A Chaotic Hyperjerk System Based on Memristive Device

Viet-Thanh Pham, Sundarapandian Vaidyanathan, Christos K. Volos, Sajad Jafari and Xiong Wang

Abstract From the mechanical system point of view, third-order derivatives of displacement or the time rate of change of acceleration is the jerk, while the fourth derivative has been known as a snap. As a result, a dynamical system which is presented by an nth order ordinary differential equation with $n > 3$ describing the time evolution of a single scalar variable is considered as a hyperjerk system. Hyperjerk system has received significant attention because of its elegant form. Motivated by reported attractive hyperjerk systems, a 4-D novel chaotic hyperjerk system has been introduced and studied in this work. Interestingly, this hyperjerk system displays an infinite number of equilibrium points because of the presence of a memristive device. In addition, an adaptive controller is proposed to achieve synchronization of such novel hyperjerk systems with two unknown parameters. In order to confirm the feasibility of the mathematical hyperjerk model, its electronic circuit is designed and implemented by using SPICE.

V.-T. Pham (✉)
School of Electronics and Telecommunications, Hanoi University of Science and Technology, Hanoi, Vietnam
e-mail: pvt3010@gmail.com

S. Vaidyanathan
Research and Development Centre, Vel Tech University, Tamil Nadu, India
e-mail: sundar@veltechuniv.edu.in

C.K. Volos
Physics Department, Aristotle University of Thessaloniki, Thessaloniki, Greece
e-mail: volos@physics.auth.gr

S. Jafari
Biomedical Engineering Faculty, Amirkabir University of Technology, Tehran, Iran
e-mail: sajadjafari@aut.ac.ir

X. Wang
Institute for Advanced Study, Shenzhen University, Guangdong, Shenzhen 518060, People's Republic of China
e-mail: wangxiong8686@szu.edu.cn

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1 Introduction

Chaotic systems have applied in several fields of science and engineering [2, 3, 7, 9, 46, 50, 66] after the vital discovery of Lorenz's model for atmospheric convection [31]. There are well-known chaotic systems such as Rössler system [42], Arneodo system [1], Chen system [7], Lü system [32] etc. In addition, various new chaotic systems have been introduced recently [16, 20, 34, 37, 40, 57, 63].

There is significant interest in investigating novel jerk chaotic systems [47]. From the view point of mathematics, a jerk system is presented by an explicit third-order ordinary differential equation which describes the time evolution of a single scale variable, for example x. Therefore, a jerk system is given as

$$\frac{d^3x}{dt^3} = f\left(\frac{d^2x}{dt^2}, \frac{dx}{dt}, x\right) \tag{1}$$

From the view point of mechanics, system (1) is called jerk system because when the scalar x represents the position of a moving object at the time t, the third derivative indicates the jerk [44]. Interestingly, well-known chaotic systems, i.e. Lorenz and Rössler systems, can be represented in jerk forms [21, 28].

Different examples of jerk systems were reported in the literature. A piecewise exponential jerk system was investigated by Sun and Sprott [52]. Another simple chaotic jerk system with exponential nonlinearity was presented in Munmuangsaen et al. [35] while its elegant electronic circuital implementation, including six resistors, three capacitors, four operational amplifiers and a silicon diode only, was introduced in Sprott [48]. A six-term 3-D novel jerk chaotic system with two hyperbolic sinusoidal nonlinearities was proposed by Vaidyanathan et al. [59]. Multi-scroll chaotic attractors could be generated in the jerk mode [30] or jerk circuits [33, 67] while multi-scroll and hypercube attractors were also achieved from a general jerk circuit using Josephson junctions [65].

By generalizing the definition of a jerk system [45], a hyperjerk system can be considered as

$$\frac{d^{(n)}x}{dt^n} = f\left(\frac{d^{(n-1)}x}{dt^{n-1}}, \ldots, \frac{dx}{dt}, x\right), \tag{2}$$

with $n \geq 4$ [47]. Hyperjerk form can described all periodically forced oscillators and many of the coupled oscillators [29] while transformation of 4-D dynamical systems to hyperjerk form was reported in Elhadj and Sprott [12]. Chaotic hyperjerk system including fourth and fifth derivatives was introduced [8]. In addition, Chlouverakis and Sprott found hyperchaotic hyperjerk flows. More recently, Sundarapandian proposed a 4-D novel hyperchaotic hyperjerk system by adding a quadratic nonlinearity to the Chlouverakis–Sprott hyperjerk system [60].

It is easy to see that reported jerk/hyperjerk systems have a finite number of equilibrium points. It is very interesting to ask naturally whether there exists a chaotic jerk/hyperjerk system without equilibria or with an unlimited equilibrium set. Some authors have recently answered this attractive question. Wang and Chen [64] constructed a jerk system with no equilibrium point, but still generated a chaotic attractor. A chaotic memory system with infinitely many equilibria was designed by using the concept of memory element [4]. Studying such jerk/hyperjerk systems with special features is still an open research direction.

