Dynamics, Synchronization and SPICE Implementation of a Memristive System with Hidden Hyperchaotic Attractor

Viet-Thanh Pham, Sundarapandian Vaidyanathan, Christos K. Volos, Thang Manh Hoang and Vu Van Yem

Abstract The realization of memristor in nanoscale size has received considerate attention recently because memristor can be applied in different potential areas such as spiking neural network, high-speed computing, synapses of biological systems, flexible circuits, nonvolatile memory, artificial intelligence, modeling of complex systems or low power devices and sensing. Interestingly, memristor has been used as a nonlinear element to generate chaos in memristive system. In this chapter, a new memristive system is proposed. The fundamental dynamics properties of such memristive system are discovered through equilibria, Lyapunov exponents, and Kaplan–York dimension. Especially, hidden attractor and hyperchaos can be observed in this new system. Moreover, synchronization for such system is studied and simulation results are presented showing the accuracy of the introduced synchronization scheme. An electronic circuit modelling such hyperchaotic memristive system is also reported to verify its feasibility.

Keywords Chaos · Hyperchaos · Lyapunov exponents · Hidden attractor · No-equilibrium · Memristor · Synchronization · Circuit · SPICE

V.-T. Pham (✉) · T.M. Hoang · V. Van Yem
School of Electronics and Telecommunications,
Hanoi University of Science and Technology, Hanoi, Vietnam
e-mail: pvt3010@gmail.com

T.M. Hoang
e-mail: thang.hoangmanh@hust.edu.vn

V. Van Yem
e-mail: yem.vuvan@hust.edu.vn

S. Vaidyanathan
Research and Development Centre, Vel Tech University,
Tamil Nadu, India
e-mail: sundar@veltechuniv.edu.in

C.K. Volos
Physics Department, Aristotle University of Thessaloniki,
Thessaloniki, Greece
e-mail: volos@physics.auth.gr

A.T. Azar and S. Vaidyanathan (eds.), *Advances in Chaos Theory and Intelligent Control*, Studies in Fuzziness and Soft Computing 337,
DOI 10.1007/978-3-319-30340-6_2

1 Introduction

After the discovery of Lorenz's model for atmospheric convection [1], there has been significant interest in chaotic systems [2–9]. In the past few decades, different chaotic systems have been reported such as Rössler system [10], Arneodo system [11], Chen system [6], Lü system [12], Vaidyanathan system [13], time-delay systems [14] and so on [15, 16]. Chaotic behaviors are useful and have been applied in many fields, for example a double-scroll chaotic attractor has been used to generate true random bits [17], chaotic path planning has been generated for autonomous mobile robots [18], fingerprint images encryption scheme based on chaotic attractors has been implemented, or applications of time delay systems in secure communication have been proposed [19] due to their complex dynamics.

In addition, hyperchaotic system was introduced and studied [20]. Hyperchaotic system is characterized by more than one positive Lyapunov exponent and, thus, presents a higher level of complexity with respect to chaotic system [21]. As a result, hyperchaos is better than conventional chaos in a variety of areas, for instance, hyperchaos increases the security of chaotic-based communication systems significantly [22, 23]. Moreover hyperchaos has used in diverse applications such as cryptosystems [24], neural networks [25], secure communications [22, 23], or laser design [26]. Especially, the intrinsic nonlinear characteristic of memristor has been expointed in designing hyperchaotic oscillators. Some recent researches show that memristor is a potential candidate for generating hyperchaos [27, 28].

In this chapter, our work introduces a memristive system which can exhibit hyperchaotic attractors. Moreover such memristive system does not have equilibrium points. This chapter is organized as follows. Section 2 summarized related works. Section 3 gives a brief representation to the memristive system. Dynamics and properties of such memristive system is introduced in Sect. 4 while the adaptive synchronization scheme is studied in Sect. 5. Section 6 presents circuital implementation of memristive system using SPICE. Finally, conclusions are drawn in Sect. 7.

2 Related Work

Motivated by complex dynamical behaviors of hyperchaotic systems and special features of memristor, some memristor-based hyerchaotic systems have been introduced, recently. Hyperchaos was generated by combining a memristor with cubic nonlinear characteristics and a modified canonical Chua's circuit [28]. This memristor-based modified canonical Chua's circuit is a five-dimensional hyperchaotic oscillator. By extending the HP memristor-based canonical Chua's oscillator, a six-dimensional hyperchaotic oscillator was designed [29]. Authors used a configuration based on two HP memristors in antiparallel [27]. Four-dimensional hyperchaotic memristive systems were discovered by Li et al. [30, 31]. A 4D memristive system with a line of equilibrium was presented in [30] while another memristive system with an uncountable infinite number of stable and unstable equilibria was reported in [31]. A memristor-based hyperchaotic system without equilibrium was introduced in [32]. These memristive systems belong to a new category of chaotic systems with hidden attractors [33, 34].

The terminology "hidden attractor" has been introduced recently although the fact that the problem of analyzing hidden oscillations and to the finding of hidden oscillations in automatic control systems were studied a long time ago. According to a new classification of chaotic dynamics proposed by Leonov and Kuznetsov [33–35], there are two types of attractors: self-excited attractors and hidden attractors. A self-excited attractor has a basin of attraction that is excited from unstable equilibria. In contrast, hidden attractor cannot be found by using a numerical method in which a trajectory started from a point on the unstable manifold in the neighbourhood of an unstable equilibrium [35]. The discovery of dynamical systems with hidden attractors is a great challenge due to their appearance in many research fields such as in mechanics, secure communication and electronics [34, 36–39]. For example, hidden attractor in smooth Chua's system was reported in [40]. Hidden oscillations in mathematical model of drilling system [41] and hidden oscillations in nonlinear control systems [42] were witnessed. Various examples of hidden attractors were summarized in [43–46]. Hidden attractors were observed in a 4-D Rikitake dynamo system [47] or 5-D hyperchaotic Rikitake dynamo system [48]. Hidden attractors in a chaotic system with an exponential nonlinear term were introduced in [49]. In addition, algorithms for searching for hidden oscillations were presented in [50, 51].

Motivated by complex dynamical behaviors of chaotic systems, noticeable characteristics of memristor, and unknown features of hidden attractors, studying memristive hyperchaotic systems with hidden attractors is still an attractive research direction [30, 31].

