

Chapter 2

Radioactivity and Penetrating Power of Nuclear Radiation

In this chapter, we primarily discuss radioactivity, the process by which an unstable nucleus loses energy by emitting radiation. First we illustrate the main types of decays (alpha, beta, gamma, internal conversion, electron capture, nucleon emission and fission), then we discuss the radioactive decay law, decay rates, mean lifetimes of isotopes, radioactive families and sequential decays.

In the second part of the chapter, we discuss the interaction between radiation and matter, the existence of an artificial radioactivity in addition to natural one, due to the development of nuclear technologies, the biological effects of ionising radiation with elements of dosimetry, and the applications of nuclear radiation in medicine, research and industry.

2.1 Nuclear Decay

As discussed in the previous Chapter, a nucleus containing a number of protons and neutrons not well balanced is unstable. It will break spontaneously—that is, without supplying external energy—in pieces or rearrange its internal structure with a release of energy. This phenomenon is called *radioactivity*. The unstable nucleus is said to be *radioactive* and the change is called *radioactive decay*. The original nucleus which decays is called the *parent nuclide*, and the product nucleus is called the *daughter nuclide*. When the parent and the daughter nuclides are different chemical elements, the decay process results in the creation of an atom of a different element. This process is known as *a nuclear transmutation*. In the decay, both the total electric charge and the number of nucleons are conserved, i.e. must remain the same before and after the decay.

There are many types of radioactive decay (see Table 2.1). A radioactive decay is classified as α -, β -, or γ -decay, according to the type of particle emitted. Other types of decay produce emissions of nucleons or nuclei lighter than the parent nucleus. All these radiation emissions are determined and measured with great precision and are typical of each radioactive nucleus.

Table 2.1 Main features of the different types of radioactivity

Radioactivity	Change in nuclear charge	Change in mass number	Characteristic of the process
α -decay	$Z - 2$	$A - 4$	Emission of an alpha particle ($A = 4$, $Z = 2$) from a nucleus
β^- -decay	$Z + 1$	A	A neutron in a nucleus transforms into a proton plus an electron and an antineutrino: $n \rightarrow p + e^- + \bar{\nu}$
β^+ -decay	$Z - 1$	A	A proton in a nucleus transforms into a neutron plus a positron and a neutrino: $p \rightarrow n + e^+ + \nu$
Electron capture	$Z - 1$	A	A nucleus captures an atomic electron and emits a neutrino. The daughter atom is left in an excited and unstable state. $p + e^- \rightarrow n + \nu$
γ -decay	Z	A	A nucleus in an excited state makes a transition to a state with lower energy by emitting a photon
Internal conversion	Z	A	An excited nucleus transfers energy to an atomic electron, which is subsequently ejected from the atom
n emission	Z	$A - 1$	Emission of a neutron from the nucleus
p emission	$Z - 1$	$A - 1$	Emission of a proton from the nucleus
Spontaneous fission	$\sim \frac{1}{2}Z$	$\sim \frac{1}{2}A$	The nucleus disintegrates, usually into two fragments of very approximately equal mass and charge (see Chap. 3)

2.1.1 α -Decay

For the heaviest nuclei whose average nuclear binding energy is relatively low, some of their nucleons can go towards a higher binding energy by emitting a helium nucleus, ${}^4\text{He}_2$, which is called α -particle in radioactive decays. An α -particle has a double positive charge $+2e$, and mass number $A = 4$. Then, the daughter nucleus has a mass number A and an atomic number Z lower than the parent by four and two units, respectively.

As we have seen in Sect. 1.8, the helium nucleus is a cluster of four nucleons—two protons and two neutrons—extraordinarily strongly bound (binding energy per nucleon $B/A = 7.06$ MeV/nucleon, a very high value for such a light nucleus). Then it is not surprising that all heavy nuclei ($A \geq 180$) are energetically unstable against α -decay. Specifically, α -decay is energetically allowed if the mass of the parent nucleus is larger than the sum of those of the daughter and helium nuclei:

$$M(Z, A) - M(Z - 2, A - 4) - M({}^4\text{He}) > 0. \quad (2.1)$$

This relation can be written in terms of the total binding energies of the nuclei, obtaining

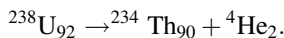
$$B({}^4\text{He}) > B(Z, A) - B(Z - 2, A - 4). \quad (2.2)$$

Therefore, in α -decay the parent nucleus releases some energy Q_α . This is given by the mass difference between the parent and the daughter nucleus multiplied by c^2 :

$$Q_\alpha = [M(Z, A) - M(Z - 2, A - 4) - M(^4\text{He})] c^2. \quad (2.3)$$

The energy Q_α goes into the kinetic energies of the α -particle and the daughter nucleus (see Problem 2.1).

An example of α -decay is the transmutation of uranium-238 into thorium-234:



The thorium nucleus has 2-protons and 2-neutrons less than the uranium: the missing nucleons are taken away by the α -particle.

Problem 2.1: Alpha decay of ^{238}U . Calculate the energy released in the α -decay of uranium-238 (rest mass $M_{\text{U}} = 238.0508$ u) into thorium-234 (rest mass $M_{\text{Th}} = 234.0436$ u). Calculate the fraction of this energy taken by the α -particle.

Solution: With clear meaning of the symbols we have

$$\Delta m = M_{\text{U}} - M_{\text{Th}} - M_{\text{He}} = 238.0508 \text{ u} - 234.0436 \text{ u} - 4.0026 \text{ u} = 0.0046 \text{ u}.$$

$$Q = \Delta m c^2 = 0.0046 \times 931.494 = 4.28 \text{ MeV}.$$

This energy is shared between the thorium nucleus and the α -particle. The nucleus ^{234}Th has a mass much larger than that of the α -particle; therefore it recoils with smaller speed. In the decay both the energy and the linear momentum are conserved, then assuming the parent nucleus to be at rest and neglecting relativistic corrections, one has:

$$M_{\text{Th}} v_{\text{Th}} = M_\alpha v_\alpha,$$

$$\frac{1}{2} M_{\text{Th}} v_{\text{Th}}^2 + \frac{1}{2} M_\alpha v_\alpha^2 = k_{\text{Th}} + k_\alpha;$$

where we have indicated with k_α and k_{Th} the kinetic energies of the helium and thorium nuclei, respectively. Clearly, $k_{\text{Th}} + k_\alpha = Q$.

From the first relationship we get $v_\alpha = \frac{M_{\text{Th}}}{M_\alpha} v_{\text{Th}}$, and then, from the second relationship, the kinetic energy of the α -particle is

$$\begin{aligned} k_\alpha &= \frac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2} M_\alpha \left(\frac{M_{\text{Th}}}{M_\alpha} v_{\text{Th}} \right)^2 = \frac{1}{2} M_{\text{Th}} v_{\text{Th}}^2 \frac{M_{\text{Th}}}{M_\alpha} = \frac{M_{\text{Th}}}{M_\alpha} k_{\text{Th}} = \frac{234.0436}{4.0026} k_{\text{Th}} \\ &= 58.47 k_{\text{Th}}. \end{aligned}$$

Then the energy Q released in the decay is equal to:

$$Q = k_{\text{Th}} + k_{\alpha} = 58.47 k_{\text{Th}} + k_{\text{Th}} = 59.47 k_{\text{Th}}.$$

From which it results

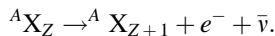
$$k_{\text{Th}} = \frac{Q}{59.47} = 0.017 Q = 0.073 \text{ MeV} = 73 \text{ keV},$$

$$k_{\alpha} = 58.47 k_{\text{Th}} = \frac{58.47}{59.47} Q \approx 0.983 Q = 4.21 \text{ MeV}.$$

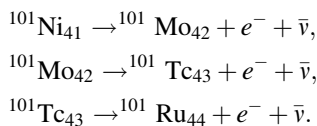
The α -particle, which is much lighter than thorium, takes $\approx 98.3\%$ of the total released energy. The thorium nucleus takes the remaining 1.7% .

2.1.2 β^{-} -Decay

A β^{-} -decay (*beta-minus decay*) occurs when the ratio of neutrons to protons in the nucleus is too high. In this case, an excess neutron transforms into a proton and an electron plus a neutral particle of negligible mass, called *antineutrino* (symbol $\bar{\nu}$). The proton stays in the nucleus and the electron is ejected. In the process, the parent nucleus transforms into a daughter one having the same mass number A , and the atomic number Z greater by one unit:



Examples of β^{-} -decay are those of nuclei up the left side of the parabola of Fig. 1.10, namely of niobium-101 into molybdenum-101, molybdenum-101 into technetium-101, and technetium-101 in ruthenium-101:



Energetically, β^{-} -decay is possible whenever the mass of the daughter atom $M(A, Z + 1)$ is smaller than the mass of its isobaric neighbour:

$$M(A, Z) > M(A, Z + 1). \quad (2.4)$$

We consider here the atomic masses and so the rest mass of the electron created in the decay is automatically taken into account. The tiny mass of the antineutrino

(<2 eV/c² [1]) is negligible in the mass balance. The decay energy of a β^- -decay is given by the mass difference between the parent and the daughter atom multiplied by c^2 :

$$Q_{\beta^-} = [M(A, Z) - M(A, Z + 1)]c^2. \quad (2.5)$$

Problem 2.2: Beta decay of ^{234}Th . Calculate the maximum energy of the electron emitted in the β^- -decay of thorium-234, $^{234}\text{Th}_{90}$ (rest mass $M_{\text{Th}} = 234.04359 \text{ u}$), into protoactinium-234, $^{234}\text{Pa}_{91}$ (rest mass $M_{\text{Pa}} = 234.04330 \text{ u}$).

Solution: The atom $^{234}\text{Pa}_{91}$ produced by the β^- -decay of $^{234}\text{Th}_{90}$ is not neutral as its nucleus has one proton more ($Z = 91$) than the parent nucleus, but the same number of bound electrons (90). Then it has one electron less. However, if we sum up the mass of the produced Pa and that of the emitted β^- particle, we get the mass of the Pa nucleus plus the mass of 91 electrons, which is exactly the atomic mass of the neutral Pa atom (except for the negligible binding energy of the atomic electron). Then the mass difference Δm corresponding to the β^- -decay is

$$\Delta m = M_{\text{Th}} - M_{\text{Pa}} = 234.04359 \text{ u} - 234.04330 \text{ u} = 0.00029 \text{ u}.$$

The equivalent energy released in the decay is

$$Q = \Delta mc^2 = (0.00029 \text{ u}) \times (931.494 \text{ MeV}/c^2) = 0.27 \text{ MeV}.$$

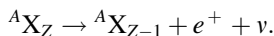
This energy is redistributed among the daughter nucleus, the electron and the antineutrino. From the exact equations of the reaction kinematics, it is possible to show that, to a very good approximation, the maximum electron energy is given by the total energy released in the reaction. Hence, 0.27 MeV is, to a good approximation, the maximum energy that the emitted electrons can have.

N.B. The neutrino emitted in the decay does not contribute to the mass balance, as its mass is negligible.

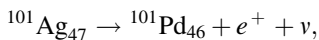
2.1.3 β^+ -Decay

A β^+ -decay (*beta-plus decay*) occurs when a proton inside a nucleus converts into a neutron by emitting a positron (i.e., a positive electron) and a neutral particle called *neutrino* (symbol ν). In such a process, the parent nucleus transforms into a

daughter nucleus having the same mass number A and the atomic number Z smaller by one unit:



Examples of β^- -decay are those of nuclei up the right side of the parabola of Fig. 1.10, namely of silver-101 in palladium-101, palladium-101 in rhodium-101, and rhodium-101 in ruthenium-101:



Energetically, β^+ -decay is possible whenever the following relationship between the masses of the parent and daughter atoms, $M(A, Z)$ and $M(A, Z - 1)$, is satisfied:

$$M(A, Z) > M(A, Z-1) + 2m_e. \quad (2.6)$$

This relationship takes into account that the final system of a β^+ decay consists of a nucleus $(Z - 1, A)$ with $(Z - 1)$ atomic electrons, together with the emitted positron as well as of the existence of an excess electron in the parent nucleus. Hence, with clear meaning of the symbols, the total mass of the final system is $M(A, Z - 1) + m_{\beta^+} + m_{e^-}$, that is, recalling that the positron has the same mass as the electron ($m_{\beta^+} = m_{e^-} = m_e$), $M(A, Z - 1) + 2m_e$.

