

Chapter 2

Prediction of the Coupled Impedance from Frequency Response Data

Ramona Fagiani, Elisabetta Manconi, and Marcello Vanali

Abstract This work deals with the common case of large mechanical systems which can be modelled considering two main sub-structures: a source structure, corresponding for example to the engine, and a receiving structure, as the accommodation area/cabin in these machines. Numerical models are highly desired for predicting the vibroacoustic behaviour of the receiver at a design stage and for design optimisation. In many cases the mechanical systems must be studied using sub-structuring techniques

Here a standard frequency response coupling technique (or impedance coupling technique) is applied for the prediction of the complex power transmitted between a source structure and a receiving structure in a multi-point-connection case. The data required in this case are the frequency response functions (FRFs) of both the source and the receiver and the velocity at the connecting points under operating conditions. Experimental aspects and issues associated with this technique are investigated considering a simplified mechanical model. This consists of a driven beam connected in two or more points to a source. Results are discussed and compared with those obtained from a numerical model.

Keywords Coupled impedance • Coupled structures • Modal analysis

2.1 Introduction

Determining the vibro-acoustic behaviour of complex mechanical structures can be very challenging, since several components be simultaneously involved [1, 2]. In order to quantify the force transmitted through substructures for structure-born noise estimation, the knowledge of mechanical mobility and impedance at the connecting points is necessary [3]. In this regard, Dynamic Substructuring (DS) [4] plays a significant role for investigating a structural system by the analysis of the single sub-structure dynamic features and the way in which they combine. The substructuring approach has some significant advantages over the consideration of the full structure, in particular it allows evaluation of the dynamic behaviour of large and complex structures when the analysis of the all structure is unrealistic. Moreover, combining analytically, numerically or experimentally single modelled parts of the whole structure, local dynamic behaviour can be recognized more easily and neglected when it has no significant impact on the assembled system. In many cases this results in a simplified and clearer representation of the structure overall dynamics and in a significant reduction of the analysis time.

Despite all the attention received in the last years, DS still requires additional research, especially concerning a number of challenges associated with the coupling of experimentally obtained substructures. Just to cite a few works, a review of the difficulties encountered in experimental DS are summarised in [4], errors related to truncation of modal degrees of freedom in [5] and the estimation of rotational degrees of freedom in [6], inclusion of rigid body modes in [7], non-linear mechanical coupling in [8], random measurement noise which may result in low signal-to-noise ratio, especially in the case of lightly damped structures in [9], inexact determination of the antiresonance modes in [10].

In this paper a frequency response coupling technique (or impedance coupling technique) is revised for the prediction of the complex impedance. Data required are the frequency response functions (FRFs) of the source and the receiver and the velocity/accelerations at the connecting points under operating conditions. These can be either experimental data or numerical data obtained from finite element analysis or modal analysis. The FRF coupling technique is first applied to a simple case study concerning two beams for validating the method numerically and experimentally and for investigating its accuracy according to the frequency range of interest.

R. Fagiani • E. Manconi • M. Vanali (✉)

Industrial Engineering Department, Università degli Studi di Parma, Parco Area delle Scienze 181/A, 43124 Parma, Italy
e-mail: marcello.vanali@unipr.it

2.2 FRF Coupling

This section is dedicated to revise a methodology for coupling a flexible structure to another structure (e.g. a source machine) using the FRF matrices, e.g. [4]. The objective is to describe an approach for evaluating the power input from a large machine to a flexible structure when forces and responses data of the separated structures can be measured and only acceleration/velocity responses data of the whole coupled structure in operation are available.