3 Model of the Memristive System

A flux-controlled memristor is considered in this work. Its memductance is a second degree polynomial function:

$$W(\varphi) = \alpha + 3\beta\varphi^2, \tag{1}$$

with $\alpha = 0.4$ and $\beta = 0.001$. Memductance (1) is similar to known memductance [28, 30, 52, 53]. Using this memristor, a four-dimensinal memristive system is proposed as

$$\begin{cases} \dot{x} = 36y - 36x \\ \dot{y} = -2xz + 20y - axW(\varphi) - b \\ \dot{z} = 2xy - 3z \\ \dot{\varphi} = 2x, \end{cases} \tag{2}$$

where a, b are parameters, and $W(\varphi)$ is the memductance as introduced in (1). It is noting that the memristive system (2) has an uncountable number of equilibrium points when $b = 0$. Moreover, system (2) generate hyperchaos for different values of the parameter a. For example, hyperchaotic attractors is obtained when $a = 30, b = 0$ and the chosen initial conditions are $(x(0), y(0), z(0), \varphi(0)) = (0.5, 0, 0.5, 0)$. In this case, memristive system (2) is similar to the reported one in [31], therefore it will be not discussed in the next sections.

4 Dynamics and Properties of the Memristive System

The memristive system (2) is investigated when $b \neq 0$. It is easy to obtain the equilibrium points for system (2) by solving $\dot{x} = 0$, $\dot{y} = 0$, $\dot{z} = 0$, and $\dot{\varphi} = 0$ that is

$$36y - 36x = 0, \tag{3}$$

$$-2xz + 20y - axW(\varphi) - b = 0, \tag{4}$$

$$2xy - 3z = 0, \tag{5}$$

$$2x = 0, \tag{6}$$

From (3), (5) and (6), we have $x = y = z = 0$. As a results, Eq. (4) reduces to $b = 0$, which is an contradiction. Hence there are not equilibrium points in memristive system (2).

In this work, the parameters are selected as $a = 30$, $b = 0.001$ and the initial conditions are

$$(x(0), y(0), z(0), \varphi(0)) = (0.5, 0, 0.5, 0). \tag{7}$$

Lyapunov exponents, which measure the exponential rates of the divergence and convergence of nearby trajectories in the phase space of the chaotic system [8, 54], are calculated using the well-known algorithm in [55]. The Lyapunov exponents of the system (2) are

$$\lambda_1 = 0.2590, \lambda_2 = 0.0658, \lambda_3 = 0, \lambda_4 = -19.3246. \tag{8}$$

It is noting that the sum of the Lyapunov exponents is negative, and so the novel memristive hyperchaotic system is dissipative. There are two positive Lyapunov exponents, one zero and one negative Lyapunov exponents. Thus, the memtistive system (2) is a four-dimension hyperchaotic system according to [20]. It is worth noting that this memristive system can be classified as a hyperchaotic system with hidden strange attractor because its basin of attractor does not contain neighbourhoods of equilibria [33, 34]. The 3-D and 2-D projections of the hyperchaotic attractors without equilibrium in this case are illustrated in Figs. 1, 2, 3, 4, 5 and 6.

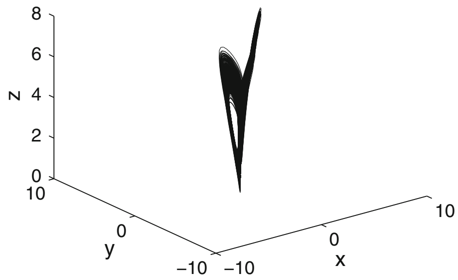

Fig. 1 3-D projection of the hyperchaotic memristive system without equilibrium (2) in the (x, y, z)-space

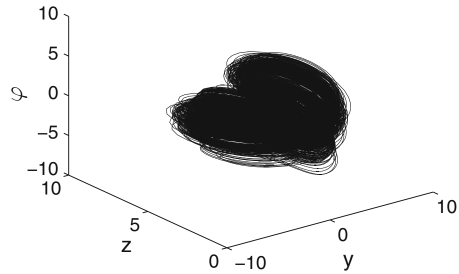

Fig. 2 3-D projection of the hyperchaotic memristive system without equilibrium (2) in the (y, z, φ)-space

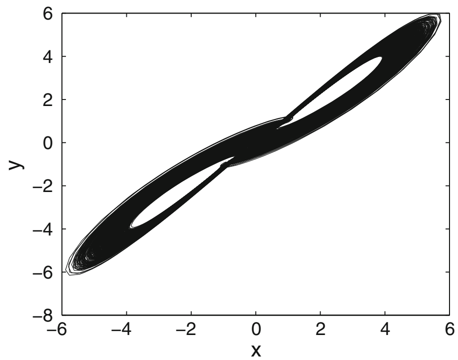

Fig. 3 2-D projection of the hyperchaotic memristive system without equilibrium (2) in the (x, y)-plane

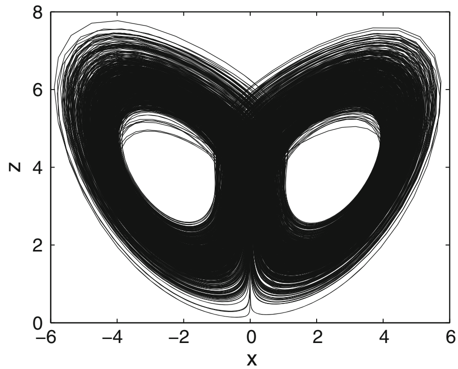

Fig. 4 2-D projection of the hyperchaotic memristive system without equilibrium (2) in the (x, z)-plane

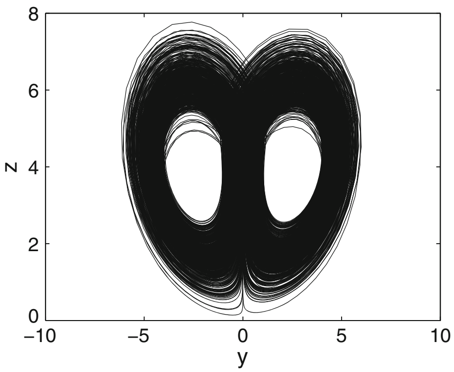

Fig. 5 2-D projection of the hyperchaotic memristive system without equilibrium (2) in the (y, z)-plane

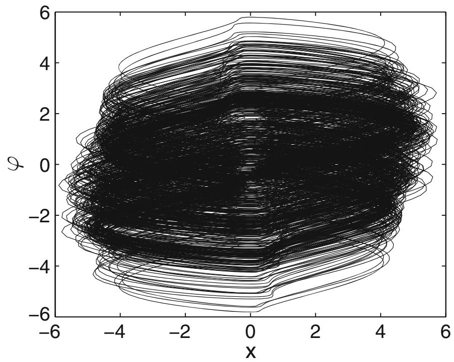

Fig. 6 2-D projection of the hyperchaotic memristive system without equilibrium (2) in the (x, φ)-plane

It is known that the Kaplan–Yorke fractional dimension, which presents the complexity of attractor [56], is defined by

$$D_{KY} = j + \frac{1}{|\lambda_{j+1}|} \sum_{i=1}^{j} \lambda_i, \tag{9}$$

where j is the largest integer satisfying $\sum_{i=1}^{j} \lambda_i \geq 0$ and $\sum_{i=1}^{j+1} \lambda_i < 0$. The calculated fractional dimension of memristive system (2) when $a = 30$, $b = 0.001$ is

$$D_{KY} = 3 + \frac{\lambda_1 + \lambda_2 + \lambda_3}{|\lambda_4|} = 3.0168. \tag{10}$$

Equation (10) indicates a strange attractor.

