Nonlinear Observer Design for Chaotic Systems

Sundarapandian Vaidyanathan

Abstract This work investigates the nonlinear observer design for chaotic systems. Explicitly, we have applied Sundarapandian's theorem (2002) for local exponential observer design for nonlinear systems to design nonlinear observers for chaotic systems with a single stable equilibrium point, viz. Wei-Wang system (2013) and Kingni-Jafari system (2014). MATLAB simulations are provided to illustrate the phase portraits and nonlinear observer design for the Wei-Wang and Kingni-Jafari chaotic systems.

Keywords Nonlinear observers · Exponential observers · Observability · Chaotic systems

1 Introduction

The observer design problem is to estimate the state of a control system when only the plant output and control input are available for measurement. The problem of designing observers for linear control systems was first introduced and fully solved by Luenberger [21]. The problem of designing observers for nonlinear control systems was proposed by Thau [59]. Over the past three decades, several techniques have been developed in the control systems literature to the construction of observers for nonlinear control systems [5].

A necessary condition for the existence of a local exponential observer for nonlinear control systems was obtained by Xia and Gao [99]. On the other hand, sufficient conditions for nonlinear observers have been derived in the control literature from an impressive variety of points of view. Kou ey al. [14] derived sufficient conditions for the existence of local exponential observers using Lyapunov-like method. In [11, 15, 16, 98], suitable coordinate transformations were found under which a nonlin-

S. Vaidyanathan (✉)
Research and Development Centre, Vel Tech University, Avadi, Chennai 600062, Tamil Nadu, India
e-mail: sundarvtu@gmail.com

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ear control systems is transferred into a canonical form, where the observer design is carried out. In [61], Tsinias derived sufficient Lyapunov-like conditions for the existence of local asymptotic observers for nonlinear systems. A harmonic analysis method was proposed in [8] for the synthesis of nonlinear observers.

A characterization of local exponential observers for nonlinear control systems was first obtained by Sundarapandian [38]. In [38], necessary and sufficient conditions were obtained for exponential observers for Lyapunov stable continuous-time nonlinear systems and an exponential observer design was provided by Sundarapandian which generalizes the linear observer design of Luenberger [21] for linear control systems. In [41], Sundarapandian obtained necessary and sufficient conditions for exponential observers for Lyapunov stable discrete-time nonlinear systems and also provided a formula for designing exponential observers for Lyapunov stable discrete-time nonlinear systems. In [37], Sundarapandian derived new results for the global observer design for nonlinear control systems.

The concept of nonlinear observers for nonlinear control systems was also extended in many ways. In [39, 40], Sundarapandian derived new results characterizing local exponential observers for nonlinear bifurcating systems. In [42, 43, 48, 49], Sundarapandian derived new results for the exponential observer design for a general class of nonlinear systems with real parametric uncertainty. In [44–47], Sundarapandian derived new results and characterizations for general observers for nonlinear systems.

Chaotic systems are defined as nonlinear dynamical systems which are sensitive to initial conditions, topologically mixing and with dense periodic orbits. Sensitivity to initial conditions of chaotic systems is popularly known as the *butterfly effect*. Small changes in an initial state will make a very large difference in the behavior of the system at future states. Chaotic behaviour was suspected well over hundred years ago in the study of three bodies problem by Henri Poincaré [4], but chaos was experimentally established by Lorenz [19] only a few decades ago in the study of 3-D weather models.

Some classical paradigms of 3-D chaotic systems in the literature are Rössler system [30], ACT system [1], Sprott systems [36], Chen system [9], Lü system [20], Liu system [18], Cai system [6], Chen-Lee system [10], Tigan system [60], etc.

Many new chaotic systems have been discovered in the recent years such as Zhou system [100], Zhu system [101], Li system [17], Wei-Yang system [97], Sundarapandian systems [51, 56], Vaidyanathan systems [68, 69, 71–74, 76, 78, 81, 92, 95], Pehlivan system [24], etc.

Synchronization of chaotic systems is a phenomenon that occurs when two or more chaotic systems are coupled or when a chaotic system drives another chaotic system. Because of the butterfly effect which causes exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, the synchronization of chaotic systems is a challenging research problem in the chaos literature [2, 3].

Major works on synchronization of chaotic systems deal with the complete synchronization of a pair of chaotic systems called the *master* and *slave* systems. The design goal of the complete synchronization is to apply the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically with time.

Pecora and Carroll pioneered the research on synchronization of chaotic systems with their seminal papers [7, 23]. The active control method [12, 31, 32, 50, 55, 62, 66, 83, 84, 87] is typically used when the system parameters are available for measurement. Adaptive control method [33–35, 52–54, 64, 70, 77, 82, 85, 86, 91, 94] is typically used when some or all the system parameters are not available for measurement and estimates for the uncertain parameters of the systems.

Backstepping control method [25–29, 58, 88, 93] is also used for the synchronization of chaotic systems, which is a recursive method for stabilizing the origin of a control system in strict-feedback form. Another popular method for the synchronization of chaotic systems is the sliding mode control method [57, 63, 65, 67, 75, 79, 80, 89, 90], which is a nonlinear control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to "slide" along a cross-section of the system's normal behavior.