Figure 2.1 shows structure 1 (source) connected to structure 2 (receiver) through multiple points together with the model the free body diagram for each substructure. Each connection is modelled using a connector system C_i . These can be a rigid connections or flexible-damped connections. Dynamics of structures 1 and 2 is often studied using a discretised model, e.g. as in standard FE or modal analysis, and generally the DOFs at which the internal forces act, $\mathbf{u}_1 = [u_{11}, u_{12}, \dots, u_{1n}]^T$; $\mathbf{u}_2 = [u_{21}, u_{22}, \dots, u_{2n}]^T$, are a subset of the DOFs used to discretise the structure, $\mathbf{y}_1 = [y_{11}, y_{12}, \dots, y_{1m}]^T$ and $\mathbf{y}_2 = [y_{21}, y_{22}, \dots, y_{2q}]^T$. The same situation arises in experimental modal analysis when the responses evaluated at the connection points are a subset of the responses used for characterizing the dynamic structure behaviour. This situation is pictured in Fig. 2.1.

In this case, the DOFs of structure j corresponding to the connected ones can be expressed in terms of the DOFs of the structure using an influence matrix $\mathbf{W}_j = [\mathbf{W}_{j1} \dots \mathbf{W}_{jn}]$. Here $[\mathbf{W}_{j1} \dots \mathbf{W}_{jn}]$ are column vectors of zeros and ones, e.g. $\mathbf{W}_{j1} = [0, 0, 1, 0 \dots 0]^T$, where the ones identifies the position of the DOFs corresponding to the DOFs of structure on which the internal forces $\mathbf{F}_{Cj}$ act. Thus the influence matrix relates displacements and internal forces of structure 2, structure 1, and connectors C_j as

$$\begin{aligned} \mathbf{F}_{C1}(\omega) &= -\mathbf{W}_1 \mathbf{F}_{1C}, \mathbf{u}_1 = \mathbf{W}_1^T \mathbf{y}_1 \\ \mathbf{F}_{C2}(\omega) &= -\mathbf{W}_2 \mathbf{F}_{2C}, \mathbf{u}_2 = \mathbf{W}_2^T \mathbf{y}_2 \end{aligned} \quad (2.1)$$

Dynamic equilibrium of each free body diagram in Fig. 2.1 can be written as

$$\begin{cases} \mathbf{F} + \mathbf{F}_{C1} = \mathbf{H}_1(\omega) \mathbf{y}_1 \\ \mathbf{F}_{1C} = \mathbf{H}_c(\omega) (\mathbf{u}_1 - \mathbf{u}_2) \\ \mathbf{F}_{C2} = \mathbf{H}_2(\omega) \mathbf{y}_2 \end{cases} \Rightarrow \begin{cases} \mathbf{F} - \mathbf{W}_1 \mathbf{F}_{1C} = \mathbf{H}_1(\omega) \mathbf{W}_1 \mathbf{u}_1 \\ \mathbf{F}_{1C} = \mathbf{H}_C(\omega) (\mathbf{u}_1 - \mathbf{u}_2) \\ \mathbf{F}_{C2} = \mathbf{H}_2(\omega) \mathbf{y}_2 \end{cases} \quad (2.2)$$

Since $\mathbf{F}_{C1} = -\mathbf{W}_1 \mathbf{F}_{1C}$, combining the first and the second of Eq. (2.6) the relative displacement becomes $(\mathbf{u}_1 - \mathbf{u}_2) = [\mathbf{H}_1(\omega) \mathbf{W}_1 + \mathbf{W}_1 \mathbf{H}_C(\omega)]^{-1} \mathbf{F} - [\mathbf{H}_1(\omega) \mathbf{W}_1 + \mathbf{W}_1 \mathbf{H}_C(\omega)]^{-1} \mathbf{H}_1(\omega) \mathbf{W}_1 \mathbf{u}_2$. The internal force transmitted to structure 2 is thus

