

# On the Identification of $\alpha$ -Asynchronous Cellular Automata in the Case of Partial Observations with Spatially Separated Gaps

Witold Bolt, Barbara Wolnik, Jan M. Baetens and Bernard De Baets

**Abstract** In this paper we present a statistical method, based on frequencies, for identifying so-called  $\alpha$ -asynchronous Cellular Automata from partial observations, i.e. pre-recorded configurations of the system with some cells having an unknown (missing) state. The presented method, in addition to finding the unknown Cellular Automaton, is able to unveil the missing state values with high accuracy.

**Keywords** Asynchronous cellular automata · Identification · Parameter estimation

## 1 Introduction

Cellular Automata (CAs) are commonly used modelling constructs for addressing a variety of problems [12]. In order to use CAs for a practical modelling task, one needs to understand the underlying mechanisms of the phenomenon at stake, and translate them into a CA rule. Additionally, the state space, tessellation and neighborhood structure need to be pinned down beforehand. This hampers the use of CAs, since there are problems for which it is hard to manually design a proper local rule. In some cases, only the initial and final states of the system are known (e.g. [2, 20, 21]).

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Besides classical deterministic CAs, Stochastic CAs (SCAs) are frequently used. Many efforts have been made in the direction of developing automated methods for constructing CAs and SCAs based on observed space-time diagrams [1, 3, 5, 6, 10, 11, 15–19, 22, 24, 26, 27]. Yet, there is only very limited literature for the case of incomplete observations in the deterministic case [7, 8]. To the best of our knowledge, the identification of CAs in the context of incomplete observations and stochastic rules has not yet been tackled by other authors.

The main goal of the research presented in this paper is to develop a method for the automated identification of a relatively simple class of SCAs, namely  $\alpha$ -asynchronous CAs ( $\alpha$ -ACAs) [14], based on partial observations. The presented method is based on statistical principles for estimating the parameters of binomial distributions based on frequencies of observed events. Moreover, we also present a method for completing those observations, i.e. filling the missing gaps into the observations. In addition to serving as a useful tool for building and analyzing models based on  $\alpha$ -ACAs, the presented method is a first step towards an effective identification of SCAs based on partial observations.

The performance of the presented method is verified with computational experiments, for the class of  $\alpha$ -ACAs corresponding to Elementary CAs (ECAs). The results show that the accuracy of the identification algorithm, when it comes to estimating the value of the synchrony rate  $\alpha$ , finding the underlying CA and filling the missing states in the observation is very high.

This paper is organized as follows. In Sect. 2, we introduce definitions and present some well-known facts on CAs and SCAs. In Sect. 3,  $\alpha$ -ACAs are formally introduced, while the formal definition of the identification problem is given in Sect. 4. Section 5 holds the description of the identification algorithm. The paper is concluded with Sect. 6, which presents the results of computational experiments.

## 2 Preliminaries

In this paper, we will concentrate on 1D, deterministic, two-state CAs with a symmetric neighborhood and a finite number of cells. Let  $r \in \mathbb{N}_0$ ,  $R = 2r + 1$  and let  $f: \{0, 1\}^R \rightarrow \{0, 1\}$  be a function, then for  $N > 0$ , we define the  $N$ -cell global CA rule  $A_N: \{0, 1\}^N \rightarrow \{0, 1\}^N$  as:

$$A_N(\dots, s_i, \dots) = (\dots, f(s_{i-r}, \dots, s_{i+r}), \dots), \quad (1)$$

where periodic boundary conditions are assumed, i.e. for any  $i \in \mathbb{Z}$  it holds that  $s_{i+N} = s_i$ . The function  $f$  used in (1) will be referred to as a local rule, and the integer  $r$  will be referred to as the radius of the neighborhood. Any local rule can be uniquely defined by a lookup table (LUT), which lists all the possible arguments of the local rule together with the corresponding function values. It is assumed that the arguments are listed in lexicographic order. The general form of such a LUT for  $r = 1$  is shown in Table 1.

**Table 1** The LUT of the local rule  $n = (l_7, l_6, l_5, l_4, l_3, l_2, l_1, l_0)_2$ 

(1,1,1)	(1,1,0)	(1,0,1)	(1,0,0)	(0,1,1)	(0,1,0)	(0,0,1)	(0,0,0)
$l_7$	$l_6$	$l_5$	$l_4$	$l_3$	$l_2$	$l_1$	$l_0$

**Table 2** The LUTs of ECAs 51 and 204

	(1,1,1)	(1,1,0)	(1,0,1)	(1,0,0)	(0,1,1)	(0,1,0)	(0,0,1)	(0,0,0)
<b>51</b> negation	0	0	1	1	0	0	1	1
<b>204</b> identity	1	1	0	0	1	1	0	0

The LUT can be used to enumerate local rules, as the coefficients  $l_i$  can be treated as digits in the binary representation of an integer  $n$ , i.e. in the case of  $r = 1$  the number of a local rule is  $n = \sum_{i=0}^7 l_i 2^i$ . Clearly, this reasoning extends to larger radii. Given that the ordering of the arguments in the LUT is fixed, only the second row needs to be known in order to uniquely define a CA, such that a LUT may be represented as a binary vector of length  $2^R$ .

