Chapter 2
Theory Overview

The SM is set up as a quantum field theory, using a Lagrangian formalism with gauge symmetry constraints to describe the matter particles and their interactions. The SM has been extraordinarily successful in describing the properties of matter and its interactions from subatomic to cosmological scales, and provides a unified view of the electromagnetic, weak and strong nuclear forces. Nevertheless, several open questions remain, such as the apparent presence of so called "dark matter" and "dark energy" in astronomical and cosmological surveys, the generation of neutrino masses and the observed matter-antimatter imbalance in the universe. A large variety of extensions of the SM have been devised to resolve these problems, though none of these have seen strong experimental confirmation yet. An overview of quantum field theories and their mathematical foundations may be found in the following textbooks: [1–5]. The description below largely follows Ref. [6].

2.1 The Standard Model Lagrangian

The Lagrangian of the SM $\mathcal{L}_{\mathrm{SM}}$ may be split into several terms, each describing a different aspect of the underlying physics of the SM.

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{ferm}} + \mathcal{L}_{\mathrm{Yuk}}. \tag{2.1}$$

The first term describes the gauge bosons and their interactions, as they arise from the gauge symmetries imposed on the Lagrangian and is accordingly denoted $\mathcal{L}_{\mathrm{YM}}$ after Chen-Ning Yang and Robert Mills, who first analysed non-abelian gauge groups in depth [7]. A bare Yang-Mills theory requires massless gauge bosons contrary to observation. Accordingly, the second term $\mathcal{L}_{\mathrm{H}}$ introduces the Higgs field, its self-interaction and interaction with the gauge bosons, which allow the gauge bosons to acquire mass in a gauge-invariant manner. The the third term subsumes the parts of the Lagrangian that describe the propagation of the matter fields and their interaction

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with the gauge bosons. Finally, $\mathcal{L}_{\text{Yuk}}$ describes the interaction of the matter fields with the Higgs boson, giving rise to the fermion masses through the Yukawa couplings.

$\mathcal{L}_{\text{YM}}$ can be written as:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} W^i_{\mu\nu} W^{i,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}, \tag{2.2}$$

where

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon^{ijk} W^j_\mu W^k_\nu, \qquad i, j, k = 1, 2, 3, \tag{2.3}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{2.4}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu, \qquad a, b, c = 1, \ldots, 8, \tag{2.5}$$

represent the field strength tensors associated to the different symmetries of the SM: $W^i_{\mu\nu}$ corresponds to the $\text{SU}(2)_\text{I}$ symmetry group of the weak isospin I^i_w, $B_{\mu\nu}$ to the U(1) symmetry of the weak hypercharge Y_w and $G^a_{\mu\nu}$ corresponding to the $\text{SU}(3)_\text{c}$ symmetry of the QCD color charge. ϵ^{ijk} and f^{abc} denote the structure constants of the SU(2) and SU(3) groups, following the conventions used in Ref. [8], respectively. g and g_s (as well as g' introduced below) denote the coupling constants for these interactions. For the B field, with its abelian U(1) symmetry, this term describes the free propagation of the field. For the W and G fields, with their non-abelian symmetries, additional terms arise, leading to interactions of the gauge fields among themselves. As the structure of these interactions is determined by the corresponding symmetry group, it is of interest to study multi-boson interactions to test the symmetry structure of the SM.

The interactions of the matter particles (i.e. fermions) with the gauge fields is described by

$$\mathcal{L}_{\text{ferm}} = \text{i}\overline{\Psi}_\text{L} \not{D} \Psi_\text{L} + \text{i}\overline{\psi}_{\ell_\text{R}} \not{D} \psi_{\ell_\text{R}} + \text{i}\overline{\Psi}_Q \not{D} \Psi_Q + \text{i}\overline{\psi}_{u_\text{R}} \not{D} \psi_{u_\text{R}} + \text{i}\overline{\psi}_{d_\text{R}} \not{D} \psi_{d_\text{R}}, \tag{2.6}$$

where Ψ_L represents left-handed lepton doublets of $\text{SU}(2)_\text{I}$, made up of the charged leptons and corresponding neutrinos, and Ψ_Q the equivalent doublets of up- and down-type quark pairs. ψ_{f_R} denotes the corresponding right handed fermion singlets ($f = \ell, u, d$, where ℓ stands for charged leptons, u for up-type quarks, and d for down-type quarks), omitting the right-handed neutrinos, which have no interactions in the SM. As we will later see, the absence of right-handed neutrinos precludes mass generation for the neutrinos through interactions with the Higgs field. In the notation of Eq. 2.6, the interactions are hidden in the definition of the covariant derivative D:

$$D_\mu = \partial_\mu + \text{i}g I^i_\text{w} W^i_\mu + \text{i}g' Y_\text{w} B_\mu + \text{i}g_s T^a_\text{c} G^a_\mu, \tag{2.7}$$

where I^i_w, Y_w and T^a_c correspond to the generators of the respective gauge groups in the representation of the fermions they act on, as detailed in Ref. [8].