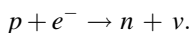
Then, the total energy available for the β^+ -decay is

$$\begin{aligned} Q_{\beta^+} &= \{M(A, Z) - [M(A, Z - 1) + 2m_e]\}c^2 \\ &= [M(A, Z) - M(A, Z-1) - 2m_e]c^2 = \Delta mc^2 - 2m_e c^2. \end{aligned} \quad (2.7)$$

Therefore, the maximum kinetic energy of the emitted positron is $2m_e c^2$ less than the mass-energy difference Δmc^2 between the parent and daughter nucleus.

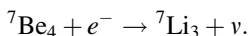
2.1.4 Electron Capture

Electron capture is a decay process in which an atomic electron interacts with the nucleus, where it combines with a proton, forming a neutron and a neutrino. The neutrino is ejected from the atomic nucleus:



During electron capture, an atom changes a proton into a neutron, so that it changes from one element to another. However, the total number of nucleons

(protons + neutrons) remains the same. For example, after undergoing electron capture, an atom of beryllium (with 4 protons and 3 neutrons) becomes an atom of lithium (with 3 protons and 4 neutrons):



Electron capture results in a neutral atom, since the loss of one atomic electron is balanced by the loss of one positive nuclear charge. The process leaves a vacancy in the electron energy level from which the electron came, and that vacancy is filled by the dropping down of a higher-level electron with the release of some energy. Although most of the time this energy is released in form of an electromagnetic radiation, called X-ray, having a wavelength of the order of 10^{-8} to 10^{-11} m, such energy can also be transferred to another atomic electron, which is ejected from the atom. This second electron is called an *Auger electron* and the process called *Auger effect* after one of its discoverers.¹ In the latter case, a positive ion is left over.

Electron capture reactions compete with β^+ -decay. It can occur when the following energy conservation condition is satisfied

$$M(A, Z)c^2 > M(A, Z - 1)c^2 + \epsilon,$$

where ϵ is the excitation energy of the atomic shell of the daughter nucleus. The condition stems from the fact that the atom of the new element formed in the reaction has a vacancy in an electron energy level, i.e. it is produced in an excited atomic energy level. Clearly, the energy available in the capture process, given by the difference $[M(A, Z) - M(A, Z - 1)]c^2$, must be greater than ϵ .

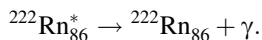
2.1.5 γ -Decay

The nucleus does not have a rigid structure and may undergo excitations and oscillations accompanied by changes of shape. A γ -decay (gamma decay) occurs when a nucleus that is in an excited state (that is, in a configuration with energy higher than the ground state) makes a transition to either an excited state with lower energy than the initial one or to the ground state (we recall that this is the most stable configuration of the nucleus). The energy resulting from the jump from one level up to the other level down is emitted as an electromagnetic wave, called γ -ray, having a wavelength of the order of 10^{-10} to 10^{-13} m, without other particles being emitted in the decay. Gamma rays are also described as packets of energy, called *photons*.

Both the number of protons and neutrons in the nucleus do not change in this process as photons are electrically neutral, so the parent and daughter atoms are the same chemical element.

¹Pierre Victor Auger (Paris, France, 1899—Paris, France, 1993) was a French physicist who has made important contributions in the fields of atomic, nuclear, and cosmic ray physics.

An example of a γ -decay is the decay of excited radon-222



The unstable parent nucleus $^{222}\text{Rn}_{86}^*$ (the asterisk is used to indicate that the nucleus is not in its energy ground state) is itself the daughter nucleus of a preceding α -decay.

2.1.6 Internal Conversion

Internal conversion is a radioactive decay process in which an excited nucleus, instead of emitting a gamma ray, de-excites by knocking out one of the electrons in the atom. The electron is emitted with a well-defined energy, the same energy that a gamma ray would have had in the same decay process (as usually, we neglect the atomic binding energy). Thus, in an internal conversion process, a high-energy electron is emitted from the radioactive atom, not from the nucleus. For this reason, the high-speed electrons resulting from internal conversion are not beta particles, since the latter come from beta decay, where they are produced in the nuclear decay process.

During internal conversion, the atomic number does not change, and thus (as is the case of gamma decay) no transmutation of one element to another takes place. However, since an electron is lost, a hole appears in an electron shell which is subsequently filled by an upper electron, producing an X-ray or an Auger electron.

2.1.7 Nucleon Emission

Highly excited neutron-rich or proton-rich nuclei, formed as the product of other types of decay, can occasionally lose energy by way of neutron and proton emission, resulting in a transition from one isotope to another of the same element, or from a nuclide of one element to a nuclide of another element, respectively.

2.1.8 Spontaneous Fission

In another type of radioactive decay, the nucleus disintegrates into other nuclei that are not well defined, but rather correspond to a range of fragments of the original nucleus. This decay, called spontaneous fission (see Chap. 3), happens when a large unstable nucleus spontaneously splits into two (and occasionally three) smaller daughter nuclei, and generally leads to the emission of gamma rays, neutrons, and beta particles.

2.1.9 Summary

Table 2.1 summarises the main features of the above different types of radioactivity.

Each unstable nuclide emits radiation of the same type and with the same total energy; for example, a γ emitter always emits γ -rays of certain fixed energies.

If a radioactive nucleus decays to another unstable nucleus, then that will also decay. Radioactive decay will continue until a stable nucleus is formed. This is the case, for example, of uranium-238, which transmutes into lead with a sequence of fourteen decays (see Sect. 2.3).

Alpha decay mainly occurs in the nuclides of the heavier elements (the lightest nuclide featuring alpha-decay being ^{105}Te , $Z = 52$ [2]), while beta decay occurs in the nuclides of all elements: precisely the nuclides that are above the stability curve of Fig. 1.7 decay β^+ , those found under the curve decay β^- . Gamma decay often accompanies an alpha or beta decay. In fact, when a radioactive nucleus decays, the daughter nucleus is not necessarily produced in the ground state. When the daughter nucleus is produced in an excited state, it decays rapidly to the ground state, either directly, or by subsequent emissions of gamma rays when passing through other intermediate excited states. Gamma rays are, therefore, the most common form in which the surplus energy of the excited products of radioactive decay is liberated.

2.2 The Radioactive Decay Law

The decay events take place completely at random; there is no way of predicting the precise instant at which a particular nucleus will decay: it can happen next moment, next day or even next century. However, we can compute decay probabilities.

Each radioactive isotope has a characteristic, well-defined probability of decaying per unit time, which is denoted by λ and takes the name of *decay constant*. This is an intrinsic property of the nucleus itself, which is independent of all physical and chemical conditions, such as temperature, pressure, concentration, age or past history of the radioactive nuclide. Radioactive nuclei decay spontaneously because of their internal dynamics and therefore their decay constants never change.

In a specimen containing N radioactive atoms at an arbitrary instant t , the number of decays dN that occur on average in a short time interval dt is proportional to dt and N , namely, according to the definition of λ :

$$dN = -\lambda N dt, \quad (2.8)$$

where the minus sign in the second member indicates that the number N of radioactive nuclei decreases with time. Obviously, being nuclear decay a random phenomenon, the number dN is subject to statistical fluctuations: it represents the average number of decays in the short time interval dt .

Equation (2.8) can be rewritten, separating the variables,

$$\frac{dN}{N} = -\lambda dt,$$

which, by integrating both sides

$$\int_{N_0}^N \frac{dN}{N} = - \int_0^t \lambda dt,$$

where N_0 is the number of radioactive nuclei present at time zero and N represents the number of the nuclides surviving until the time t , gives

$$\ln \frac{N}{N_0} = -\lambda t.$$

Taking the anti-logs of both sides of this equation, we get:

$$N(t) = N_0 e^{-\lambda t}, \quad (2.9)$$

which is called the *radioactive decay law*.

To measure the rate of decay the *half-life* or *half-period*—usually denoted by $\tau_{1/2}$ —is often used, which is the time interval over which, on average, half of the radionuclides present in the radioactive material disintegrate.

Equation (2.9) indicates that the number of radioactive nuclei present in a sample decreases with time with the exponential trend shown in Fig. 2.1: after each $\tau_{1/2}$ -period there remains half the amount of radionuclides that existed at the beginning of that period. Therefore, after a time of n half-lives the amount of radioactive isotope is reduced to $1/2^n$ of the initial quantity. Again, we have to take the decay law and the halving of the number of radionuclides as representing average behaviours because of the statistical nature of the decay phenomenon.

According to its definition, from the survival Eq. (2.9) it follows that the half-life is given by:

$$\tau_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}. \quad (2.10)$$

Often used is also the *mean-life* of an isotope $\tau = 1/\lambda$, and the previous relation becomes:

$$\tau_{1/2} = 0.693 \times \tau. \quad (2.11)$$

It should be noted that the two variables, mean-life and half-life, have different numerical values. Then, confusing them may result in serious errors.

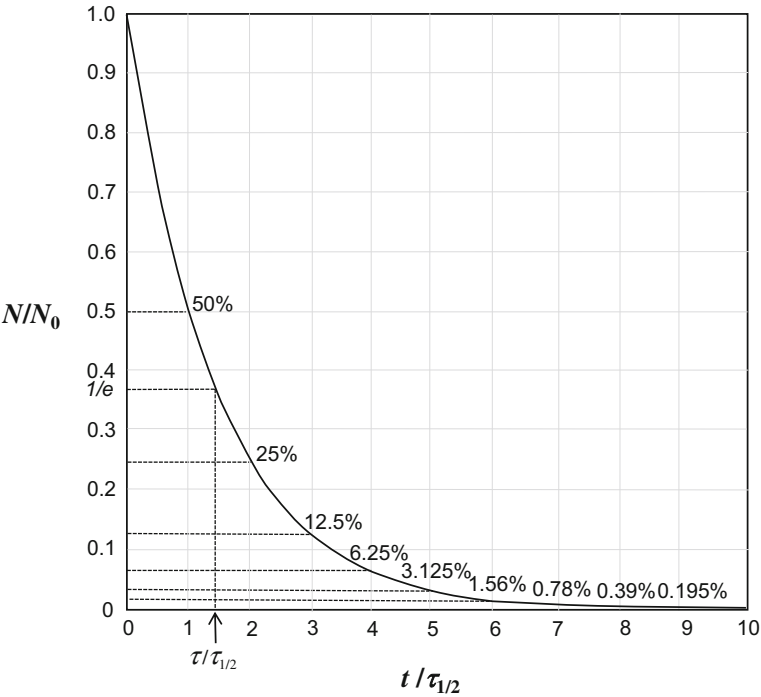


Fig. 2.1 Curve of radioactive decay. The ordinate shows the fraction number of radionuclides surviving until the time $t/\tau_{1/2}$, over an interval of ten half-life periods. After seven half-lives, the amount of radionuclides has been reduced to less than 0.8 % of the initial value. After ten half-lives, to less than 0.1 %

Substitution of τ in Eq. (2.9) shows that the mean-life is the time required for the number of atoms to fall to $e^{-1} = 0.368$ of any initial value.

Table 2.2 shows the half-life and decay mode of some radionuclides. The half-lives of different radioactive nuclei vary within very wide limits, from over one hundred thousand billion years for indium-115, to eight days for iodine-131, down to less than a millisecond for polonium-214. Uranium-238 has a half-life approximately equal to the age of the Earth, which is estimated to be 4.5 billion years. Therefore, about half of the uranium-238 that existed at the time of the formation of the Earth is still present today.