CAs for which there exists a local rule with radius  $r = 1$  will be referred to as ECAs [25]. Due to their simplistic definition and rich dynamics, ECAs form a well-studied class of CAs. For that reason, the examples and experiments presented in this paper are based on ECAs and their asynchronous counterparts.

*Example 1* In Table 2 the LUTs of ECA rules 51 and 204 are shown, which are known as negation and identity CAs, respectively. Both of these CAs can be expressed with a local rule of radius zero, but the ECA description is more commonly used.

With  $\{0, 1\}^*$  we will denote the set of all binary sequences of finite length, i.e.  $\{0, 1\}^* = \bigcup_{M=1}^{\infty} \{0, 1\}^M$ . The function  $A: \{0, 1\}^* \rightarrow \{0, 1\}^*$ , defined by  $A(X) = A_M(X)$  if  $X \in \{0, 1\}^M$ , with every global rule  $A_M$  being defined with the same local rule  $f$ , will be referred to as a generalized global CA rule. We will simply refer to such functions as global rules or rules. In this paper, a CA will be identified by its global rule, and by referring to a CA, we therefore always refer to its global rule in this generalized sense.

Every CA  $A$  can be uniquely defined by its local rule  $f$  with neighborhood radius  $r \geq 0$ . Every local rule can be uniquely described with a set of neighborhood configurations  $\mathcal{C}(f)$ , for which the local rule agrees with identity CA, i.e.  $(x_1, \dots, x_R) \in \mathcal{C}(f)$  if and only if  $f(x_1, \dots, x_R) = x_{r+1}$ . As a consequence of the binary nature of the state set, it further holds that  $f(x_1, x_2, \dots, x_R) = 1 - x_{r+1}$  when  $(x_1, \dots, x_R) \notin \mathcal{C}(f)$ .

Let  $A$  be a CA,  $X \in \{0, 1\}^M$  for some  $M$  and  $T > 0$ . The finite sequence of vectors given by:

$$(X, A(X), A^2(X), \dots, A^{T-1}(X)),$$

**Table 3** pLUT of a stochastic ECA local rule

(1,1,1)	(1,1,0)	(1,0,1)	(1,0,0)	(0,1,1)	(0,1,0)	(0,0,1)	(0,0,0)
$p_7$	$p_6$	$p_5$	$p_4$	$p_3$	$p_2$	$p_1$	$p_0$

where  $A^t$  denotes the result of applying the rule  $A$  to  $A^{t-1}(X)$ , will be referred to as the space-time diagram covering  $T$  time steps. Each of its elements will be referred to as a configuration of the CA  $A$ , while the first element will be referred to as the initial configuration. For any  $t = 0, 1, \dots, T - 1$  and  $m = 1, \dots, M$ ,  $A^t(X)[m]$  refers to the state of the  $m$ th cell in the  $t$ th row of the space-time diagram.

The CAs defined above are deterministic and are fully governed by their local rule. However, there are also Stochastic CAs (SCAs), for which the local rule is a random function that can be uniquely defined by a probability lookup table (pLUT). The pLUT lists all possible neighborhood configurations and maps them to the probabilities of transition to state 1. The general form of a pLUT for the stochastic counterparts of ECAs is shown in Table 3.

Formally, the meaning of the pLUT is the following. Let  $\tilde{f}$  be the local rule of an SCA with unit neighborhood radius, and let  $(x_1, x_2, x_3) \in \{0, 1\}^3$ . Let  $i$  be an integer such that the vector  $(x_1, x_2, x_3)$  is its binary representation, then the entries in Table 3 are given by:

$$\mathbb{P}(\tilde{f}(x_1, x_2, x_3) = 1) = p_i .$$

Obviously, from this it follows that:

$$\mathbb{P}(\tilde{f}(x_1, x_2, x_3) = 0) = 1 - p_i .$$

In the case of SCAs, it is hard to define the space-time diagram in a strict, formal way. So if  $A$  is an SCA, any sequence of configurations, which can be obtained by simulating  $A$ , starting from some given initial configuration, is a space-time diagram. Formally it means that if  $p_i \in ]0, 1[$  for  $i = 0, \dots, 7$ , any sequence of binary configurations makes up a space-time diagram. Yet, the likelihood of observing a given space-time diagram is uniform only in case  $p_i = 0.5$  for all  $i = 0, \dots, 7$ . In other cases, the probability distribution over the space of space-time diagrams might be more complex.