In this chapter, our work has concentrated on a hyperjerk system based on a memristive device which can exhibit chaotic attractors. Moreover, such hyperjerk system has an infinite number of equilibrium points. This research work is organized as follows. Section 2 gives a brief introduction to the memristive device. The memristive hyperjerk system is presented in Sect. 3 while its qualitative properties are analyzed in Sect. 4. In Sect. 5, we describe the adaptive synchronization design for achieving global chaos synchronization of the identical novel hyperjerk systems with two unknown parameters. Section 6 shows the circuital implementation of our memristive hyperjerk system. Finally, conclusions are drawn in the last section.

2 Model of Memristive Device

Memristor was proposed by L.O. Chua as the fourth basic circuit element beside the three conventional ones (the resistor, the inductor and the capacitor) [10]. Memristor presents the relationship between two fundamental circuit variables, the charge (q) and the flux (φ). Hence, there are two kinds of memristor: charge-controlled memristor and flux-controlled memristor [10, 54]. A charge-controlled memristor is described by

$$v_M = M(q)\, i_M, \tag{3}$$

where v_M is the voltage across the memristor and i_M is the current through the memristor. Here the memristance (M) is defined by

$$M(q) = \frac{d\varphi(q)}{dq}, \tag{4}$$

while the flux-controlled memristor is given by

$$i_M = W(\varphi)\, v_M, \tag{5}$$

where $W(\varphi)$ is the memductance, which is defined by

$$W(\varphi) = \frac{dq(\varphi)}{d\varphi}. \tag{6}$$

Moreover, by generalizing the original definition of a memristor [11, 54], a memristive system is given as:

$$\begin{cases} \dot{x} = F(x, u, t) \\ y = G(x, u, t)\, u, \end{cases} \tag{7}$$

where u, y, and x denote the input, output and state of the memristive system, respectively. The function F is a continuously differentiable, n-dimensional vector field and G is a continuous scalar function.

Based on the definition of memristive system [4, 11, 38, 54], a memristive device is introduced in this section and used in our whole chapter. The memristive device is described by the following form:

$$\begin{cases} \dot{x}_1 = x_2 \\ y = (1 - x_1)\, x_2. \end{cases} \tag{8}$$

Here x_2, y, and x_1 are the input, output and state of the memristive device, respectively.

An external bipolar periodic signal is applied across terminals of memristive device (8) to investigate its fingerprint [51, 54, 55]. The external sinusoidal stimulus is given by

$$x_2 = X_2 \sin(2\pi f t), \tag{9}$$

where X_2 is the amplitude and f is the frequency. From the first equation of (8), the state variable of the memristive device is obtained as

$$\begin{aligned} x_1(t) &= \int_{-\infty}^{t} x_2(\tau)\, d\tau = x_1(0) + \int_{0}^{t} X_2 \sin(2\pi f \tau)\, d\tau \\ &= x_1(0) + \frac{X_2}{2\pi f}\left(1 - \cos(2\pi f t)\right), \end{aligned} \tag{10}$$

with $x_1(0)$ is the initial condition of the internal state in the memristive device. Thus, the initial condition of the internal state variable is given by

$$x_1(0) = \int_{-\infty}^{0} x_2(\tau)\, d\tau. \tag{11}$$

Substituting (9) and (10) into (8), it is easy to derive the output of the memristive device

$$\begin{aligned} y(t) &= \left[1 - x_1(0) - \frac{X_2}{2\pi f}\left(1 - \cos(2\pi f t)\right)\right] X_2 \sin(2\pi f t) \\ &= \left(1 - x_1(0) - \frac{X_2}{2\pi f}\right) X_2 \sin(2\pi f t) + \frac{X_2^2}{4\pi f t} \sin(4\pi f t). \end{aligned} \tag{12}$$

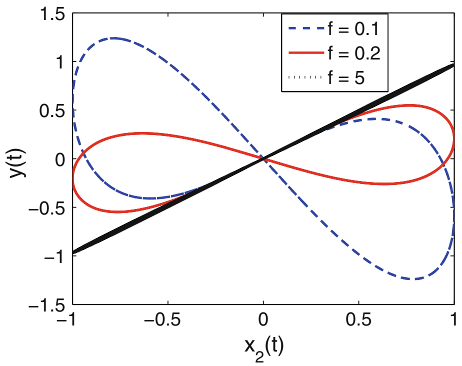

Fig. 1 Hysteresis loops of the proposed memristive device (8) driven by a sinusoidal stimulus (9) when changing the frequency f

From Eq. (12), it is easily seen that the output y depends on the frequency of the applied input stimulus. Hysteresis loop of the memristive device (8) when driven by a periodic signal (9) with different frequencies are shown in Fig. 1. Exhibited "pinched hysteresis loop" in the input–output plane indicates the vital fingerprint of memristive device (8).