5 Adaptive Anti-synchronization of the Memristive System

The possibility of synchronization of two coupled chaotic systems [57–60] is one of the most vital characteristics relating to chaotic systems and their applications. Different research activities using synchronization of nonlinear systems have been investigated in literature [38, 61–69]. For instant, synchronized states in a ring of mutually coupled self-sustained nonlinear electrical oscillators were considered in [70], ragged synchronizability of coupled oscillators was observed in [71], various synchronization phenomena in bidirectionally coupled double-scroll circuits were reported in [72], or observer for synchronization of chaotic systems with application to secure data transmission was studied in [73]. Many synchronization schemes have been introduced such as lag synchronization [74], frequency synchronization [75], projective-anticipating synchronization [76], anti-synchronization [77], adaptive synchronization [78], or hybrid chaos synchronization [63], etc. Here we consider the adaptive synchronization of identical memristive hyperchaotic systems with two unknown parameters.

In this section, we consider the memristive sytem (2) as the master system as follows

$$\begin{cases} \dot{x}_1 = 36y_1 - 36x_1 \\ \dot{y}_1 = -2x_1z_1 + 20y_1 - ax_1W(\varphi_1) - b \\ \dot{z}_1 = 2x_1y_1 - 3z_1 \\ \dot{\varphi}_1 = 2x_1. \end{cases} \tag{11}$$

The states of the master system (11) are x_1, y_1, z_1, φ_1, and $W(\varphi_1)$ is the memductance as given in (1). The slave system is considered as the controlled memristive system and its dynamics is given as

$$\begin{cases} \dot{x}_2 = 36y_2 - 36x_2 + u_x \\ \dot{y}_2 = -2x_2z_2 + 20y_2 - ax_2W(\varphi_2) - b + u_y \\ \dot{z}_2 = 2x_2y_2 - 3z_2 + u_z \\ \dot{\varphi}_2 = 2x_2 + u_\varphi, \end{cases} \tag{12}$$

where x_2, y_2, z_2, φ_2, are the states of the slave system while u_x, u_y, u_z, u_φ are the adaptive controls. These controls will be constructed for the anti-synchronization of the master and slave systems. In order to estimate unknown parameters a and b, $A(t)$ and $B(t)$ are used.

The anti-synchronization error between memristive systems (11) and (12) is described by the following relation

$$\begin{cases} e_x = x_1 + x_2 \\ e_y = y_1 + y_2 \\ e_z = z_1 + z_2 \\ e_\varphi = \varphi_1 + \varphi_2. \end{cases} \tag{13}$$

Therefore, the anti-synchronization error dynamics is determined by

$$\begin{cases} \dot{e}_x = 36e_x - 36e_y + u_x \\ \dot{e}_y = -2\left(x_1z_1 + x_2z_2\right) + 20e_y - a\left(x_1W\left(\varphi_1\right) + x_2W\left(\varphi_2\right)\right) - 2b + u_y \\ \dot{e}_z = 2\left(x_1y_1 + x_2y_2\right) - 3e_z + u_z \\ \dot{e}_\varphi = 2e_x + u_\varphi. \end{cases} \tag{14}$$

Our goal is to find the appropriate controllers u_x, u_y, u_z, u_φ to stabilize the system (14). Thus, we propose the following controllers for system (14):

$$\begin{cases} u_x = -36e_x + 36e_y - k_xe_x \\ u_y = 2\left(x_1z_1 + x_2z_2\right) - 20e_y + A\left(t\right)\left(x_1W\left(\varphi_1\right) + x_2W\left(\varphi_2\right)\right) + 2B\left(t\right) - k_ye_y \\ u_z = -2\left(x_1y_1 + x_2y_2\right) + 3e_z - k_ze_z \\ u_\varphi = -2e_x - k_\varphi e_\varphi, \end{cases} \tag{15}$$

where k_x, k_y, k_z, k_φ are positive gain constants for each controllers and $A(t)$, $B(t)$ are the estimate values for unknown system parameters. The update laws for the unknown parameters are defined as

$$\begin{cases} \dot{A} = -e_y\left(x_1W\left(\varphi_1\right) + x_2W\left(\varphi_2\right)\right) \\ \dot{B} = -2e_y. \end{cases} \tag{16}$$

Next, the main result of this section will be presented and proved.

Theorem 5.1 *If the adaptive controller (15) and the updating laws of parameter (16) are chosen, the anti-sychronization between the master system (11) and the slave system (12) is achieved.*

Proof Here $e_a(t)$ and $e_b(t)$ are the parameter estimation errors given as

$$\begin{cases} e_a\left(t\right) = a - A\left(t\right) \\ e_b\left(t\right) = b - B\left(t\right). \end{cases} \tag{17}$$

Differentiating (17) with respect to t, we obtain

$$\begin{cases} \dot{e}_a\left(t\right) = -\dot{A}\left(t\right) \\ \dot{e}_b\left(t\right) = -\dot{B}\left(t\right). \end{cases} \tag{18}$$

Substituting adaptive control law (15) into (14), the closed-loop error dynamics is determined as

$$\begin{cases} \dot{e}_x = -k_xe_x \\ \dot{e}_y = -\left(a - A\left(t\right)\right)\left(x_1W\left(\varphi_1\right) + x_2W\left(\varphi_2\right)\right) - 2\left(b - B\left(t\right)\right) - k_ye_y \\ \dot{e}_z = -k_ze_z \\ \dot{e}_\varphi = -k_\varphi e_\varphi \end{cases} \tag{19}$$

Then substituting (17) into (19), we have

$$\begin{cases} \dot{e}_x = -k_x e_x \\ \dot{e}_y = -e_a \left(x_1 W(\varphi_1) + x_2 W(\varphi_2) \right) - 2e_b - k_y e_y \\ \dot{e}_z = -k_z e_z \\ \dot{e}_\varphi = -k_\varphi e_\varphi \end{cases} \quad (20)$$

We consider the Lyapunov function as

$$\begin{aligned} V(t) &= V\left(e_x, e_y, e_z, e_\varphi, e_a, e_b\right) \\ &= \tfrac{1}{2}\left(e_x^2 + e_y^2 + e_z^2 + e_\varphi^2 + e_a^2 + e_b^2\right). \end{aligned} \quad (21)$$

The Lyapunov function is clearly definite positive.

