

Chapter 2

Fiber Optic Sensor Arrays for Real-Time Virtual Instrumentation and Control of Flexible Structures

Hector Gutierrez, Behzad Shariati Javani, Daniel Kirk, Weihua Su, Michael Wolf, and Edwin Griffin

Abstract Launch vehicles and aerospace structures trend towards slender and more lightweight designs to reduce mass and aerodynamic drag. Slender structures are more affected by structural vibrations that interfere with both inertial sensing and navigation performance. Slender aerospace structures (launch vehicles, aircraft, and others) would greatly benefit from means for real-time monitoring of the flexible dynamics: real time modal sensing, as well as estimation of acceleration and rate of rotation at any point in the structure. This paper proposes the use of fiber-Bragg (FBG) sensor arrays for real-time monitoring and control of a flexible subscale test article representative of a launch vehicle. A virtual sensor for estimating acceleration and rate of rotation at any point in the structure is presented: the FBG Inertial Measurement Unit. The FBG-IMU is a real-time estimator based on a finite element model of the structure, the state-space equations of the flexible dynamics, and a finite number of strain measurements provided by the FBG array. The FBG-IMU allows accurate estimation of the states of the flexible structure with no need of estimation of load forces or damping matrix, as current structural models of flexible structures are forced to assume. The FBG-IMU is a contribution to the real-time estimation of the flexible dynamics in structures without the need of estimating loads or damping.

Keywords Fiber optic strain gage • Real-time modal analysis • Structural dynamics • Real-time estimation and control of flexible structures

2.1 Introduction

Next generation launch vehicles will trend towards slender and more lightweight designs. This natural trend has been limited due to the associated increase in flexible effects on the structure. Undesired vibrations affect structural health and also degrade control performance, in particular since the Inertial Measurement Unit (IMU) onboard the launch vehicle is affected by both the flexible dynamics as well as the rigid body motion, and feeds both signals into the control loop. Another important challenge in launch vehicle simulation and control is given by the time-varying mass and inertia of the vehicle, and the corresponding changes in modal frequencies and modal shapes of the structure as propellant is exhausted. To stabilize the trajectory of a rocket while considering its flexible dynamics, the estimation of displacement, rate of rotation and acceleration at any point on launch vehicle is of great value for both mission planning and controller design [1].

Fiber Optic Bragg Grating (FBG) sensor arrays are lightweight, insensitive to EM noise, and can be embedded in large numbers in a single fiber with no additional penalty in weight or complexity. Large numbers of FBG sensors can be multiplexed at the speed of light at the interrogator, enabling the potential use of FBG sensor arrays to develop systems which can accurately predict the dynamic characteristics of the structure in real time. This can be coupled with various control systems techniques for real time estimation and vibration control in launch vehicles.

Until recently, the use of FBG sensor systems was limited to semi-static measurements of strain in multiple points of a structure due to the fact that optical interrogation equipment had limited bandwidth (typically below 10 Hz) and used output buffers that further delayed the availability of updated readings. Recently, fiber optic sensor array interrogation systems with

H. Gutierrez (✉) • B.S. Javani • D. Kirk

Department of Mechanical and Aerospace Engineering and Department of Electrical and Computer Engineering,
Florida Institute of Technology, Melbourne, FL 32905, USA
e-mail: hgutier@fit.edu

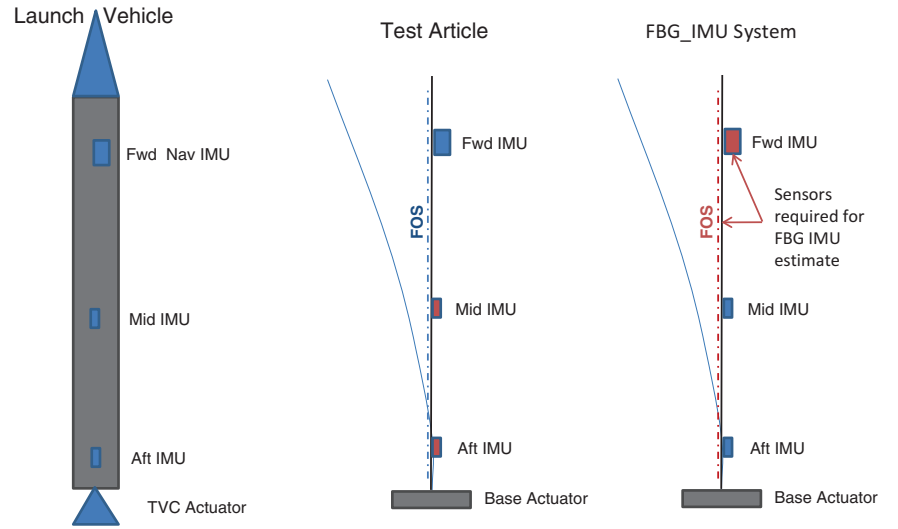
W. Su

Department of Aerospace Engineering and Mechanics, The University of Alabama, Tuscaloosa, AL 35487, USA

M. Wolf • E. Griffin

NASA Kennedy Space Center, Launch Services Program, Washington, DC 20546, USA

Fig. 2.1 FBG-IMU system vs. standard launch vehicle body bending configuration



capabilities in excess of 1000 Hz have become available. Such systems, combined with the fact that multiplexed interrogation of optical gratings occurs at the speed of light, provides a potential breakthrough in dynamic monitoring of large structures: the strain at hundreds of points within a structure can now be monitored almost simultaneously at high sampling rates. Fiber optic sensor technology with high interrogation rate and unbuffered output therefore has the potential to provide real-time estimation of flexible dynamics. This, combined with distributed auxiliary actuators and advanced control algorithms opens the path for the real time control of the flexible dynamics of future aerospace structures [2].

A typical standard launch vehicle body-bending sensor configuration consists of three ring laser gyros (RLG). The primary objective of this paper is to propose two novel approaches for the estimation of acceleration and rate of rotation at any point in the structure based on a FBG strain sensor array plus a forward placed IMU, compared to the standard launch vehicle body bending sensor configuration shown in Fig. 2.1.

The proposed algorithms enable prediction of deflection, rate of rotation and acceleration of any point in the flexible structure relative to the rigid body frame, and are based on modal matrices $\{[M], [k], [\Phi], [\Theta]\}$ that are calculated using only a finite element model of the structure. The estimation is based on direct measurements of strain in multiple points in the structure, which allows accurate estimation of the states of the flexible structure with no need of estimation of load forces or a damping matrix, or no need to assume that the damping is linear and viscous, as current structural models of flexible structures are forced to assume (Fig. 2.2).