3 A 4-D Novel Memristive Hyperjerk System

In this chapter, a novel 4-D memristive system is proposed by using the memristive device (8) and the reported approach in Bao et al. [4]. The novel memristive system is given in system form as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_3 - ax_4 - bx_3x_4 - y, \end{cases} \tag{13}$$

where a, b are positive parameters and $y = (1 - x_1)\, x_2$ is the output of memristive device (8).

The novel memristive system (13) can be rewritten by

$$\frac{d^4x_1}{dt^4} = f\left(\frac{d^3x_1}{dt^3}, \frac{d^2x_1}{dt^2}, \frac{dx_1}{dt}, x_1\right), \tag{14}$$

where

$$f = -\frac{d^2x_1}{dt^2} - a\frac{d^3x_1}{dt^3} - b\frac{d^2x_1}{dt^2}\frac{d^3x_1}{dt^3} - (1 - x_1)\frac{dx_1}{dt}. \tag{15}$$

Therefore, memristive system (13) is called a hyperjerk system because it involves time derivatives of a jerk function [45, 47]. In this chapter, the memristive system (13) is chaotic when the parameters a, and b take the values

$$a = 0.5, \quad b = 0.4. \tag{16}$$

For the selected parameter values in (16), the Lyapunov exponents of the novel memristive system (13) are obtained as

$$L_1 = 0.0730, \quad L_2 = 0.0018, \quad L_3 = 0, \quad L_4 = -0.5755. \tag{17}$$

For numerical simulations, we take the initial conditions of the novel memristive system (13) as $x_1(0) = 0.06$, $x_2(0) = 10^{-6}$, $x_3(0) = 0$, and $x_4(0) = 0$. Here the initial condition of the input of the memristive device $x_2(0)$ should be tiny to guarantee an appropriate value of the internal state variable of the memristive device. Figures 2, 3 and 4 illustrate the 2-D projections and 3-D projections of the new memristive system (13).

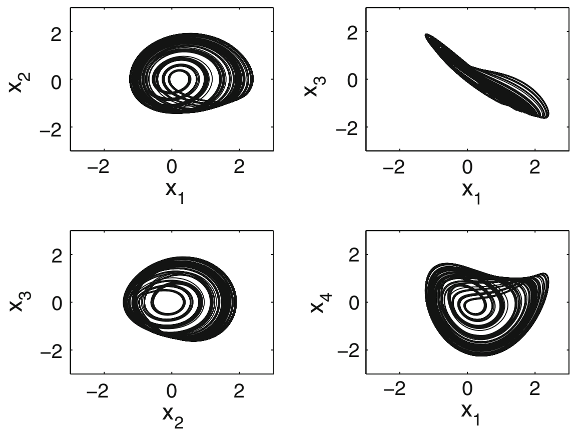

Fig. 2 2-D projections of the novel chaotic hyperjerk system (13) in (x_1, x_2)-plane, (x_1, x_3)-plane, (x_2, x_3)-plane, and (x_1, x_4)-plane

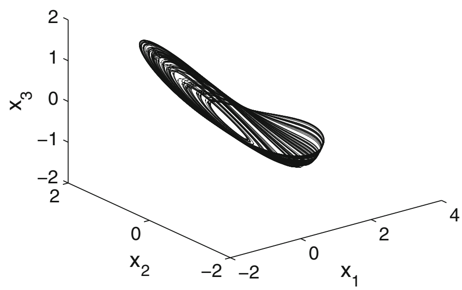

Fig. 3 Strange attractor of the novel chaotic hyperjerk system (13) in (x_1, x_2, x_3)-space

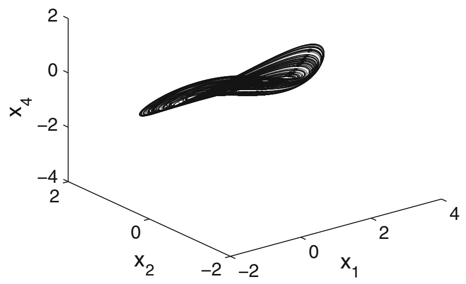

Fig. 4 Strange attractor of the novel chaotic hyperjerk system (13) in (x_1, x_2, x_4)-space

4 Analysis of the 4-D Novel Memristive Hyperjerk System

4.1 Equilibrium Points

The equilibrium points of the 4-D novel memristive hyperjerk system (13) are obtained by solving the equations

$$\begin{cases} f_1(x_1, x_2, x_3, x_4) = x_2 & = 0 \\ f_2(x_1, x_2, x_3, x_4) = x_3 & = 0 \\ f_3(x_1, x_2, x_3, x_4) = x_4 & = 0 \\ f_4(x_1, x_2, x_3, x_4) = -x_3 - ax_4 - bx_3x_4 - y & = 0 \end{cases} . \tag{18}$$

