

# Preface

It is our privilege and great honor to pay our tribute to our friend, collaborator, and teacher Rabi N. Bhattacharya in the form of this volume of selected papers, collecting some of his most influential papers together with commentaries of colleagues from around the world. For more than three decades, we have been able to see Rabi's knowledge, insight, and influence in probability, statistics, and their applications grow; we hope this volume will shed some light on this fact and the reader will enjoy reading about Rabi's scientific developments and the state of research his name represents.

The book is divided into three parts, *Modes of Approximation*, *Large Times for Markov Processes*, and *Stochastic Foundations in Applied Sciences*, representing the main scientific contributions of Professor Bhattacharya. It ranges from theoretical statistics via analytical probability theory, Markov processes, and random dynamics to applied topics in statistics, economics, and geophysics. Such a wide range of interests is hardly overseen by a single person, so we are especially glad that so many of our colleagues representing these fields were willing and eager to contribute to this volume. Their articles help to position Rabi's work in the light of other achievements, further developments, and directions of research. An explicit goal is to help researchers who may wish to embark on any of these many varied paths. The reader will find a list of Rabi's PhD students and scientific writings just before the table of content.

This volume would not have been put together without the help of several people and numerous publishers. First of all, Anirban DasGupta from Purdue University made an initial proposal to Springer for a volume like this one. We thank Lizhen Li from the University of Texas at Austin for her support in collecting data, and Christopher Tommich and Benjamin Levitt from Birkhäuser/Springer for their help in the editorial process. The technical support by Suresh Kumar from Birkhäuser is greatly appreciated.

We also thank the publishers *American Mathematical Society*, *World Scientific Publishing Com.*, *Institute of Mathematical Statistics*, *Society for Industrial and Applied Mathematics (SIAM)*, *János Bolyai Mathematical Society*, *Elsevier*, and *Springer Nature* for generously granting the copyrights of those articles of R.N. Bhattacharya we had chosen to be reproduced in this volume. A list of these papers is provided under Acknowledgments.

## Dedicated to Random Processes

The name “Bhattacharya” is a combination of two words: Bhatta, meaning Vedic priest, generally known as sages or seers during the Vedic period, and the word Acharya, with the Sanskrit meaning of teacher. Although Rabindra Nath (Rabi) Bhattacharya has no formal religious affiliation, he exemplifies the best of the human spirit through his steadfast concern for the well-being of others throughout his career. This comes across most clearly in his professorial role via a sincere dedication to teaching, research, and service. Rabi has been a remarkable “seer” of unknown mathematical and statistical truths and always happy to share those insights with students and collaborators.

“But this can be treated as a Markov process” is a standing phrase of Rabi’s that exemplifies a fundamentally important mathematical structure that attracted Rabi’s attention shortly after completing his PhD thesis on a classic unsolved problem pertaining to rates of convergence in the central limit theorem for independent and identically distributed random vectors. An interest in Markov processes and their applications endured throughout his scientific career and is also well documented by this *Selecta* volume. An important part of this volume is dedicated to Rabi’s extraordinary contributions to the theory and application of Markov dynamics in their many guises, ranging from discrete parameter Markov chains and/or i.i.d. iterated random maps to continuous parameter Markov processes and diffusions.

However, this volume covers more than one or two such topics from Rabi’s work and interest in mathematics. There are two achievements connected to his name forever, the aforementioned clarification of rates of convergence to the normal distribution, the topic he started in his dissertation that made him well known immediately. This culminated in his providing the complete mathematical framework to justify the Edgeworth expansion of distributions during a collaboration with J.K. Ghosh in the 1970s. A statistician might argue that its value is overcome by computing power. In fact, Rabi and his students have substantiated this in a comparative analysis with Efron’s bootstrap method. However, the intrinsic mathematical and statistical value of Rabi’s contribution is far from being even a topic of discussion when viewed on its own right. Such results gain their importance in mathematical applications and generally serve as the final word on this question in the foundations of probability theory and mathematical statistics.