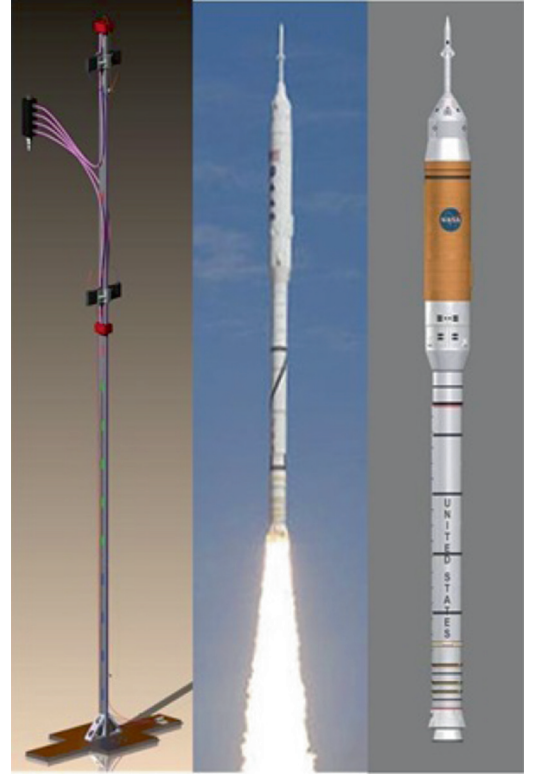
2.2 Approach

To validate the proposed algorithms on a setup with only one degree of freedom, the flexible launch vehicle is first simplified to a clamped-free uniform flexible cantilever beam that only deflects transversely. A linear motor air-bearing stage is used to introduce vibrations as base motion, as shown in Fig. 2.3. Although cantilever beam equations predict the bending shape of the cantilever beam correctly, these simplified mathematical equations fall short when additional components are added to the bare beam, such as thrusters, the wiring harness and the sensor arrays. ANSYS finite element analysis software was used to create a more accurate model of the test article that can be transferred to and implemented in MATLAB/Simulink to develop a state-space model of the structure.

2.3 Finite Element Analysis

The goal of developing a finite element analysis of the test article is to define the damping, stiffness and mass matrices that can be used to create a benchmark model of the structure that can be used for comparison, namely:

Fig. 2.2 Test article design analogous to slender flexible launch vehicle



$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = 0 \quad (2.1)$$

$$[I]\{\ddot{\theta}\} + [C]\{\dot{\theta}\} + [K]\{\theta\} = 0 \quad (2.2)$$

Where $[M]$, $[C]$, $[K]$, $[I]$ are the mass, damping, stiffness and rotational inertia matrices and $\{x\}$, $\{\theta\}$ are the translational and rotational degrees of freedom respectively. Dimensions of the test article were chosen based on the first three weak natural frequencies. The structural model reflects the integrated test article including every component, such as gas thrusters, FBG sensors, IMUs, plumbing and wiring harness. The mass of all items installed on the beam was considered in the FEM model, and the ANSYS model was designed such that each FBG sensor is collocated with a FEM node as shown in Fig. 2.4.

Although the proposed models in this paper do not require estimation of damping, this was done as part of the benchmark state-space model of the test article. Damping measurements were done using the half power method to estimate damping ratio at each natural frequency. The damping ratio for each modal frequency is estimated as shown in Eq. (2.3):

$$\zeta = \frac{f_2 - f_1}{2f_{res}} \quad (2.3)$$

Where f_{res} is the corresponding resonant peak and f_1 , f_2 are the frequencies corresponding to the peak amplitude divided by the square root of two. According to Raleigh damping theory, the damping matrix $[C]$ can be calculated based on two coefficients $\{\alpha, \beta\}$ that are calculated using a two-equation with two-unknown system based on the first two natural frequencies of a the test article as shown in Eq. (2.4).

$$\zeta = \frac{1}{2} \left(\frac{\alpha}{\omega_r} + \beta\omega_r \right) \Rightarrow [C] = \alpha[M] + \beta[K] \quad (2.4)$$

After $\{\alpha, \beta\}$ are updated in the ANSYS model, the C matrix can be extracted from ANSYS to provide the set of beam model matrices $\{M$ (mass), K (stiffness), C (damping) $\}$. The beam model matrices were verified in the frequency domain by comparing against the measured frequency response function (FRF) as shown in Fig. 2.5. The experimental FRF was measured by using input base motion with band limited noise displacement (PRBS), 0.01–30 Hz, and displacement, rate of rotation and acceleration output at node 69/80.