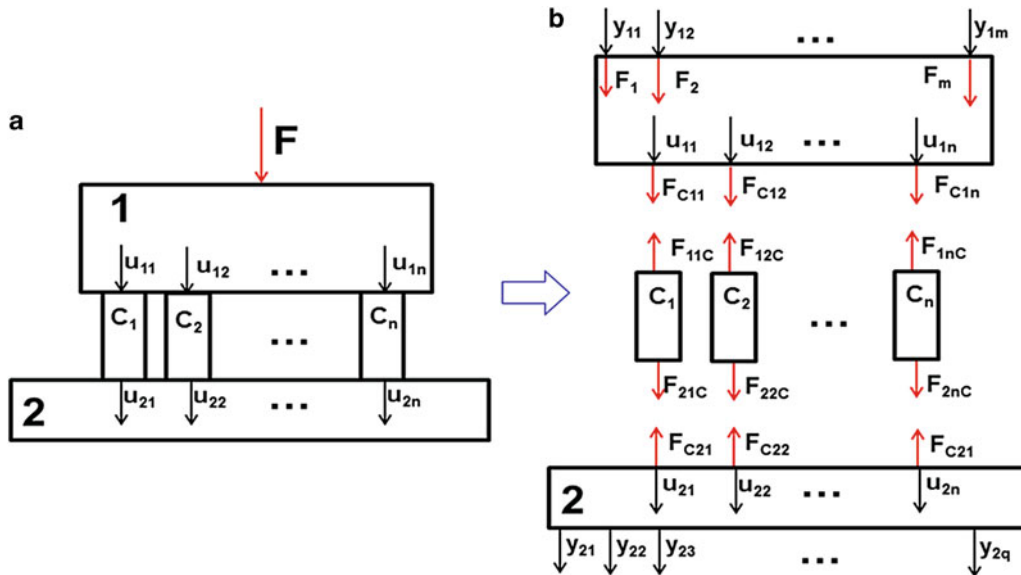


Fig. 2.1 Schematic representation of a machine-source (structure 1) connected to a receiving structure (structure 2) through multiple points: (a) coupled system, (b) uncoupled system

$$\mathbf{F}_{2C} = -\mathbf{F}_{1C} = -\mathbf{H}_C(\omega)\mathbf{U}_1(\omega)^{-1}\mathbf{F} + \mathbf{H}_C(\omega)\mathbf{U}_1(\omega)^{-1}\mathbf{H}_1(\omega)\mathbf{W}_1\mathbf{u}_2 \quad (2.3)$$

where $\mathbf{U}_1(\omega) = \mathbf{H}_1(\omega)\mathbf{W}_1 + \mathbf{W}_1\mathbf{H}_C(\omega)$. Using Eq. (2.5)

$$\mathbf{F}_{C2} = \mathbf{W}_2\mathbf{H}_C(\omega)\mathbf{U}_1(\omega)^{-1}\mathbf{F} - \mathbf{W}_2\mathbf{H}_C(\omega)\mathbf{U}_1(\omega)^{-1}\mathbf{H}_1(\omega)\mathbf{W}_1\mathbf{W}_2^T\mathbf{y}_2 \quad (2.4)$$

Given $\mathbf{F}_{C2} = \mathbf{H}_2(\omega)\mathbf{y}_2$, Eq. (2.8) becomes

$$\mathbf{F} = \left[\mathbf{U}_1(\omega)\mathbf{H}_C(\omega)^{-1}\mathbf{W}_2^T\mathbf{H}_2 + \mathbf{H}_1(\omega)\mathbf{W}_1\mathbf{W}_2^T \right] \mathbf{y}_2 \quad (2.5)$$

Hence the frequency response matrix of the coupled structure evaluated at the receiver is

$$\mathbf{H}(\omega) = \mathbf{U}_1(\omega)\mathbf{H}_C(\omega)^{-1}\mathbf{W}_2^T\mathbf{H}_2 + \mathbf{H}_1(\omega)\mathbf{W}_1\mathbf{W}_2^T \quad (2.6)$$

If structure 2 is excited by the external force $\mathbf{F}$ while structure 1 is free to vibrate, the transfer matrix modifies as

$$\mathbf{H}(\omega) = \mathbf{H}_2 + \mathbf{W}_2\mathbf{H}_C(\omega)\mathbf{U}_1(\omega)^{-1}\mathbf{H}_1(\omega)\mathbf{W}_1\mathbf{W}_2^T \quad (2.7)$$

Once the transfer matrix of the coupled structure is known, the impedance and inertance matrices are $\mathbf{Z}(\omega) = \mathbf{H}(\omega)/i\omega$ and $\mathbf{I}(\omega) = -\mathbf{H}(\omega)/\omega^2$.