### 3 $\alpha$ -Asynchronous CAs

Classically, states in CAs are updated synchronously, i.e. a new state is assigned to all cells simultaneously at every time step according to the local rule. Yet, different approaches of breaking the synchronicity of CAs have been proposed [23]. Interest-

ingly, the choice of the update scheme, which defines the order or timing of cell state updates, has very important repercussions on the dynamical properties of CAs [4]. Here, we focus on one of such schemes, namely  $\alpha$ -asynchronous CAs ( $\alpha$ -ACAs). A detailed definition of  $\alpha$ -ACAs is presented below, while the description of their most important properties and applications can be found in [13].

Any  $\alpha$ -ACA can be defined by a deterministic CA  $A$  and a probability  $\alpha$ , called the synchrony rate, which controls whether or not its cells are updated. More precisely,  $\alpha$  is the probability of applying the local rule  $f$  of  $A$ . Let  $\tilde{f}$  be the random function (local rule) corresponding to an  $\alpha$ -ACA, then for any  $x_1, \dots, x_R, y \in \{0, 1\}$  it holds that:

$$\mathbb{P}(\tilde{f}(x_1, \dots, x_R) = y) = \begin{cases} 0, & \text{if } (x_1, \dots, x_R) \in \mathcal{C}(f) \wedge y \neq x_{r+1}, \\ 1, & \text{if } (x_1, \dots, x_R) \in \mathcal{C}(f) \wedge y = x_{r+1}, \\ \alpha, & \text{if } (x_1, \dots, x_R) \notin \mathcal{C}(f) \wedge y \neq x_{r+1}, \\ 1 - \alpha, & \text{if } (x_1, \dots, x_R) \notin \mathcal{C}(f) \wedge y = x_{r+1}. \end{cases}$$

Note that if  $\alpha = 0$ , such a system stays at its initial configuration, whereas the system is equivalent to a deterministic CA  $A$  if  $\alpha = 1$ .

The essential property of  $\alpha$ -ACAs as defined here is that they may equivalently be considered as SCAs for which the local rule  $f$  is selected with probability  $\alpha$ , while the identity rule is selected with probability  $1 - \alpha$ . Hence, we may say that CA  $A$  becomes stochastically mixed with the identity rule. In the remainder we will write  $A_\alpha$  to denote the  $\alpha$ -ACA which is defined with the use of CA  $A$  and synchrony rate  $\alpha$ .

Let us assume that a local rule of an ECA  $A$  is defined by the LUT  $(l_i)_{i=0}^7$ . Clearly,  $\alpha$ -ACAs form a special class of SCAs, therefore, we can represent  $A_\alpha$  in terms of a pLUT. If  $\alpha \in [0, 1]$  and  $\bar{\alpha} = 1 - \alpha$ , the pLUT of  $A_\alpha$  is given by Table 4.

As can be inferred from the LUT shown in Table 4,  $A_\alpha$  is deterministic on those neighborhood configurations belonging to the set  $\mathcal{C}(f)$ , where  $f$  is the local rule of  $A$ . For remaining neighborhood configurations, where  $f$  agrees with the negation rule,  $A_\alpha$  is stochastic. This simple property is important in the construction of the identification algorithm.