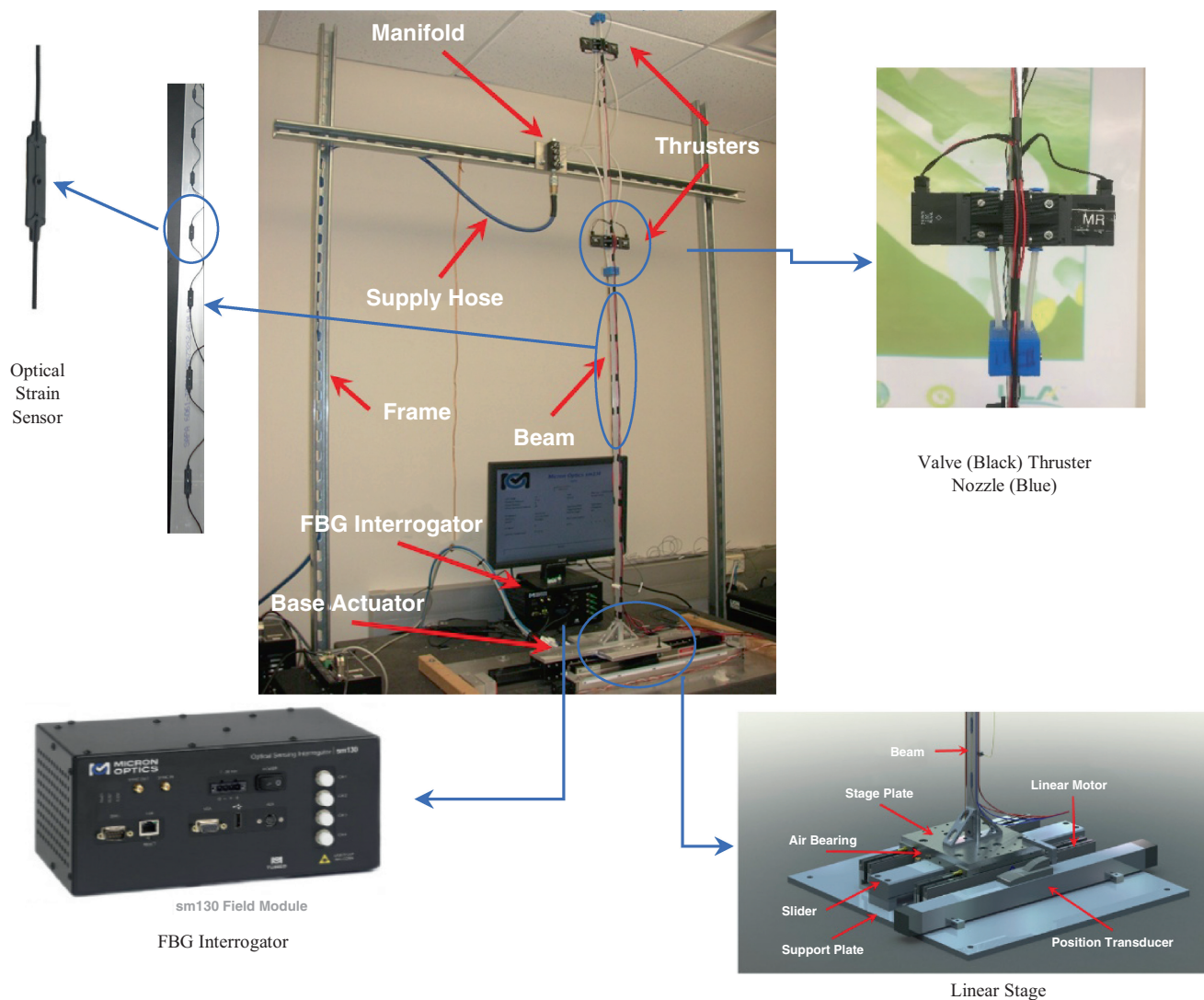


Fig. 2.3 Full test article setup

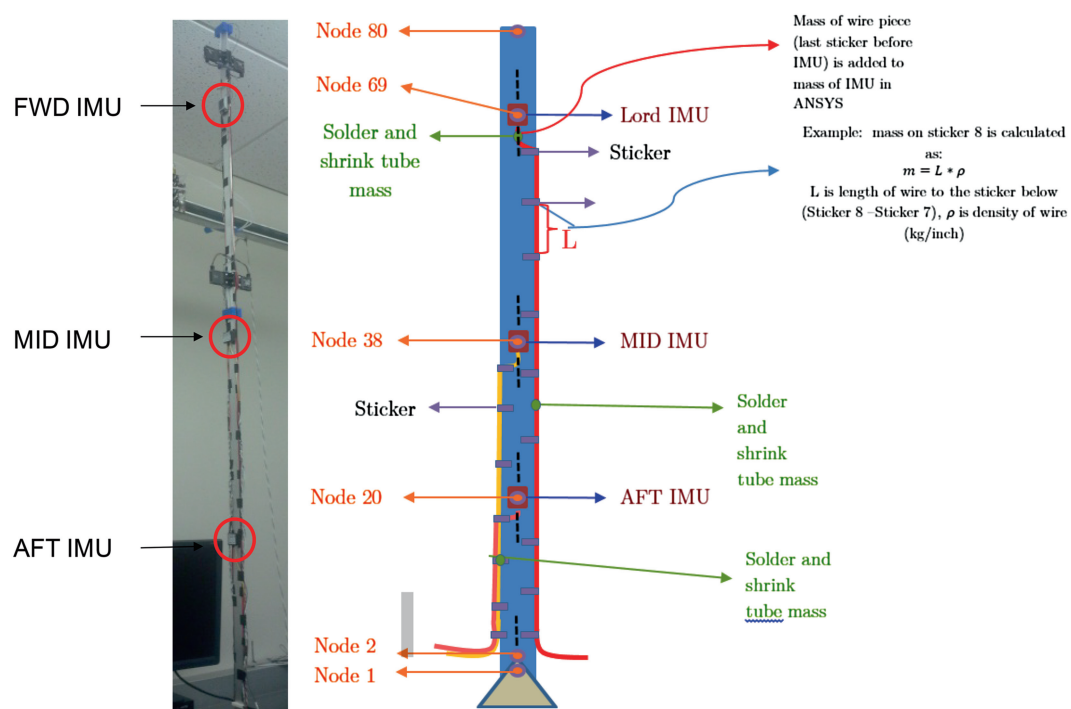


Fig. 2.4 Test article assembly and FEM node location on test article

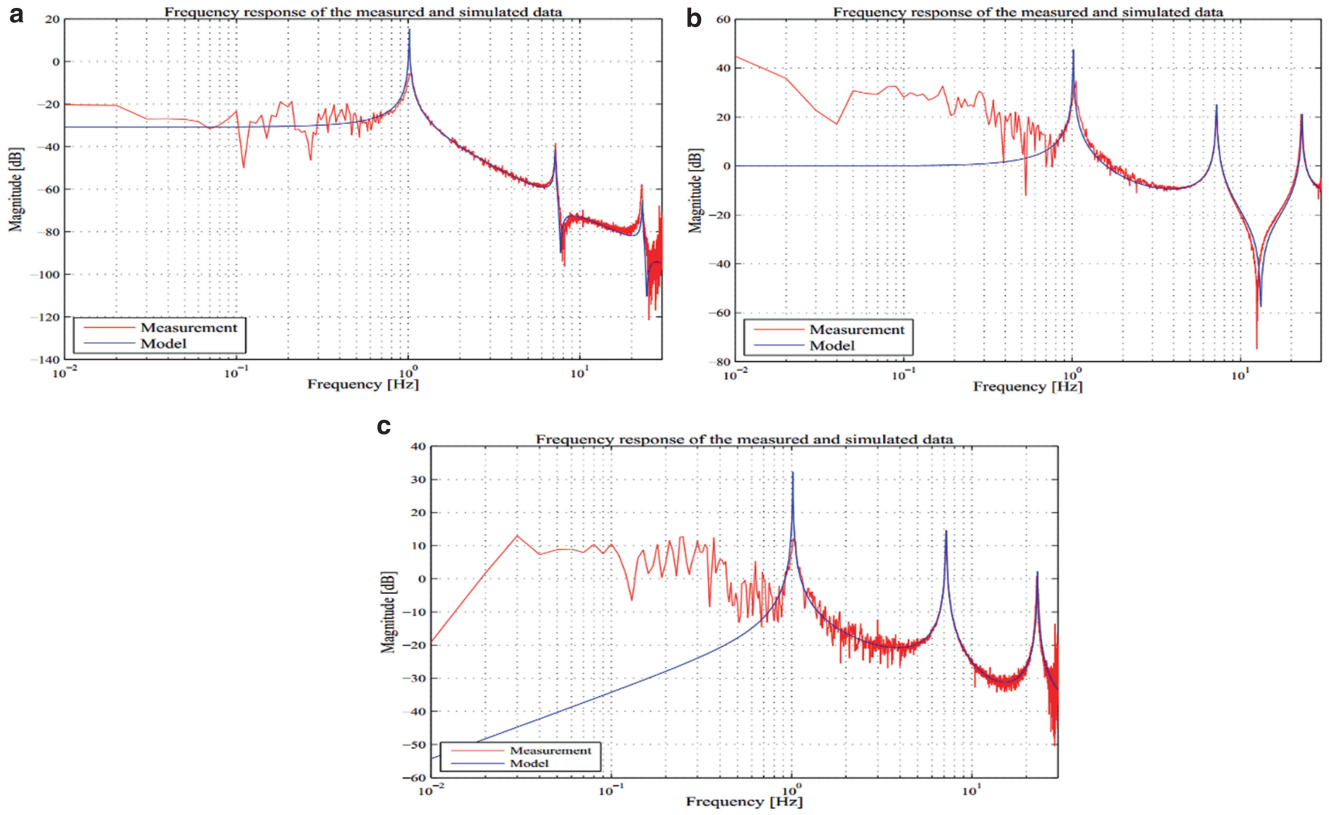


Fig. 2.5 Measured vs. predicted FRF, input base motion to output (a) deflection (b) rate of rotation (c) acceleration for node 69

2.4 Model Formulation

Two different novel modeling algorithm are discussed for developing FBG-IMU. Both take inputs from the FBG array and forward IMU time-domain data along with the launch vehicle's FEM (Finite Element Method) structural model and desired location(s) on launch vehicle, and output the estimated IMU data (Rate of rotation, acceleration) and deflection at any desired locations within the structure. While mode-shapes in the first method are calculated using $[M]$ and $[K]$ matrices the from finite element analysis of the test article, in the second method the mode-shapes are estimated by the use of Legendre polynomials.