The above formulation has been applied to a simple experimental case and results are discussed in the next section.

2.3 Experimental Setup

The example shown in this section consists of a simplified model. The model is used for validating the coupling technique. Two steel beams are considered as depicted in Fig. 2.2. The first beam, named 'source', is linked to the other one, called 'receiver', at two points (points 1 and 2 in the figure). In the real case these are welded, thus the connection in the numerical case is modelled as rigid. Material properties are: Young modulus 210 GPa, density 7860 kg/m³, Poisson's coefficient 0.33. The receiver is 3240 mm long and has a square cross section of 25 mm. The source is 400 mm long and has a cross section of 25 × 50 mm. Both the coupled and uncoupled source and receiver are suspended by two rubber bands in order to simulate free-free boundary conditions.

Source and receiver accelerance matrices, $\mathbf{G}_S$ and $\mathbf{G}_R$ respectively are evaluated experimentally or numerically. These are obtained exciting the structure and evaluating the acceleration at the connection points $\ddot{\mathbf{y}}_1 = \mathbf{G}_S\mathbf{F}_1$ and $\ddot{\mathbf{y}}_2 = \mathbf{G}_R\mathbf{F}_2$. Thus, according to the scheme in Fig. 2.1, the impedance matrices are $\mathbf{H}_S = -\omega^2\mathbf{G}_S^{-1}$ $\mathbf{H}_R = -\omega^2\mathbf{G}_R^{-1}$. Equation (2.7) is then

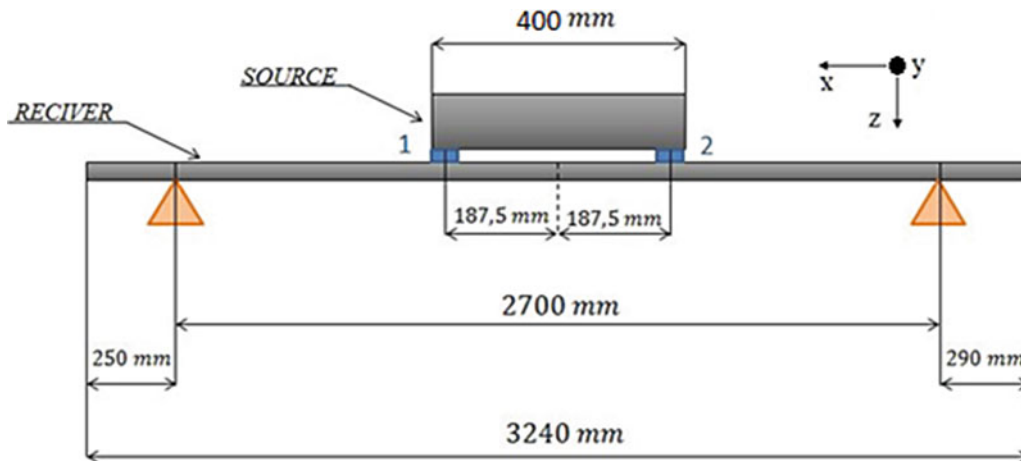


Fig. 2.2 Schematic representation of the case study

applied to obtain the FRFs matrix of the coupled structure $\mathbf{H}_{RS}$. Since only the degrees of freedom corresponding to points 1 and 2 are coupled and the connections are modelled as rigid, becomes Eq. (2.7) $\mathbf{H}_{RS} = \mathbf{H}_R + \mathbf{H}_S$. The accelerance matrix $\mathbf{G}_{RS}$ of the coupled structure can be also rewritten explicitly in terms of $\mathbf{G}_S$ and $\mathbf{G}_R$ using the Woodbury matrix identity as shown in [11, 13, 14]

$$\mathbf{G}_{RS} = \mathbf{G}_R - \mathbf{G}_R(\mathbf{G}_R + \mathbf{G}_S)^{-1}\mathbf{G}_R \quad (2.8)$$

Due the symmetry of the structure, the matrices are expected to be symmetric and the elements on the diagonal corresponding to point 1 and 2 are expected to be the same.