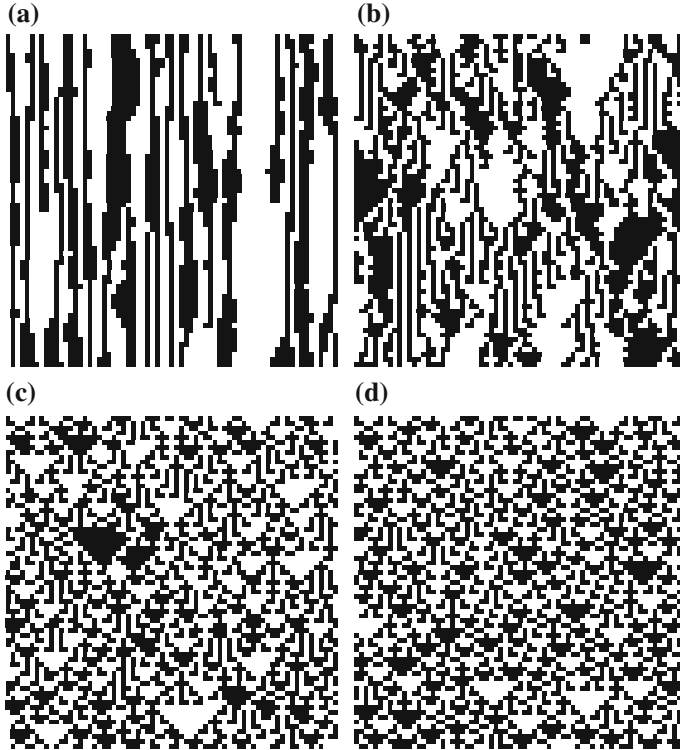
*Example 2* Let  $A$  be the ECA 150. The pLUT of  $A_\alpha$  is given by Table 5. The space-time diagrams evolved for the same initial configuration for: (a)  $\alpha = 0.1$ , (b)  $\alpha = 0.5$ , (c)  $\alpha = 0.9$  and (d)  $\alpha = 1$  are shown in Fig. 1. As can be inferred from the plots, the behavior of the dynamical system is greatly affected by  $\alpha$ .

**Table 4** pLUT of an  $\alpha$ -ACA local rule

(1,1,1)	(1,1,0)	(1,0,1)	(1,0,0)	(0,1,1)	(0,1,0)	(0,0,1)	(0,0,0)
$\alpha l_7 + \bar{\alpha}$	$\alpha l_6 + \bar{\alpha}$	$\alpha l_5$	$\alpha l_4$	$\alpha l_3 + \bar{\alpha}$	$\alpha l_2 + \bar{\alpha}$	$\alpha l_1$	$\alpha l_0$

**Table 5** pLUT of  $A_\alpha$  for A being ECA 150

(1,1,1)	(1,1,0)	(1,0,1)	(1,0,0)	(0,1,1)	(0,1,0)	(0,0,1)	(0,0,0)
1	$\bar{\alpha}$	0	$\alpha$	$\bar{\alpha}$	1	$\alpha$	0

**Fig. 1** Space-time diagrams of  $A_\alpha$  evolved for the same initial configuration, for different synchrony rates, where  $A$  is ECA 150. **a**  $\alpha = 0.1$ . **b**  $\alpha = 0.5$ . **c**  $\alpha = 0.9$ . **d**  $\alpha = 1$ 

## 4 Identification Problem

In this section we define the identification problem. Our formulation is based on the concept of an observation of a space-time diagram, which is assumed to be incomplete, i.e. it contains only partial information on the states of the underlying  $\alpha$ -ACA.

Let  $I$  be an  $N \times M$  array containing symbols belonging to the set  $\{0, 1, ?\}$ , where the symbols 0 and 1 denote valid states of an unknown  $\alpha$ -ACA, while the symbol ? denotes an unknown state belonging to the set  $\{0, 1\}$ . Additionally, let the first row  $I[1] \in \{0, 1\}^M$  represent the initial configuration of an  $\alpha$ -ACA. Such an array  $I$  will be referred to as an observation. If an observation  $I$  does not contain the symbol ?,

we refer to it as complete, while it will be referred to as partial otherwise. Entries in the observations occupied by the symbol ? will be denoted as gaps.

Note that the assumption of a completely observed initial configuration, i.e.  $I[1] \in \{0, 1\}^M$  is crucial for the construction of the presented method and cannot be relaxed easily. Yet, this condition can be met in many practical applications as there are often means of controlling the initial state of the system in question.

Let  $r \in \mathbb{N}_0$ , then the vector  $(I[n, m - r], \dots, I[n, m + r])$  will be denoted by  $I[n, m|r]$ , for any  $n, m \in \mathbb{Z}$ , where periodic boundary conditions are assumed. Furthermore, we assume that there are strict limitations for what concerns the occurrence of gaps in observations. If for some  $n, m$  it holds that  $I[n, m] = ?$ , then  $I[n - 1, m|r] \in \{0, 1\}$ ,  $I[n + 1, m|r] \in \{0, 1\}$  and in  $I[n, m|2r]$  only for the pair  $(n, m)$  it holds that  $I[n, m] = ?$ , i.e. for  $m' = m - 2r, \dots, m - 1, m + 1, \dots, m + 2r$  it holds that  $I[n, m'] \in \{0, 1\}$ . Such a condition of spatial separation of gaps allows to consider each of the gaps separately in the gap filling process.