This description does not reduce trivially to the well established theory of quantum electrodynamics, where the fermions interact with the photon field A_μ in a manner that is parity-blind and proportional to $Q\overline{\psi}\not{A}\psi$. However, using the Gell-Mann–Nishijima relation for the electric charge $Q = I^3_\mathrm{w} + Y_\mathrm{w}/2$, it is possible to recover the structure of quantum electrodynamics by constructing A_μ as a linear combination of the W^3_μ and B_μ fields:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_\mathrm{w} & -s_\mathrm{w} \\ s_\mathrm{w} & c_\mathrm{w} \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \tag{2.8}$$

Keeping the total normalizations constant, the linear combination is parameterized as a rotation by an angle θ_w, the so called weak mixing- or Weinberg angle. θ_w is determined by relating the unit charge e to the coupling constants g and g' as follows:

$$\cos\theta_\mathrm{w} = c_\mathrm{w} = \sqrt{1 - s^2_\mathrm{w}} = \frac{g}{\sqrt{g^2 + g'^2}}, \qquad e = \frac{gg'}{\sqrt{g^2 + g'^2}}. \tag{2.9}$$

Through these relations, quantum electrondynamics is recovered as part of the SM, where in addition to the photon a second neutral gauge field, the Z boson arises. The remaining W^1_μ adn W^2_μ gauge fields have no definite electric charge and different linear combinations

$$W^\pm_\mu = (W^1_\mu \mp \mathrm{i} W^2_\mu)/\sqrt{2} \tag{2.10}$$

are chosen to represent the phyiscal fields with unit charge.

Considering only $\mathcal{L} = \mathcal{L}_\mathrm{YM} + \mathcal{L}_\mathrm{ferm}$, we arrive at a self-consistent theory, though all the involved particles, bosons as well as fermions, are massless. Masses cannot be easily introduced for either the bosons or the fermions, as the naive mass terms, $W^i_\mu W^{i,u}$ and $(\overline{\psi}_{f_\mathrm{L}} \psi_{f_\mathrm{R}} + \overline{\psi}_{f_\mathrm{R}} \psi_{f_\mathrm{L}})$ for the bosons and fermions, respectively, are not gauge invariant. For the fermions such a mass term would be a valid addition if left- and right-handed fermions behaved equivalently under $\mathrm{SU}(2)_\mathrm{I} \times \mathrm{SU}(1)_\mathrm{Y}$ transformations, but the absence of right-handed neutrinos in the SM spoils the symmetry. The generation of particle masses while preserving gauge invariance requires a more complex scheme.

2.1.1 Electroweak Symmetry Breaking and the Higgs Mechanism

The most commonly proposed mechanism to generate the masses of the SM particles is via the introduction of an additional symmetry which is spontaneously broken in the so called Higgs mechanism. It is introduced into the Lagrangian as

$$\mathcal{L}_\mathrm{H} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi), \tag{2.11}$$

where Φ is a complex scalar $\mathrm{SU}(2)_\mathrm{I}$ doublet $(\phi^+, \phi^0)^\mathrm{T}$ with $Y_{\mathrm{w},\Phi} = 1$, leading to a positive electric charge for ϕ^+ and a neutral ϕ^0. The potential $V(\Phi)$ governs the self-interaction of the newly introduced field and may be freely chosen under the constraint that resulting Lagrangian is gauge invariant and renormalizable. The simplest form that fulfill these constrains and also allows for the generation of particle masses is:

$$V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2, \tag{2.12}$$

where μ^2 and λ are free real parameters. The condition $\lambda > 0$ guarantees a stable vacuum state. The sign on the first term is chosen to give the potential its characteristic "mexican hat" shape, which drives the spontaneous symmetry breaking: The ground state takes on a non-vanishing vacuum expectation value (vev) Φ_0. The vev is computed by minimizing $V(\Phi)$:

$$\Phi_0^\dagger\Phi_0 = \frac{v^2}{2}, \qquad v = 2\sqrt{\frac{\mu^2}{\lambda}}. \tag{2.13}$$