2.3.1 Experimental Case Study

The accelerance matrices of the single beams and of the coupled structure have been measured by impact testing (impact hammer PCB 086C03) in the nearby area of the triaxial accelerometers (PCB M352C66) placed at points 1 and 2. The points correspond to the connection points of the coupled structure (Figs. 2.2, 2.3, and 2.4). All data were sampled at 6400 Hz.

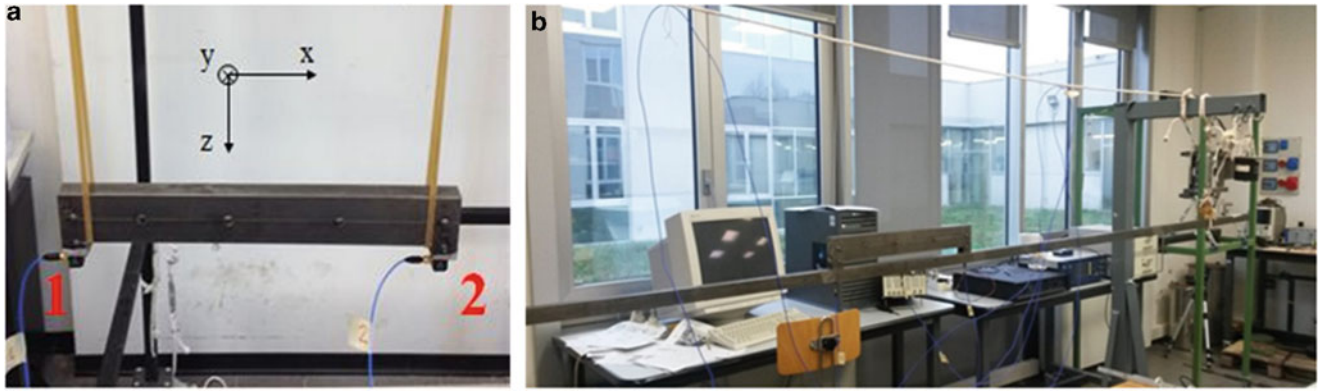


Fig. 2.3 Experimental case study: (a) source, (b) coupled structure

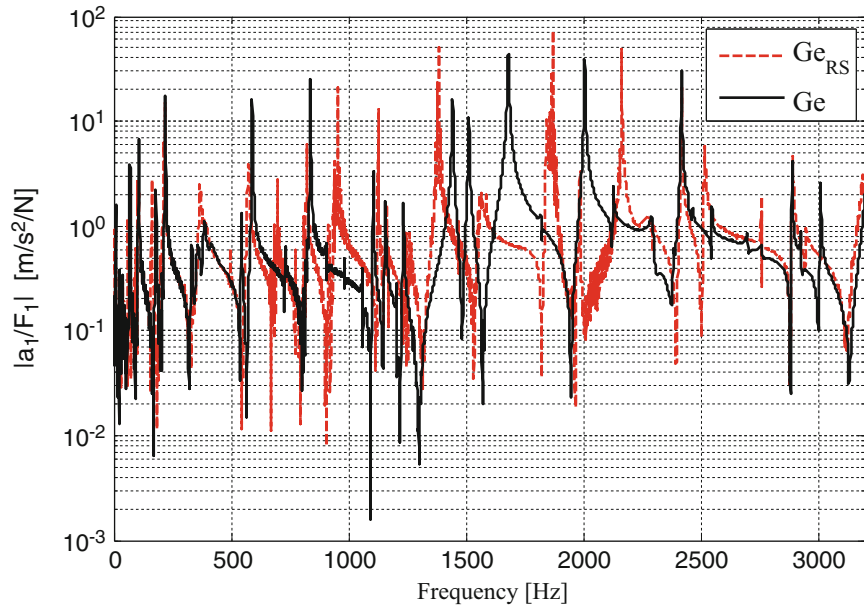


Fig. 2.4 Experimental case study. Comparison between measured FRF for the coupled structure $\mathbf{G}_e$ and those obtained coupling the FRF matrices of the source and the receiver $\mathbf{G}_{eRS}$ using Eq. (2.12)

The structures were excited in the all x, y, z and accelerations were measured in the same directions at point 1 and point 2. The measure of rotations for a simple structure like this is very difficult and becomes impossible for more realistic structures, thus moments and rotational responses were disregarded. All the cross combination of input force and output acceleration were measured. Therefore the size of the FRF matrices is 6×6 . The experimental data were processed and H1 method was used as the FRF estimator [12].

