Chapter 2
Theoretical Frameworks and School Context

Abstract The rationale and Spatial Operational Capacity theoretical framework that support the project are described along with the design research methodology that evolved over the 7 years of research, the school context, and a pre-interview used to support the researchers' perspectives on each participant child's beginning spatial ability.

2.1 Why Are Visualization Skills Important?

Visual and spatial thinking and reasoning "generally refers to the ability to represent, transform, generalize, communicate, document, and reflect on visual information" (Hershkowitz et al. 1989, p. 75). Pittalis and Constantinou (2010) summarize spatial thinking as "a form of mental activity that enables individuals to create spatial images and to manipulate them in solving various practical and theoretical problems" (p. 191). This includes finding meaning in the shape, size, orientation, location, direction or trajectory, of objects, processes or phenomena, or the relative positions in space of multiple objects, processes or phenomena. The importance of visual thinking and reasoning has been expressed by researchers and standards organizations across mathematical and scientific disciplines. Researchers have shown that imagistic processing, in balance with algebraic thinking, is an essential component in developing proficiency with abstract axiomatic mathematics (e.g., Dreyfus 1991; Presmeg 1992; Tall et al. 2001). Outside of mathematics, e.g., in the physical and geosciences, a pertinent question of interest to researchers is, "How do people combine information gathered from multiple viewpoints into a single integrated mental model of the three-dimensional object or process, and how can that inherent human ability be harnessed to help students interpret 1D or 2D data sets in terms of 3D processes?" (Science Education Research Center 2009). Researchers have shown that spatial abilities can be learned through appropriate learning experiences (e.g., Clements and Battista 1992; Ganesh et al. 2009; Yakimanskaya 1991).

J. Sack and I. Vazquez, *A 3D Visualization Teaching-Learning Trajectory for Elementary Grades Children*, SpringerBriefs in Education,
DOI 10.1007/978-3-319-29799-6_2

2.2 The Spatial Operational Capacity Framework

The Spatial Operation Capacity (SOC) theoretical framework (see Fig. 2.1), originally designed by Van Niekerk (1997), consists of four main categories of variables that can contribute to the complexity of a visual image as a stimulus in task design namely:

1. Perception: The stimulus with which visual information is presented to the learner is grouped among four different categories: (i) full-scale images, (ii) virtual real images, (iii) conventional graphic images, or (iv) iconic images. These categories are differentiated by the closeness of the representation to reality in both a visual and a tactile sense.
2. Dimensionality: The objects, which are presented via the visual information that the learner perceives, processes or acts on, can be (i) one-dimensional (points and lines), (ii) two-dimensional (e.g., triangles, quadrilaterals), or (iii) three-dimensional images (e.g., prisms, pyramids) and may be a part of or the entire presented stimulus.
3. Transformations: A critically important cognitive process that must be addressed during visual processing, while acting on the object/s represented by the image, is the ability to comprehend the nature of the changes that objects and situations can undergo during perception. In other words, this is the ability of the learner to keep track of what is fixed and what changes when manipulating objects and