The control and synchronization of chaotic systems is based on the full knowledge of the states of the systems. When some of the chaotic systems are not available for measurement, exponential observer design for chaotic systems can be used in lieu of the states of the systems Thus, observer design for chaotic systems has important applications in the control literature.

This work is organized as follows. Section 2 reviews the definition and results of local exponential observers for nonlinear systems. Section 3 details the dynamic analysis and phase portraits of the Wei-Wang chaotic system [96]. Section 4 details the nonlinear observer design for the Wei-Wang chaotic system. Section 5 details the dynamic analysis and phase portraits of the Kingni-Jafari chaotic system [13]. Section 6 details the nonlinear observer design for the Kingni-Jafari chaotic system. Section 7 provides the conclusions of this work.

2 Review of Nonlinear Observer Design for Nonlinear Systems

By the concept of a *state observer* for a nonlinear system, it is meant that from the observation of certain states of the system considered as *outputs* or *indicators*, it is desired to estimate the state of the whole system as a function of time. Mathematically, observers for nonlinear systems are defined as follows.

Consider the nonlinear system described by

$$\dot{x} = f(x) \tag{1a}$$

$$y = h(x) \tag{1b}$$

where $x \in \mathbf{R}^n$ is the *state* and $y \in \mathbf{R}^p$ is the *output*.

It is assumed that $f : \mathbf{R}^n \to \mathbf{R}^n$, $h : \mathbf{R}^n \to \mathbf{R}^p$ are C^1 mappings and for some $x^\star \in \mathbf{R}^n$, the following hold:

$$f(x^\star) = 0, \quad h(x^\star) = 0 \tag{2}$$

Remark 1 We note that the solutions $x^\star$ of $f(x) = 0$ are called the *equilibrium points* of the plant dynamics (1a). Also, the assumption $h(x^\star) = 0$ holds without any loss of generality. Indeed, if $h(x^\star) \neq 0$, then we may define a new output function as

$$\psi(x) = h(x) - h(x^\star) \tag{3}$$

and it is easy to see that $\psi(x^\star) = 0$. ■

The linearization of the nonlinear system (1a) and (1b) at $x = x^\star$ is given by

$$\dot{x} = Ax \tag{4a}$$

$$y = Cx \tag{4b}$$

where

$$A = \left[\frac{\partial f}{\partial x}\right]_{x=x^\star} \quad \text{and} \quad C = \left[\frac{\partial h}{\partial x}\right]_{x=x^\star} \tag{5}$$

Definition 1 ([38]) A C^1 dynamical system defined by

$$\dot{z} = g(z, y), \quad (z \in \mathbf{R}^n) \tag{6}$$

is called a **local asymptotic** (respectively, **exponential**) **observer** for the nonlinear system (1a)–(1b) if the following two requirements are satisfied:

(i) If $z(0) = x(0)$, then $z(t) = x(t)$, for all $t \geq 0$.
(ii) There exists a neighbourhood V of the equilibrium $x^\star$ of $\mathbf{R}^n$ such that for all $z(0), x(0) \in V$, the estimation error

$$e(t) = z(t) - x(t) \tag{7}$$

decays asymptotically (respectively, exponentially) to zero as $t \to \infty$. ■

Theorem 1 ([38]) *Suppose that the nonlinear system dynamics (1a) is Lyapunov stable at the equilibrium $x = x^\star$ and that there exists a matrix K such that $A - KC$ is Hurwitz. Then the dynamical system defined by*

$$\dot{z} = f(z) + K[y - h(z)] \tag{8}$$

is a local exponential observer for the nonlinear system (1a)–(1b). ■

Remark 2 The estimation error is governed by the dynamics

$$\dot{e} = f(x+e) - f(x) - K[h(x+e) - h(x)] \tag{9}$$

Linearizing the error dynamics (9) at $x = x^\star$, we get the linear system

$$\dot{e} = Ee, \quad \text{where} \;\; E = A - KC \tag{10}$$

If (C, A) is observable, then the eigenvalues of $E = A - KC$ can be arbitrarily placed in the complex plane [22] and thus a local exponential observer of the form (8) can always be found such that the transient response of the error decays quickly with any desired speed of convergence. ■

3 Dynamic Analysis of the Wei-Wang Chaotic System

The Wei-Wang chaotic system [96] is described by the 3-D dynamics

$$\begin{aligned} \dot{x}_1 &= ax_1 + x_2x_3 \\ \dot{x}_2 &= -x_2 + x_1^2 \\ \dot{x}_3 &= 1 - 4x_1 \end{aligned} \tag{11}$$

where a is a constant, positive parameter.

The system (11) exhibits a chaotic attractor when $a = 0.03$.

For numerical simulations, we take the initial conditions as

$$x_1(0) = -0.6, \;\; x_2(0) = 0.9, \;\; x_3(0) = -1.7 \tag{12}$$

Figure 1 shows the 3-D phase portrait of the Wei-Wang chaotic system (11). Figures 2, 3, and 4 show the 2-D projections of the Wei-Wang chaotic system (11) on the (x_1, x_2), (x_2, x_3) and (x_1, x_3) coordinate planes respectively.