2.2.1 Activity

The amount of radiation emitted by a substance containing radioactive isotopes depends on two factors: the number of unstable nuclei present and their half-life. In order to quantify the amount of radiation emitted, the *activity* dN/dt of a radioactive nuclide is used. This is defined as the number of disintegrations per unit time.

Table 2.2 Half-life and decay modes of certain radioactive nuclei [3]

Isotope	Symbol	Half-life	Decay mode
Polonium-214	$^{214}\text{Po}_{84}$	$1.64 \times 10^{-4} \text{ s}$	α, γ
Oxygen-15	$^{15}\text{O}_8$	2.04 min	β^+, EC
Krypton-89	$^{89}\text{Kr}_{36}$	3.15 min	β^-, γ
Nitrogen-13	$^{13}\text{N}_7$	9.97 min	β^+, EC
Carbon-11	$^{11}\text{C}_6$	20.39 min	β^+, EC
Uranium-239	$^{239}\text{U}_{92}$	23.45 min	β^-, γ
Fluorine-18	$^{18}\text{F}_9$	109.8 min	β^+, EC
Neptunium-239	$^{239}\text{Np}_{93}$	2.36 d	β^-, γ
Radon-222	$^{222}\text{Rn}_{86}$	3.82 d	α, γ
Iodine-131	$^{131}\text{I}_{53}$	8.03 d	β^-, γ
Tritium	$^3\text{H}_1$	12.32 a	β^-
Strontium-90	$^{90}\text{Sr}_{38}$	28.79 a	β^-
Cesium-137	$^{137}\text{Cs}_{55}$	30.1 a	β^-, γ
Radium-226	$^{226}\text{Ra}_{88}$	1600 a	α, γ
Carbon-14	$^{14}\text{C}_6$	5700 a	β^-
Americium-243	$^{243}\text{Am}_{95}$	7370 a	α, γ
Plutonium-239	$^{239}\text{Pu}_{94}$	24,110 a	α, γ
Neptunium-237	$^{237}\text{Np}_{93}$	$2.14 \times 10^6 \text{ a}$	α, γ
Curium-247	$^{247}\text{Cm}_{96}$	$1.56 \times 10^7 \text{ a}$	α, γ
Uranium-235	$^{235}\text{U}_{92}$	$7.04 \times 10^8 \text{ a}$	α, γ
Potassium-40	$^{40}\text{K}_{19}$	$1.25 \times 10^9 \text{ a}$	$\beta^-, \beta^+, \text{EC}$
Uranium-238	$^{238}\text{U}_{92}$	$4.47 \times 10^9 \text{ a}$	α, γ
Thorium-232	$^{232}\text{Th}_{90}$	$1.41 \times 10^{10} \text{ a}$	α, γ
Rubidium-87	$^{87}\text{Rb}_{37}$	$4.81 \times 10^{10} \text{ a}$	β^-
Indium-115	$^{115}\text{In}_{49}$	$4.41 \times 10^{14} \text{ a}$	β^-

Using Eqs. (2.8) and (2.10) we have:

$$\left| \frac{dN}{dt} \right| = \lambda N = \frac{0.693}{\tau_{1/2}} N, \quad (2.12)$$

where the absolute value is used because the variation dN is negative.

The unit of activity is the becquerel (Bq), so named in honour of the French physicist Antoine Henri Becquerel (Paris, France, 1852—Le Croisic, France, 1908. Nobel laureate in Physics in 1903), who first discovered the phenomenon of radioactivity. One becquerel is equal to one disintegration or nuclear transformation per second.

A historical unit still widely used is the curie (Ci), so named in honour of the Poland-born French physicist Marie Skłodowska Curie (Warsaw, Poland, 1867—Passy, France, 1934. Nobel laureate in Physics in 1903). One curie is equal to 3.70×10^{10} disintegrations per second. This is the activity of 1.0 g of radium-226. Clearly, $1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq}$.

Problem 2.3: Survival of uranium isotopes. The ^{238}U and ^{235}U isotopes have half-lives of 4.47 and 0.7 billion years, respectively. Calculate the fractions of the two isotopes initially present at the formation of the Earth that still exist, knowing that our solar system is considered to be 4.5 billion year old. Calculate the $^{235}\text{U}/^{238}\text{U}$ ratio in natural uranium at that date, considering that today it is 0.7 %.

Solution: The ^{238}U isotope has half-life equal to the age of the Earth and then about half of all the ^{238}U initially present still exists today. For the isotope ^{235}U , the age of the Earth corresponds to more than 6 half-lives. Therefore, there is still only about

$$\frac{1}{2^6} = \frac{1}{64} = 0.0156 = 1.56\% \text{ of the original uranium-235.}$$

Using these data it is easy to calculate that the $^{235}\text{U}/^{238}\text{U}$ ratio in natural uranium when the Earth was formed was about 22 %. Using the correct values for the half-lives of the two isotopes the correct value of this percentage is 23.1 %.

Problem 2.4: Activity of antimony. A radioactive isotope ^{124}Sb (antimony), with initial activity $R_0 = 7.4 \times 10^7$ Bq, has a half-life of 60 d. Calculate its residual activity R after one year.

Solution: the decay constant of the sample of antimony is

$$\lambda = \frac{0.693}{60 \text{ d}} = 0.01155 \text{ d}^{-1}.$$

Using Eqs. (2.9) and (2.12) and calling N_0 and N the number of nuclei of Sb at the time zero, and after 1 year (= 365.25 d), respectively, we get:

$$R_0 = \left| \frac{dN}{dt} \right|_0 = \lambda N_0;$$

$$\begin{aligned} R &= \left| \frac{dN}{dt} \right| = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t} = 7.4 \times 10^7 e^{-0.01155 \times 365.25} \\ &= 1.09 \times 10^6 \text{ Bq.} \end{aligned}$$

Problem 2.5: Half-life and decay constant of a radionuclide. The activity of a radioactive isotope decreases from 350 to 275 disintegrations per minute in 3.5 h. Calculate the half-life of the radionuclide and its decay constant.

Solution: From the previous problem we have

$$R/R_0 = e^{-\lambda t}.$$

Extracting the natural logarithm of the first and second member of this equation, one gets:

$$\ln \frac{R}{R_0} = -\lambda t.$$

From which it follows

$$\lambda = -\frac{1}{t} \ln \frac{R}{R_0} = \frac{1}{t} \ln \frac{R_0}{R}.$$

That is, noting that 3.5 h = 12,600 s:

$$\lambda = \frac{1}{12,600} \ln \frac{350}{275} = 1.91 \times 10^{-5} \text{ s}^{-1}.$$

The half-life is then:

$$\tau_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1.91 \times 10^{-5}} = 0.3628 \times 10^5 \text{ s} = 10.08 \text{ h}.$$

2.3 Radioactive Families

A decay chain is a sequence of radioactive decay processes, in which the decay of one nucleus creates a new nucleus that is itself radioactive, and this sequence continues until eventually a stable nucleus is reached. For example, uranium-238 decays into thorium-234, which in turn decays into protactinium-234, and so on until stable lead-206 is produced at the end of the chain. The group of nuclides within a series of decays is called *radioactive family*.

All known natural radionuclides are grouped in three families having as the nucleus at the beginning of the series respectively thorium-232 (^{232}Th , $Z = 90$), uranium-238 (^{238}U , $Z = 92$), and uranium-235 (^{235}U , $Z = 92$). All isotopes of these elements are radioactive and the above three are the heaviest ones with a half-life so long that they still exist in nature.

The mass numbers of every isotope in these chains have values of the type $A = 4k$, $A = (4k + 2)$, and $A = (4k + 3)$, with k integer, respectively. A radioactive family of the type $A = (4k + 1)$ is missing in nature, but it has been produced artificially.² This family is known as the *neptunium family* from its longest-lived member $^{237}\text{Np}_{93}$, that has a half-life of 'only' 2.14×10^6 years. Its absence in nature is explained by the fact that all its members have lifetimes much shorter than the age of the solar system;

²Also the elements beyond uranium and thorium in the three other families have been produced artificially.

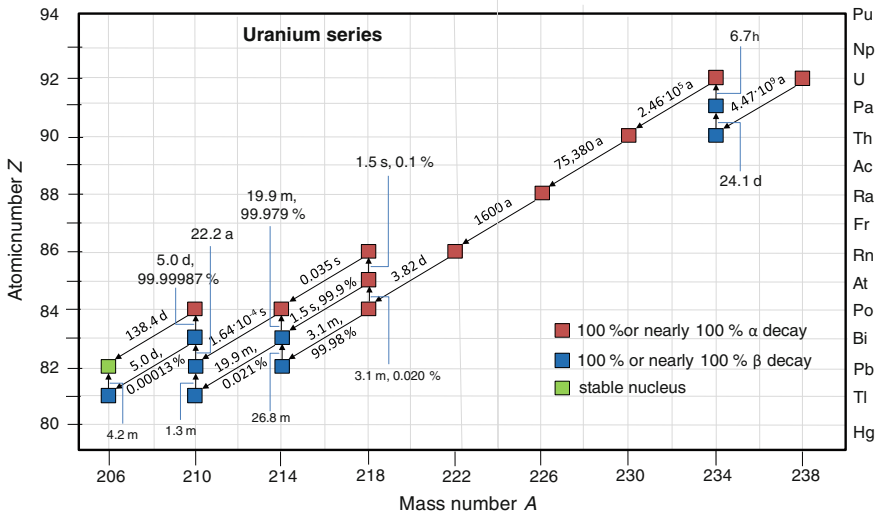


Fig. 2.2 The uranium decay series ($4k + 2$) in the A - Z plane. It is the longest-known decay chain. The half-life of each nucleus is given, together with its decay mode and relative probability (where the latter is not indicated it is equal to 100 %). Values have been rounded off for convenience. Diagonal arrows represent alpha decays, while vertical arrows represent beta decays. Red and blue squares are 100 % or nearly 100 % alpha- and beta-emitting nuclei, respectively. The green square corresponds to the final stable isotope, lead-206

so, even if these radionuclides existed at the time of the formation of the Earth, they are now completely transmuted into the final stable nuclide of the family, which is thallium-205 ($^{205}\text{Tl}_{81}$). Some older sources quote bismuth-209 as the final isotope, but it was recently discovered that it is radioactive, with a half-life of 1.9×10^{19} years. This is the only series that does not end with an isotope of lead. For all the other three families indicated above, the series of subsequent decays continues until it forms a stable isotope of lead ($^{206}\text{Pb}_{82}$, $^{207}\text{Pb}_{82}$ or $^{208}\text{Pb}_{82}$).

Figure 2.2 shows the uranium decay series ($4k + 2$) which has as parent the ^{238}U nucleus and ends with ^{206}Pb . The nuclei are represented by red or blue squares at the corresponding values of Z and A . The diagonal arrows indicate an α -decay (A and Z decrease by 4 and 2 units, respectively), and the vertical ones indicate a β -decay (A remains the same and Z increases by 1).

The initial nucleus ^{238}U decays by α -emission into ^{234}Th , but this daughter nucleus is itself unstable and undergoes β -decay into the protactinium nucleus, ^{234}Pa . This nucleus is also unstable and decays into ^{234}U , and so on.

For some radionuclides, two alternative decay modes, or *branches*, are possible. It is the case of ^{218}Po in the figure, which can undergo both α - and β -decay leading to the daughter nuclei ^{214}Pb and ^{218}At , respectively. Then, after another decay of the daughter nuclei (β - and α -decay, respectively), both branches lead to ^{214}Bi . Similar cases are those of the isotopes ^{218}At , ^{214}Bi , and ^{210}Bi . When such nuclei

decay, they can do so following one path or the other, with different probabilities called *branching ratios*. However, the half-life assigned to those isotopes is just one number referred to a generic decay of one or the other type.