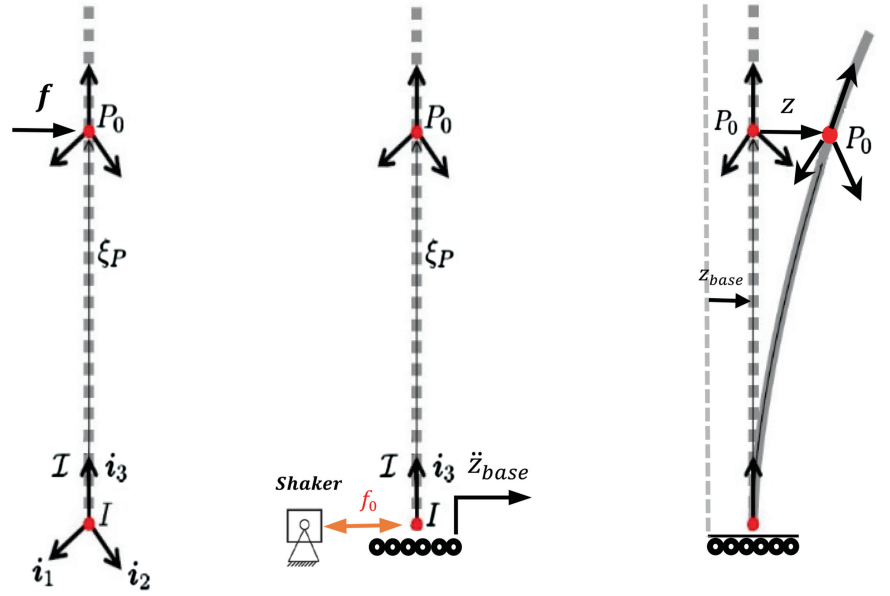
2.5 FBG-IMU System Development (First Approach)

Consider a linear structure subject to a point force f , and a structural excitation given by a single axis base acceleration $\ddot{z}_{base}$ shown in Fig. 2.6. The system may be considered as placed on a fictitious shaking table excited by an (unknown) reaction force f_0 . Hamilton's principle shown in Eq. (2.5) states that the variational indicator vanishes for arbitrary virtual variations of the path between two instants t_1 and t_2 compatible with the kinematics, and such that the configuration is fixed at t_1 and t_2

$$\int_{t_1}^{t_2} (\delta L + \delta W_{nc}) dt = \int_{t_1}^{t_2} [\delta(T - V) + \delta W_{nc}] dt = 0 \quad (2.5)$$

In (Eq. (2.5)) T is the kinetic energy, V the strain energy and δW_{nc} is the virtual work associated with the non-conservative forces (the external force f and the support reaction f_0 in this case). One can decomposes the global displacements Z (generalized displacements including translational as well as rotational degrees of freedom) into the rigid body mode and the flexible motion relative to the base as shown in Eq. (2.6).

Fig. 2.6 Flexible structure with sensor at point P, input force f and base acceleration $\ddot{z}_{base}$



$$Z = 1z_0 + z \quad (2.6)$$

Where z_0 is the support motion, 1 is the unit rigid body mode (all translational d.o.f. along the axis of excitation are equal to 1 and the rotational d.o.f. are equal to 0). The virtual displacements satisfy Eq. (2.7).

$$\delta Z = 1\delta z_0 + \delta z \quad (2.7)$$

Where δz_0 is arbitrary and δz satisfies the clamped boundary conditions at the base. Let b be the influence vector of the external loading f (for a point force, b contains all 0 except 1 at the d.o.f. where the load is applied), so that $b^T Z$ is the generalized displacement of the d.o.f. where f is applied. The various energy terms involved in Hamilton's principle are:

$$T = \frac{1}{2} \dot{Z}^T M \dot{Z} \quad (2.8)$$

$$T = \frac{1}{2} \dot{z}^T M \dot{z} \quad (2.9)$$

$$\delta W_{nc} = f_0 \delta z_0 + f b^T \delta Z = (f_0 + f b^T) \delta z_0 + f b^T \delta z \quad (2.10)$$

One notices that the kinetic energy depends on the absolute velocity while the strain energy depends on the flexible motion alone that is the motion relative to the base. For a point force, $b^T 1 = 1$. Substituting into the variational indicator, one finds:

$$\int_{t_1}^{t_2} \left[\dot{Z}^T M \delta \dot{Z} - z^T K \delta z + (f_0 + f b^T) \delta z_0 + f b^T \delta z \right] dt = 0 \quad (2.11)$$

And upon integrating the first term of Eq. (2.11) By parts and taking into account that $\delta Z(t_1) = \delta Z(t_2) = 0$, for arbitrary δx_0 and δy . It follows that:

$$M \ddot{Z} + K z = b f \quad (2.12)$$

$$f_0 = 1^T M \ddot{Z} - f \quad (2.13)$$

The presence of f in the right side of this equation is necessary to achieve static equilibrium; it is often ignored in the formulation. Combining Eqs. (2.12) and (2.13), one can compute the classical equation expressed in rigid body coordinate frame:

$$M\ddot{z} + C\dot{z} + Ky = -M1\ddot{Z} + bf \quad (2.14)$$

$$f_0 = -1^T(Kz + C\dot{z}) \quad (2.15)$$

From now on, we will assume $f = 0$ to focus on the response to the support acceleration. Now let $\omega_1, \dots, \omega_n$ and $\phi_z = (\phi_{1z}, \phi_{2z}, \dots, \phi_{nz})$ be natural frequencies and mode shapes of the structure, from mode superposition theory one can have:

$$\vec{z} = \phi_z \vec{q} \quad (2.16)$$

Equation (2.16) performs a change of variables from physical coordinates z (relative to rigid body frame) to modal coordinates q . Taking second derivative of Eq. (2.16) with respect to time results in [3]:

$$\ddot{\vec{z}} = \phi_z \ddot{\vec{q}} \quad (2.17)$$

$$\ddot{\vec{q}} = \ddot{\vec{q}} \quad (2.18)$$

To consider the relationship between strain, deflection and motion relative to the inertial frame, consider first the case of pure bending on the vertical plane. Notice that accelerometer readings, which are relative to the inertial frame but expressed on the flexible body frame (FBF), include rigid body rotation, rotation due to bending, and gravity. FBG sensors measure the motion of individual points in the structure relative to the rigid body frame (RBF). The FBG acceleration (second derivative of deflection at point P) is therefore the acceleration of P relative to the RBF. The vector of displacements in the RBF is calculated from the vector of strain measurements by the strain to deflection conversion [] which is shown in Eq. (2.19). One can rewrite Eqs. (2.17) and (2.18) in a state-space form, this is shown in Eqs. (2.20) and (2.21). $[C_{out}]$ and $[D_{out}]$ in Eq. (2.21) can be chosen differently depending on the desired output. one important point to notice is that as mentioned earlier The FEM model of the modified test article has 80 nodes and 79 mass elements. Since each element has 6 degrees of freedom, the mass $[M]$, stiffness $[K]$ and all d.o.f mode shape matrix $[\Phi]$ are 474×474 matrices. $[\Phi]$ matrix includes three deflection mode shape matrices : $[\phi_x]_{79 \times 79}$, $[\phi_y]_{79 \times 79}$ and $[\phi_z]_{79 \times 79}$ and three rotation mode shape matrices: $[\theta_x]_{79 \times 79}$, $[\theta_y]_{79 \times 79}$ and $[\theta_z]_{79 \times 79}$. The deflection and rotation mode shape matrix $[\phi_z]_{79 \times 79}$ and $[\theta_y]_{79 \times 79}$ are extracted from the 474×474 ANSYS modal matrix and are used in the both first and second FBG-IMU system modeling approaches (Fig. 2.7).