It is known that the Wei-Wang chaotic system (11) has a stable equilibrium at

$$x^\star = \begin{bmatrix} -0.6 \\ 0.9 \\ -1.7 \end{bmatrix} \tag{13}$$

Also, the Lyapunov exponents of the Wei-Wang chaotic system (11) for the parameter value $a = 0.03$ and for the initial values (12) are numerically found as

$$L_1 = 0.0340, \quad L_2 = 0, \quad L_3 = -1.0002 \tag{14}$$

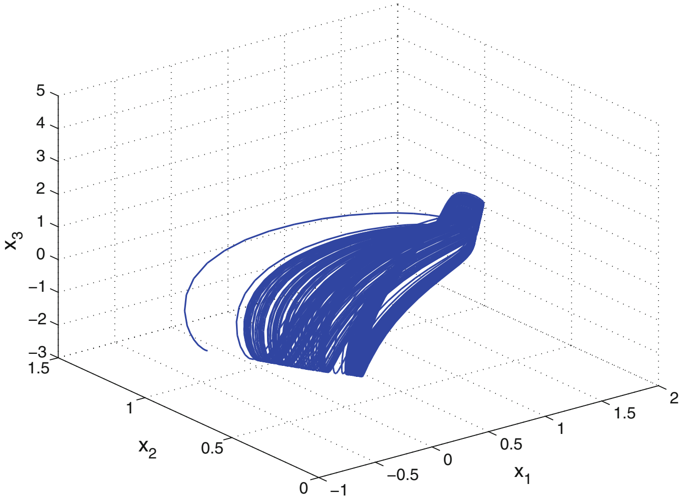

Fig. 1 3-D phase portrait of the Wei-Wang chaotic system

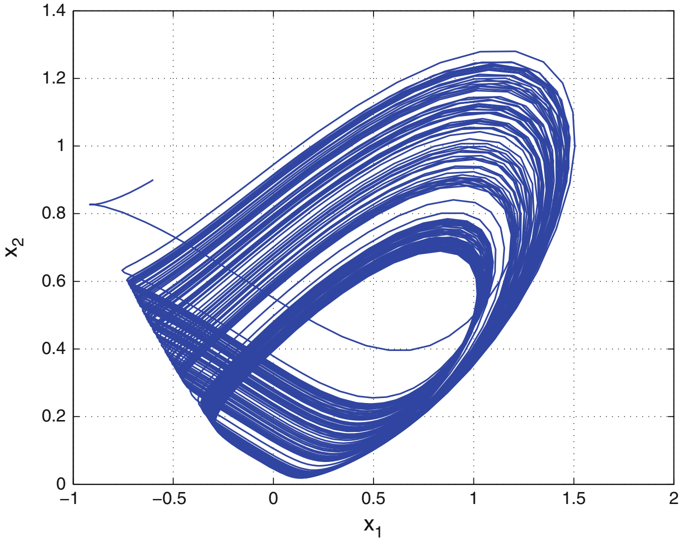

Fig. 2 2-D projection of the Wei-Wang chaotic system on the (x_1, x_2) plane

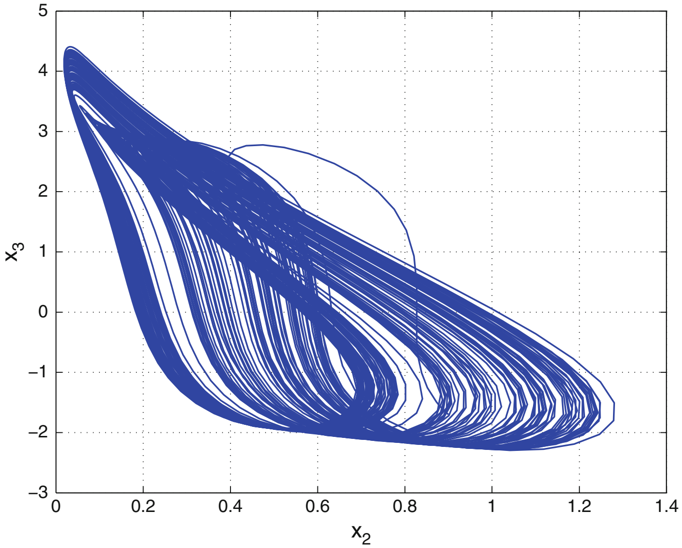

Fig. 3 2-D projection of the Wei-Wang chaotic system on the (x_2, x_3) plane

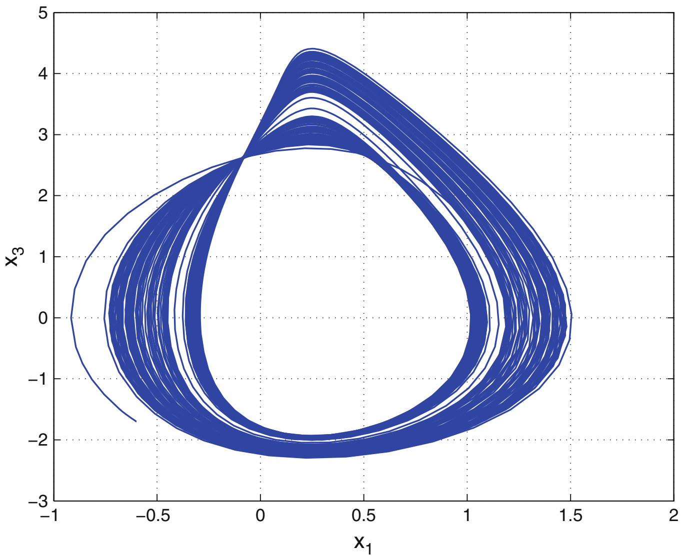

Fig. 4 2-D projection of the Wei-Wang chaotic system on the (x_1, x_3) plane

Thus, the Kaplan–Yorke dimension of the Wei-Wang chaotic system (11) is found as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0340 \tag{15}$$