Thanks to these decay chains, it is possible to find in nature radioactive elements with short half-lives that would otherwise not exist. For example, the radium isotope $^{226}\text{Ra}_{88}$ has a half-life so short (1600 years) that all the radionuclides of this species that were produced when the Earth was formed, about 4.5 billion years ago, have now disappeared. However, the decay chain of the uranium-238 family ensures a continuous supply of $^{226}\text{Ra}_{88}$. The same occurs for many other radioactive nuclei.

In a radioactive family, the decay of the parent nucleus acts as a bottleneck for the decay of the daughter nucleus. For example, in comparison to the half-life of 1600 years of the $^{226}\text{Ra}_{88}$, the daughter nucleus radon, $^{222}\text{Rn}_{86}$, decays almost immediately (3.82 days), but cannot decay before being formed. Therefore, it is possible to observe the decay of a $^{222}\text{Rn}_{86}$ nucleus, when one of its parents $^{226}\text{Ra}_{88}$ happens to decay after many years of survival.

2.4 Sequential Decays

If a parent nucleus A decays to a daughter nucleus B that is also unstable, the equation governing growth and depletion of the two species A and B are:

$$\frac{dN_A}{dt} = -\lambda_A N_A, \quad (2.13)$$

$$\frac{dN_B}{dt} = -\lambda_B N_B + \lambda_A N_A, \quad (2.14)$$

where N_A and N_B are the numbers of nuclei of parent and daughter, λ_A and λ_B their respective decay constants.

The general solution to this equation for the nucleus B is

$$N_B(t) = N_B(0)e^{-\lambda_B t} + \frac{\lambda_A}{\lambda_B - \lambda_A} N_A(0)(e^{-\lambda_A t} - e^{-\lambda_B t}), \quad (2.15)$$

where $N_A(0)$ and $N_B(0)$ are the numbers of nuclei of parent and daughter at the initial time ($t = 0$). The first term of the right side of this equation represents the number of nuclei B surviving at the time t from the initial number $N_B(0)$, while the second term represents the number of nuclei B produced from the decay of nuclei A and not yet decayed at the time t .

If initially no nuclei B are present, i.e. $N_B(0) = 0$, Eq. (2.15) reduces to

$$N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} N_A(0)(e^{-\lambda_A t} - e^{-\lambda_B t}). \quad (2.16)$$

We note at once that $N_B(t)$ is zero both at $t = 0$ and $t = \infty$, when all nuclei of both A and B have decayed. Clearly, N_B will grow as it is fed by the decay of the parent nucleus, but later as the rate of supply decreases the growth of N_B will be halted and its own decay will become dominant and will set up a decrease in N_B . Then, at some intermediate time t_m the number of nuclei B, N_B , will pass through a maximum value. This is the time t_m for which $dN_B(t)/dt = 0$.

The derivative of Eq. (2.16) with respect to time is zero when

$$\lambda_A e^{-\lambda_A t_m} = \lambda_B e^{-\lambda_B t_m}, \quad (2.17)$$

from which it follows that

$$t_m = \frac{\ln(\lambda_A/\lambda_B)}{(\lambda_A - \lambda_B)}. \quad (2.18)$$

The activity of the nucleus B is $\lambda_B N_B$ (not dN_B/dt). From Eq. (2.16) it follows that

$$\lambda_B N_B = \frac{\lambda_B \lambda_A}{\lambda_B - \lambda_A} N_A(0) (e^{-\lambda_A t} - e^{-\lambda_B t}), \quad (2.19)$$

or, since the activity of A at the time t is $\lambda_A N_A = \lambda_A N_A(0) e^{-\lambda_A t}$,

$$\lambda_B N_B = \lambda_A N_A \frac{\lambda_B}{\lambda_B - \lambda_A} (1 - e^{-(\lambda_B - \lambda_A)t}). \quad (2.20)$$

It is interesting to consider three special cases.

- (a) **Daughter nucleus longer-lived than parent.** If $\tau_B > \tau_A$, i.e. $\lambda_B < \lambda_A$, then after a long time ($t \gg 1/\lambda_A$)

$$e^{-\lambda_A t} \ll e^{-\lambda_B t},$$

so Eq. (2.16) may be written

$$N_B = \frac{\lambda_A}{\lambda_A - \lambda_B} N_A(0) e^{-\lambda_B t}. \quad (2.21)$$

Therefore, the amount of nuclei B after a long time is determined only by its own half-life. The initial stock of short-lived nuclei $N_A(0)$ has, in effect, quickly become an initial stock of long-lived nuclei $N_B(0)$ with $N_B(0) \cong N_A(0)$, which decay exponentially like $e^{-\lambda_B t}$. The activity of the daughter nucleus eventually becomes effectively independent of the residual activity of the parent. This case is shown schematically in Fig. 2.3a.

- (b) **Daughter nucleus shorter-lived than parent.** If $\tau_B < \tau_A$, i.e. $\lambda_B > \lambda_A$, then after a long time ($t \gg 1/\lambda_B$)

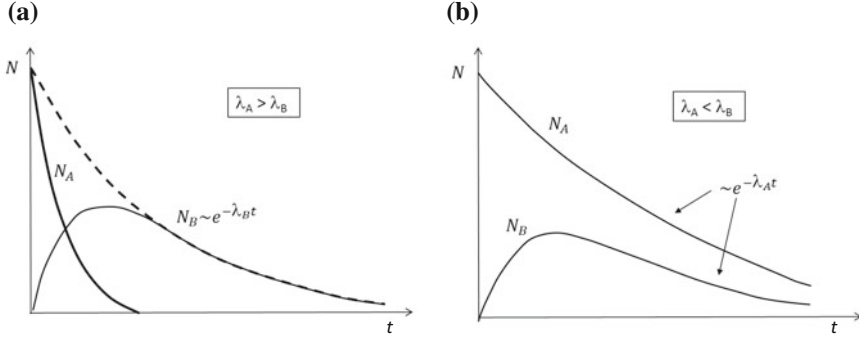


Fig. 2.3 **a** Decay of a radioactive daughter longer-lived than parent and **b** of a radioactive daughter shorter-lived than parent. In the latter case, at large t , the parent and daughter nuclei decay with the same exponential law, maintaining a constant ratio. The area under a curve in each figure (that is its integral from $t = 0$ to $t = \infty$) represents the total number of disintegrations, hence the total number of radioactive nuclei originally available. In each figure, the curves are not to scale as the areas under the two curves must be equal because the same nuclei are involved, only at different stages in their disintegration series

$$N_B = \frac{\lambda_A}{\lambda_B - \lambda_A} N_A(0) e^{-\lambda_A t}. \quad (2.22)$$

The amount of nuclei B after a long time is determined only by the half-life of the parent nucleus A, as shown in Fig. 2.3b.

From Eq. (2.22) it follows that, for large t :

$$\frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B - \lambda_A} = \frac{\tau_B}{\tau_A - \tau_B}, \quad (2.23)$$

and recalling that the activity of A and B are $\lambda_A N_A$ and $\lambda_B N_B$ this can be written

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{\lambda_B}{\lambda_B - \lambda_A} = \frac{\tau_A}{\tau_A - \tau_B}, \quad (2.24)$$

meaning that the ratio of the activities of daughter and parent is constant.

This case, where, for large t , the ratio of the activity of daughter to that of parent is greater than unity, is called *transient equilibrium*.

- (c) **Daughter nucleus much shorter-lived than parent.** When $\tau_B \ll \tau_A$, i.e. $\lambda_B \gg \lambda_A$, Eq. (2.20) takes on a particularly simple form

$$\lambda_B N_B(t) = \lambda_A N_A(1 - e^{-\lambda_B t}). \quad (2.25)$$

The activity of the daughter nucleus increases according to the simple exponential growth curve governed by its own decay constant, λ_B . In these cases, the equilibrium ratio of the activities becomes substantially unity,

$$\lambda_B N_B = \lambda_A N_A. \quad (2.26)$$

This result can be also found from Eq. (2.24) by letting $\lambda_A \rightarrow 0$ in the right-hand side. This condition is called *secular equilibrium*.

If we have a decay chain $A \rightarrow B \rightarrow C \rightarrow D \rightarrow \dots$, with $\tau_B, \tau_C, \tau_D, \dots \ll \tau_A$, i.e. $\lambda_B, \lambda_C, \lambda_D, \dots \gg \lambda_A$, we can repeat the same steps and write

$$\begin{aligned} \frac{dN_A}{dt} &= -\lambda_A N_A, \\ \frac{dN_C}{dt} &= -\lambda_C N_C + \lambda_B N_B = -\lambda_C N_C + \lambda_A N_A, \end{aligned}$$

because B is in secular equilibrium with A and we can apply (2.26). It is therefore easy to demonstrate that we also have

$$\lambda_C N_C = \lambda_D N_D = \dots = \lambda_A N_A,$$

so that all isotopes generated by a decay chain starting with a longer-lived isotope will reach a secular equilibrium where all activities are equal. This is what is found for instance in the decay chain of natural uranium, whose two isotopes 235 and 238 are in secular equilibrium with all 11 and 14 daughter nuclei, respectively.

2.5 Accumulation of Decay Products in a Series of Decays

Let us consider the series of decays



that involves n different nuclide-to-nuclide decays. For simplicity, we assume that at time $t = 0$ there are N_0 nuclei of A present and no nuclei of its series of decay products B, C, ..., M, N (that is, $N_B, N_C, \dots, N_M, N_N = 0$). Let the decay constants of A and its products be $\lambda_A, \lambda_B, \lambda_C, \dots, \lambda_M, \lambda_N$. Then at a time t , the number N_N of nuclei N present will be given by the integral of

$$\frac{dN_N}{dt} = N_M \lambda_M - N_N \lambda_N, \quad (2.27)$$

where N_M is evaluated from a series of equations similar to Eq. (2.27) for the amounts of the preceding products. This is entirely analogous to Eq. (2.14) for dN_B/dt but the solution requires more mathematical steps, since N_M is a more complicated function than N_B . The result of this integration is

$$N_N = N_0(h_A e^{-\lambda_A t} + h_B e^{-\lambda_B t} + h_C e^{-\lambda_C t} + \dots + h_M e^{-\lambda_M t} + h_N e^{-\lambda_N t}), \quad (2.28)$$

in which the coefficients are dimensionless functions of the decay constants and have the following systematic values

$$\begin{aligned} h_A &= \frac{\lambda_A}{\lambda_N - \lambda_A} \frac{\lambda_B}{\lambda_B - \lambda_A} \frac{\lambda_C}{\lambda_C - \lambda_A} \dots \frac{\lambda_M}{\lambda_M - \lambda_A}, \\ h_B &= \frac{\lambda_A}{\lambda_A - \lambda_B} \frac{\lambda_B}{\lambda_N - \lambda_B} \frac{\lambda_C}{\lambda_C - \lambda_B} \dots \frac{\lambda_M}{\lambda_M - \lambda_B}, \\ &\dots \\ h_M &= \frac{\lambda_A}{\lambda_A - \lambda_M} \frac{\lambda_B}{\lambda_B - \lambda_M} \frac{\lambda_C}{\lambda_C - \lambda_M} \dots \frac{\lambda_N}{\lambda_N - \lambda_M}, \\ h_N &= \frac{\lambda_A}{\lambda_A - \lambda_N} \frac{\lambda_B}{\lambda_B - \lambda_N} \frac{\lambda_C}{\lambda_C - \lambda_N} \dots \frac{\lambda_M}{\lambda_M - \lambda_N}. \end{aligned} \quad (2.29)$$

The initial condition, that $N_N = 0$ at $t = 0$, requires that the sum of the coefficients be zero, and these coefficients do satisfy the condition that

$$h_A + h_B + h_C + \dots + h_M + h_N = 0. \quad (2.30)$$

The above expressions are valid also if the final nucleus N is stable, in which case it is sufficient to put $\lambda_N = 0$, which immediately implies $h_N = 1$.