Taking time derivative of (21) along the trajectories of (13) and (17) we get

$$\dot{V}(t) = e_x \dot{e}_x + e_y \dot{e}_y + e_z \dot{e}_z + e_\varphi \dot{e}_\varphi + e_a \dot{e}_a + e_b \dot{e}_b. \quad (22)$$

From (18), (20), and (22) we have

$$\begin{aligned} \dot{V}(t) = &-k_x e_x^2 - e_a \left[e_y \left(x_1 W(\varphi_1) + x_2 W(\varphi_2) \right) + \dot{A} \right] \\ &- e_b \left(2e_y + \dot{B} \right) - k_y e_y^2 - k_z e_z^2 - k_\varphi e_\varphi^2. \end{aligned} \quad (23)$$

Then by applying the parameter update law (16), Eq. (23) become

$$\dot{V}(t) = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2 - k_\varphi e_\varphi^2. \quad (24)$$

Obviously, derivative of the Lyapunov function is definite negative. According to the Lyapunov stability [79, 80] we obtain $e_x(t) \to 0$, $e_y(t) \to 0$, $e_z(t) \to 0$, $e_\varphi(t) \to 0$, $e_a(t) \to 0$, $e_b(t) \to 0$ exponentially when $t \to 0$ that is, anti-synchronization between master and slave system. This completes the proof. □

We illustrate the proposed anti-synchronization scheme with a numerical example. In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the systems. The parameters of the memristive hyperchaotic systems are selected as $a = 30$, $b = 0.001$ and the positive gain constant as $k = 4$. The initial conditions of the master system (11) and the slave system (12) have been chosen as $x_1(0) = 0.5$, $y_1(0) = 0$, $z_1(0) = 0.5$, $\varphi_1(0) = 0$ and $x_2(0) = -0.9$, $y_2(0) = -0.4$, $z_2(0) = 0.8$, $\varphi_2(0) = 0.5$, respectively. We assumed that the initial values of the parameter estimates are $A(0) = 29$ and $B(0) = 0.5$.

It is easy to see that when adaptive control law (15) and the update law for the parameter estimates (16) are applied, the anti-synchronization of the master (11) and slave system (12) occurred as illustrated in Figs. 7, 8, 9 and 10. It is noting that time series of master states are denoted as blue solid lines while corresponding slave states are plotted as red dash-dot lines in such figures. In addition, the time-history of the anti-synchronization errors e_x, e_y, e_z, and e_φ is presented in Fig. 11.

The anti-synchronization errors converge to the zero, which implies that the chaos anti-synchronization between the memristive systems is realized.

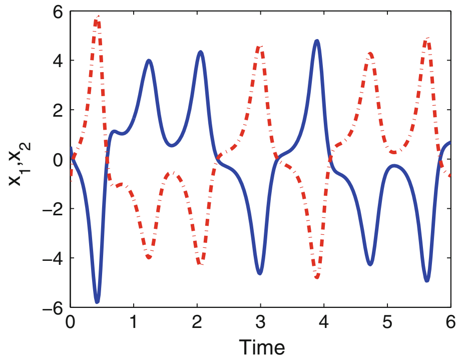

Fig. 7 Anti-synchronization of the states $x_1(t)$ and $x_2(t)$

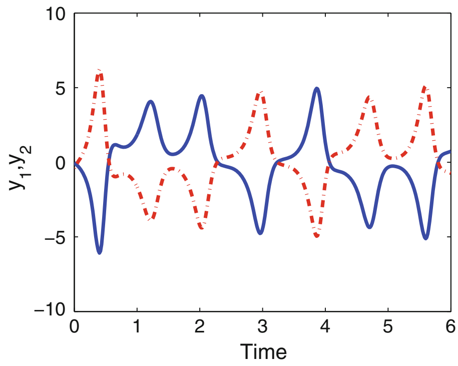

Fig. 8 Anti-synchronization of the states $y_1(t)$ and $y_2(t)$

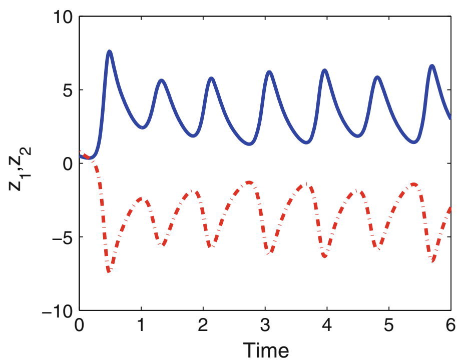

Fig. 9 Anti-synchronization of the states $z_1(t)$ and $z_2(t)$

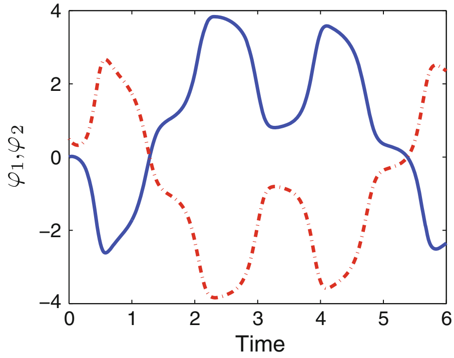

Fig. 10 Anti-synchronization of the states $\varphi_1(t)$ and $\varphi_2(t)$

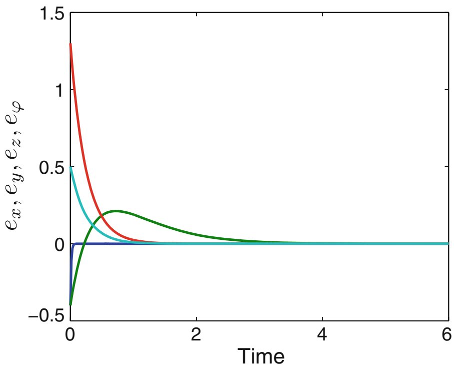

Fig. 11 Time series of the anti-synchronization errors e_x, e_y, e_z, and e_φ

6 Circuit Realization of the Memristive System

Circuital design of chaotic/hyperchaotic systems plays an important role on the field of nonlinear science due to its applications in secure communication, signal processing, random bit generator, or path planning for autonomous mobile robot etc. [17, 18, 22, 62, 81, 82]. In addition, circuital implementation of chaotic/hyperchaotic systems is also provide an effective approach for investigating dynamics of such theoretical models [61, 83]. For example, chaotic attractors can be observed on the oscilloscope easily or experimental bifurcation diagrams can be obtained by varying the values of variable resistors [84, 85].