Figure 2.4 shows the comparison between the modulus of the measured acceleration FRF function $\mathbf{G}_e$ of the coupled structure at point and those obtained applying Eq. (2.10) $\mathbf{G}_{eRS}$, viz. coupling the measured FRF matrices of the source $\mathbf{G}_{eS}$ and the receiver $\mathbf{G}_{eR}$. Only the component along the z -axis is showed as an example but the same results apply to all other directions. As expected, this first sets of experimental results shows that the coupling model gives good prediction at low frequency. At higher frequency results become less and less reliable. Differences can be due to several factors such as a lack in modelling the connections between the source and the receiver, uncertainties in measuring, e.g. force applied in z direction is not perfectly aligned along the z axis and so on. However, it is expected that the main errors have been introduced neglecting moments and rotational responses. This is investigated in the next section.

2.4 Numerical Model

The aim of this section is validating numerically the coupling technique and investigating the influence of the moments and rotational displacements in the FRF coupling model. The structure depicted in Fig. 2.2 is discretised using BEAM elements with 6DOFs per node, three translations and three rotations. The receiver is discretised using 324 elements while the source using 40 elements. Source and receiver are solved separately to obtain their FRF matrices, $\mathbf{G}_{nS}$ and $\mathbf{G}_{nR}$. These are coupled using Eq. (2.8) to obtain $\mathbf{G}_{nRS}$. The FRF matrix, $\mathbf{G}_n$, obtained from an FE model of the coupled structure, is also evaluated to model the real coupled structured and compare the results. Figure 2.5 shows the comparison between the direct frequency responses function G_{nRS} and G_n at point 1. It can be seen that the curves are in perfect agreement and thus coupling approach theoretically predicts the dynamics accurately when all the forcing and moments and displacements and rotations are considered. The experimental measure of the rotational displacements can be so difficult that in many practical cases only translation forces and responses are considered.

In order to simulate numerically this realistic situation, the FRF matrices of the source and the receiver are evaluated only for the translational degrees of freedom. Figure 2.6 shows the comparison between FRF for the coupled structure $\mathbf{G}_n$ and those obtained coupling the FRF matrices of the source and the receiver $\mathbf{G}_{nRS}$ when only the translational degrees of

