a light on as many branches of the theory of ordered sets as possible, with the main obstacle being my limited cranial capacity. Indeed, although in this text I can present the most I have ever known about ordered sets, writing the text has also shown me how much I don't know. However, that should not be a problem for anyone, as that which we don't know will always exceed that which we do. Writing the second edition also was quite humbling in another way. Thorough re-reading revealed more typos than were reported on the posted errata, and some of them were rather embarrassing.

The presentation remains modular. Specifically, we have the following:

- Chapters 1, 2, 3, and Sections 4.1 and 4.2 form the core of the text. If you are new to ordered sets, you should read this part in the order in which it is presented here. (Skip Section 3.5 if you are not focusing on analysis.)
- The remaining chapters can be read in just about any order.

There have been some content rearrangements and additions. The automorphism problem (see Open Question 2.14), of which nothing appears to be known beyond the references given here, has been moved into Chapter 2 to feature it more prominently. Similarly, the chapter on algorithms has been “promoted” to Chapter 5 and expanded to focus even more strongly on constraint satisfaction problems. I believe that more results similar to the “ \Leftarrow ” direction of Theorem 5.57 can be proved for the fixed point property and for other constraint satisfaction problems. Chapter 6 is new and serves to separate the fixed clique property from the more fundamental fixed simplex property and to give an idea about graph homomorphisms and their connections to and differences from order-preserving maps. Finally, the future importance of discrete Morse functions for the fixed point property for ordered sets is indicated in Appendix B. Overall, I have shifted the primary focus toward finite ordered sets, with results on infinite ordered sets moved to the back of each chapter whenever possible. The references [21, 23, 25, 31, 41, 42, 54, 56, 91, 98, 102, 112,

120, 246, 247, 251, 252, 254, 311] should provide ample opportunity for further study of various aspects of ordered sets.

Although the goal was and is to have a self-contained exposition, the first edition's appendix on ordered L^p -spaces has been turned into Section 3.5 and Exercises 2-50, 3-32, 3-33, 3-34, 4-37, 8-11, 8-12, 8-13, and 8-18. If you are familiar with L^p -spaces, the exercises will be natural; if you are not, the appendix probably would have felt quite uncomfortable anyway. So the simple advice here is that if L^p -spaces are not part of your repertoire, then these exercises should probably be skipped.

Other additions are technical in nature, but important nonetheless. The text is now available as an ebook with live links for the internal references. This can make reading easier at times, but the usefulness of a paper copy should not be underestimated. I have read and written thousands of pages on screen. However, when I really want to learn something well, I read a paper copy.

I hope you will enjoy reading this text, in any of its forms, as much as I enjoyed writing it. Let me conclude with a final request/recommendation for readers who use this text as part of an effort to improve proof-writing skills. The transition to doing proofs is hard. To me, it is the hardest challenge that I have ever successfully met. Looking up solutions does not facilitate this transition. So, as you work the exercises, do *not* use the library, Google, or other resources to find solutions. (I maintain that I learned how to do proofs partly because I was too lazy to look up solutions, preferring hours of thinking over a half hour in the library.) Use your brain, your whole brain and nothing but your brain, aided by paper and pencil.

Hattiesburg, MS, USA
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Ordered Sets

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Topology

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