We consider the identification problem with the assumption that  $I$  was generated by some unknown  $\alpha$ -ACA, denoted by  $A_\alpha$ . Solving the identification problem means finding the CA  $A$  and  $\hat{\alpha}$ , as an approximation of  $\alpha$ , such that  $\hat{\alpha} \in ]\alpha_1, \alpha_2[$ , where  $\alpha_2 - \alpha_1$  is as small as possible, and it is very likely that  $\alpha \in ]\alpha_1, \alpha_2[$ . More formally, we select  $L \in ]0, 1[$  and assume a confidence level  $1 - L$ . We build an estimate for  $A$ ,  $\alpha_1$  and  $\alpha_2$ , based on observation  $I$ , such that observation  $I$  is a space-time diagram of  $A_\alpha$  for some  $\alpha \in ]\alpha_1, \alpha_2[$  with probability  $1 - L$ . In those cases where the observation  $I$  is incomplete, the identification algorithm should yield the most likely values of the missing states.

We will consider the identification problem in the context of observation sets  $\mathcal{I}$  containing one or more observation  $I_j$  for  $j = 1, \dots, |\mathcal{I}|$ , of the behavior of some  $\alpha$ -ACA, where  $I_j \in \mathcal{I}$  contains  $M_{I_j}$  columns and  $N_{I_j}$  rows.

It should be mentioned that the identification problem presented here, has limited applicability when it comes to real-world modeling tasks. This is mostly due to the fact that binary CAs are typically too simple to mimic real-world processes as a consequence of their limited state set. Yet, the identification problem becomes of more direct importance in the case of multi-state and multi-dimensional CAs. Although the presented solution algorithm is tailored towards the binary case, it is possible to generalize it. This is one of the topics of future research in this area.

## 5 Identification Algorithm

### 5.1 Complete Observations

In this section we describe the algorithm for solving the identification problem in the case of complete observations. Besides, we assume that  $\alpha$  is bounded between known bounds  $a > 0$  and  $b < 1$ .

Based on a set of complete observations  $\mathcal{I}$ , we define frequency tables  $N = (N_0, \dots, N_{2^R-1})$  and  $K = (K_0, \dots, K_{2^R-1})$ , where  $N_i$  denotes the number of occurrences of the  $i$ th neighborhood configuration in all of the observations  $I \in \mathcal{I}$ , where the last row of each observation is discarded, i.e. we count the occurrences of the neighborhoods in every row except for the last one, and store the results in  $N$ . To build table  $K$ , we additionally check the state of the central cell in the next row for each of the visited neighborhoods, and we count the number of cases where the value of the central cell changed, i.e. cases where the unknown  $\alpha$ -ACA changed the value of a cell to its complement, and thus acted like the negation CA. It is obvious that for all  $i$  it holds that  $K_i \leq N_i$ .

**Proposition 1** *Let  $N^* = \sum_{i:K_i>0} N_i$ ,  $K^* = \sum_{i=0}^{2^R-1} K_i$  and  $\hat{\alpha} = \frac{K^*}{N^*}$ . The proportion  $\hat{\alpha}$  is a random variable following a binomial distribution with success probability equal to  $\alpha$ .*

Following [9] there are various methods to estimate the confidence interval for  $\alpha$  using  $\hat{\alpha}$ . Here, we choose the normal distribution approximation, even though the authors of [9] advice against it. This choice is motivated by the fact that this method leads to an algorithm with a reasonable accuracy and at the same time its implementation is straightforward. Assuming that  $1 - L$  is the selected confidence level, then the following holds with probability  $1 - L$ :

$$\hat{\alpha} - z_L \sqrt{\frac{\hat{\alpha}(1 - \hat{\alpha})}{N^*}} \leq \alpha \leq \hat{\alpha} + z_L \sqrt{\frac{\hat{\alpha}(1 - \hat{\alpha})}{N^*}}, \quad (2)$$

where  $z_L$  is the argument at which the cumulative standard normal distribution function takes the value of  $1 - \frac{L}{2}$ . The above holds if  $N^*$  is large enough, for example if both  $N^*\alpha$  and  $N^*(1 - \alpha)$  are greater than five [9]. Since  $\alpha$  is unknown, we impose a bit stronger condition  $N^* > \max(\frac{5}{\alpha}, \frac{5}{1-\alpha})$ , which can be easily verified.