The full form of Eq. (2.28), in which the initial concentrations of all daughters nuclei at time $t = 0$ are different from zero (that is, $N_B, N_C, \dots, N_M, N_N \neq 0$), are called the *Bateman equations* and they appear whenever one wants to describe the full evolution of the nuclides in a decay series.

2.5.1 Approximate Method for Short Accumulation Times

If we consider a time t which is short compared with the mean-life of any member of a series of decays (that is, $t \ll 1/\lambda_i$ for any $i = A, B, C, \dots, N$) approximate solutions can be based on series expansions of the exponential in the general solutions (2.28). In this case, again assuming for simplicity that at $t = 0$ only the A nucleus be present with a number of nuclei N_0 , initially for nucleus B we can write

$$dN_B = N_0 \lambda_A dt,$$

($N_0\lambda_A$ being the activity of A). Therefore, by simple integration, one has

$$N_B = \int_0^t dN_B = \int_0^t N_0\lambda_A dt = N_0\lambda_A t, \quad (2.31)$$

which is equivalent to saying that the initial growth of B is linear with time.

In turn, nucleus C comes from the decay of B, so that we can write

$$dN_C = N_B\lambda_B dt = (N_0\lambda_A t)\lambda_B dt, \quad (2.32)$$

which, by simple integration, gives

$$N_C = \int_0^t dN_C = \int_0^t (N_0\lambda_A t)\lambda_B dt = N_0\lambda_A\lambda_B \frac{t^2}{2}. \quad (2.33)$$

We can arrive to the same result by taking the average activity of B in the time from 0 to t , $\langle N_B\lambda_B \rangle t = (N_B\lambda_B/2)t = (N_0\lambda_A\lambda_B t/2)$ and multiplying by t , $N_C = (N_0\lambda_A\lambda_B t/2)t = (N_0\lambda_A\lambda_B t^2/2)$. In a similar way we can derive the growth of D, which is given by

$$dN_D = N_C\lambda_C dt = \left(N_0\lambda_A\lambda_B \frac{t^2}{2} \right) \lambda_C dt, \quad (2.34)$$

which, by performing one further integration, gives

$$N_D = \int_0^t dN_D = \int_0^t \left(N_0\lambda_A\lambda_B \frac{t^2}{2} \right) \lambda_C dt = N_0\lambda_A\lambda_B\lambda_C \frac{t^3}{6}, \quad (2.35)$$

and so on and so forth. Note that for each nucleus, under these approximations its growth depends only on the decay constants of its precursors, i.e. the growth of B depends only on λ_A , that of C depends only on λ_A and λ_B , that of D depends only on λ_A , λ_B and λ_C , and so on and so forth.

Problem 2.6: Growth of the radioactive daughter. A radioactive source contains only a pure isotope $^{210}\text{Bi}_{83}$. This decays by β^- emission (with half-life $\tau_{1/2A} = 5.0$ d) to $^{210}\text{Po}_{84}$, which in turn decays by α -particle emission (with half-life $\tau_{1/2B} = 138.4$ d) to $^{206}\text{Pb}_{82}$. Calculate at which time T the rate of α -particle emission will reach a maximum.

Solution: The rate of α -decays evolves according to Eq. (2.16). Calling N_A and N_B the numbers of nuclei Bi and Po, and $\lambda_A = \ln 2/\tau_{1/2A}$ and $\lambda_B = \ln 2/\tau_{1/2B}$ the relevant decay constants, the maximum value of α -particle rate is obtained by equating to zero the derivative of that equation.

$$\begin{aligned}\frac{dN_B}{dt} &= \frac{\lambda_A}{\lambda_B - \lambda_A} N_A(0) \frac{d}{dt} (e^{-\lambda_A t} - e^{-\lambda_B t}) \\ &= \frac{\lambda_A}{\lambda_B - \lambda_A} N_A(0) (-\lambda_A e^{-\lambda_A t} + \lambda_B e^{-\lambda_B t}) = 0.\end{aligned}$$

This gives

$$-\lambda_A e^{-\lambda_A T} + \lambda_B e^{-\lambda_B T} = 0,$$

from which it follows

$$\frac{\lambda_A}{\lambda_B} = e^{-(\lambda_B - \lambda_A)T},$$

which gives

$$\begin{aligned}T &= \frac{\ln(\lambda_A/\lambda_B)}{\lambda_A - \lambda_B} = \frac{1}{\ln 2} \frac{\tau_{1/2A} \tau_{1/2B}}{\tau_{1/2B} - \tau_{1/2A}} \ln \frac{\tau_{1/2B}}{\tau_{1/2A}} = \frac{1}{0.693} \frac{5.0 \times 138.4}{138.4 - 5} \ln \frac{138.4}{5} \\ &= 24.9 \text{ d}.\end{aligned}$$

Problem 2.7: Accumulation of a stable end-product in a decay series.

Consider the series disintegration $A \rightarrow B \rightarrow C \rightarrow D$ (stable), with decay constants λ_A , λ_B , and λ_C . Show that, if A is a very long-lived source, the number N_D of nuclei D at a time t is given by

$$N_D(t) = N_A(0) \left[1 - e^{-\lambda_A t} + \frac{\lambda_A \lambda_C}{\lambda_B (\lambda_C - \lambda_B)} e^{-\lambda_B t} + \frac{\lambda_A \lambda_B}{\lambda_C (\lambda_B - \lambda_C)} e^{-\lambda_C t} \right].$$

Solution: Using Eqs. (2.28) and (2.29) one has

$$N_D(t) = N_A(0) (h_A e^{-\lambda_A t} + h_B e^{-\lambda_B t} + h_C e^{-\lambda_C t} + h_D e^{-\lambda_D t}),$$

with

$$\begin{aligned}h_A &= \frac{\lambda_A}{\lambda_D - \lambda_A} \frac{\lambda_B}{\lambda_B - \lambda_A} \frac{\lambda_C}{\lambda_C - \lambda_A}, \\ h_B &= \frac{\lambda_A}{\lambda_A - \lambda_B} \frac{\lambda_B}{\lambda_D - \lambda_B} \frac{\lambda_C}{\lambda_C - \lambda_B}, \\ h_C &= \frac{\lambda_A}{\lambda_A - \lambda_C} \frac{\lambda_B}{\lambda_B - \lambda_C} \frac{\lambda_C}{\lambda_D - \lambda_C}, \\ h_D &= \frac{\lambda_A}{\lambda_A - \lambda_D} \frac{\lambda_B}{\lambda_B - \lambda_D} \frac{\lambda_C}{\lambda_C - \lambda_D}.\end{aligned}$$

Observing that $\lambda_D = 0$ these become

$$h_A = -\frac{\lambda_B}{\lambda_B - \lambda_A} \frac{\lambda_C}{\lambda_C - \lambda_A}; \quad h_B = -\frac{\lambda_A}{\lambda_A - \lambda_B} \frac{\lambda_C}{\lambda_C - \lambda_B};$$

$$h_C = -\frac{\lambda_A}{\lambda_A - \lambda_C} \frac{\lambda_B}{\lambda_B - \lambda_C}; \quad \text{and } h_D = 1.$$

Assuming $\lambda_A \ll \lambda_B$ and $\lambda_A \ll \lambda_C$, one can approximate the above equations

$$h_A = -1; \quad h_B = \frac{\lambda_A}{\lambda_B} \frac{\lambda_C}{\lambda_C - \lambda_B}; \quad h_C = \frac{\lambda_A}{\lambda_C} \frac{\lambda_B}{\lambda_B - \lambda_C}; \quad \text{and } h_D = 1.$$

So that one has

$$N_D(t) = N_A(0) \left[1 - e^{-\lambda_A t} + \frac{\lambda_A \lambda_C}{\lambda_B (\lambda_C - \lambda_B)} e^{-\lambda_B t} + \frac{\lambda_A \lambda_B}{\lambda_C (\lambda_B - \lambda_C)} e^{-\lambda_C t} \right].$$

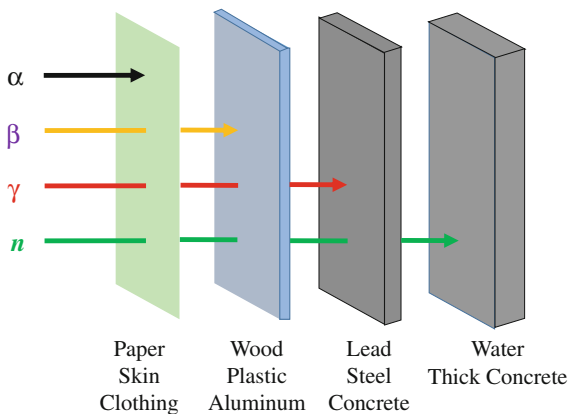
2.6 Penetrating Power of Nuclear Radiation

Radiation from radioactive nuclei produces specific physical effects when it crosses materials. In particular, the interaction between radiation and matter can result in damages to the crossed materials. For instance, electronic circuits can be easily damaged by certain types of radiation. This is also true for the cells of living beings, in which case a health hazard is implied.

The knowledge of the activity of a radioactive material is not sufficient to evaluate entirely the effects produced by the specific radiation. This stems from the fact that the interactions of nuclear radiation with matter are different and depend on the type and energy of the radiation, as schematically shown in Fig. 2.4. By knowing the ability of the different types of radiation to penetrate matter one can gain an understanding on how to design an appropriate shielding in order to protect people, the environment and sensitive equipment.

Alpha particles can travel only a few centimetres in the air and can be stopped by a sheet of paper or a layer of skin. Beta particles can travel metres in the air and several millimetres in the human body; they can be stopped by a sheet of metal of a few millimetre thickness. Gamma rays, X-rays, and neutrons are more penetrating; thick barriers of dense metal like lead or thick walls of concrete best stop gammas, while neutrons are best shielded by thick layers of concrete or by materials rich in hydrogen atoms, such as water or paraffin. Neutron absorbers like boron or cadmium are also used for their ability to capture neutrons. When shielding neutrons, a special consideration must be given to secondary gamma rays that are produced when neutrons scatter off nuclei or are captured by them.

Fig. 2.4 The penetrating power of different radiations



Such a different behaviour depends on the various ways of interaction with matter of the various types of radiation.

Alpha and beta particles, which are charged particles, when passing through matter, collide with atoms and molecules encountered along their path. In these collisions, they can either excite atomic levels of atoms and molecules or knock electrons out of them (that is, they can ionise them). In both processes, alpha and beta particles transfer a fraction of their energy to the atomic electrons. Because of this energy loss, they slow down and eventually come to rest. At the end of their path, when they stop, their initial energy has been entirely transferred to the crossed material. Consequently, α - and β -particles have a characteristic average traveling distance (so-called *range*) beyond which they are effectively fully absorbed. This range depends on the energy of the ionising particles and the nature of the crossed material. Typically the energy loss per unit length-of-path of alpha-particles in crossing matter is very high and that of beta-particles is lower than alphas but still significant; this is why relatively thin layers of light material (air, paper, wood, plastic, human skin) are highly effective in stopping them and can therefore perform a shielding function.

Gamma rays and X-rays are both forms of electromagnetic radiation, like light and radio waves, but of much higher frequency (and hence shorter wavelength). They can also be thought of as beams of quantum particles, *photons*, each carrying a discrete quantity of energy but having no mass. Gamma and X-rays have the same basic properties, but differ in their origin. X-rays are emitted by atomic electrons, that is, from processes outside the nucleus, while γ -rays originate inside the nucleus. X-rays are also generally lower in energy and, therefore, less penetrating than γ -rays. Gamma rays and X-rays also ionise the material they pass through, but the way they lose energy is quite different from alpha particles. A photon can cross significant thicknesses of material retaining its entire energy, until it interacts with an atom and knocks out an electron, transferring to it a good fraction of its energy, or being completely absorbed in the process. The extracted electron, in turn, transfers the received energy to the material, with the same mechanism of β -rays.