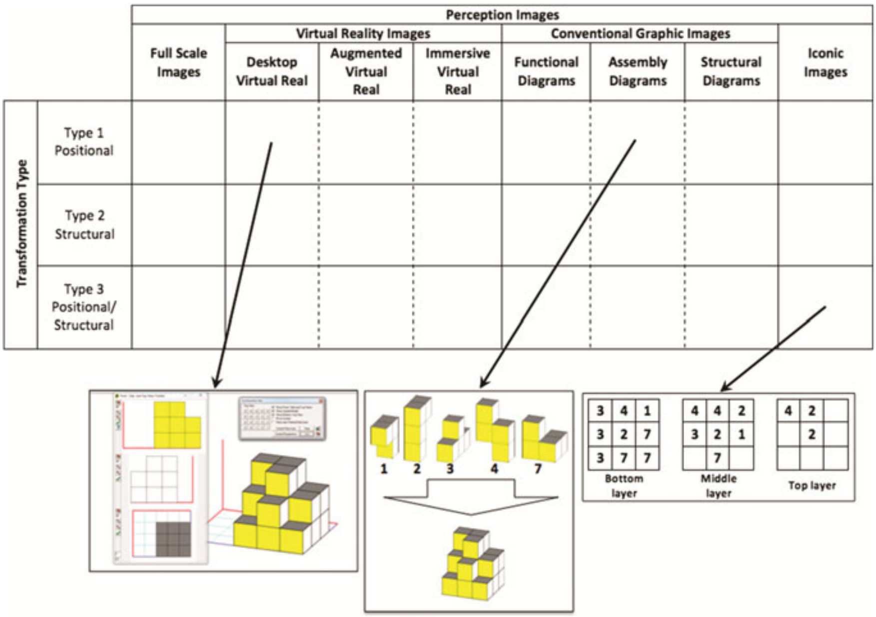

Fig. 2.1 The spatial operational capacity (SOC) model

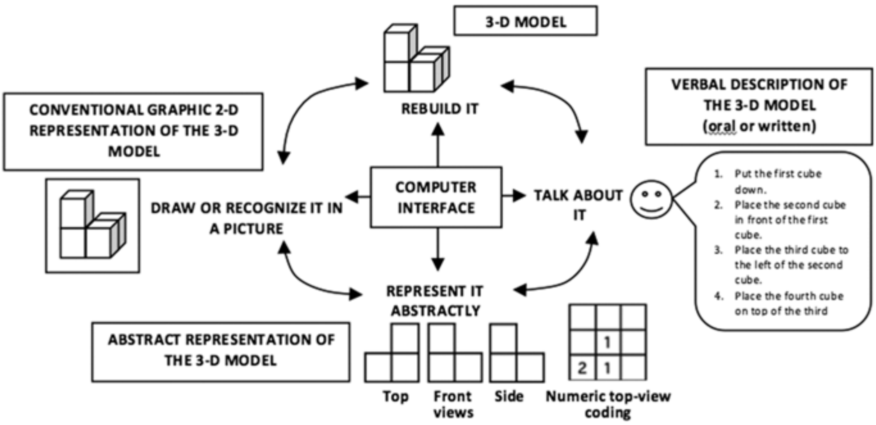

Fig. 2.2 Adapted SOC model

situations. The three different kinds of transformations that objects, which are represented via visual images, can undergo are (i) positional, (ii) structural, or (iii) combined positional-structural changes.

4. Mobility: The visual images contemporary learners encounter that can be represented with respect to mobility are determined by the nature of the visual image per se. This variable reflects the importance that the authors give to the role of the body in visual imaging (Hansen 2004). This mobility aspect can be represented as a continuum between a static medium (printed/typographic materials) and a potentially dynamic medium (digital and electronic materials) (Rückriem 2009, pp. 97–99). These different kinds of mobilities are represented in this model as (i) static (print) (ii) semi dynamic (e.g., PowerPoint slides, photo slides), or (iii) dynamic (video/film/television) images.

For this particular study, the SOC framework was adapted as shown in Fig. 2.2. Activities utilize actual 3D models, namely, loose cubes and puzzle figures, made from unit cubes glued together in different 3- or 4-cube arrangements (see Chap. 3); conventional 2D pictures of the 3D models that have hidden components since only up to 3 views can be shown in such pictures; abstract representations of the 3D models such as top-plan numeric views or front-side-top views that do not obviously resemble only one 3D figure; and the Geocadabra Construction Box dynamic computer interface that integrates these representations in real time (see Chap. 4).

2.3 Research Methodology

Design research principles guided the methodology for the entire project. While some design researchers focus their analyses on whole-group sense making (e.g., Cobb et al. 2001), others (e.g., Simon et al. 2010) focus on fine-grained individual participant's conceptualizations. Each lesson was part of a larger design experiment followed by a retrospective analysis in which the research team determined the actual outcomes and then planned the next lesson. This may have been an iteration of the last lesson to improve the outcomes, a rejection of the last lesson if it failed to produce adequate progress toward the desired outcomes, or a change in direction if unexpected, but interesting, outcomes arose that were deemed worthy of more attention. The overarching question that guided this research team's analysis was, how does the research team attend to individual children's sense making and to their collective understandings, in order to move to more complex and deeper challenges for the entire class? The data corpus included formal and informal interviews, video-recordings and transcriptions, field notes, student products and lesson notes.

2.4 School and Classroom Context

The study was conducted over a 7 year period beginning Fall 2007 in a dual-language urban elementary school serving approximately four hundred students within one of the largest public school districts in the mid-southwestern United States. Approximately 70 % of the students are Hispanic, 20 % are African American and the remaining 10 % are White or Asian. Three-quarters of the students are designated "At-Risk" and 55 % are English-language learners. The participants in the study represented a typical cross-section of the larger school community. Mathematics instruction is conducted in Spanish for students below fourth grade during the academic day but this project was conducted in English within an established after-school program at the school.