4 Nonlinear Observer Design for the Wei-Wang Chaotic System

This section investigates the problem of nonlinear observer design for the Wei-Wang chaotic system described by the dynamics

$$\begin{aligned} \dot{x}_1 &= ax_1 + x_2x_3 \\ \dot{x}_2 &= -x_2 + x_1^2 \\ \dot{x}_3 &= 1 - 4x_1 \end{aligned} \tag{16}$$

where a is a positive parameter.

The system (16) is chaotic when $a = 0.03$. In the chaotic case, the system (16) has a stable equilibrium point

$$x^\star = \begin{bmatrix} -0.6 \\ 0.9 \\ -1.7 \end{bmatrix} \tag{17}$$

We consider the output function as

$$y = x_1 \tag{18}$$

The linearization of the plant dynamics (16) at $x = x^\star$ is given by

$$A = \frac{\partial f}{\partial x}\left(x^\star\right) = \begin{bmatrix} a & x_3^\star & x_2^\star \\ 2x_1^\star & -1 & 0 \\ -4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.03 & -1.7 & 0.9 \\ -1.2 & -1 & 0 \\ -4 & 0 & 0 \end{bmatrix} \tag{19}$$

Also, the linearization of the output function (18) at $x = x^\star$ is given by

$$C = \frac{\partial h}{\partial x}\left(x^\star\right) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{20}$$

From (19) and (20), the observability matrix for the Wei-Wang system (16) with the output (18) is given by

$$\mathbf{O}(C, A) = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.03 & -1.7 & 0.9 \\ -1.5591 & 1.649 & 0.027 \end{bmatrix} \tag{21}$$

We find that

$$\det[\mathbf{O}(C,A)] = -1.53 \neq 0 \tag{22}$$

which shows that $\mathbf{O}(C,A)$ has full rank. Thus, (C,A) is completely observable [22].

Since the equilibrium $x = x^\star$ is Lyapunov stable, by Sundarapandian's theorem (Theorem 1), we obtain the following result, which gives a construction of nonlinear observer for the Wei-Wang chaotic system.

Theorem 2 *The Wei-Wang chaotic system (16) with the output (18) has a local exponential observer of the form*

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} az_1 + z_2 z_3 \\ -z_2 + z_1^2 \\ 1 - 4z_1 \end{bmatrix} + K\left[y - z_1\right] \tag{23}$$

where K is a matrix chosen such that $A - KC$ is Hurwitz. Since (C,A) is observable, a gain matrix K can be found such that the error matrix $E = A - KC$ has arbitrarily assigned set of stable eigenvalues. ■

For numerical simulations, we find an observer gain matrix K so that

$$\text{eig}(A - KC) = \{-6, -6, -6\} \tag{24}$$

Using MATLAB, we get

$$K = \begin{bmatrix} 17.0300 \\ 72.3294 \\ 236.0000 \end{bmatrix} \tag{25}$$

Thus, a local exponential observer for the Wei-Wang system (16) is given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} az_1 + z_2 z_3 \\ -z_2 + z_1^2 \\ 1 - 4z_1 \end{bmatrix} + \begin{bmatrix} 17.0300 \\ 72.3294 \\ 236.0000 \end{bmatrix} \left[y - z_1\right] \tag{26}$$

where $a = 0.03$ (as in the chaotic case).

For numerical simulations, the initial conditions are chosen as

$$x(0) = \begin{bmatrix} 1.5 \\ 2.4 \\ 1.8 \end{bmatrix} \quad \text{and} \quad z(0) = \begin{bmatrix} 2.6 \\ 4.7 \\ 2.9 \end{bmatrix} \tag{27}$$

Figures 5, 6 and 7 depict the exponential convergence of the observer states z_1, z_2, z_3 of the system (26) to the plant states x_1, x_2, x_3 of the Wei-Wang system (16).

Figure 8 depicts the exponential convergence of the estimation errors e_1, e_2, e_3.