So passing through matter, gamma and X-rays eventually disappear by being absorbed by the atoms, instead of losing energy by slowing down (as they are forms of electromagnetic radiation, there is only one possible speed for them: the speed of light). Consequently, gamma and X-rays do not have a specific penetration depth, rather their intensity decreases exponentially toward zero as the depth of the crossed medium increases. Instead of the average penetration depth in a medium, for photons one speaks of *attenuation length*, which is the distance over which their intensity is reduced by a factor $e = 2.71828$.

Neutrons are particles which are also very penetrating. On Earth, they can be produced in the spontaneous fission of nuclei, in the induced fission of nuclei in a nuclear reactor, or by energetic cosmic rays that break up nuclei into their proton and neutron constituents. They have null electric charge and, therefore, they interact weakly with matter and do not directly ionise it. Consequently, they are very penetrating, and cannot be slowed down easily.

Neutrons lose speed in collisions with atomic nuclei of the material they pass through. At low neutron energies collisions are predominantly *elastic*, i.e. are processes in which the total kinetic energy of the interacting bodies is conserved. In Sect. 3.7 it will be shown by simple kinematics considerations that the lighter the atomic nucleus, the higher the energy lost in the collision. This results in a progressive loss of their kinetic energy. To stop neutrons one therefore uses thick layers of concrete or materials that contain hydrogen (like water or paraffin), which absorb their energy more effectively. If there is sufficient material for the neutrons to undergo a sufficient number of collisions, this energy loss goes on until neutrons are “thermalized”, i.e. reach equilibrium with the thermal energy of atoms in the medium they move. In this condition, they have a mean energy $\sim kT$ (where $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$ is the Boltzmann’s constant) given by the temperature T of the medium; at ordinary temperature, $T = 300^\circ\text{C}$, $kT \approx 0.025 \text{ eV}$.

If the incident neutron energy is high enough, an *inelastic* collision can occur. In this case, the recoiling nuclei can be left in an excited state, whose energy is later released by emitting γ -rays, or can be broken up with emission of other particles (photons, neutrons, protons and α -particles). Neutrons can also be captured by certain nuclei, a process that is typically accompanied by the emission of γ -rays, and the isotope formed in the capture process can be unstable and undergo β -decay. In all the above collision processes, emitted gamma rays and charged particles can ionise matter, thereby transferring energy to it.

Like gamma radiation, the range of neutrons is not well defined. The distance they travel depends on the probability of interaction in a particular material. Also for them, one speaks of attenuation length.

In Table 2.3 are given the penetration depth of various types of ionising radiation when entering the human tissue, in increasing order of penetration and for two different kinetic energies of 1 and 10 MeV. For α and β , the penetration depth refers to their range. For neutrons and γ -rays, it is the distance travelled until the number of incoming particles is halved.

For a given specific energy, alpha particles, which have a much greater mass than beta particles (about eight thousand times larger), travel much slower and have

Table 2.3 The penetration depth of various types of ionising radiation in human tissue for two given energies

Energy (MeV)	Penetration depth (mm)			
	α	β	n	γ
1	0.005	5	25	100
10	0.2	50	100	300

more time to interact with the atoms of the material, so they lose energy much more quickly. On top of that, alpha particles have two units of electric charge and therefore the Coulomb force between them and the atoms is twice the one felt by beta particles. As a consequence of both these phenomena, alpha particles are stopped by small material thicknesses, featuring the smallest penetration depth. Neutrons and gamma rays are not hindered by charge and so their penetrating power in our body is large.

By using the same ‘interaction-time argument’, we understand that higher energy means deeper penetration, hence the difference between the two rows in Table 2.3.

2.7 Dosimetry

The main effect of exposure to radiation of any material—such as human tissue—is the deposition of energy in the material. So the unit used to measure radiation exposure is based on the amount of radiation energy absorbed.

Radiation exposure, usually called *absorbed dose*, is measured in *gray* (Gy), so-called after the British physicist Louis H. Gray (Richmond upon Thames, England, 1905—Hillingdon, England, 1965), a pioneer in the field of X-ray and radium radiation measurement, and their effects on living tissue. One gray is defined as the deposition of 1 J of energy in 1 kg of material; 1 Gy = 1 J/kg.

An old and still used, mainly in the USA, unit of absorbed dose is the *rad*, abbreviation for *radiation absorbed dose*. One rad is equivalent to one hundredth of a gray (1 rad = 0.01 Gy).

To assess the effects of radiation in biological tissues it is not sufficient to know the absorbed dose; one must also take into account the quality of the radiation (some types of radiation are more harmful than others) and the fact that certain tissues are more sensitive than others. For an equal absorbed dose, α -particles are much more potentially harmful than β - and γ -rays because they deposit their energy within shorter distances (see Table 2.3), i.e. into a smaller volume; then the resulting cell damage may be more difficult to repair.

Furthermore, different tissues have different sensitivities to radiation damage. In general, the radiation sensitivity of a tissue is proportional to the rate of proliferation of its cells, and inversely proportional to the degree of cell differentiation. For example, for an equal dose from a given type of radiation, the damage caused to the lungs and reproductive organs is greater than that produced to the bones and teeth.

This also means that a developing embryo is most sensitive to radiation during the early stages of differentiation.

The potential radiation damage also depends on the mode of introduction into the body; for example, α radiation is very dangerous if the emitting radionuclide is inhaled. Instead, for external radiation, the danger is much less, because α -particles can be easily shielded and, indeed, the thickness of the skin is sufficient to stop them.

To account for the quality of radiation, different types of radiation are given different weighting factors, that are used to relate the energy they deposit to the biological significance of the damage they cause; the higher the factor, the greater the damage.

To obtain a quantity that expresses, on a common scale for all ionising radiation, the biological damage to the exposed tissue the *equivalent dose* is used, which is obtained by multiplying the absorbed dose by a dimensionless, weighting factor, w_R , of the radiation considered. Table 2.4 reports the weighting factors w_R of the different types of radiation.

The equivalent dose is measured in *sievert* (symbol Sv), named after the Swedish medical physicist Rolf M. Sievert (Stockholm, Sweden, 1986—Stockholm 1966), renowned for work on radiation dose measurements and research on the biological effects of radiation.

The sievert is defined as the absorbed dose from any radiation that produces the same biological damage as 1 gray of X-rays. Therefore, 1 Sv, unlike 1 Gy, produces the same biological effects regardless of the type of radiation considered. For example, Table 2.4 shows that 1 Gy of beta or gamma radiation has 1 Sv of biological effect; 1 Gy of alpha radiation has 20 Sv effect; 1 Gy of 10–100 keV neutron radiation will cause 10 Sv of biological effect.

Moreover, to account for different sensitivities to radiation damage of the different tissues, one uses the *effective dose*, which is obtained by multiplying the equivalent dose for a dimensionless weighting factor, w_T , of the relevant tissue. Also the effective dose is measured in sievert. Table 2.5 shows the weighting factors w_T of certain tissues and organs.

Table 2.4 Radiation weighting factors w_R of various radiation types

Radiation type and energy range	w_R
X- and γ -rays of any energy	1
Electrons and positrons of any energy	1
Protons of energy >2 MeV	5
α particles, fission fragments, heavy nuclei	20
Neutrons of energy <10 keV	5
10–100 keV	10
100–2000 keV	20
2–20 MeV	10
>20 MeV	5

Table 2.5 Weighting factors of some tissues and organs

Tissue/organ	w_T
Gonads	0.20
Red bone marrow, colon, stomach, lung	0.12
Bladder, breast, liver, oesophagus, thyroid	0.05
Skin, bone surfaces	0.01
Other	0.05

A sievert is a fairly large dose of radiation, so the millisievert and microsievert (abbreviated as mSv and μ Sv) are often used for the average doses commonly encountered.

An older unit of equivalent dose and effective dose is the *rem* (Roentgen³ Equivalent in Man), symbol rem. One rem is equal to one hundredth of a Sievert (1 rem = 0.01 Sv, = 10 mSv, or equivalently, 1 Sv = 100 rem).

Both short- and long-lived radioactive materials can present serious hazards, but for somewhat different reasons. Radioisotopes with short half-lives are dangerous for the straightforward reason that they have an extremely high number of decays per second per mass unit. Therefore, they can expose people to very high doses in a short time. On the other hand, the shorter the lifetime the faster the radioactive substance decays to natural levels of radioactivity (see Problem 2.9 and Sect. 2.8) and, therefore, for short-lived radioactive materials a protective shielding is required for a shorter interval of time. Isotopes with mid-to-long half-lives do not expose people as heavily (they have a relatively small number of decays per second per mass unit), but they can keep an entire area significantly radioactive for a very long time, e.g. hundreds or thousands or even tens of thousands of years. That is the main reason why disposing of reactor wastes, which often contain such isotopes, is a non-trivial challenge. At the extreme end are isotopes that are so long-lived that their hazard levels are close to zero. Uranium-238, the isotope left after the fissile uranium-235 isotope is removed, falls into this category with its 4.5 billion years half-life.

Problem 2.8: Residual activity. A hospital has bought a specimen of $^{60}\text{Co}_{27}$ for medical irradiation treatments. Calculate the residual activity of the specimen after three half-lives (the half-life of ^{60}Co is about 1925 days).

Solution: According to the definition of half-life, the residual activity of the specimen after $3\tau_{1/2}$ amounts to $1/2^3 = 1/8$ of the initial value.

³The roentgen was the most commonly used unit of radiation exposure in the 1930s. It was named after German physicist Wilhelm Roentgen (Remscheid, Germany, 1845—Munich, Germany, 1923) the early pioneer of medical X-rays. This unit is obsolete and no longer clearly defined (though as a rule of thumb: 1 roentgen is approximately 10 mSv). The roentgen is essentially a measure of how many ion pairs are formed in a given volume of air when it is exposed to radiation. Therefore, it is not a measure of energy absorbed, or dose.

When the activity of a medical source falls down that much, it may become ineffective for treatment but would still be hazardous in terms of potential exposure and should be disposed of. Disposal is usually performed by special transportation to dedicated repositories.

Problem 2.9: Potential hazard of radioactive materials. As regards the hazard, the most potentially harmful radioactive materials are often neither those with long half-life nor those with short half-life, but those in between. This is because our body would be exposed only to the decays that occur in the course of our life and radionuclides with short half-life decay sufficiently rapidly on a human scale, while those with long half-life produce only very few decays per second.

Evaluate the potential hazard of three beta emitters A, B and C with half-life 3 a, 30 a and 300 a, respectively, for an integrated total exposure of 3 years. Suppose, for simplicity, that the three radioactive substances A, B and C have the same initial number N_0 of radionuclides.

Solution: To estimate the potential danger of the three substances one has to evaluate how many nuclei of the three radioisotopes have decayed after 3 years. Then, using Eq. (2.9), with clear meaning of symbols, one gets that in 3 years

- half of the initial nuclei of the radionuclide A (with half-life 3 a) will have decayed

$$\frac{N_0 - N_A}{N_0} = 1 - e^{-3 \times 0.693/3} = 1 - e^{-0.693} = 1 - \frac{1}{2} = 0.5, \text{ by definition.}$$

- less than 1/10 of the initial nuclei of the radionuclide B (with half-life 30 a) will have decayed

$$\frac{N_0 - N_B}{N_0} = 1 - e^{-3 \times 0.693/30} = 1 - e^{-0.0693} = 1 - 0.933 = 0.067,$$

but the material will remain radioactive for a time interval ten times longer than the substance A.