The resulting ground state is not unique, but degenerate in three of its four dimensions. As the ground state remains symmetric under the unbroken $\mathrm{U}(1)_\mathrm{em}$ symmetry, the vev is only determined up to a complex phase, which we choose to give a real lower component to Φ_0. For the upper component, we choose a description that provides a vanishing value to obtain an electrically neutral vacuumm, i.e. $\Phi_0 = (0, v)^\mathrm{T}$. The field Φ can thus be reparameterized in terms of perturbations around the vev:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 = (v + H + \mathrm{i}\chi)/\sqrt{2} \end{pmatrix}, \tag{2.14}$$

where H represents the real scalar Higgs field, which can be understood as a vacuum excitation and correspondingly carries the vacuum quantum numbers The additional fields ϕ^+ and χ are complex and real, respectively, and bear formal resemblance to Goldstone bosons. However, these three Goldstone-like modes are not physical, as a gauge transformation can always be found that will let them vanish. Using this so called "unitary gauge" Eq. 2.14 may be substituted together with the definition of the covariant derivative (Eq. 2.7) in Eq. 2.11, yielding

$$\mathcal{L}_{\mathrm{H,U\text{-}gauge}} = \frac{1}{2}(\partial H)^2 + \frac{g^2}{4}(v+H)^2 W^+_\mu W^{-,\mu} + \frac{g^2}{8c_\mathrm{w}^2}(v+H)^2 Z_\mu Z^\mu \tag{2.15}$$

$$+ \frac{\mu^2}{2}(v+H)^2 - \frac{\lambda}{16}(v+H)^4, \tag{2.16}$$

using $I^i_{\mathrm{w},\Phi} = \sigma^i/2$, $Y_{\mathrm{w},\Phi} = 1$, $T^a_{\mathrm{c},\Phi} = 0$.

The results show the origin of mass for the electroweak gauge bosons: bilinear terms in the W and Z fields appear, proportional to the vev. This conserves the degrees of freedoms of the theory, the three Goldstone-like modes, which fell away in the unitary gauge, reappear as the additional degrees of freedom in the now massive gauge boson fields. A similar bilinear term in H is responsible for the mass of the Higgs boson itself. The masses may be expressed in terms of the previously defined parameters as:

$$M_{\mathrm{W}} = \frac{gv}{2}, \qquad M_{\mathrm{Z}} = \frac{M_{\mathrm{W}}}{c_{\mathrm{w}}}, \qquad M_{\mathrm{H}} = \sqrt{2\mu^2}. \tag{2.17}$$

In addition to the masses, Eq. 2.16 also introduces interactions between the gauge bosons and the Higgs field as well as Higgs boson self interactions.

2.1.2 Fermion Masses

The generation of fermion masses can also be associated to the Higgs field, but proceeds by a fundamentally different mechanism. This is achieved by extending the SM Lagrangian by the so-called Yukawa term $\mathcal{L}_{\mathrm{Yuk}}$ that intermixes the fermions and the Higgs field in a gauge-invariant manner:

$$\mathcal{L}_{\mathrm{Yuk}} = -\overline{\Psi}_{\mathrm{L}} G_\ell \psi_{\ell_{\mathrm{R}}} \Phi - \overline{\Psi}_Q G_u \psi_{u_{\mathrm{R}}} \tilde{\Phi} - \overline{\Psi}_Q G_d \psi_{d_{\mathrm{R}}} \Phi + \mathrm{h.c.}, \tag{2.18}$$

where "h.c." denotes hermitian conjugates and $\tilde{\Phi} = \mathrm{i}\sigma^2\Phi^* = ((\phi^0)^*, -\phi^-)^{\mathrm{T}}$ the charge-conjugate Higgs doublet with quantum numbers opposite to Φ. The G_f represent complex 3×3 matrices, which are free parameters of the theory. At first sight this appears to introduce a large number of free parameters into the SM. However, through appropriate field redefinitions a majority of these parameters may be eliminated.

The Yukawa term (Eq. 2.18) generates the fermion masses as it contains terms bilinear in the fermion fields. The fermion masses are encoded in the matrices G_f and are free parameters in contrast to the W and Z boson mass, which are strictly related to the weak couplings and the Higgs field parameters. Off-diagonal elements of the G_f induce oscillations between the fermion generations during free propagation. For the leptons, an appropriate basis can be chosen that diagonalizes G_ℓ, providing mass eigenstates of definite generation. This is not possible for the quarks, where the G_f for up- and down-type quarks cannot be diagonalized simultaneously. This leads to the mixing of quark generations in the weak interaction [9].

The absence of right-handed neutrinos in this setup prevents the generation of neutrino masses through this mechanism. However, since the initial work on the Higgs mechanism, neutrino oscillations have been discovered [10], indicating that neutrinos are massive, if light. The neutrino masses are commonly explained in terms of the so called see-saw mechanism [11, 12], though in the context of this work, neutrinos may effectively be treated as massless.

Using the mass eigenstates, each fermion couples to the Higgs boson with strength $y_f = M_f/v$. This coupling structure (i.e. purely scalar couplings proportional to the fermion mass) is a strong prediction of the SM Higgs mechanism and provides empirical means to distinguish it from alternative mass generation mechanisms, where other couplings structures and strengths may occur.