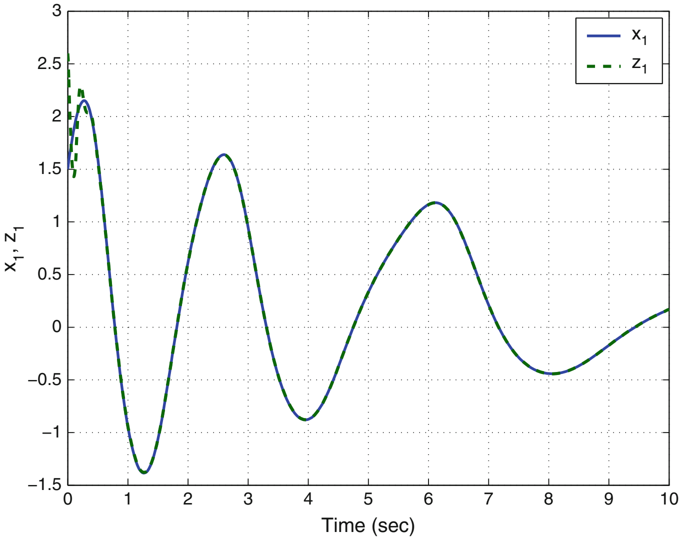

Fig. 5 Synchronization of the states x_1 and z_1

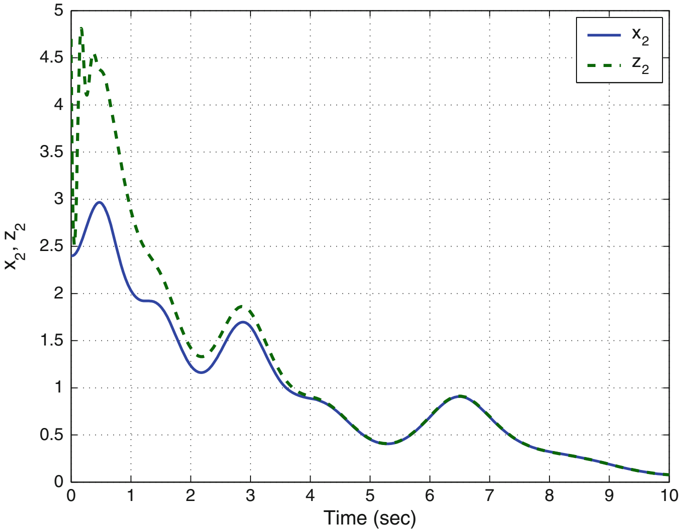

Fig. 6 Synchronization of the states x_2 and z_2

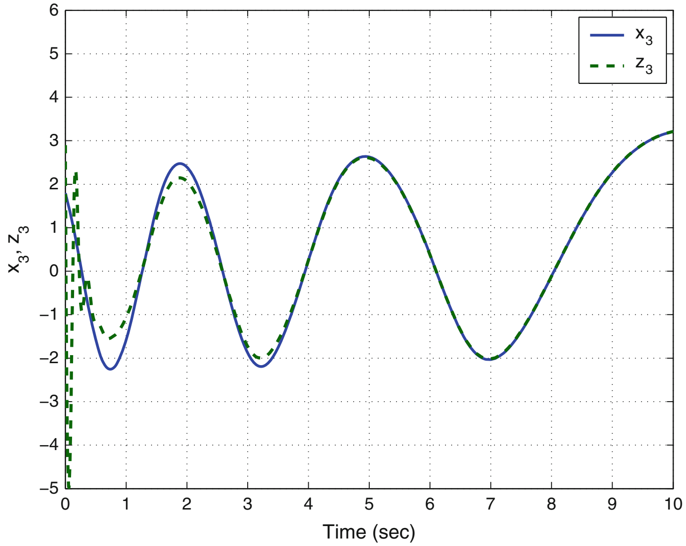

Fig. 7 Synchronization of the states x_3 and z_3

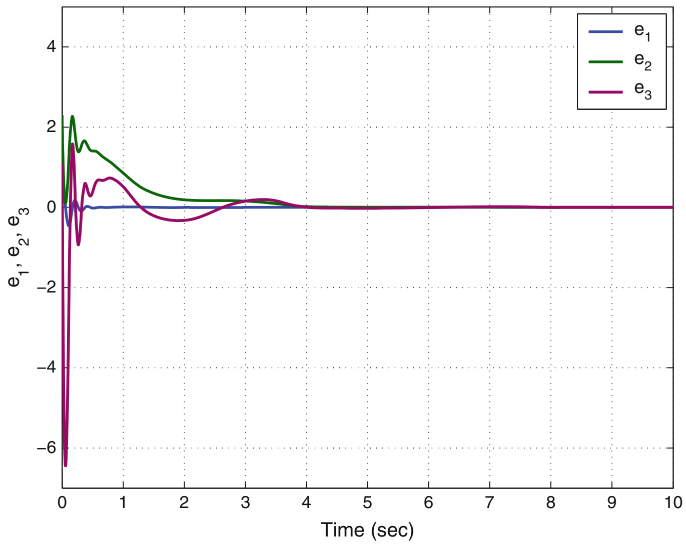

Fig. 8 Time-history of the estimation errors e_1, e_2, e_3

5 Dynamic Analysis of the Kingni-Jafari Chaotic System

The Kingni-Jafari chaotic system [13] is described by the 3-D dynamics

$$\begin{aligned} \dot{x}_1 &= -x_3 \\ \dot{x}_2 &= -x_1 - x_3 \\ \dot{x}_3 &= 3x_1 - ax_2 + x_1^2 - x_3^2 - x_2x_3 + b \end{aligned} \tag{28}$$

where a, b are constant, positive parameters.

The system (28) exhibits a chaotic attractor when $a = 1.3$ and $b = 1.01$.

For numerical simulations, we take the initial conditions as

$$x_1(0) = 0.1, \quad x_2(0) = 0.1, \quad x_3(0) = 0.1 \tag{29}$$

Figure 9 shows the 3-D phase portrait of the Kingni-Jafari chaotic system (28). Figures 10, 11 and 12 show the 2-D projections of the Kingni-Jafari chaotic system (28) on the (x_1, x_2), (x_2, x_3) and (x_1, x_3) coordinate planes respectively.