- less than 1/100 of the initial nuclei of the radionuclides C (with half-life 300 a) will have decayed

$$\frac{N_0 - N_C}{N_0} = 1 - e^{-3 \times 0.693/300} = 1 - e^{-0.00693} = 1 - 0.993 = 0.007;$$

but the material will remain radioactive for a time interval hundred and ten times longer than the substance A and B, respectively. If not suitably shielded, the substance A with the shortest lifetime would be most harmful. On the contrary, if the substance A is well-shielded based on its high activity, but B and C are not, based on their longer lifetime, substance B will pose a higher

hazard as almost 1/10 of its nuclei will decay during a 3-year exposure, as opposed to substance C for which less than 1 % of its nuclei will decay during the same time span. On the other hand, substance C will remain radioactive for much longer time, which is also a concern.

2.8 Natural and Artificial Radioactivity

There are many unstable isotopes in nature and their radioactivity is called *natural radioactivity*. In the laboratory, we can produce many other unstable isotopes as a result of various nuclear reactions; we say then that these isotopes are produced artificially and one speaks in these cases of *artificial radioactivity*.

It is worth to point out that from the physical point of view natural and artificial radiation are perfectly equal: the properties of a given isotope are independent of the method by which it was obtained. Therefore, artificial radioactivity is not more dangerous or beneficial than natural radiation.

2.8.1 Natural Radioactivity

Radiation is always a component of the environment and, therefore, a large part of the radiation dose we receive is of natural origin and inevitable. Consequently, life has evolved in, and adapted to an environment that has significant levels of ionising radiation.

For example, in our daily life we are exposed to cosmic rays—mainly originating outside the Solar system and consisting primarily of very high energy protons and atomic nuclei—raining down on Earth. They are thought to be generated through various processes, including the birth and death of stars, and maybe those taking place in the so-called active galactic nuclei. The energies of the primary cosmic rays range from around 1 GeV to as much as 10^{11} GeV. The rate at which these particles arrive at the top of the atmosphere falls off with increasing energy, from about 10,000 per square metre per second at 1 GeV to less than one per square kilometre per century for the highest energy particles. When they arrive at the Earth, cosmic rays collide with the nuclei of atoms in the upper layers of the atmosphere, creating showers of secondary particles, mainly *pions*. The charged pions can swiftly decay to another type of subatomic particle, called *muon*. Unlike pions, these do not interact strongly with matter, and can travel through the atmosphere and penetrate below ground. The rate of muons arriving at the surface of the Earth is such that about one per second passes through a volume the size of a person's head. The Earth's atmosphere and magnetic field act as shield against cosmic radiation, reducing the amount that reaches the Earth's surface. With that in mind, it

is easy to understand that the annual dose we get from cosmic radiation depends on what altitude we live at.

Naturally occurring radioactive materials (the so-called *NORM*) are present everywhere: in the soil we walk on and in the buildings we live in, in the food we eat and the water we drink. Radioactive gases are in the air we breathe and even our body is made weakly radioactive by the presence of natural radioactive substances. There is no way on Earth one can get away from natural radiation.

Levels of natural radiation, the so-called *background radiation*, as it is usually called, are not constant everywhere; they vary greatly from one location to another, and at different heights above sea level. For example, people resident in Colorado (USA) are exposed to more natural radiation than residents of the east or west coast of the USA, because Colorado has more cosmic radiation, being located at a higher altitude, and more terrestrial radiation from its soils enriched in naturally occurring uranium. Similarly, people standing in St. Peter's Square in Rome, Italy, are exposed to more natural radiation than people staying in other Roman squares, because of the natural thorium content in the porphyry rocks (the famous “cobblestones”), with which St. Peter's Square is paved.

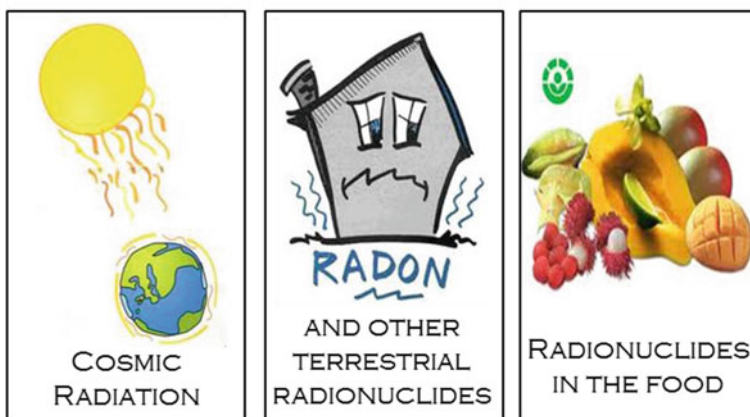
2.8.2 Artificial Radioactivity

The humankind is not only exposed to natural radiation throughout life, but also to various new sources of radiation created in human activities. They comprise radioactive nuclei produced in the various stages of electricity generation by nuclear fission plants (largely from the reprocessing of the fuel discharged from nuclear plants—so-called *spent fuel*—and to a lesser extent from nuclear fuel manufacturing and energy production, see Chap. 6), or in nuclear weapon tests carried out by a few countries in the 1940s and 1950s, or by irradiation of stable nuclei. The production of the latter is linked to their extended use in medicine, for diagnosis and treatment, and numerous other industrial applications. X-ray generators and particle accelerators used in diagnostic and therapeutic medicine, industry and scientific research are another source of artificial radiation.

Figure 2.5 is a cartoon-view summary of the above.

From all the above it is clear that the amount of radiation we receive depends to some extent also on our way of life, our diet, occupation and use of medical services. For example, dentists administering X-rays to their patients, or miners working underground in certain mines, will receive extra radiation. Airline crews, who spend more time than most people at high altitudes, also receive extra radiation from the effects of cosmic rays. Astronauts in space are subject to a similar exposure. Moreover, the use of X-rays and other medical imaging techniques such as computerised tomography (CT) and positron emission tomography (PET) involve significant exposure to ionising radiation. However, the risks involved in these practices are considered to be outweighed by the benefits to the patient. In some countries, the use of such techniques has grown rapidly in recent years (see Fig. 2.6).

NATURAL RADIOACTIVITY



ARTIFICIAL RADIOACTIVITY

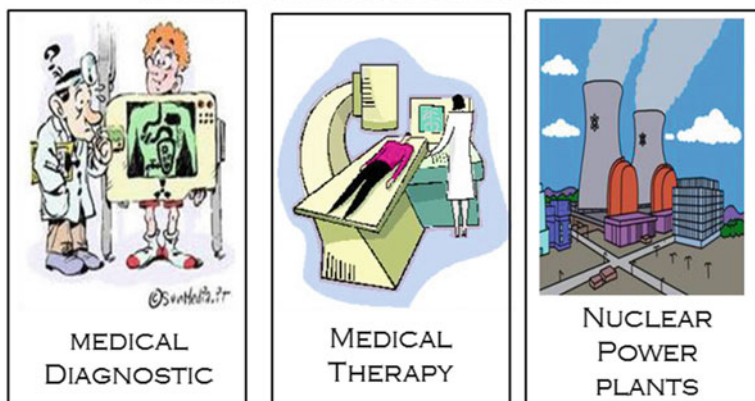
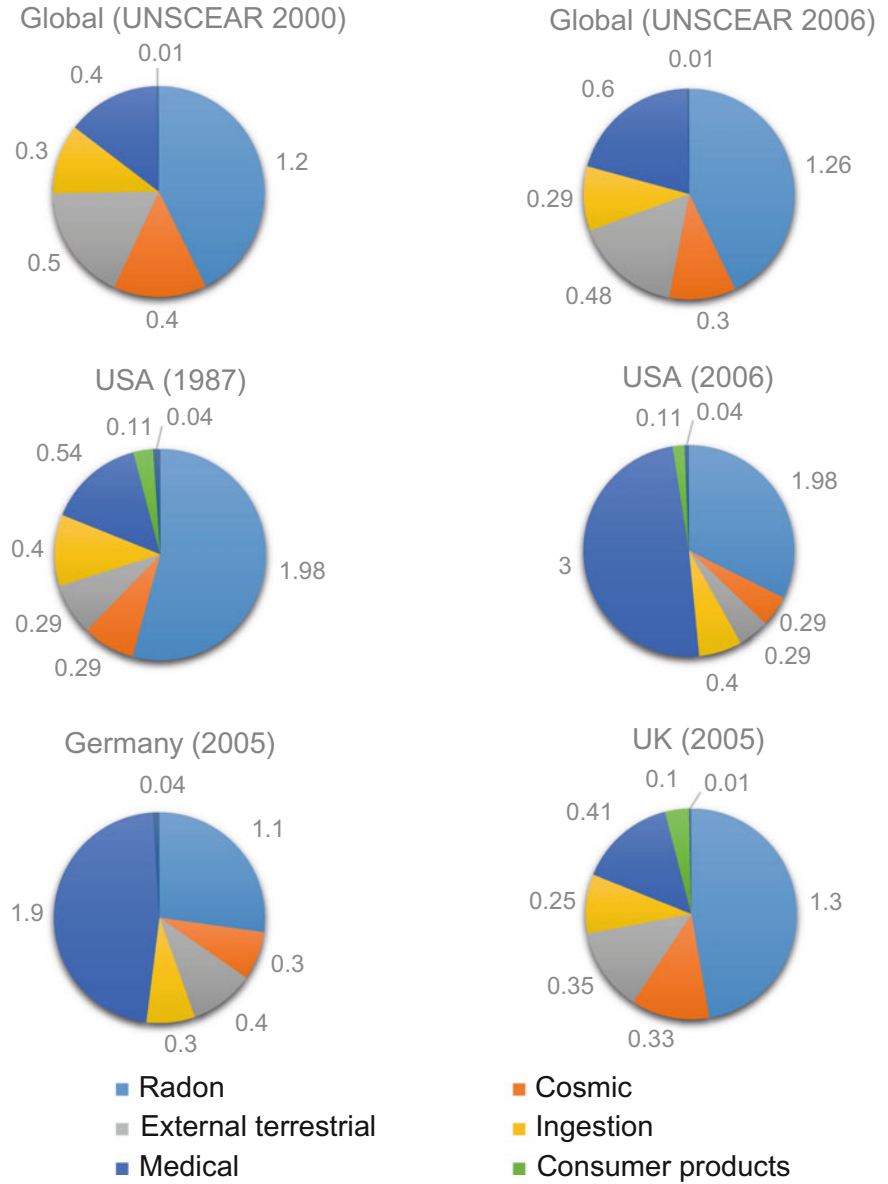


Fig. 2.5 The humankind has always been subject to natural radiation that is all around us. It comes from the outer space (cosmic radiation), the ground (terrestrial radiation) and even from food. Human beings are also exposed to various artificial sources of radiation. X-ray and other types of radiation used in diagnostic and therapeutic medicine, the radioactive fallout caused by experiments with nuclear weapons or radioactive materials generated during the production of nuclear energy are some examples of these artificial radiation sources

2.9 Average Annual Radiation Dose

As exposure to natural radiation sources is unavoidable, it is interesting to know the average annual effective dose that each individual receives from natural sources of radiation. This is an interesting reference level for every consideration about the dose absorption from man-made radiation sources.



◀ **Fig. 2.6** Estimated contribution in millisievert to public annual exposure from different radiation sources for USA, Germany and United Kingdom and UNSCEAR estimates of the worldwide average annual exposures. For most individuals in the world about 80–85 % of the dose is from natural radiation. Different distributions can be expected for other countries, as all countries considered here have a high level of development. Obtained with data from Ref. [4]

The United Nations Scientific Committee on the Effects of Atomic Radiation (UNSCEAR)⁴ has gathered information since 1955 on the typical levels and most important sources of human radiation exposure, and produces a report every few years summarizing the average exposures from all sources. Only occasionally doses to members of the public are directly measured. Usually these doses are assessed on the basis of environmental or effluent⁵ monitoring data, using models to simulate environmental exposure scenarios. Moreover, data have different sources of uncertainties reflecting the different way the information is provided by different countries.