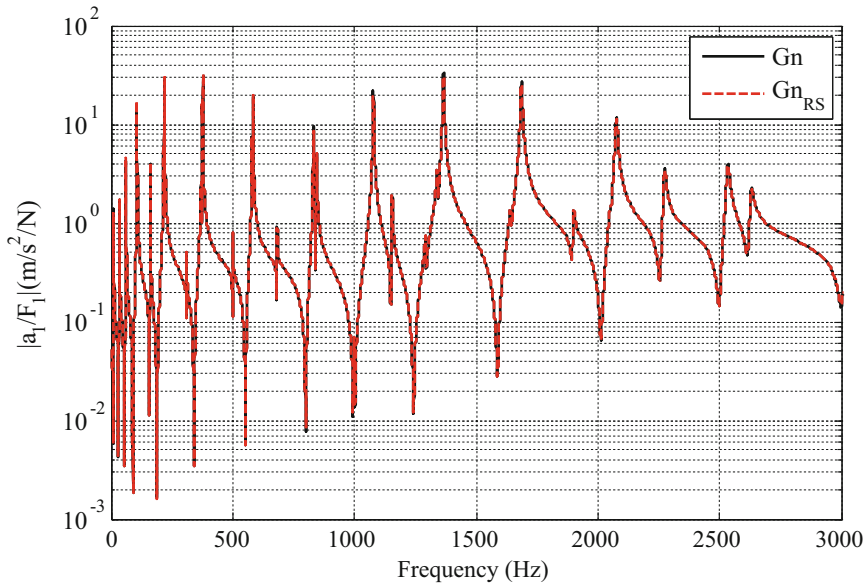


Fig. 2.5 Numerical case study. Comparison between numerical FRF for the coupled structure $\mathbf{G}_n$ and those obtained coupling the FRF matrices of the source and the receiver $\mathbf{G}_{nRS}$ using Eq. (2.12)

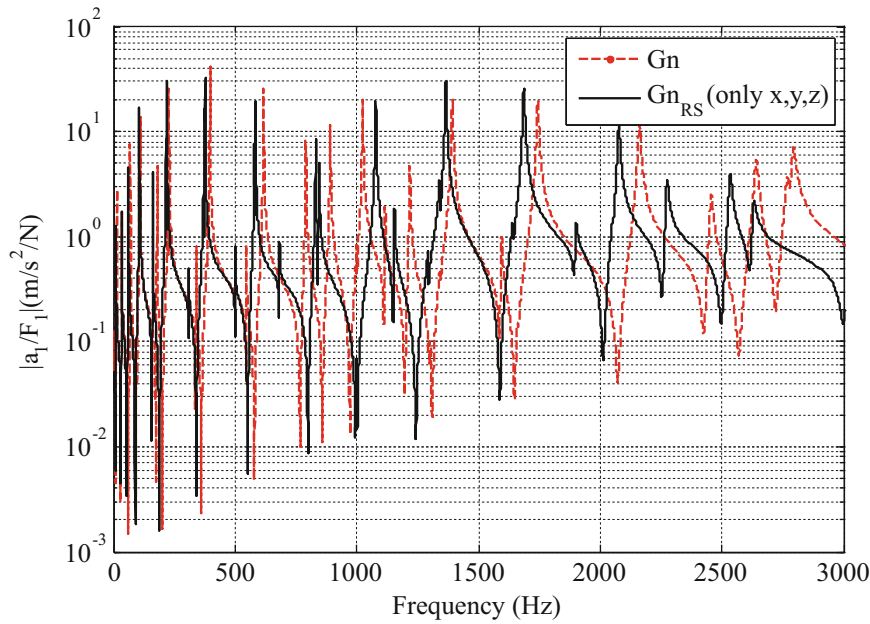


Fig. 2.6 Numerical case study. Comparison between numerical FRF for the coupled structure $\mathbf{G}_n$ and those obtained coupling the FRF matrices of the source and the receiver $\mathbf{G}_{nRS}$ using Eq. (2.12) when rotational responses were neglected

freedom were considered. This implies that the matrices used in Eq. (2.7) $\mathbf{G}_{nR}$ and $\mathbf{G}_{nS}$ are now 6×6 matrices, as measured in the experimental case, instead of 12×12 . As expected the curves are now different showing that neglecting moments and rotational degrees of freedom lead to inaccurate results.

2.5 Conclusions

In this paper a frequency response coupling technique (or impedance coupling technique) was revised for the prediction of the overall impedance matrix of a coupled structure. The data required are the frequency response functions (FRFs) of the source and the receiver and the velocity/accelerations at the connecting points under operating conditions. These can be either experimental data, or numerical data obtained from finite element analysis or modal analysis, or both experimental and numerical results for different substructures. A simplified model was considered for investigating the validity of the procedure numerically and experimentally. Numerical results showed very good agreement between the FRF functions of the coupled structure and those obtained coupling the impedance matrices of the source and the receiver, while experimental results showed significant differences between at higher frequencies. These were investigated using the numerical model. Although some experimental errors were introduced due to lack in modelling the connections between the source and the receiver and uncertainties in measuring (e.g. force applied in z direction is not perfectly aligned along the z axis and so on) it was found that the main errors is introduced in neglecting moments and rotational responses.

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