$$\ddot{\vec{z}}_{N_s \times 1} = \gamma [\phi_z]_{N_s \times N_m} \left([\Psi_z]_{N_m \times N_s}^T [\Psi_z]_{N_s \times N_m} \right)^{-1} [\Psi_z]_{N_m \times N_s}^T \ddot{\vec{z}}_{N_s \times 1} \quad (2.19)$$

$$\begin{bmatrix} \dot{\vec{q}}_{N_m \times 1} \\ \ddot{\vec{q}}_{N_m \times 1} \end{bmatrix} = \begin{bmatrix} 0_{N_m \times N_m} & I_{N_m \times N_m} \\ 0_{N_m \times N_m} & 0_{N_m \times N_m} \end{bmatrix} \begin{bmatrix} \vec{q}_{N_m \times 1} \\ \dot{\vec{q}}_{N_m \times 1} \end{bmatrix} + \begin{bmatrix} 0_{N_m \times N_s} \\ (\phi_z^T \phi_z)^{-1} \phi_z^T N_m \times N_s \end{bmatrix} \begin{bmatrix} \ddot{\vec{z}}_{N_s \times 1} \end{bmatrix} \quad (2.20)$$

$$y_{out} = [C_{out}] \begin{bmatrix} \dot{\vec{q}}_{N_m \times 1} \\ \ddot{\vec{q}}_{N_m \times 1} \end{bmatrix} + [D_{out}] \begin{bmatrix} \ddot{\vec{z}}_{N_s \times 1} \end{bmatrix} \quad (2.21)$$

$$y_{out} \rightarrow \text{Deflection} \rightarrow [C_{out}] = \begin{bmatrix} \phi_z(\zeta_p)_{1 \times N_m} & 0_{1 \times N_m} \end{bmatrix} \quad (2.22)$$

$$y_{out} \rightarrow \text{rate of rotation} \rightarrow [C_{out}] = \begin{bmatrix} 0_{1 \times N_m} & \theta_y(\zeta_p)_{1 \times N_m} \end{bmatrix} \quad (2.23)$$

$$y_{out} \rightarrow \text{acceleration} \rightarrow [C_{out}] = [0_{1 \times 2N_m}], [D_{out}] = [\phi_z(\zeta_p) \phi_z^+ 1 \times N_s] \quad (2.24)$$

Fig. 2.7 Relationship between deflection and motion relative to the inertial frame

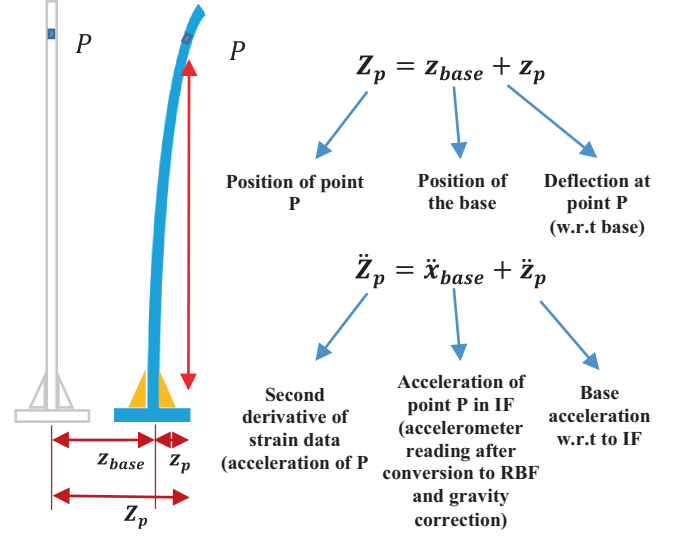
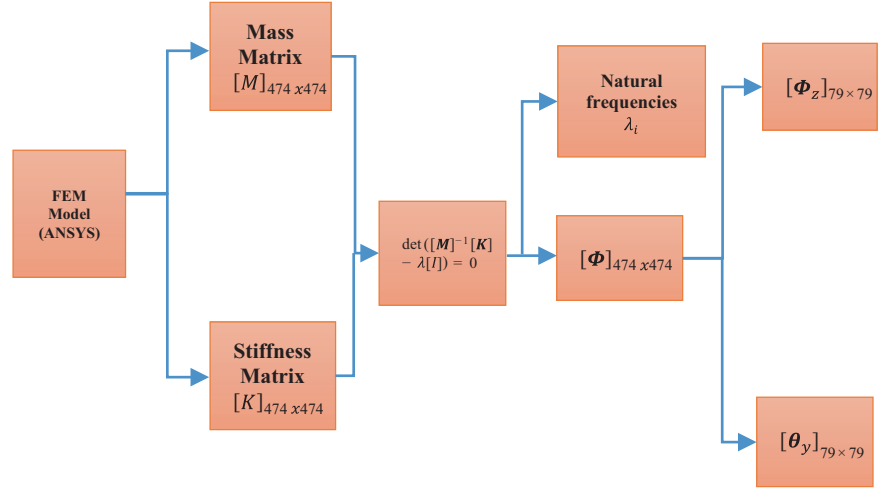


Fig. 2.8 Extraction of mode shape matrices from the ANSYS FEM Model



$[\phi_z]$ and $[\Psi_z]$ are matrix of deflection mode shape along z axis and matrix of strain mode shape both evaluated at N_s FBG sensor locations and for N_m number of modes; γ is a constant factor to convert strain from the beam's neutral axis to the beam's surface respectively. $\vec{z}_{N_s \times 1}$ and $\vec{\epsilon}_{N_s \times 1}$ are vectors of deflection and strain at N_s FBG sensor locations along the beam.

$\begin{bmatrix} \vec{q}_{N_m \times 1} \\ \dot{\vec{q}}_{N_m \times 1} \end{bmatrix}$ in Eq. (2.20) is a column vector of modal amplitudes and velocities and is the state vector of the state-space model. $\phi_z(\zeta_p)$ and $\theta_z(\zeta_p)$ In Eqs. (2.23) and (2.24) are row vectors generated by polynomial interpolation from N_m mode shapes at the desired output location (coordinate ζ along the beam) where one is interested to compute deflection, rate of rotation or acceleration at that point. The displacement mode shape matrix in z bending plane $[\phi_z]$, and the rotation mode shape matrix in the axis perpendicular to the bending plane $[\theta_y]$ are 79×79 matrices and are extracted from the all D.O.F. mode shape matrix Φ computed by ANSYS. An outline of the extraction of model and modal matrices from the ANSYS FEM model is shown in Fig. 2.8.