The estimated interval given by (2) can be adjusted, taking into account the assumption that  $\alpha \in [a, b]$ . For that purpose, let:

$$\alpha_1 = \max\left(a, \hat{\alpha} - z_L \sqrt{\frac{\hat{\alpha}(1 - \hat{\alpha})}{N^*}}\right), \quad \alpha_2 = \min\left(b, \hat{\alpha} + z_L \sqrt{\frac{\hat{\alpha}(1 - \hat{\alpha})}{N^*}}\right),$$

then it holds that  $\alpha \in [\alpha_1, \alpha_2]$  with probability  $1 - L$ . Note that  $\alpha_2 - \alpha_1 \leq \frac{z_L}{\sqrt{N^*}}$  and for commonly used confidence levels it holds that  $z_L < 3$ . Thus, if  $N^*$  is sufficiently large, we are sure that the interval  $[\alpha_1, \alpha_2]$  narrows.

Summing up, we have formulated the estimation method for the confidence interval of  $\alpha$ . If a point estimation is desired, the value of  $\frac{\alpha_1 + \alpha_2}{2}$ , which in most cases is equal to  $\hat{\alpha}$ , shall be used. We now turn to the method for constructing the local rule of CA A.

Let  $f$  be the unknown local rule given by a LUT  $(l_i)_{i=0}^{2^R-1} \in \{0, 1\}^{2^R}$ , where the  $l_i$ 's are unknown. Deciding on these might be seen as picking the same value as the identity rule or taking the opposite value. If  $K_i > 0$  and  $N_i > 0$  we pick the value



opposite to identity, i.e. we select the value from the LUT of the negation rule. When  $K_i = 0$  and  $N_i > 0$ , we are not sure if  $f$  agrees with the identity CA on the  $i$ th neighborhood, or too few samples were observed. Yet, if it holds that:

$$(1 - a)^{N_i} \leq \frac{L}{2^{|i:K_i=0|}}, \quad (3)$$

we may assume that  $f$  agrees with the identity. This assures that the total probability of picking the wrong value, over all the neighborhood configurations for which  $K_i = 0$ , is not higher than  $L$ . Otherwise if Eq. (3) is not fulfilled or  $K_i = N_i = 0$ , we are not able to select the value of  $l_i$ . Still, if we can simulate the unknown  $\alpha$ -ACA starting from an arbitrary configuration, this bottleneck is eliminated. In the remainder we assume that such cases do not happen.

The computational complexity of the identification algorithm is linear, in the sense that it is proportional to the number of observed cells. Consequently, the algorithm is applicable even for relatively big observation sets.

## 5.2 Gap Filling Procedure

Having defined the estimates  $\alpha_1, \alpha_2, \hat{\alpha}$  and  $A$  in the case of complete observations, we now turn to the case of partial observations obeying the spatial separation condition formulated in Sect. 4, which guarantees that we can treat each of the gaps separately. We can find the estimates for  $\alpha$  and  $A$  following the method described in Sect. 5.1, with the only change that  $K_i$  and  $N_i$  are calculated discarding those entries that contain the symbol ?. In other words, we ignore entries with gaps in the first step of estimating the parameters of  $\alpha$ -ACA. Then, for every  $I \in \mathcal{I}$  and  $(n, m)$  such that  $I[n, m] = ?$ , we follow the procedure outlined below to find the missing state. Let the function  $f: \{0, 1\}^R \rightarrow \{0, 1\}$  be the local rule of CA  $A$ , and  $\tilde{f}$  be the corresponding random local rule of  $A_\alpha$ .

If  $f(I[n-1, m|r]) = I[n-1, m]$ , then  $f$  agrees with the identity CA on the neighborhood configuration  $I[n-1, m|r]$ , and we may replace the ? at  $(n, m)$  by  $I[n-1, m] \in \{0, 1\}$ . Otherwise, we need to inspect values in the  $n+1$ th row of observation  $I$  to find the most likely value for  $I[n, m]$ . In this case, from the definition of  $\alpha$ -ACAs, we know that:

$$p_y = \mathbb{P}(I[n, m] = y) = \begin{cases} 1 - \alpha, & \text{if } I[n-1, m] = y, \\ \alpha, & \text{if } I[n-1, m] = 1 - y. \end{cases} \quad (4)$$

The informal meaning of  $p_y$  is that it is the probability of  $I[n, m]$  being  $y$ , as calculated by only examining the  $n-1$ th row of observation. For  $h \in \{-r, \dots, r\}$ , let  $F_h$  denote the random event that, starting from the neighborhood configuration