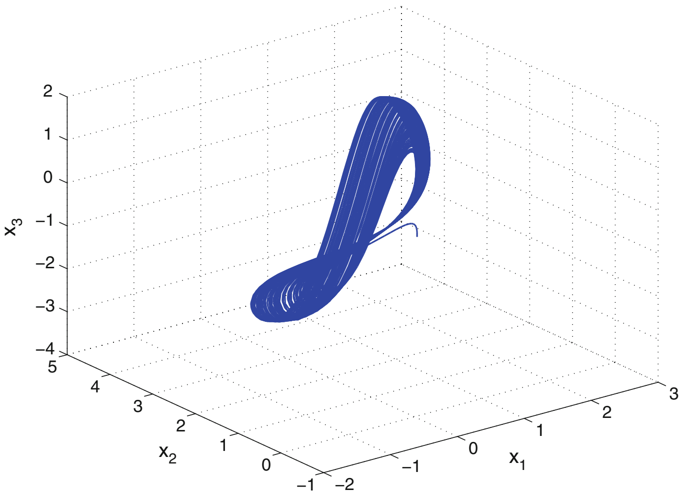

Fig. 9 3-D phase portrait of the Kingni-Jafari chaotic system

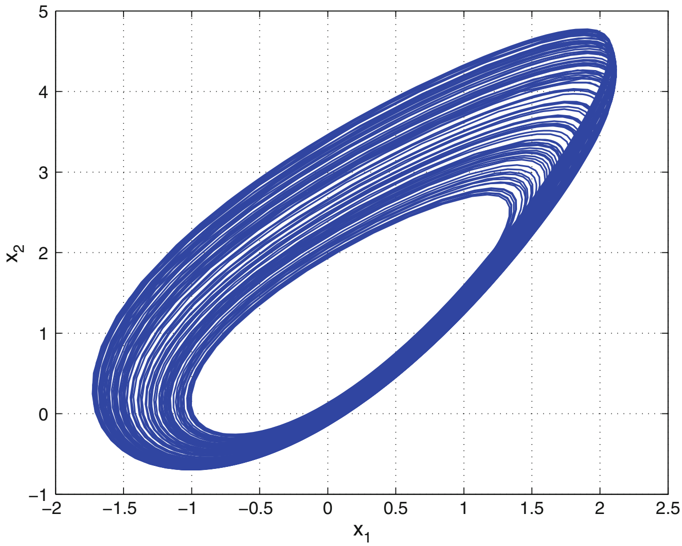

Fig. 10 2-D projection of the Kingni-Jafari chaotic system on the (x_1, x_2) plane

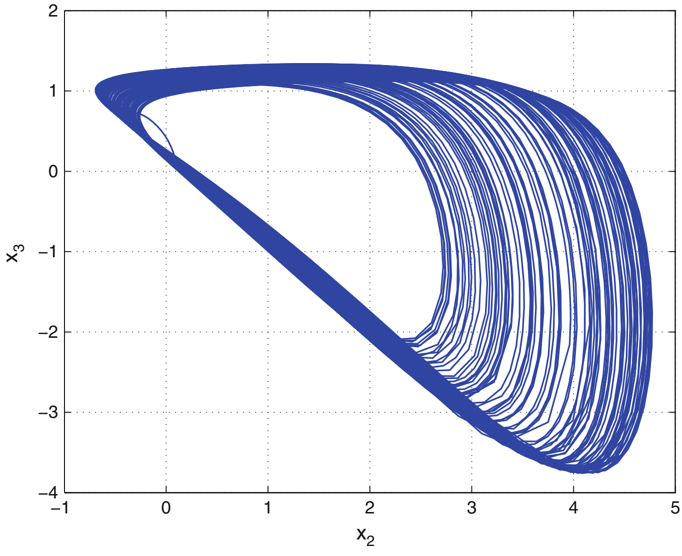

Fig. 11 2-D projection of the Kingni-Jafari chaotic system on the (x_2, x_3) plane

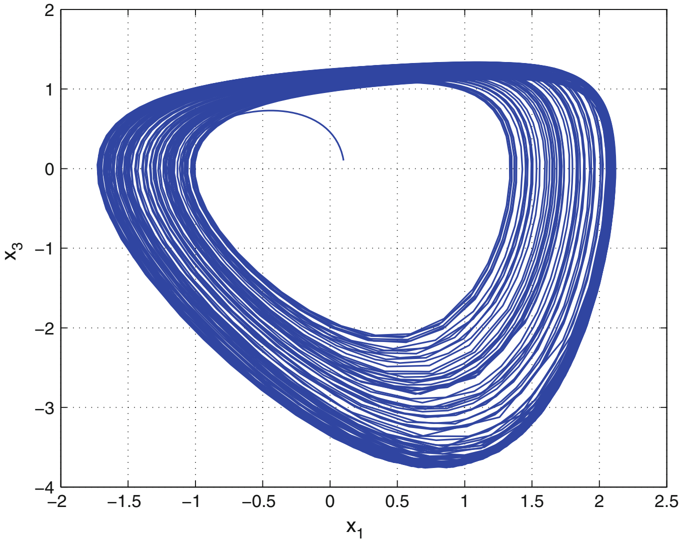

Fig. 12 2-D projection of the Kingni-Jafari chaotic system on the (x_1, x_3) plane