Thus, the equilibrium points of the system (13) are characterized by the equations

$$y = (1 - x_1)x_2 = 0, \quad x_2 = 0, \quad x_3 = 0, \quad x_4 = 0 \tag{19}$$

Solving the system (19), we get the equilibrium points of the hyperjerk system (13) as

$$E_c = \begin{bmatrix} c \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{20}$$

where c is a real constant. Interestingly, the novel hyperjerk system (13) displays an infinite number of equilibrium points because of the presence of a memristive device (8). According to a new classification of chaotic dynamics [24–27], there are two kinds of attractors: self-excited attractors and hidden attractors. A self-excited attractor has a basin of attraction that is excited from unstable equilibria. In contrast, a hidden attractor cannot be discovered by using a numerical approach where a trajectory started from a point on the unstable manifold in the neighbourhood of an unstable equilibrium [15, 22, 23]. Therefore, hyperjerk system (13) can be considered as a chaotic memristive system with hidden attractor.

In order to discover the stability type of the equilibrium points E_c the Jacobian matrix of the novel memristive hyperjerk system (13) is calculated at any point $\mathbf{x}$ as

$$J(\mathbf{x}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ x_2 & x_1 - 1 & -1 - bx_4 & -a - bx_3 \end{bmatrix}, \tag{21}$$

It is noting that

$$J_0 \triangleq J(E_c) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & c - 1 & -1 & -0.5 \end{bmatrix}, \tag{22}$$

which has the characteristic equation is

$$\lambda \left(\lambda^3 + 0.5\lambda^2 + \lambda + 1 - c \right) = 0. \tag{23}$$

When $c = 0.06$ the characteristic Eq. (23) has a zero eigenvalue and three nonzero eigenvalues

$$\lambda_1 = 0, \quad \lambda_2 = -0.7749, \quad \lambda_{3,4} = 0.1375 \pm 1.0928i \tag{24}$$

This shows that the equilibrium point E_c is an unstable saddle-focus point.

4.2 Lyapunov Exponents and Kaplan–Yorke Dimension

For the parameter values $a = 0.5$, $b = 0.4$ and $c = 0.06$, the Lyapunov exponents of the novel memristive hyperjerk system (13) are obtained using MATLAB as

$$L_1 = 0.0730, \quad L_2 = 0.0018, \quad L_3 = 0 \text{ and } L_4 = -0.5755 \tag{25}$$

There is one positive Lyapunov exponents in the LE spectrum (25), thus the novel memristive hyperjerk system (13) exhibits chaotic behavior.

In addition, since $L_1 + L_2 + L_3 + L_4 = -0.5007 < 0$, it indicates that the novel memristive system (13) is dissipative.

The Kaplan–Yorke fractional dimension, that presents the complexity of attractor [46, 50], is defined by

$$D_{KY} = j + \frac{1}{|L_{j+1}|} \sum_{i=1}^{j} L_i \tag{26}$$

where j is the largest interger satisfying $\sum_{i=1}^{j} L_i \geq 0$ and $\sum_{i=1}^{j+1} L_i < 0$. Therefore, the Kaplan–Yorke dimension of the novel memristive hyperjerk system (13) is calculated as

$$D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.130, \tag{27}$$

which is fractional.

5 Adaptive Synchronization for the Hyperjerk Memristive System

One of the most important characteristics relating to chaotic systems and their applications is the possibility of synchronization of two chaotic ones [5, 13, 17, 36]. A wide range of research activities based on synchronization of nonlinear systems has been studied [6, 14, 18, 39, 49, 58]. For example, various synchronization phenomena in bidirectionally coupled double scroll circuits were reported in Volos et al. [61] or image encryption process based on chaotic synchronization phenomena was presented in [62]. Different synchronization schemes have proposed such as anti-synchronization [56], adaptive sycnchronization [59], or hybrid chaos synchronization [18], etc. Here we consider the adaptive synchronization of identical 4-D memristiive hyperjerk systems with two unknown parameters.

The master system is considered as the 4-D novel memristive hyperjerk system given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_3 - ax_4 - bx_3x_4 - x_2 + x_1x_2 \end{cases} \tag{28}$$

where x_1, x_2, x_3, x_4 are the states of the system, and a, b are unknown constant parameters.

The slave system is considered as the 4-D novel memristive hyperjerk system given by

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = y_4 \\ \dot{y}_4 = -y_3 - ay_4 - by_3y_4 - y_2 + y_1y_2 + u \end{cases} \tag{29}$$

where y_1, y_2, y_3, y_4 are the states of the system, and u is a backstepping control to be determined using estimates $\hat{a}(t)$ and $\hat{b}(t)$ for a and b, respectively.