2.2 Predictions in Hadron Collisions

Perturbation theory can be used to compute the scattering matrix of processes involving the fundamental particles of the SM. However, color-charged particles like quarks and gluons are hidden from direct observation by the phenomenon of confinement [13]: at the LHC protons are accelerated and hadron jets detected in the experiments. These complex initial and final states are ultimately modeled to conform to our knowledge of strong interactions but must necessarily depend on ingredients derived from measurements to properly describe the low-energy, non-perturbative aspects of proton collisions.

The initial state protons may be envisioned in the parton model to consist of three valence quarks as well as a sea of virtual quarks and gluons. The composition of the proton is described by a parton desnsity function (PDF), that gives the probability to find a given parton carrying a momentum fraction x of the proton. Due to the non-perturbative effects prevalent in low energy QCD, the PDFs cannot be derived from first principles, but have to be extracted from data (see for example [14–16]). The PDFs are used to compute scattering cross sections, by integrating over all possible initial state momentum fractions:

$$\sigma_{pp\to X} = \sum_{i,j} \int \mathrm{d}x_1 \mathrm{d}x_2 \cdot pdf(x_1)pdf(x_2) \cdot \hat{\sigma}_{ij\to X}(x_1 P_1, x_2 P_2)\,, \tag{2.19}$$

where x_1 and x_2 denote the momentum fractions of the two interacting parton in their respective protons, P_1 and P_2 the proton momenta and $\hat{\sigma}_{ij\to X}$ the cross sections for two partons of type i and j to scatter to the final state X. The sum is taken over the types of partons in the proton, i, j. This naive approach suffers from two major issues: the cross section computed as described above is not stable against initial state QCD effects, as there is no well defined separation of scales between the PDFs and the hard matrix element. Additionally It is not clear whether such a simply factorized approach is possible at all.

The separation between the PDFs and matrix element can be made explicit by introducing a factorization scale Q^2, such that processes at an energy scale below Q^2 are implicitly included in the PDF definition, which gain a dependence on Q^2. The separating scale Q^2 is artificial, so that the theory can ultimately not depend on its value. This condition leads to a set of integro-differential equations (the so called DGLAP equations [17–19]), similar to the renormalization group equations, fixing the evolution in Q^2 of the PDF. This evolution describes the emission of gluons as well as gluon splitting starting from an initial parton in the proton. Though traditionally, the PDFs are defined for quarks and gluons, it is possible to also absorb initial state photon radiation into the PDFs, effectively resulting in a photon density of the proton [20]. The contribution of these photon induced processes is typically small for final states that can also be reached from quark- or gluon initial states. However, they may make up a significant fraction of the total cross section in processes involving electrically charged, but color-neutral particles, i.e. W bosons.

The applicability of the factorization approach shown in Eq. 2.19 can only be shown for some classes of all possible processes [21]. Generally, factorization is expected to hold in the limit of the production of very heavy or very high p_{T} particles. Nevertheless, experience shows that the factorization approach produces excellent predictions in the kinematic regime probed at the LHC.

Just as non-perturbative QCD effects complicate the initial state of hadron colliders, they are also responsible for complications in hadronic final states. Similar to the difference in scales between the scale of initial state proton mass and the hard scale of the scattering process, that is bridged by the DGLAP evolution, there is a difference in scales between the hard process and the scale of the final state hadron masses. The difference is treated analogously, producing gluon emissions off color-charged particles as well as gluons splitting in a process usually called parton shower. While a comprehensive treatment of the these emissions would be exceedingly complex, it turns out that color coherence effects suppress a large number of possible emission patterns, so that simple topologies dominate [22]. These simpler patterns, i.e. ordered in emission angle or transverse momentum, are the only ones used in the common simulation programs, greatly simplifying the computation.

Due to the confining nature of QCD, free color-charged particles are not observed, but rather color-neutral hadrons reach the detector. The transition between the partonic and hadronic regimes ("hadronization") is modeled empirically, with inspirations from the underlying theory. The simplest approach is based on so called fragmentation functions, which describe the probability to observe a given hadron carrying a certain momentum fraction of the parton. Current simulation programs use more sophisticated methods to model the hadronization process. The most prominent of these are the Lund string model [23] and the cluster fragmentation model [24]. In the Lund string model, the outgoing quarks are connected by strings, representing the confined color fields between the quarks. Potential energy stored in the string is used to iteratively create quark-antiquark pairs, forming color-neutral hadrons. In this picture gluons are envisioned as kinks in the connecting color strings. In contrast, the cluster model groups the final state partons into minimal color-neutral groups (the eponymous clusters), which are assumed to decay in a similar way as excited hadrons.

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