It is known that the Kingni-Jafari chaotic system (28) has a stable equilibrium at

$$x^\star = \begin{bmatrix} 0 \\ b/a \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.7769 \\ 0 \end{bmatrix} \tag{30}$$

Also, the Lyapunov exponents of the Kingni-Jafari chaotic system (28) for the parameter values $a = 1.3$, $b = 1.01$ and for the initial values (29) are numerically found as

$$L_1 = 0.0933, \quad L_2 = 0, \quad L_3 = -1.2969 \tag{31}$$

Thus, the Kaplan–Yorke dimension of the Kingni-Jafari chaotic system (28) is found as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0719 \tag{32}$$

6 Nonlinear Observer Design for the Kingni-Jafari Chaotic System

This section investigates the problem of nonlinear observer design for the Kingni-Jafari chaotic system described by the dynamics

$$\begin{aligned} \dot{x}_1 &= -x_3 \\ \dot{x}_2 &= -x_1 - x_3 \\ \dot{x}_3 &= 3x_1 - ax_2 + x_1^2 - x_3^2 - x_2x_3 + b \end{aligned} \tag{33}$$

where a is a positive parameter.

The system (33) is chaotic when $a = 1.3$ and $b = 1.01$. In the chaotic case, the system (33) has a stable equilibrium point

$$x^{\star} = \begin{bmatrix} 0 \\ 0.7769 \\ 0 \end{bmatrix} \tag{34}$$

We consider the output function as

$$y = x_1 \tag{35}$$

The linearization of the plant dynamics (33) at $x = x^{\star}$ is given by

$$A = \frac{\partial f}{\partial x}\left(x^{\star}\right) = \begin{bmatrix} a & x_3^{\star} & x_2^{\star} \\ 2x_1^{\star} & -1 & 0 \\ -4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & -1 \\ 3 & -1.3 & -0.7769 \end{bmatrix} \tag{36}$$

Also, the linearization of the output function (35) at $x = x^{\star}$ is given by

$$C = \frac{\partial h}{\partial x}\left(x^{\star}\right) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{37}$$

From (36) and (37), the observability matrix for the Kingni-Jafari system (33) with the output (35) is given by

$$\mathbf{O}(C, A) = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ -3 & 1.3 & 0.7769 \end{bmatrix} \tag{38}$$

We find that

$$\det[\mathbf{O}(C, A)] = 1.3 \neq 0 \tag{39}$$

which shows that $\mathbf{O}(C, A)$ has full rank. Thus, (C, A) is completely observable [22].

Since the equilibrium $x = x^{\star}$ is Lyapunov stable, by Sundarapandian's theorem (Theorem 1), we obtain the following result, which gives a construction of nonlinear observer for the Kingni-Jafari chaotic system.

Theorem 3 *The Kingni-Jafari chaotic system (33) with the output (35) has a local exponential observer of the form*

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -z_3 \\ -z_1 - z_3 \\ 3z_1 - az_2 + z_1^2 - z_3^2 - z_2 z_3 + b \end{bmatrix} + K \left[y - z_1 \right] \tag{40}$$

where K is a matrix chosen such that $A - KC$ is Hurwitz. Since (C, A) is observable, a gain matrix K can be found such that the error matrix $E = A - KC$ has arbitrarily assigned set of stable eigenvalues. ■

For numerical simulations, we find an observer gain matrix K so that

$$\text{eig}(A - KC) = \{-6, -6, -6\} \tag{41}$$

Using MATLAB, we get

$$K = \begin{bmatrix} 17.2231 \\ 182.3769 \\ -92.9194 \end{bmatrix} \tag{42}$$

Thus, a local exponential observer for the Wei-Wang system (16) is given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -z_3 \\ -z_1 - z_3 \\ 3z_1 - az_2 + z_1^2 - z_3^2 - z_2 z_3 + b \end{bmatrix} + \begin{bmatrix} 17.2231 \\ 182.3769 \\ -92.9194 \end{bmatrix} \left[y - z_1 \right] \tag{43}$$

where $a = 1.3$ and $b = 1.01$ (as in the chaotic case).

For numerical simulations, the initial conditions are chosen as

$$x(0) = \begin{bmatrix} 3.7 \\ 5.2 \\ 2.8 \end{bmatrix} \quad \text{and} \quad z(0) = \begin{bmatrix} 1.6 \\ 2.4 \\ 1.9 \end{bmatrix} \tag{44}$$

Figures 13, 14 and 15 depict the exponential convergence of the observer states z_1, z_2, z_3 of the system (43) to the plant states x_1, x_2, x_3 of the Kingni-Jafari system (33).

Figure 16 depicts the exponential convergence of the estimation errors e_1, e_2, e_3.