The synchronization errors between the states of the master system (28) and the slave system (29) are defined as

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \\ e_4 = y_4 - x_4 \end{cases} \tag{30}$$

Thus, the error dynamics is easily obtained as follows

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = e_4 \\ \dot{e}_4 = -e_3 - ae_4 - e_2 - b(y_3y_4 - x_3x_4) + y_1y_2 - x_1x_2 + u \end{cases} \tag{31}$$

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \end{cases} \tag{32}$$

Differentiating (32) with respect to t, we obtain the following equations:

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \end{cases} \tag{33}$$

Next, the main result of this section will be presented and proved.

Theorem 5.1 *The identical 4-D novel memristive hyperjerk systems (28) and (29) with unknown parameters a and b are globally and exponentially synchronized by the adaptive control law*

$$\begin{cases} u(t) = -5e_1 - 9e_2 - 8e_3 - [4 - \hat{a}(t)]e_4 + \hat{b}(t)\,(y_3y_4 - x_3x_4) \\ \qquad\quad - (y_1y_2 - x_1x_2) - kz_4 \end{cases} \tag{34}$$

where $k > 0$ *is a gain constant,*

$$z_4 = 3e_1 + 5e_2 + 3e_3 + e_4, \tag{35}$$

and the update law for the parameter estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t)$ *is given by*

$$\begin{cases} \dot{\hat{a}}(t) = -e_4z_4 \\ \dot{\hat{b}}(t) = -\,(y_3y_4 - x_3x_4)\,z_4 \end{cases} \tag{36}$$

Proof We prove this result via backstepping control method and Lyapunov stability theory.

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2}\, z_1^2 \tag{37}$$

where

$$z_1 = e_1 \tag{38}$$

Differentiating V_1 along the error dynamics (31), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = e_1 e_2 = -z_1^2 + z_1(e_1 + e_2) \tag{39}$$

Here, we define

$$z_2 = e_1 + e_2 \tag{40}$$

Using (40), we can simplify the Eq. (39) as

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \tag{41}$$

Secondly, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2}\, z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \tag{42}$$

Differentiating V_2 along the error dynamics (31), we get

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3) \tag{43}$$

Now, we define

$$z_3 = 2e_1 + 2e_2 + e_3 \tag{44}$$

Using (44), we can simplify the Eq. (43) as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3 \tag{45}$$

Thirdly, we define a quadratic Lyapunov function

$$V_3(z_1, z_2, x_3) = V_2(z_1, z_2) + \frac{1}{2}\, z_3^2 = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2\right) \tag{46}$$

Differentiating V_3 along the error dynamics (31), we get

$$\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 + z_3(3e_1 + 5e_2 + 3e_3 + e_4) \tag{47}$$

Now, we define

$$z_4 = 3e_1 + 5e_2 + 3e_3 + e_4 \tag{48}$$

Using (48), we can simplify the Eq. (47) as

$$\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 + z_3 z_4 \tag{49}$$

Finally, we define a quadratic Lyapunov function

$$V(z_1, z_2, z_3, z_4, e_a, e_b) = V_3(z_1, z_2, z_3) + \frac{1}{2}z_4^2 + \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2 \tag{50}$$

which is a positive definite function on R^6.

Differentiating V along the error dynamics (31), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4(z_4 + z_3 + \dot{z}_4) - e_a\dot{\hat{a}} - e_b\dot{\hat{b}} \tag{51}$$

Equation (51) can be written compactly as

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4 S - e_a\dot{\hat{a}} - e_b\dot{\hat{b}} \tag{52}$$

where

$$S = z_4 + z_3 + \dot{z}_4 = z_4 + z_3 + 3\dot{e}_1 + 5\dot{e}_2 + 3\dot{e}_3 + \dot{e}_4 \tag{53}$$

A simple calculation gives

$$S = 5e_1 + 9e_2 + 8e_3 + (4 - a)e_4 - b\,(y_3 y_4 - x_3 x_4) + (y_1 y_2 - x_1 x_2) + u \tag{54}$$

Substituting the adaptive control law (34) into (54), we obtain

$$S = -\left[a - \hat{a}(t)\right] e_4 - \left[b - \hat{b}(t)\right] (y_3 y_4 - x_3 x_4) - k z_4 \tag{55}$$

Using the definitions (33), we can simplify (55) as

$$S = -e_a e_4 - e_b\,(y_3 y_4 - x_3 x_4) - k z_4 \tag{56}$$

Substituting the value of S from (56) into (52), we obtain

$$\left\{ \begin{aligned} \dot{V} &= -z_1^2 - z_2^2 - z_3^2 - (1 + k)z_4^2 + e_a(-e_4 z_4 - \dot{\hat{a}}) \\ &\quad + e_b\left[-\,(y_3 y_4 - x_3 x_4)\, z_4 - \dot{\hat{b}}\right] \end{aligned} \right. \tag{57}$$

Substituting the update law (36) into (57), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - (1+k)z_4^2, \tag{58}$$

which is a negative semi-definite function on R^6. Therefore, according to the Lyapunov stability theory [19, 43] we obtain $e_1(t) \to 0$, $e_2(t) \to 0$, $e_3(t) \to 0$, $e_4(t) \to 0$, $e_a(t) \to 0$, $e_b(t) \to 0$ exponentially when $t \to 0$ that is, synchronization between master and slave system.