Therefore, in this work, an electronic circuit is introduced to implement memristive system (2). By using the operational amplifiers approach [85], the circuit is proposed as shown in Fig. 12. Here the variables x, y, z, φ of memristive system (2) are the voltages across the capacitor C_1, C_2, C_3, and C_4, respectively. It is noted

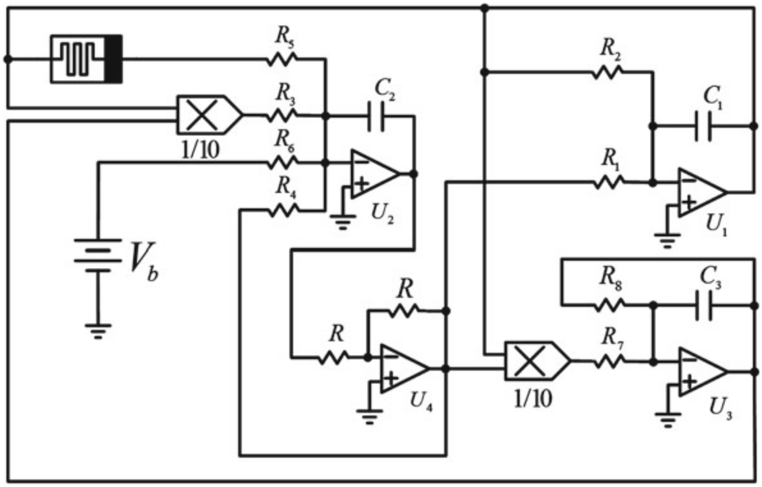

Fig. 12 Schematic of the circuit which modelling hyperchaotic system (2) with the presence of the memristor

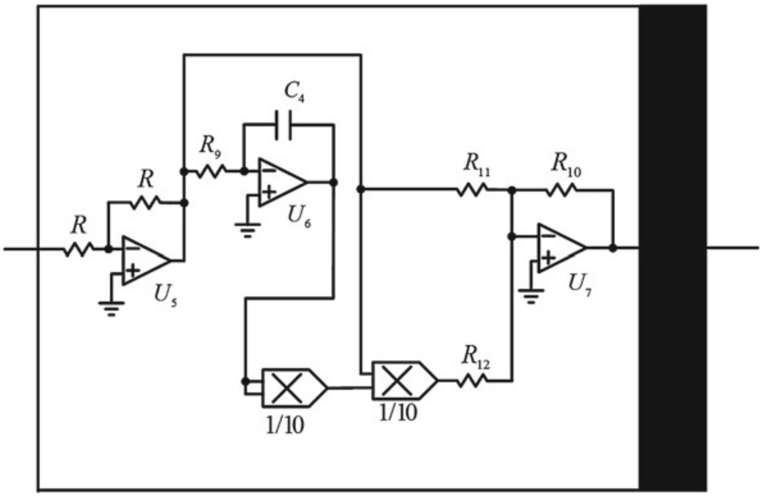

Fig. 13 Schematic of the circuit emulating the memristor

that the detailed schematic of the memristor in Fig. 12 is presented in Fig. 13. This sub-circuit of memristor emulates the memristive device only due to the fact that there are not any commercial off-the-shelf memristive device in the market at the moment [86]. The corresponding circuital equations of circuit can be described as

$$\begin{cases} \frac{dv_{C_1}}{dt} = \frac{1}{R_1C_1}v_{C_2} - \frac{1}{R_2C_1}v_{C_1} \\ \frac{dv_{C_2}}{dt} = -\frac{1}{10R_3C_2}v_{C_1}v_{C_3} + \frac{1}{R_4C_2}v_{C_2} - \frac{1}{R_6C_2}V_b \\ \qquad - \frac{1}{R_5C_2}v_{C_1}\left(\frac{R_{10}}{R_{11}} + \frac{R_{10}}{100R_{12}}v_{C_4}^2\right) \\ \frac{dv_{C_3}}{dt} = \frac{1}{10R_7C_3}v_{C_1}v_{C_2} - \frac{1}{R_8C_3}v_{C_3} \\ \frac{dv_{C_4}}{dt} = \frac{1}{R_9C_4}v_{C_1}, \end{cases} \tag{25}$$

where v_{C_1}, v_{C_2}, v_{C_3}, and v_{C_4} are the voltages across the capacitors C_1, C_2, C_3, and C_4, respectively.

The power supplies of all active devices are $\pm 15V_{DC}$ and the operational amplifiers TL084 are used in this work. The values of components in Figs. 12 and 13 are chosen as follows: $R_1 = R_2 = 1\,\text{k}\Omega$, $R_3 = R_4 = R_7 = 1.8\,\text{k}\Omega$, $R_5 = 1.2\,\text{k}\Omega$, $R_6 = 3.6\,\text{M}\Omega$, $R_8 = 12\,\text{k}\Omega$, $R_9 = 18\,\text{k}\Omega$, $R_{10} = R = 36\,\text{k}\Omega$, $R_{11} = 90\,\text{k}\Omega$, $R_{12} = 120\,\text{k}\Omega$, $V_b = 0.1V_{DC}$, and $C_1 = C_2 = C_3 = C_4 = 4.7\,\text{nF}$.

The designed circuit is implemented in SPICE. The obtained results are displayed in Figs. 14, 15, 16 and 17 which show the hyperchaotic attractors of the designed circuit in different phase planes (v_{C_1}, v_{C_2}), (v_{C_1}, v_{C_3}), (v_{C_2}, v_{C_3}), and (v_{C_1}, v_{C_4}), respectively. Theoretical attractors (see Figs. 3, 4, 5 and 6) are similar with the circuital ones (see Figs. 14, 15, 16 and 17). Moreover, the designed circuit confirms the feasibility of the memristive system.