$I[n - 1, m|r]$ , the  $\alpha$ -ACA evolution leads to the state  $I[n + 1, m + h]$  after two time steps. For any  $y \in \{0, 1\}$ , let  $p_{h,y}$  be defined as:

$$p_{h,y} = \mathbb{P}(F_h \mid I[n, m] = y). \quad (5)$$

We may calculate  $p_{h,y}$  according to the following formula, where it is assumed that  $I[n, m] = y$ , and thus  $I[n, m + h|r]$  depends on  $y$ :

$$p_{h,y} = \begin{cases} 0, & \text{if } I[n, m + h|r] \in \mathcal{C}(f) \wedge I[n, m + h] \neq I[n + 1, m + h], \\ 1, & \text{if } I[n, m + h|r] \in \mathcal{C}(f) \wedge I[n, m + h] = I[n + 1, m + h], \\ \alpha, & \text{if } I[n, m + h|r] \notin \mathcal{C}(f) \wedge I[n, m + h] \neq I[n + 1, m + h], \\ 1 - \alpha, & \text{if } I[n, m + h|r] \notin \mathcal{C}(f) \wedge I[n, m + h] = I[n + 1, m + h]. \end{cases} \quad (6)$$

Since  $\alpha$  is not known, we can only get an approximation of  $p_{h,y}$ . Yet it suffices for our purposes. The value  $p_{h,y}$  is the probability of obtaining the  $n + 1$ th row, assuming that  $y$  is the missing value in the  $n$ th row. By combining those probabilities, we will find the most likely value for  $I[n, m]$ . More formally, according to the Total Probability theorem, it holds that:

$$\begin{aligned} \mathbb{P}\left(\bigcap_{h=-r}^r F_h\right) &= \sum_{y=0}^1 \mathbb{P}(I[n, m] = y) \mathbb{P}\left(\bigcap_{h=-r}^r F_h \mid I[n, m] = y\right) \\ &= \sum_{y=0}^1 p_y \mathbb{P}\left(\bigcap_{h=-r}^r F_h \mid I[n, m] = y\right) \\ &= \sum_{y=0}^1 p_y \prod_{h=-r}^r \mathbb{P}(F_h \mid I[n, m] = y) \\ &= \sum_{y=0}^1 p_y \prod_{h=-r}^r p_{h,y}. \end{aligned}$$

This is justified as for  $h_1 \neq h_2$ , the events  $F_{h_1}$  and  $F_{h_2}$  are independent if  $I[n, m]$  is known. The probability  $\mathbb{P}(I[n, m] = y \mid \bigcap_{h=-r}^r F_h)$  is the probability of  $I[n, m] = y$  assuming that all of the transitions from  $I[n - 1, m|r]$  to  $n + 1$ th row happened according to values recorded in observation  $I$ . Due to Bayes' theorem it holds that:

$$\mathbb{P}\left(I[n, m] = y \mid \bigcap_{h=-r}^r F_h\right) = \frac{p_y \prod_{h=-r}^r p_{h,y}}{\mathbb{P}(\bigcap_{h=-r}^r F_h)}. \quad (7)$$

Therefore, the most likely value for  $I[n, m]$  is the one that maximizes the probability  $\mathbb{P}(I[n, m] = y \mid \bigcap_{h=-r}^r F_h)$ , and to find it, we only need to examine the numerators of the fractions in Eq. (7) for different values of  $y$  since the denominator does not depend on  $y$ .

Finally, the method for filling the gap in  $I[n, m]$  works as follows. Firstly, we check whether the value can be selected deterministically according to the identity rule, which happens when  $I[n-1, m|r] \in \mathcal{C}(f)$ . Otherwise, we calculate the two numerators from Eq. (7), for  $y = 0$  and  $y = 1$ , using  $\widehat{\alpha}$  instead of  $\alpha$ . If the results differ, we pick the  $y$  for which the numerator is the largest. If both numerators are equal, we compare the probabilities  $p_0$  and  $p_1$ , and pick this  $y$  for which the probability is greater. Finally, if  $p_y = p_{1-y}$ , we randomly assign a value to  $I[n, m]$ .

## 6 Experimental Results

For assessing the performance of the identification algorithm we evaluated 255 ECAs (all but ECA 204 which is the identity CA) for synchrony rates equal to  $\alpha = 0.05, 0.01, \dots, 0.95$ . For each ECA and  $\alpha$ , a set of 100 observations, each consisting of 49 time steps and 49 cells, was constructed by simulating the  $\alpha$ -ACA and storing the resulting space-time diagrams. A common set of 100 randomly generated initial configurations was used. The 95 % confidence level was set, i.e.  $L = 0.05$ . The bounds for  $\alpha$  were defined as  $a = 0.05$  and  $b = 0.95$ . In each of the observation sets, 2500 gaps were introduced at random positions in randomly selected observations, but still such that the separation condition was fulfilled.