In order to confirm and demonstrate the effectiveness of the proposed synchronization scheme, we consider a numerical example. In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the systems. The parameters of the memristive hyperjerk systems are selected as $a = 0.5$, $b = 0.4$ and the positive gain constant as $k = 6$. The initial conditions of the master system (28) and the slave system (29) have been chosen as $x_1(0) = 0.06$, $x_2(0) = 10^{-6}$, $x_3(0) = 0$, $x_4(0) = 0$ and $y_1(0) = 0.02$, $y_2(0) = 10^{-4}$, $y_3(0) = 0$, $y_4(0) = 0$, respectively. We assumed that the initial values of the parameter estimates are $\hat{a}(0) = 0.46$ and $\hat{b}(0) = 0.01$.

When adaptive control law (34) and the update law for the parameter estimates (36) are applied, the master (28) and slave system (29) are synchronized completely

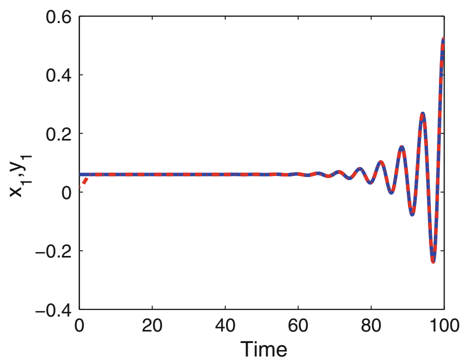

Fig. 5 Synchronization of the states $x_1(t)$ and $y_1(t)$

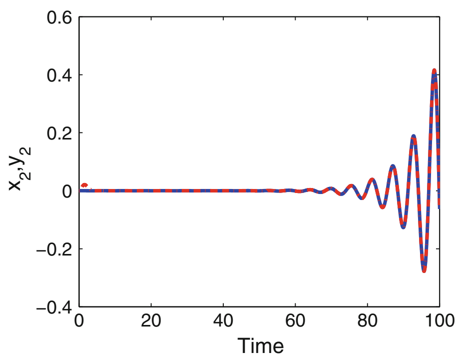

Fig. 6 Synchronization of the states $x_2(t)$ and $y_2(t)$

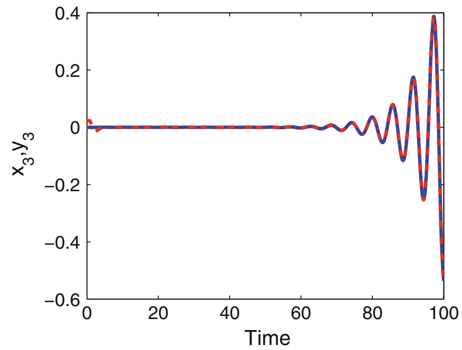

Fig. 7 Synchronization of the states $x_3(t)$ and $y_3(t)$

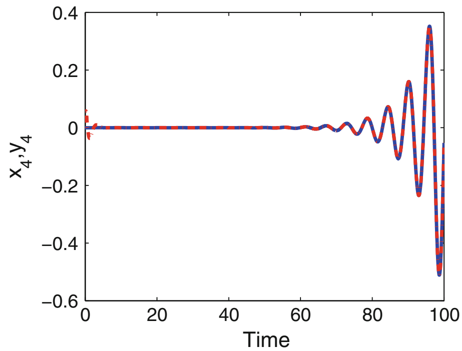

Fig. 8 Synchronization of the states $x_4(t)$ and $y_4(t)$

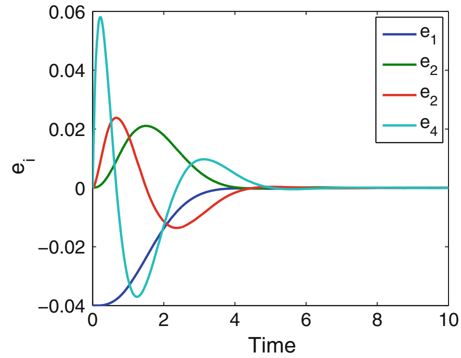

Fig. 9 Time series of the synchronization errors e_1, e_2, e_3, and e_4

as shown in Figs. 5, 6, 7 and 8. In such figures, time series of master states are denoted as blue solid lines while corresponding slave states are plotted as red dash-dot lines. In addition, the time-history of the complete synchronization errors e_1, e_2, e_3, and e_4 are reported in Fig. 9. The obtained results illustrate the correctness of used approach.