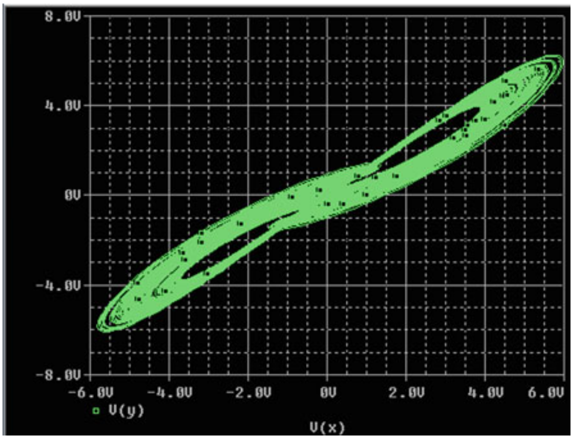

Fig. 14 Hyperchaotic attractor of the designed circuit obtained from SPICE in the (v_{C_1}, v_{C_2}) phase plane

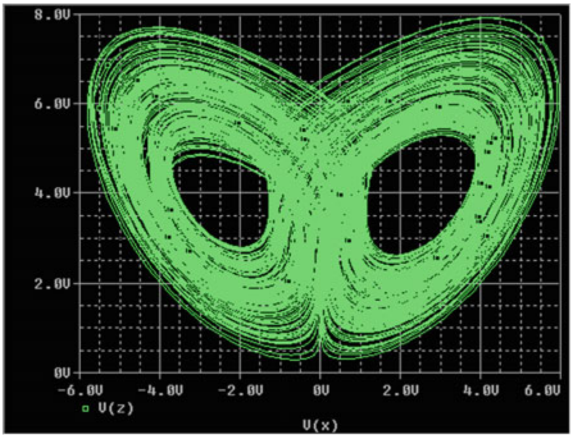

Fig. 15 Hyperchaotic attractor of the designed circuit obtained from SPICE in the (v_{C_1}, v_{C_3}) phase plane

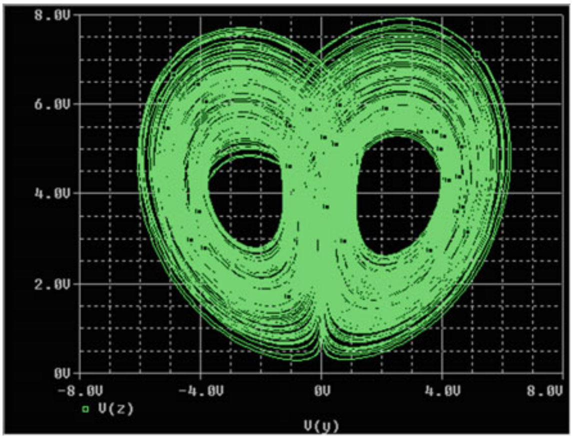

Fig. 16 Hyperchaotic attractor of the designed circuit obtained from SPICE in the (v_{C_2}, v_{C_3}) phase plane

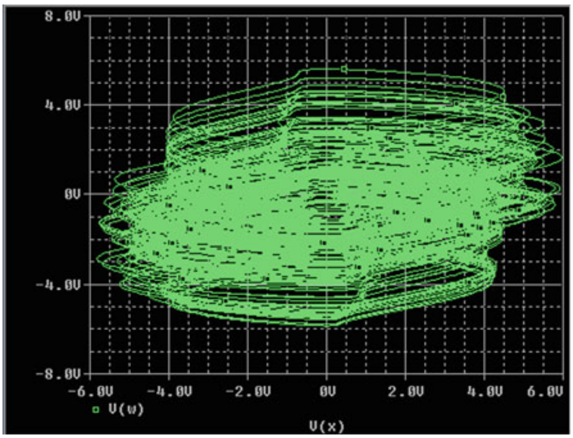

Fig. 17 Hyperchaotic attractor of the designed circuit obtained from SPICE in the (v_{C_1}, v_{C_4}) phase plane

7 Conclusion

A memristive system, which is built by using a memristor, is proposed in this work. The presence of the memristor creates the special features of such as hyperchaos, the absence of equilibrium points, and hidden attractors. Fundamental dynamical behaviors of the memristive hyperchaotic system are investigated through calculating equilibrium points, phase portraits of chaotic attractors, Lyapunov exponents and Kaplan–Yorke dimension. In addition, the capacity of synchronization of memristive systems and the feasibility of such memristive system without equilibrium are verified through anti-synchronization scheme and circuital implementation, respectively.

It is worth noting that the memristive system can exhibit double-scroll hyperchaotic attractor despite the equilibrium points have disappeared. It has been known that equilibrium points of a dynamical system, especially a chaotic one, play an important role when generating multi-scroll attractors. Therefore, investigating no-equilibrium memristive systems with multi-scroll attractors will be studied in future works.

Moreover, the memristive system has potential applications in secure communications and cryptography because of its hyperchaos and feasibility. Further studies in this research direction will be presented in future works.

Acknowledgments This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 102.02-2012.27

References

1. Lorenz EN (1963) Deterministic non-periodic flow. J Atmos Sci 20:130–141
2. Azar AT, Vaidyanathan S (2015) Chaos modeling and control systems design. Springer, Germany
3. Azar AT, Vaidyanathan S (2015) Computational intelligence applications in modeling and control. Springer, Germany
4. Azar AT, Vaidyanathan S (2015) Handbook of research on advanced intelligent control engineering and automation. IGI Global, USA
5. Chen G, Yu X (2003) Chaos control: theory and applications. Springer, Berlin
6. Chen GR (1999) Controlling chaos and bifurcations in engineering systems. CRC Press, Boca Raton
7. Sprott JC (2003) Chaos and times-series analysis. Oxford University Press, Oxford
8. Strogatz SH (1994) Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. Perseus Books, Massachusetts
9. Yalcin ME, Suykens JAK, Vandewalle J (2005) Cellular neural networks, multi-scroll chaos and synchronization. World Scientific, Singapore
10. Rössler OE (1976) An equation for continuous chaos. Phys Lett A 57:397–398
11. Arneodo A, Coullet P, Tresser C (1981) Possible new strange attractors with spiral structure. Comm Math Phys 79:573–579
12. Lü J, Chen G (2002) A new chaotic attractor coined. Int J Bif Chaos 12:659–661
13. Vaidyanathan S (2013) A new six-term 3-D chaotic system with an exponential nonlineariry. Far East J Math Sci 79:135–143