2.6 Second Approach of FBG-IMU

The finite element (FE) model of the Euler-Bernoulli beam modeling is well established. The discrete FE equation of motion is:

$$[M]\ddot{\vec{u}} + [K]\{\vec{u}\} = \vec{Q} \quad (2.25)$$

Each node has a displacement (w_z) and a rotation (θ_y) degree of freedom, i.e.

$$\{u_i\} = \{w_{zi} \ \theta_{yi}\}^T \quad (2.26)$$

One of the major benefits of using a finite element is that the finite element method is compatible with both uniform and non-uniform beams. The finite element solution gives the nodal displacements w and rotations θ . The strain due to bending is given by Eq. (2.27).

$$\epsilon_x(x, t) = y_0 \frac{\partial^2 w_z(x, t)}{\partial x^2} \quad (2.27)$$

Where y_0 is the distance from the beam reference line to the location of the Fiber Optic Strain Sensor (FBG). The linear acceleration in lateral direction is given by Eq. (2.28):

$$a_z(x, t) = \ddot{w}_z(x, t) = \frac{\partial^2 w_z(x, t)}{\partial t^2} \quad (2.28)$$

The angular velocity can be derived from the rotation, i.e.

$$\omega_y(x, t) = \dot{\theta}_y(x, t) = \frac{\partial \theta_y(x, t)}{\partial t} \quad (2.29)$$

Since integration of Eq. (2.25) is not possible due to the unknown excitations $\vec{Q}$, a modal-based solution approach will be applied, i.e. the deformation of the beam is described as a linear combination of the mode shapes

$$w_z(x, t) = \sum_{i=1}^{\infty} \varphi_{iz}(x) q_i(t) \quad (2.30)$$

x is the normalized position along the beam, i.e. $x \in [0, 1]$, which is a necessary condition for using the Legendre polynomials to estimate mode shapes. To approximate the solution the mode shapes are truncated by only selecting the N_m natural modes, i.e.

$$w_z(x, t) = \sum_{i=1}^{\infty} \varphi_{iz}(x) q_i(t) = \begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ \varphi_{1z}(x) & \varphi_{2z}(x) & \dots & \varphi_{zN_m}(x) \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_{N_m}(t) \end{bmatrix} = \phi_z(x) \vec{q}(t) \quad (2.31)$$

Where $\varphi_{iz}(x)$ are the mode shapes of the beam evaluated at the normalized positions along the beam, $q_i(t)$ are the magnitudes of the corresponding modal deflection, N_n denotes the number of nodes in FEM model and $\phi_z \in \mathbb{R}^{N_n \times N_m}$ and $q \in \mathbb{R}^{N_n \times 1}$. The approximation of the strain is given by:

$$\vec{\epsilon}_x(x, t) = y_0 \sum_{i=1}^{\infty} \frac{\partial^2 \varphi_{iz}(x)}{\partial x^2} \vec{q}_i(t) = y_0 \frac{\partial}{\partial x^2} \begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ \varphi_{1z}(x) & \varphi_{2z}(x) & \dots & \varphi_{zN_m}(x) \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_{N_m}(t) \end{bmatrix} = y_0 \frac{\partial^2 \phi_z(x)}{\partial x^2} \vec{q}(t) \quad (2.32)$$

Where $\vec{\epsilon}_x(x, t) \in \mathbb{R}^{N_n \times 1}$. The linear acceleration in the lateral direction z is obtained by

$$a_z(x, t) = \sum_{i=1}^{\infty} \varphi_{iz}(x) \ddot{q}_i(t) = \sum_{i=1}^{\infty} -\omega_i^2 \varphi_{iz}(x) q_i(t) \quad (2.33)$$

In order to calculate the second spatial derivative of each mode $\frac{\partial^2 \phi_z(x)}{\partial x^2}$ as required in Eq. (2.32), an analytical expression for each mode is required. These expressions are found using the shifted Legendre polynomials, a set of complete and orthogonal polynomials which take input in the domain $[0, 1]$ and maps the output to the domain $[-1, 1]$. The general equations for the Legendre polynomial are given as

$$P_0(x) = 1 \quad (2.34)$$

$$P_1(x) = 2x - 1 \quad (2.35)$$

$$P_{i+1}(x) = \frac{(2i+1)(2x-1)P_i(x) - iP_{i-1}(x)}{i+1} \quad (2.36)$$

A discrete mode shape $\phi_j(x)$ is approximated by using linear combinations of the shifted Legendre polynomials

$$\phi_j(x) = a_{0j}P_0(x) + a_{1j}P_1(x) + \dots + a_{Mj}P_M(x) \quad (2.37)$$

Where M is the number of Legendre polynomials. To solve for the coefficients a_{ij} the following matrix equality is solved

$$\begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ \phi_{1z}(x) & \phi_{2z}(x) & \dots & \phi_{zN_m}(x) \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} = \begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ P_0(x) & P_1(x) & \dots & P_M(x) \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} \begin{bmatrix} a_{01} & a_{02} & \dots & a_{0N} \\ a_{11} & a_{01} & \dots & a_{1N} \\ a_{21} & a_{01} & \dots & a_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ a_{M1} & a_{M2} & \dots & a_{MN} \end{bmatrix} \quad (2.38)$$

Which can be rewritten as

$$\phi_z = PA \quad (2.39)$$

Where $P \in \mathbb{R}^{N_n \times (M+1)}$ and $A \in \mathbb{R}^{(M+1) \times N}$. The coefficients matrix A can be solved as

$$A = P^{-1}\phi_z = \begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ P_0(x) & P_1(x) & \dots & P_M(x) \\ \vdots & \vdots & \dots & \vdots \end{bmatrix}^{-1} \begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ \phi_{1z}(x) & \phi_{2z}(x) & \dots & \phi_{zN_m}(x) \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} \quad (2.40)$$

Note that a pseudo-inverse of the P matrix is required in order to solve the previous equation. Now that the coefficients matrix A is known the first and second order spatial derivatives of the mode shape matrix are given by Eqs. (2.41) and (2.42)

$$\frac{d\phi_z(x)}{dx} = \frac{dP(x)}{dx}A \quad (2.41)$$

$$\frac{d^2\phi_z(x)}{dx^2} = \frac{d^2P(x)}{dx^2}A \quad (2.42)$$

Now let's define the number of FBG sensors as N_s and the number of IMU units as N_{IMU} . The locations of the normalized FOSS measurements along the beam are given by

$$\vec{x}_{FBG} = [x_{FBG,1} \ x_{FBG,2} \ \dots \ x_{FBG,i} \ \dots \ x_{FBG,N_s}]^T, \ x_{FBG,i} \in [0, 1] \quad (2.43)$$

The locations of the normalized IMU measurements along the beam are given by

$$\vec{x}_{IMU} = [x_{IMU,1} \ x_{IMU,2} \ \dots \ x_{IMU,i} \ \dots \ x_{IMU,N_{IMU}}]^T, \ x_{IMU,i} \in [0, 1] \quad (2.44)$$

The Legendre polynomial matrix and the mode shape matrix can now be evaluated at all the measurement points

$$P(\vec{x}_{FBG}, \vec{x}_{IMU}) = \begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ P_0(\vec{x}_{FBG}, \vec{x}_{IMU}) & P_1(\vec{x}_{FBG}, \vec{x}_{IMU}) & \dots & P_M(\vec{x}_{FBG}, \vec{x}_{IMU}) \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} \quad (2.45)$$

$$\phi_z(\vec{x}_{FBG}, \vec{x}_{IMU}) = \begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ \phi_1(\vec{x}_{FBG}, \vec{x}_{IMU}) & \phi_2(\vec{x}_{FBG}, \vec{x}_{IMU}) & \dots & \phi_N(\vec{x}_{FBG}, \vec{x}_{IMU}) \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} \quad (2.46)$$

Where $P(\vec{x}_{FBG}, \vec{x}_{IMU}) \in \mathbb{R}^{(N_{FBG}+N_{IMU}) \times (M+1)}$ and $\phi_z(\vec{x}_{FBG}, \vec{x}_{IMU}) \in \mathbb{R}^{(N_{FBG}+N_{IMU}) \times (N)}$. Note that it is possible that $N_{FBG} + N_{IMU} \neq N$, i.e. the number of measurement points is not equal to the number of nodes from the finite element method or the positions corresponding to the nodes of the finite element model are not equal to the positions of the measurement points, if this is the case one should approximate the mode shapes at the desired measurement points. The coefficient matrix A is then given by

$$A = P(\vec{x}_{FBG}, \vec{x}_{IMU})^{-1} \phi_z(\vec{x}_{FBG}, \vec{x}_{IMU}) \quad (2.47)$$