It is very hard to have a final judgment on Rabi’s more recent interest and achievements in statistics for data observed on manifolds. The absence of an obvious notion of a mean was overcome by Rabi and his former Indiana PhD student Vic Patrangenaru. Rabi had conceived of the proper formulation for statistical inference in this context as far back as the 1990s. Today, as we witness the emergence of a relatively new and deep area of statistics, it is not hard to guess that results of Rabi and his students will play an important role. The new theory and methods are supported by high computational power and we have been seeing this for quite some time in other areas of statistics as well. Rabi’s ideas follow traditional lines but also take excursions into unknown territory for statistical inference, such as data represented through the differential geometry of manifolds and cotangent spaces. As we see it today, one of the outstanding problems for statistical inference on manifolds is that of uniqueness of the Fréchet mean when there is positive curvature. Beyond Rabi’s initial proof for the case of the circle published in his monograph with his student Abhishek Bhattacharya, such results under conditions of practical value for statistics

remains one of the central obstructions to the development of a proper theory of inference on manifolds having positive curvature.

It is one of Rabi's firmly held tenets that mathematics can have a significant impact on applied sciences and many other disciplines, as he has demonstrated in diverse ways. Fair play in economics is one of his beliefs, and he tried in several publications to clarify the time evolution of market data. This is mainly done in the framework of random dynamics, which may be a good approximation to observed time series data. Although we do not yet know this, it is natural to follow this mathematical path to see where it leads. Another of Rabi's main contributions from an application point of view originates in the dispersion of solutes in fluid flows. In a long-term collaboration with hydrologist Vijay Gupta, Rabi was among the first to recognize the role of central limit theorems, Brownian motion, and the role of multiple-scale hierarchies in explaining observed transport behavior from laboratory to field scales. Striking results have been achieved here which made Rabi a much cited author in hydrology, geophysics, and applied probability.

Rabindra Nath Bhattacharya was born January 11, 1937, in his ancestral home of Porgola, District of Barisal, in the present country of Bangladesh. His family was uprooted by the partition of India and moved to Calcutta in 1947, where Rabi studied in Presidency College, receiving Bachelor of Science and Master of Science degrees, respectively, in 1956 and 1959. He then secured a research scholar appointment at the Indian Statistical Institute from 1959 to 1960. Rabi joined the mathematics faculty at the University of Kalyani, teaching there from 1961 to 1964. In 1964, Rabi obtained a fellowship from the Statistics Department at the University of Chicago, where he completed his PhD in 1967 under direction of Patrick Billingsley. Upon graduation, he returned to India to marry Bathika (Gouri) Banerjee, followed by acceptance of a position as Assistant Professor in the Statistics Department at the University of California in Berkeley in 1967. Rabi and Gouri have two children and four grandchildren.

In 1972, Rabi was recruited to join the Mathematics Department at the University of Arizona as an Associate Professor and was promoted to Full Professor in 1977, where he remained until 1982. In 1982, he accepted a professorship in the Mathematics Department at Indiana University and remained there until retirement in 2002. Upon retirement from Indiana University, Rabi returned to the University of Arizona where he is once again a tenured Professor of Mathematics.

Rabi received many honors and awards during his career. Well known are his Special Invited Papers in the *Annals of Probability* 1977 and the *Annals of Applied Probability* 1999. He was a specially invited lecturer at the German Mathematical Meeting 1989 and the IMS Annual Meeting 1996 in Chicago (Medallion Lecture). He received the prestigious Humboldt Senior Award in 1994/1995 and the Guggenheim Fellowship in 2002. The book *Probability, Statistics and Their Applications: Papers in Honor of Rabi Bhattacharya*<sup>1</sup> attests to the utmost appreciation by his friends, collaborators, and colleagues.

Rio de Janeiro, Brazil  
Corvallis, OR, USA  
October 2015

Manfred Denker  
Edward C. Waymire

---

<sup>1</sup> IMS Lecture Notes Monogr. Ser., 41, Institute of Mathematical Statistics, Beachwood, OH, 2003.

Rabi N. Bhattacharya

Selected Papers

Denker, M.; Waymire, E.C. (Eds.)

2016, XXI, 711 p. 1 illus., Hardcover

ISBN: 978-3-319-30188-4

A product of Birkhäuser Basel