14. Barnerjee T, Biswas D, Sarkar BC (2012) Design and analysis of a first order time-delayed chaotic system. Nonlinear Dyn 70:721–734
15. Pham V-T, Volos C, Vaidyanathan S (2015b) Multi-scroll chaotic oscillator based on a first-order delay differential equation. In: Azar AT, Vaidyanathan S (eds) Chaos modelling and control systems design, vol 581., Studies in computational intelligenceSpringer, Germany, pp 59–72
16. Vaidyanathan S, Azar AT (2015b) Analysis, control and synchronization of a nine-term 3-D novel chaotic system. In: Azar AT, Vaidyanathan S (eds) Chaos modelling and control systems design, vol 581., Studies in computational intelligenceSpringer, Germany, pp 19–38
17. Yalcin ME, Suykens JAK, Vandewalle J (2004) True random bit generation from a double-scroll attractor. IEEE Trans Circuits Syst I Regul Papers 51:1395–1404
18. Volos CK, Kyprianidis IM, Stouboulos IN (2012) A chaotic path planning generator for autonomous mobile robots. Robot Auto Syst 60:651–656
19. Hoang TM, Nakagawa M (2008) A secure communication system using projective-lag and/or projective-anticipating synchronizations of coupled multidelay feedback systems. Chaos Solitions Fractals 38:1423–1438
20. Rössler OE (1979) An equation for hyperchaos. Phys Lett A 71:155–157
21. Vaidyanathan S, Azar AT (2015a) Analysis and control of a 4-D novel hyperchaotic system. In: Azar AT, Vaidyanathan S (eds) Chaos modeling and control systems design, vol 581., Studies in computational intelligenceSpringer, Germany, pp 19–38
22. Sadoudi S, Tanougast C, Azzaz MS, Dandache A (2013) Design and FPGA implementation of a wireless hyperchaotic communication system for secure realtime image transmission. EURASIP J Image Video Process 943:1–18
23. Udaltsov VS, Goedgebuer JP, Larger L, Cuenot JB, Levy P, Rhodes WT (2003) Communicating with hyperchaos: the dynamics of a DNLF emitter and recovery of transmitted information. Optics Spectrosc 95:114–118
24. Grassi G, Mascolo S (1999) A system theory approach for designing cryptosystems based on hyperchaos. IEEE Trans Cir Sys I: Fund Theory Appl 46:1135–1138
25. Huang Y, Yang X (2006) Hyperchaos and bifurcation in a new class of four-dimensional hopfield neural networks. Neurocomputing 69:1787–1795
26. Vicente R, Dauden J, Colet P, Toral R (2005) Analysis and characterization of the hyperchaos generated by a semiconductor laser subject to a delayed feedback loop. IEEE J Quantum Electr 41:541–548
27. Buscarino A, Fortuna L, Frasca M, Gambuzza LV (2012) A chaotic circuit based on Hewlett-Packard memristor. Chaos 22:023136
28. Fitch AL, Yu D, Iu HHC, Sreeram V (2012) Hyperchaos in an memristor-based modified canonical chua's circuit. Int J Bif Chaos 22:1250133–1250138
29. Buscarino A, Fortuna L, Frasca M, Gambuzza LV (2012) A gallery of chaotic oscillators based on hp memristor. Int J Bif Chaos 22:1330014–1330015
30. Li Q, Hu S, Tang S, Zeng G (2014) Hyperchaos and horseshoe in a 4D memristive system with a line of equilibria and its implementation. Int J Cir Theory Appl 42:1172–1188
31. Li Q, Zeng H, Li J (2015) Hyperchaos in a 4D memristive circuit with infinitely many stable equilibria. Nonlinear Dyn 79:2295–2308
32. Pham VT, Volos CK, Vaidyanathan S, Le TP, Vu VY (2015c) A memristor-based hyperchaotic system with hidden attractors: dynamics, sychronization and circuital emulating. J Eng Sci Tech Rev 8:205–214
33. Leonov GA, Kuznetsov NV (2013) Hidden attractors in dynamical systems: from hidden oscillation in Hilbert-Kolmogorov, Aizerman and Kalman problems to hidden chaotic attractor in Chua circuits. Int J Bifurc Chaos 23:1330002
34. Leonov GA, Kuznetsov NV, Kuznetsova OA, Seldedzhi SM, Vagaitsev VI (2011) Hidden oscillations in dynamical systems. Trans Syst Contr 6:54–67
35. Jafari S, Sprott JC (2013) Simple chaotic flows with a line equilibrium. Chaos Solitons Fractals 57:79–84

36. Kuznetsov NV, Leonov GA, Seledzhi SM (2011) Hidden oscillations in nonlinear control systems. IFAC Proc 18:2506–2510
37. Pham V-T, Jafari S, Volos C, Wang X, Golpayegani SMRH (2014a) Is that really hidden? The presence of complex fixed-points in chaotic flows with no equilibria. Int J Bifur Chaos 24:1450146
38. Pham V-T, Volos CK, Jafari S, Wei Z, Wang X (2014b) Constructing a novel no-equilibrium chaotic system. Int J Bifurc Chaos 24:1450073
39. Sharma PR, Shrimali MD, Prasad A, Kuznetsov NV, Leonov GA (2015) Control of multistability in hidden attractors. Eur Phys J Special Topics 224:1485–1491
40. Leonov GA, Kuznetsov NV, Vagaitsev VI (2012) Hidden attractor in smooth Chua system. Phys D 241:1482–1486
41. Leonov GA, Kuznetsov NV, Kiseleva MA, Solovyeva EP, Zaretskiy AM (2014) Hidden oscillations in mathematical model of drilling system actuated by induction motor with a wound rotor. Nonlinear Dyn 77:277–288
42. Leonov GA, Kuznetsov NV (2011) Analytical-numerical methods for investigation of hidden oscillations in nonlinear control systems. IFAC Proc 18:2494–2505
43. Brezetskyi S, Dudkowski D, Kapitaniak T (2015) Rare and hidden attractors in van der polduffing oscillators. Eur Phys J Special Topics 224:1459–1467
44. Jafari S, Sprott JC, Nazarimehr F (2015) Recent new examples of hidden attractors. Eur Phys J Special Topics 224:1469–1476
45. Shahzad M, Pham VT, Ahmad MA, Jafari S, Hadaeghi F (2015) Synchronization and circuit design of a chaotic system with coexisting hidden attractors. Eur Phys J Special Topics 224:1637–1652
46. Sprott JC (2015) Strange attractors with various equilibrium types. Eur Phys J Special Topics 224:1409–1419
47. Vaidyanathan S, Volos CK, Pham VT (2015c) Analysis, control, synchronization and spice implementation of a novel 4-d hyperchaotic rikitake dynamo system without equilibrium. J Eng Sci Tech Rev 8:232–244
48. Vaidyanathan S, Pham VT, Volos CK (2015b) A 5-d hyperchaotic rikitake dynamo system with hidden attractors. Eur Phys J Special Topics 224:1575–1592
49. Pham VT, Vaidyanathan S, Volos CK, Jafari S (2015a) Hidden attractors in a chaotic system with an exponential nonlinear term. Eur Phys J Special Topics 224:1507–1517
50. Leonov GA, Kuznetsov NV (2011) Algorithms for searching for hidden oscillations in the Aizerman and Kalman problems. Dokl Math 84:475–481
51. Leonov GA, Kuznetsov NV, Vagaitsev VI (2011) Localization of hidden Chua's attractors. Phys Lett A 375:2230–2233
52. Bao B, Liu Z, Xu B (2010) Dynamical analysis of memristor chaotic oscillator. Acta Physica Sinica 59:3785–3793
53. Muthuswamy B (2010) Implementing memristor based chaotic circuits. Int J Bif Chaos 20:1335–1350
54. Sprott JC (2010) Elegant chaos: algebraically simple chaotic flows. World Scientific, Singapore
55. Wolf A, Swift JB, Swinney HL, Vastano JA (1985) Determining Lyapunov exponents from a time series. Phys D 16:285–317
56. Frederickson P, Kaplan JL, Yorke ED, York J (1983) The lyapunov dimension of strange attractors. J Differ Equ 49:185–207
57. Boccaletti S, Kurths J, Osipov G, Valladares DL, Zhou CS (2002) The synchronization of chaotic systems. Phys Rep 366:1–101
58. Fortuna L, Frasca M (2007) Experimental synchronization of single-transistor-based chaotic circuits. Chaos 17:043118-1–043118-5
59. Kapitaniak T (1994) Synchronization of chaos using continuous control. Phys Rev E 50: 1642–1644
60. Pecora LM, Carroll TL (1990) Synchronization in chaotic signals. Phys Rev A 64:821–824
61. Buscarino A, Fortuna L, Frasca M (2009) Experimental robust synchronization of hyperchaotic circuits. Phys D 238:1917–1922