Where $A \in \mathbb{R}^{(M+1) \times N}$. Now it's possible to solve for the modal solution, the strains are related to the modal deflections as follows:

$$\sum_{i=1}^{\infty} \frac{\partial^2 \varphi_{iz}(\vec{x}_{FBG})}{\partial x^2} \vec{q}_i(t) = \frac{\vec{\epsilon}_x(\vec{x}_{FBG}, t)}{y_0} \quad (2.48)$$

The modes are approximated by using the second spatial derivative of the shifted Legendre polynomial

$$\frac{\partial^2 P(\vec{x}_{FBG})}{\partial x^2} A \vec{q}(t) = \frac{\vec{\epsilon}_x(\vec{x}_{FBG}, t)}{y_0} \quad (2.49)$$

Which can be written as

$$B_1 \vec{q}(t) = C_1 \quad (2.50)$$

Where $B_1 \in \mathbb{R}^{(N_{FBG}) \times N}$ and $C_1 \in \mathbb{R}^{(N_{FBG}) \times 1}$. The linear acceleration at IMU locations are given by:

$$\sum_{i=1}^{\infty} \varphi_{iz}(\vec{x}_{IMU}) \ddot{q}_i(t) = \sum_{i=1}^{\infty} -\omega_i^2 \varphi_{iz}(\vec{x}_{IMU}) q_i(t) = \vec{a}_y(\vec{x}_{IMU}, t) \quad (2.51)$$

Which can be written as

$$B_2 \vec{q}(t) = C_2 \quad (2.52)$$

Where $B_2 \in \mathbb{R}^{(N_{IMU}) \times N}$ and $C_2 \in \mathbb{R}^{(N_{IMU}) \times 1}$. The system can now be solved by stacking the matrices and vectors and solve for $\vec{q}(t)$, i.e.

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \vec{q}(t) = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \rightarrow B \vec{q}(t) = C \quad (2.53)$$

The solution for $\vec{q}(t)$ is then given by:

$$\vec{q}(t) = B^{-1} C \quad (2.54)$$

2.7 Results

Both FBG-IMU modeling approaches were implemented in MATLAB and benchmarked using both synthetic data and FOSS measured strain data. FBG-IMU prediction for synthetic data for deflection and the acceleration prediction are shown in Figs. 2.9 and 2.10. Figure 2.9 shows the result for 1st mode beam excitation and Fig. 2.10 shows the result for beam excitation consisted of a combination of 1st, 2nd and 3rd modes. Figures 2.11, 2.12 and 2.13 correspond to FBG-IMU Predictions with FOSS strain measurements as input to the model. Three experiments were done in which beam's base was excited with 1st, 2nd and frequency sweep excitations and then both modeling approaches were compared.

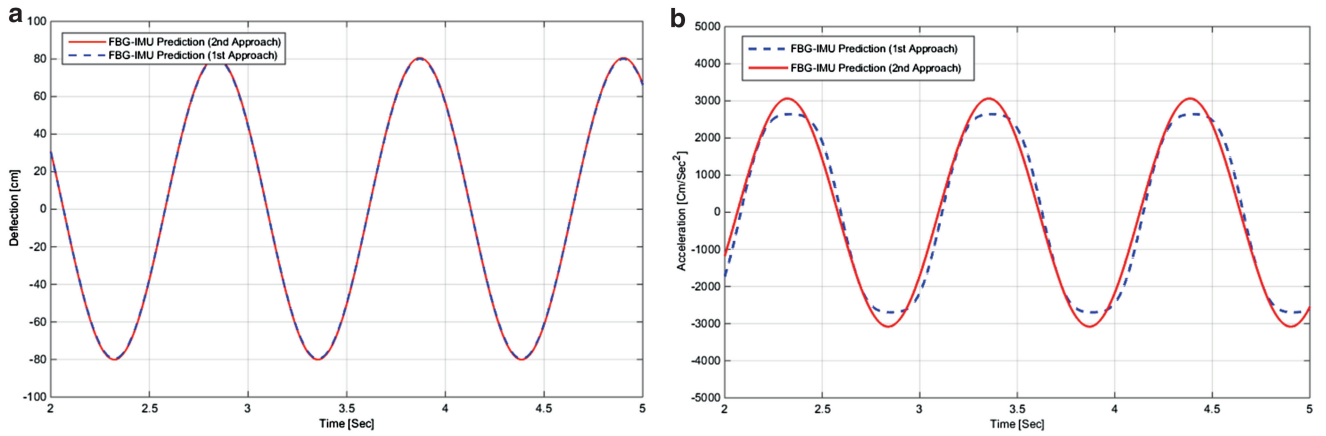


Fig. 2.9 FBG-IMU prediction, 1st approach vs. 2nd approach for deflection (*right*) and acceleration (*left*) using synthesized measurements at LORD IMU Location, 1st mode beam excitation

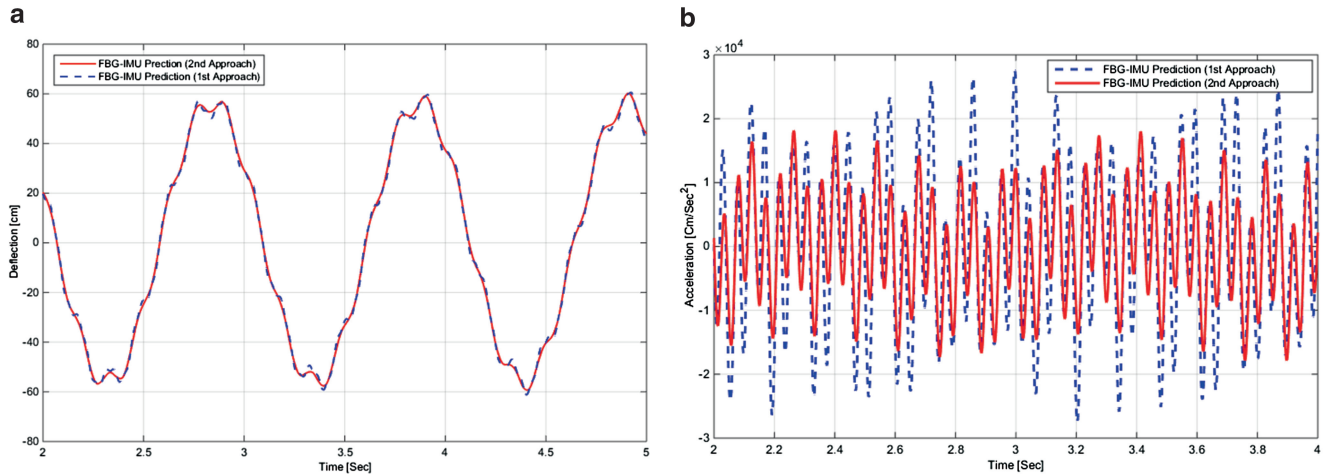


Fig. 2.10 FBG-IMU prediction for, 1st approach vs. 2nd approach for deflection (*right*) and acceleration (*left*) using synthesized measurements at LORD IMU location, beam excitation: combination of first three modes