In all of the considered cases, the unknown ECA was discovered. To verify whether the estimation of  $\alpha$  was reliable, we measured the relative error  $E$  defined as:

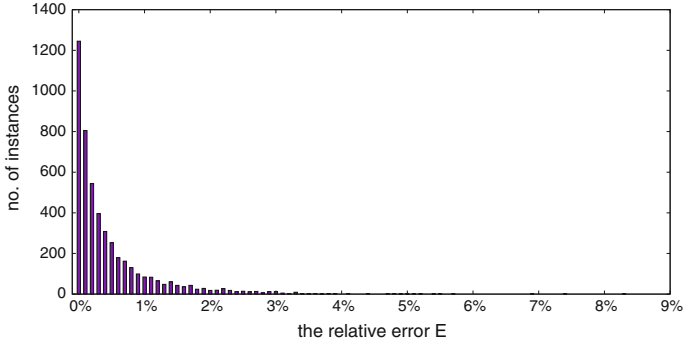
$$E = E(A_\alpha) = \frac{|\widehat{\alpha} - \alpha|}{\alpha} \times 100 \% , \quad (8)$$

where  $\widehat{\alpha}$  is the estimate obtained for  $A_\alpha$ . We obtained the following statistics of  $E(A_\alpha)$  across the ECAs and synchrony rates (where the values were truncated to two significant digits):

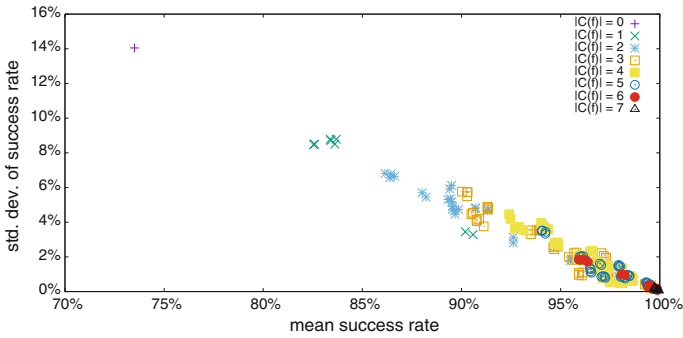
$$\min(E) = 0.00 \% , \quad \langle E \rangle = 0.51 \% , \quad \max(E) = 8.37 \% , \quad \sigma(E) = 0.68 \% ,$$

where  $\langle E \rangle$  denotes the mean error, and  $\sigma(E)$  is the standard deviation of the error. The histogram of  $E$  values with a bin width of 0.1 % is shown in Fig. 2. As can be inferred from this figure, the identification algorithm is able to find very good estimates of the synchrony rate. Not only was the maximum relative error 8.37 %, but more importantly the relative error was below 1 % in 85 % of the cases.

We now assess the performance of the second step of the identification algorithm, namely gap filling. We measured this as the percentage of correctly filled gaps in observations for a given case. For each ECA we averaged the success rate over the considered synchrony rates and the resulting quantity is denoted as  $S$ . The overall



**Fig. 2** Histogram of the relative error  $E$  with a bin width 0.1 %



**Fig. 3** Mean success rate versus the standard deviation for each of the ECAs. Different plot markers are used to indicate different sizes of the set  $\mathcal{C}(f)$

statistics of  $S$  are shown below (where the values were truncated to two significant digits):

$$\min(S) = 73.50 \%, \langle S \rangle = 96.02 \%, \max(S) = 99.95 \%, \sigma(S) = 4.10 \% .$$

Figure 3 shows both the mean success rate  $S$  and the standard deviation of the success rate for each ECA. The points on the plot close to the lower-right corner (1, 0) correspond to the most successful cases. The results are labeled according to the size of the set  $\mathcal{C}(f)$ , where  $f$  is the local rule of the considered ECA. As can be inferred from the plot, ECAs for which the set  $\mathcal{C}(f)$  is bigger, give rise to slightly better results, possible due to the fact that more transitions happened deterministically. Moreover, the outlier with a relatively low value of  $S$  originated from ECA 51 which is the negation CA, for which the  $\alpha$ -ACA reduces to a weighted, random bit flip. Yet, even in this case we were able to correctly fill in more than 70 % of the gaps. The presented results indicate that the algorithm is very effective and accurate when it comes to gap filling.

## 7 Summary

In this paper the identification of  $\alpha$ -ACAs was discussed. An algorithm for identifying the underlying CA and estimating the synchrony rate  $\alpha$  was presented. Moreover, a method for gap filling was put forward. The experimental results presented in Sect. 6 for ECAs are very promising. In all cases the algorithm was able to find the correct CA, and a good estimate for the synchrony rate was obtained. Also the rate of correctly filled gaps was very high. The algorithm will be extended to a more general setting where the observations have less separated gaps and to richer classes of SCAs.

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