62. Gamez-Guzman L, Cruz-Hernandez C, Lopez-Gutierrez R, Garcia-Guerrero EE (2009) Synchronization of Chua's circuits with multi-scroll attractors: application to communication. Commun Nonlinear Sci Numer Simul 14:2765–2775
63. Karthikeyan R, Vaidyanathan S (2014) Hybrid chaos synchronization of four-scroll systems via active control. J Electr Eng 65:97–103
64. Srinivasan K, Senthilkumar DV, Murali K, Lakshmanan M, Kurths J (2011) Synchronization transitions in coupled time-delay electronic circuits with a threshold nonlinearity. Chaos 21:023119
65. Vaidyanathan S (2014) Analysis and adaptive synchronization of eight-term novel 3-D chaotic system with three quadratic nonlinearities. Eur Phys J Special Topics 223:1519–1529
66. Vaidyanathan S, Azar AT (2015c) Anti-synchronization of identical chaotic systems using sliding mode control and an application to Vaidhyanathan-Madhavan chaotic systems. Stud Comput Intell 576:527–547
67. Vaidyanathan S, Azar AT (2015d) Hybrid synchronization of identical chaotic systems using sliding mode control and an application to Vaidhyanathan chaotic systems. Stud Comput Intell 576:549–569
68. Vaidyanathan S, Idowu BA, Azar AT (2015a) Backstepping controller design for the global chaos synchronization of Sprott's jerk systems. Stud Comput Intell 581:39–58
69. Zhu Q, Azar AT (2015) Complex system modelling and control through intelligent soft computations. Springer, Germany
70. Woafo P, Kadji HGE (2004) Synchronized states in a ring of mutually coupled self-sustained electrical oscillators. Phys Rev E 69:046206
71. Stefanski A, Perlikowski P, Kapitaniak T (2007) Ragged synchronizability of coupled oscillators. Phys Rev E 75:016210
72. Volos CK, Kyprianidis IM, Stouboulos IN (2011) Various synchronization phenomena in bidirectionally coupled double scroll circuits. Commun Nonlinear Sci Numer Simul 71:3356–3366
73. Aguilar-Lopez R, Martinez-Guerra R, Perez-Pinacho C (2014) Nonlinear observer for synchronization of chaotic systems with application to secure data transmission. Eur Phys J Special Topics 223:1541–1548
74. Rosenblum MG, Pikovsky AS, Kurths J (1997) From phase to lag synchronization in coupled chaotic oscillators. Phys Rev Lett 78:4193–4196
75. Akopov A, Astakhov V, Vadiasova T, Shabunin A, Kapitaniak T (2005) Frequency synchronization in clusters in coupled extended systems. Phys Lett A 334:169–172
76. Hoang TM, Nakagawa M (2007) Anticipating and projective–anticipating synchronization of coupled multidelay feedback systems. Phys Lett A 365:407–411
77. Vaidyanathan S (2012) Anti-synchronization of four-wing chaotic systems via sliding mode control. Int J Auto Comput 9:274–279
78. Vaidyanathan S, Volos C, Pham VT, Madhavan K, Idowo BA (2014) Adaptive backstepping control, synchronization and circuit simualtion of a 3-D novel jerk chaotic system with two hyperbolic sinusoidal nonlinearities. Arch Cont Sci 33:257–285
79. Khalil H (2002) Nonlinear systems. Prentice Hall, New Jersey
80. Sastry S (1999) Nonlinear systems: analysis, stability, and control. Springer, USA
81. Barakat M, Mansingka A, Radwan AG, Salama KN (2013) Generalized hardware post processing technique for chaos-based pseudorandom number generators. ETRI J 35:448–458
82. Volos CK, Kyprianidis IM, Stouboulos IN (2013) Image encryption process based on chaotic synchronization phenomena. Signal Process 93:1328–1340
83. Sundarapandian V, Pehlivan I (2012) Analysis, control, synchronization, and circuit design of a novel chaotic system. Math Comp Model 55:1904–1915
84. Bouali S, Buscarino A, Fortuna L, Frasca M, Gambuzza LV (2012) Emulating complex business cycles by using an electronic analogue. Nonlinear Anal Real World Appl 13:2459–2465
85. Fortuna L, Frasca M, Xibilia MG (2009) Chua's circuit implementation: yesterday, today and tomorrow. World Scientific, Singapore
86. Tetzlaff R (2014) Memristors and memristive systems. Springer, New York

Springer

http://www.springer.com/978-3-319-30338-3

Advances in Chaos Theory and Intelligent Control
Azar, A.T.; Vaidyanathan, S. (Eds.)
2016, XII, 873 p. 420 illus., 127 illus. in color.,
Hardcover
ISBN: 978-3-319-30338-3