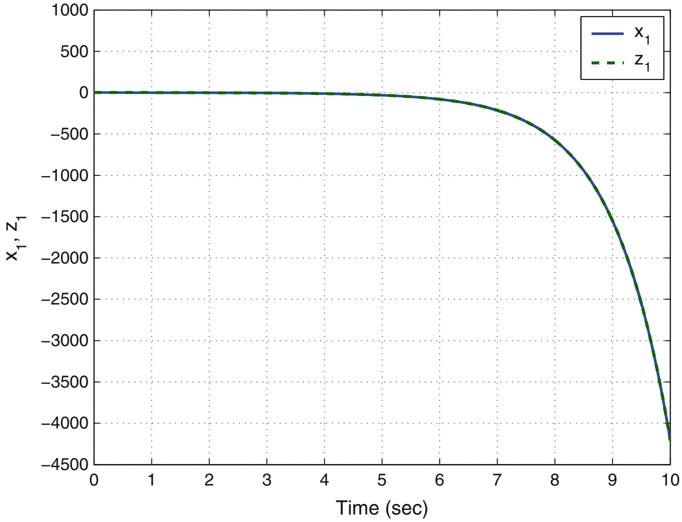

Fig. 13 Synchronization of the states x_1 and z_1

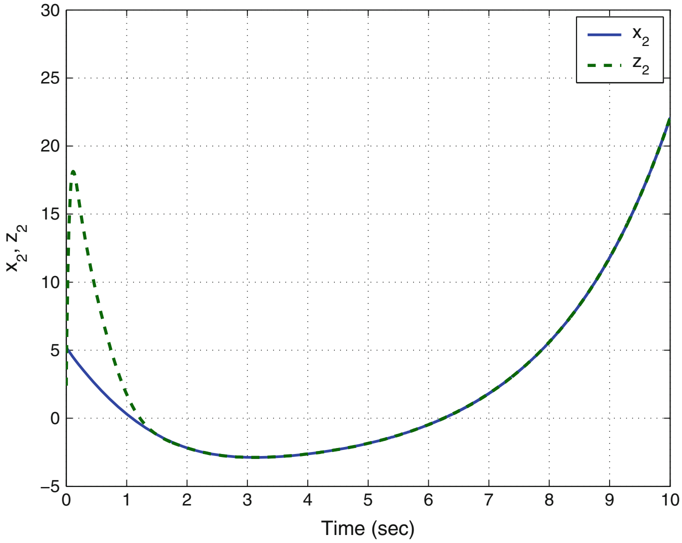

Fig. 14 Synchronization of the states x_2 and z_2

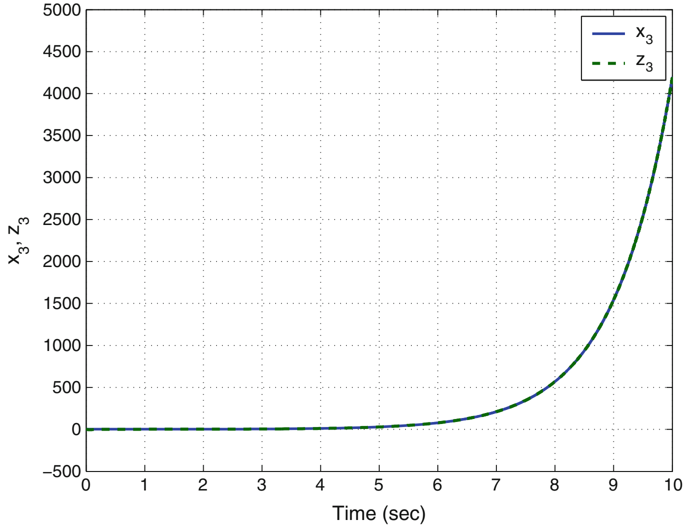

Fig. 15 Synchronization of the states x_3 and z_3

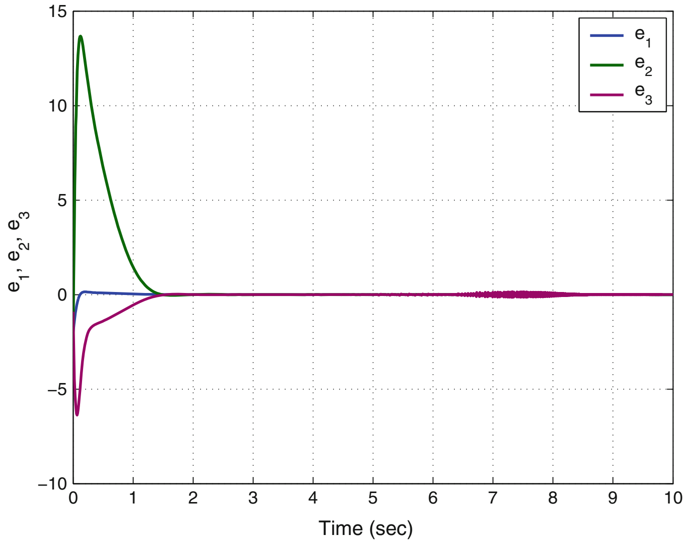

Fig. 16 Time-history of the estimation errors e_1, e_2, e_3

7 Conclusions

For many real world problems of chaotic systems, an efficient monitoring system is of great importance. In this work, the methodology based on Sundarapandian's theorem (2002) for exponential observer design is applied for the monitoring of chaotic systems with stable equilibria such as Wei-Wang system (2013) and Kingni-Jafari system (2014). MATLAB simulations have been shown to illustrate the phase portraits and nonlinear observer design for the Wei-Wang and Kingni-Jafari chaotic systems.

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