6 SPICE Implementation of the Memristive Hyperjerk System

In this section, an electronic circuit is proposed to implement memristive hyperjerk system (13). The circuit in Fig. 10 has been designed by applying the general approach with operational amplifiers [41, 53]. Thus, the variables x_1, x_2, x_3, x_4 of memristive system (13) are the voltages across the capacitor C1, C2, C3, and C4, respectively. As shown in Fig. 10 the memristive system is realized by common electronic components. Indeed the sub-circuit of memristive device in Fig. 10 only emulates the memristive device because there are not any commercial off-the-shelf memristive devices in the market yet. By applying Kirchhoff's circuit laws, the corresponding circuital equations of designed circuit can be written as:

$$\begin{cases} \frac{dv_{C_1}}{dt} = \frac{1}{R_1C_1}v_{C_2} \\ \frac{dv_{C_2}}{dt} = \frac{1}{R_2C_2}v_{C_3} \\ \frac{dv_{C_3}}{dt} = \frac{1}{R_3C_3}v_{C_4} \\ \frac{dv_{C_4}}{dt} = -\frac{1}{R_4C_4}v_{C_3} - \frac{1}{R_5C_4}v_{C_4} - \frac{1}{R_6C_4}v_{C_3}v_{C_4} - \frac{1}{R_7C_4}y, \end{cases} \tag{59}$$

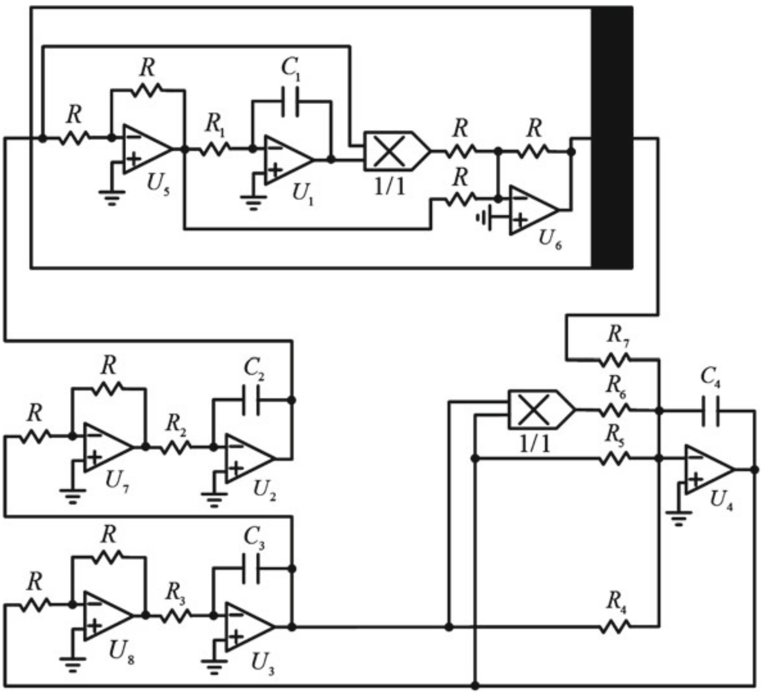

Fig. 10 Schematic of the circuit which emulating novel hyperjerk system (8) with the presence of the memristive device

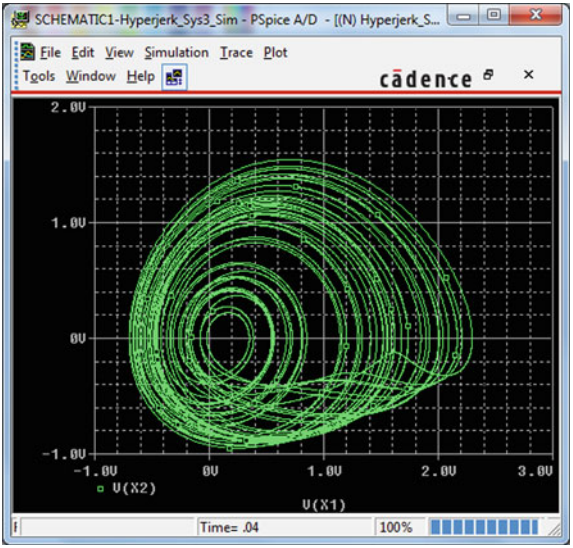

Fig. 11 Chaotic attractor of the designed circuit obtained from Cadence OrCAD in the (v_{C_1}, v_{C_2}) phase plane