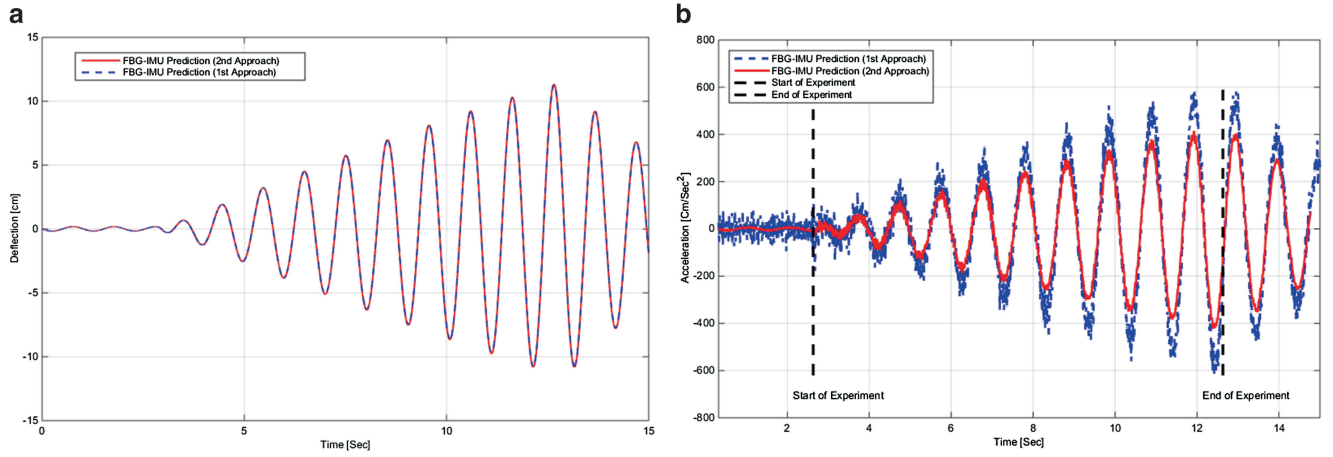


Fig. 2.11 FBG-IMU prediction for, 1st approach vs. 2nd approach for deflection (*right*) and acceleration (*left*) using FOSS measurements at LORD IMU location 1st mode base excitation

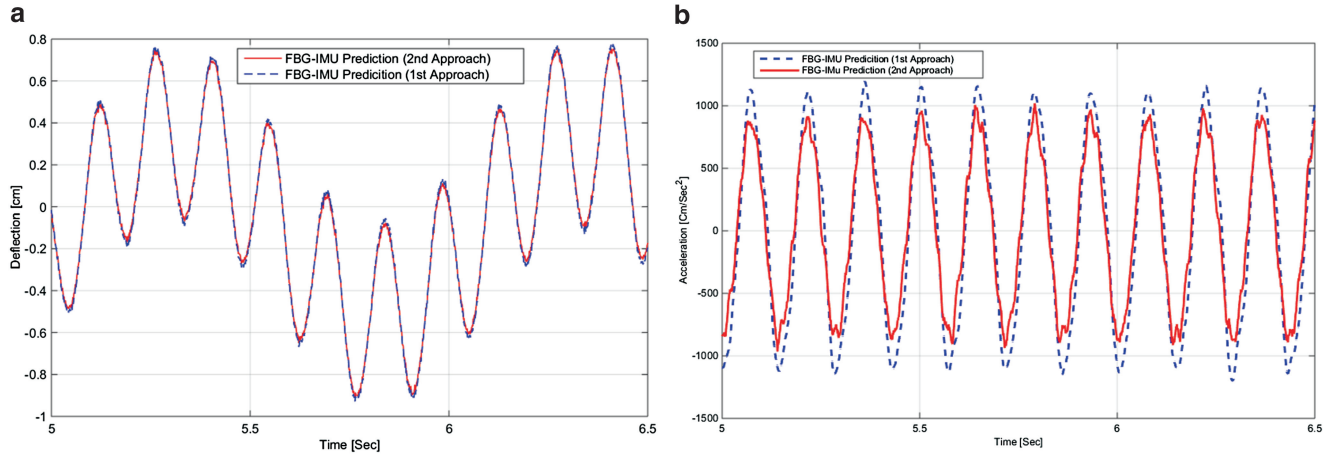


Fig. 2.12 FBG-IMU prediction for, 1st approach vs. 2nd approach for deflection (*right*) and acceleration (*left*) using FOSS measurements at LORD IMU location, 2nd mode base excitation

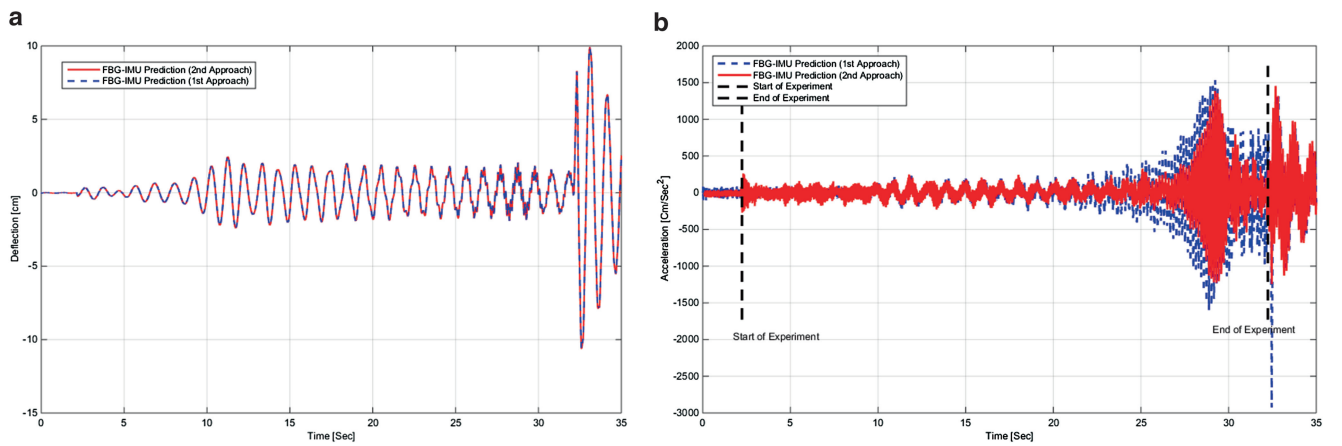


Fig. 2.13 FBG-IMU prediction for, 1st approach vs. 2nd approach for deflection (*right*) and acceleration (*left*) using FOSS measurements at LORD IMU location, frequency sweep sine (0.5–10 Hz) base excitation

2.8 Conclusions and Future Work

The FBG-IMU system is an innovative algorithm that enables accurate prediction of deflection, rate of rotation and acceleration at any point in a flexible structure relative to the rigid body frame. The FBG-IMU algorithm is based on modal matrices $\{[M], [k], [\Phi], [\Theta]\}$ that are calculated using only a finite element model of the structure. The FBG-IMU system uses an array of strain measurements at multiple points in the structure, which allows accurate estimation of the states of the flexible structure with no need of estimation of load forces or a damping matrix, or no need to assume that the damping is linear and viscous, as current structural models of flexible structures are forced to assume. The proposed algorithms are well suited for implementation on real-time, and have the potential to provide future tools for real-time estimation and control of the flexible dynamics of aerospace structures.

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