The research/instructional team initially consisted of university-based researcher, Sack, who had over 15 years of classroom experience, and two teacher-researchers, author Vazquez and another teacher, each with at least 8 years of classroom experience, who taught full-time at the third-grade and kindergarten levels in the school's dual-language program collaborating closely during the school day on matters related to their academic programs. During Year 2, the second teacher taught third grade but moved to a different school in Year 3. Sack and Vazquez continued to collaborate on the research aspects of the project over a total of 7 years. The project has become institutionalized within the school's after-school program under the direction of authors Vazquez and Sack.

For Year 1 of the study, at the beginning of the fall semester, all after-school third- and fourth-graders were invited to participate. This afforded the research team a classroom setting without the ongoing curricular and assessment pressures that have come to dominate the daily lives of school. Furthermore, the children who participated did so by choice. Fourteen fourth-graders and eleven third-graders started out in the study. However, various conflicting activities resulted in attrition and approximately eleven fourth-graders and eight third-graders attended the program consistently throughout the year. Teacher Vazquez had taught mathematics and science to all fourth-grade participants during their entire third-grade year. In Years 2–7, only third graders participated. However, during Year 2, some of the children who had participated as fourth-graders during Year 1 returned to get help with concepts that were being taught in very abstract ways.

Within this study, students worked independently or in small groups of two to four and all students were expected to ask each other for help or support before asking the teacher. Mutual respect was fostered in the classroom environment in which students felt comfortable expressing their understandings knowing they were safe to express their confusion or frustration in front of their peers. They were expected to explain and provide justification for their mathematical conclusions.

2.5 Pre-program Interview

A pre-program interview was designed to informally assess each participating child's ability to visualize the number of cubes in various conventional pictures shown in Fig. 2.3. The children were interviewed one-on-one with the researcher, who showed only 1 cube and asked how many would be needed to build the structures shown in each of the pictures. These were presented one at a time in the order shown. On average, for 10 participants, 9 out of 10 correctly determined the number of cubes

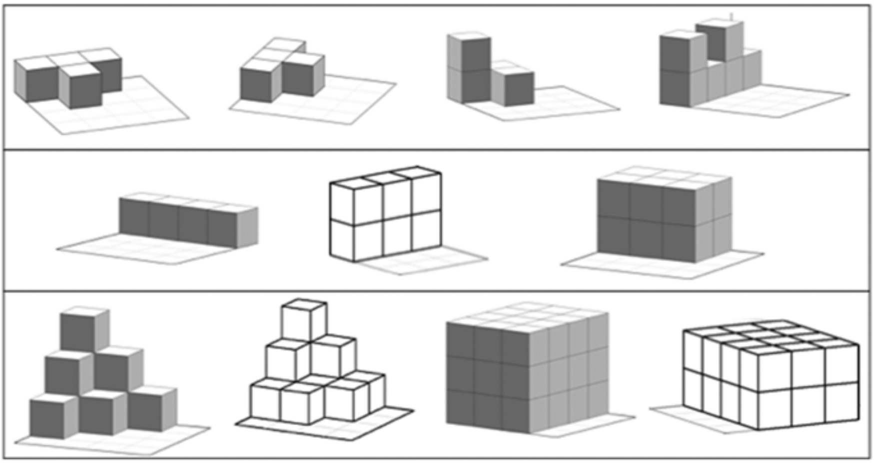

Fig. 2.3 Pre-interview figures

in the first row, where none of the cubes are invisible, and in the second row, where the structures are relatively simple. For the structures shown in the third row, only 1 or 2 out of 10 participants were able to correctly determine the number of cubes in each figure since several are hidden. Battista (1999) showed that for such structures children revert to counting visible faces or visible cubes since they are aware of but are unable to accurately count hidden cubes. The interview results aligned perfectly with Battista's findings and provided the research team a reference marker for each child before the program began.

In Chap. 3 we share introductory activities that move between loose cubes, the set of Soma figures and 2D conventional pictures of assemblies of two Soma figures. Then, in Chap. 4, we show how the children learn to use the Geocadabra Construction Box. In Chap. 5, abstract top-view plans and top, front, and side views are integrated with the 2D conventional and 3D model representations of various block figures. Chapter 6 shows how visualization activities integrated with numeracy development, in particular, with multiplication skills that are typically developed in 3rd grade.

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http://www.springer.com/978-3-319-29798-9

A 3D Visualization Teaching-Learning Trajectory for
Elementary Grades Children
Sack, J.; Vazquez, I.
2016, VI, 44 p. 31 illus., Softcover
ISBN: 978-3-319-29798-9