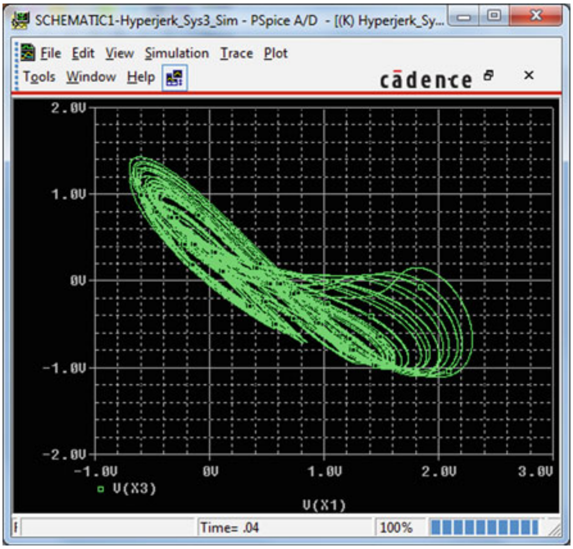

Fig. 12 Chaotic attractor of the designed circuit obtained from Cadence OrCAD in the (v_{C_1}, v_{C_3}) phase plane

where v_{C_1}, v_{C_2}, v_{C_3}, and v_{C_4} are the voltages across the capacitors C_1, C_2, C_3, and C_4, respectively. Here the memristive device is described by the following circuital equations:

$$\begin{cases} \frac{dv_{C_1}}{dt} = \frac{1}{R_1 C_1} v_{C_2} \\ y = v_{C_2} - v_{C_1} v_{C_2}. \end{cases} \tag{60}$$

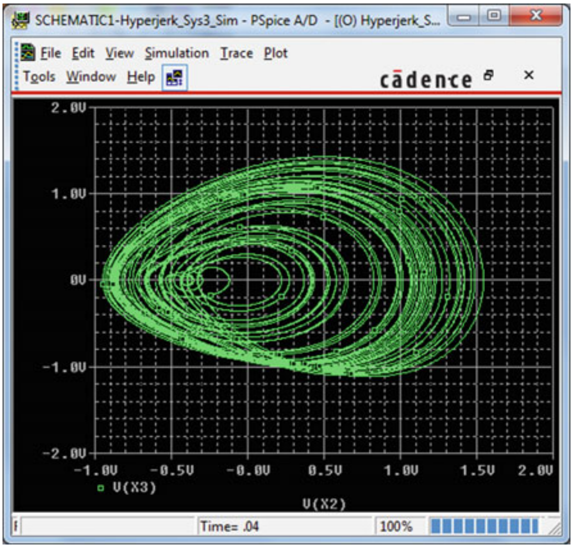

Fig. 13 Chaotic attractor of the designed circuit obtained from Cadence OrCAD in the (v_{C_2}, v_{C_3}) phase plane

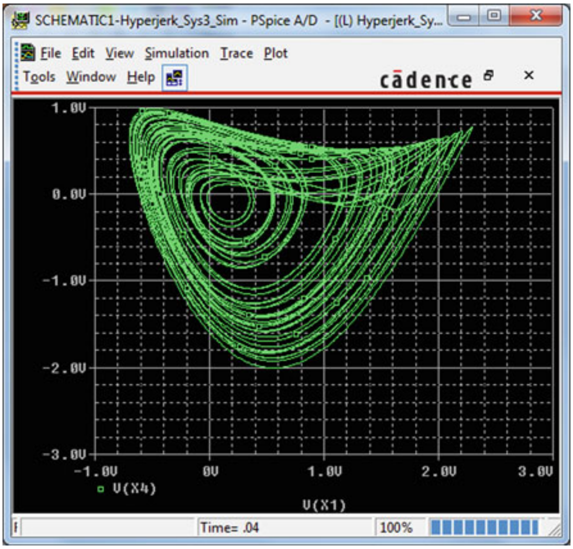

Fig. 14 Chaotic attractor of the designed circuit obtained from Cadence OrCAD in the (v_{C_1}, v_{C_4}) phase plane

The power supplies of all active devices are $\pm 15 V_{DC}$ and the operational amplifiers TL084 are used in this work. The values of components are selected as follows: $R_1 = R_2 = R_3 = R_4 = R_7 = R = 100\,\text{k}\Omega$, $R_5 = 200\,\text{k}\Omega$, $R_6 = 250\,\text{k}\Omega$, and $C_1 = C_2 = C_3 = C_4 = 1\,\text{nF}$.

The designed circuit is implemented in the electronic simulation package Cadence OrCAD and the obtained results are reported in Figs. 11, 12, 13 and 14. Theoretical attractors (see Fig. 2) are similar with the circuital ones (see Figs. 11, 12, 13 and 14).

7 Conclusion

A 4-D hyperjerk system is introduced in this work. The hyperjerk system is constructed by using a memristive device which creates the special feature of such hyperjerk system, possessing an infinite number of equilibrium points. This special feature is rarely observed in other chaotic hyperjerk systems. Dynamical behaviors of the memristive hyperjerk system are investigated through equilibrium points, projections of chaotic attractors, Lyapunov exponents and Kaplan–Yorke dimension. In addition, the capacity of synchronization scheme of memristive hyperjerk systems is shown via backstepping control approach. To verify the feasibility of such hyperjerk system, we present its circuital implementation. Because the designed circuit modeling the hyperjerk system can generate chaos, it can applied into potential applications in various fields of chaos-based engineering, such as secure communications, random bit generation, liquid mixing or path planning for mobile robot, etc.

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