The pie charts of Fig. 2.6 show the estimated contribution to public exposure from different radiation sources for different countries, and the UNSCEAR estimates of the worldwide average exposures. Although there is a distinction between natural and man-made radiation, they both affect people in the same way.

Levels of natural background typically range from about 1.5 to 3.5 mSv/a, but are more than 50 mSv/a in several places in Iran, India and Europe and up to 260 mSv/a at Ramsar in Iran, where consequently lifetime doses from natural radiation range up to several thousand millisievert. The highest known level of background radiation affecting a substantial population is in Kerala and Madras States in India where some 140,000 people receive doses which average over 15 mSv per year from gamma radiation, in addition to a similar dose from radon. Comparable levels occur in Brazil and Sudan, with average exposures up to about 40 mSv/a to many people. However, there is no evidence of increased cancer cases or other health problems arising from these high natural levels.

Figure 2.6 shows that the 2006 world natural background radiation exposure is an average of 2.33 mSv/a, while values for some individual countries are: USA 2.96 mSv/a; Germany 2.10 mSv/a, United Kingdom 2.23 mSv/a.

For most individuals, exposure to natural background radiation is the most significant part of their exposure to radiation (79 % for the 2006 world average). Radon⁶ is usually the largest natural source of radiation contributing to the exposure of members of the public, sometimes accounting for half the total exposure from all sources.

⁴The UNSCEAR was established by the General Assembly of the United Nations in 1955 with the mandate to assess and report levels and effects of exposure to ionising radiation. Governments and organizations throughout the world rely on the Committee's estimates as the scientific basis for evaluating radiation risk and for establishing protective measures.

⁵By effluent here it is meant any gaseous or liquid emission from nuclear installations.

⁶Radon is a radioactive noble gas. Its most relevant isotope is ²²²Rn, which appears in the uranium decay chain and has a half-life of 3.8 days. Being a heavy gas, it tends to accumulate in underground spaces, such as the basement of houses.

Figure 2.6 also shows that artificial sources of radiation from medical, commercial, and industrial activities contribute about 0.61 mSv/a to the 2006 world annual radiation exposure. The relevant value for the USA is 3.15 mSv/a, Germany 1.94 mSv/a (year 2005), United Kingdom 0.52 mSv/a (2005). Medical exposure, which is by far the largest of these sources of exposure, has grown very rapidly over the last three decades in some industrialised countries. This is clearly shown by the figures for the USA, where the increase in medical uses in the period 1987–2006 resulted in an increase in total annual effective dose per person from 0.54 to 3.0 mSv/a, making medical exposure comparable with the exposure due to natural background radiation. Clearly, the 3.0 mSv/a average is misleading, because the majority of people have only X-rays during their lifetime, whereas a small percentage of people have CT (computed tomography) scans, cancer treatments with radioactive isotopes, angiograms, stent implants, etc. These people have exposures several times greater than 3.0 mSv/a during their treatment periods. At the contrary, the worldwide average increase in the medical exposure is much smaller, increasing only from 0.4 mSv/a in 2000 to 0.6 mSv/a in 2006.

Figure 2.7 gives an estimate of the average radiation dose deposited in the human body by some medical imaging techniques. As it is seen, the radiation dose from a pelvis computed tomography is about three to five times that of the annual naturally occurring background radiation dose; from a chest computed tomography is from twice to more than three times the annual background radiation dose; from a

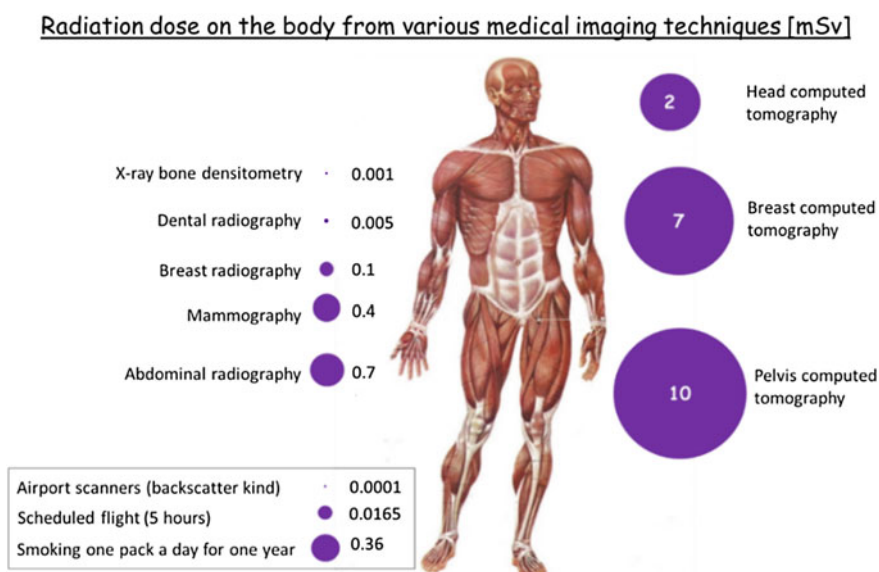


Fig. 2.7 Average radiation doses on the body, in mSv, from various medical imaging techniques. The area of the disks is proportional to the absorbed dose. Also shown are the doses from airport scanners, 5 h of scheduled flight, and smoking one pack a day for one year. Obtained with data from Ref. [5]

mammography is 0.4 mSv, and from a dental radiography is 0.005 mSv. The figure also gives the average absorbed dose from airport X-ray backscatter scanners, five hours of scheduled flight, and smoking one pack of cigarettes a day for one year.

2.10 Biological Effects of Radiation

When one is exposed to radiation, that is when one absorbs some radiation energy, it is said that one gets a *dose of radiation*. This does not make the person radioactive or cause him or her to become contaminated.⁷

Radiation that causes ionisation within human tissues can have very harmful effects. However, like with many other substances we introduce in our body (coffee, alcohol, smoke, prescription drugs, etc.), possible effects of radiation on human health can be evaluated correctly when one knows the intensity of the radiation received and the time-span over which exposure occurs.

For example, one can drink a shot of whiskey without suffering significant side effects. The situation may be different if, instead, one drinks a whole bottle of whiskey. In this case, it is also necessary to know whether it is drunk in a few minutes, in a day or over a longer period of time.

It must also be noted that for many natural substances, threshold values exist below which the substances are beneficial, or even essential, for human life, while they become dangerous above such values. This is, for example, the case of many metals (such as zinc, iron, selenium, and others), vitamins, and also of oxygen itself.

Radiation damage to living tissues results from the transfer of energy to atoms and molecules in the cellular structure, which is the fundamental component of biological tissues. It is composed for 80 % by water (H₂O) and for the remaining 20 % by complex biological structures. Every cell of the body contains molecules of deoxyribonucleic acid (DNA) which store genetic information and control cell growth, functions, development and reproduction. Ionising radiation causes atoms and molecules to become excited or ionised. These excitations and ionisations can cause harmful biological effects, depending on the part of the cell affected. If the ionisation takes place in the DNA, its functions can be altered with serious physical harm up to cancer or genetic modifications.

Our cells have effective repair mechanisms in place against some types of damages, including those induced by radiation (in fact, chromosome aberrations

⁷Radioactive contamination is the deposition of, or presence of radioactive substances on surfaces or within solids, liquids or gases (including the human body), where their presence is unintended or undesirable. While radiation exposure and radiation dose are referred to a situation where the person absorbed energy in the form of gamma rays, alpha particles, beta particles or neutrons, contamination occurs when some quantity of radioactive material is transferred on the person's body or on their clothes or shoes. This can happen for instance when the radioactive material is in a form easily transferrable from one place to another, i.e. when it is in the form of a fluid, dust, shavings, airborne particulate, etc.

occur constantly in our bodies). Therefore, injured or damaged cells can repair themselves, resulting in no residual damage. If unable to repair themselves, the cells may die or may perform their functions incorrectly. In particular, they may start to reproduce at an uncontrolled rate, which could lead to cancer.

Millions of body cells die every day, being replaced through normal biological processes. Consequently, cell death due to radiation exposure is an issue only if the number of cells involved becomes so significant as to destroy tissues or impair the functionality of some organs. Broadly speaking, for low doses (such as that received every day from background radiation) damage to the DNA of the cells is the dominant effect, while for high doses cell death prevails.

It is difficult to estimate cancer induction risks, because most of the radiation exposures that humans receive are very close to background levels. At low dose levels of less than 100 mSv spread out over long periods of time, the risk of radiation-induced cancers is so low that it is not readily distinguishable from normal levels of cancer occurrence. In addition, leukaemia or solid tumours induced by radiation are indistinguishable from those that result from other causes. On the contrary, high radiation doses (above 100 mSv in a short period of time, also called *acute doses*) can kill so many cells that tissues and organs are damaged immediately, in what is called *Acute Radiation Syndrome*. The higher the radiation dose, the sooner the effects of radiation will appear, and the higher the probability of death.

A linear, no-threshold (LNT) dose response relationship is used to describe the relationship between radiation dose and the probability for occurrence of cancer. This dose-response hypothesis assumes that there is no threshold, that is to say there is no exposure level below which the risk is zero. It also assumes that any increase in dose, no matter how small, results in an incremental increase in risk: if the exposure doubles, the risk also doubles.

Some scientists strongly dispute the no-threshold assumption. Because life has successfully evolved in the presence of significant levels of natural radiation, they suggest a threshold hypothesis according to which very small exposures are harmless. Other scientists even assert that low levels of radiation are beneficial to health (this idea is known as *hormesis*) [6, 7]. Available data are not clearly enough in favour of any model.

Although generally considered conservative, the LNT model is used worldwide for radiation protection regulations. Using a LNT risk model, the US National Academy of Sciences Committee on the Biological Effects of Ionising Radiation (the BEIR Committee)⁸ has estimated that a dose of 0.1 mSv creates an additional risk of death from cancer of approximately one in a million [8].

Clearly, acceptance of a risk is a highly personal matter that requires a certain degree of information to allow for aware decisions. Table 2.6 compares the

⁸The National Academy of Sciences (NAS) is a private, non-profit organization of distinguished scholars of the USA. Established in 1863, the NAS is charged with providing independent, objective advice to the nation on matters related to science and technology. In particular, the NAS has been periodically tasked to evaluate the overall state of our knowledge about radiation risk since the first BEIR Committee, which issued its report in July of 1956.

Table 2.6 List of some activities calculated to have a one-in-a-million chance of causing death [6]

Activity	Risk
Smoking 1.4 cigarettes	Lung cancer
Eating 40 tablespoons of peanut butter	Liver cancer
Eating charcoal broiled steaks	Cancer
Spending two days in New York City	Air pollution
Driving 65 km in a car	Accident
Flying 4000 km in a jet	Accident
Canoeing for 6 min	Accident
Receiving a dose of 0.1 mSv of radiation	Cancer

potential risk of radiation to other common activities that also create a risk of a one-in-a-million chance of death [9].

It is therefore clear how the risk associated to a very low radiation dose (0.1 mSv) is comparable to other activities in everyday life, although such a radiation dose is already about 10 times more than what we receive in a day from natural background.

2.11 Applications of Ionising Radiation in Medicine, Research and Industry

Today, ionising radiation is widely used in medicine, research and industry, as well as for generating electricity. In addition, it has useful applications in agriculture, archaeology, space exploration, law enforcement, geology (including mining), environment, conservation and preservation of objects of cultural heritage significance and many others. These beneficial uses of radiation are often less known and publicized than its potential to do harm.

Below we list some of the many beneficial uses of radiation.

2.11.1 Medical Applications

Modern medicine benefits tremendously from the use of ionising radiations, both for diagnosis and therapy.

Diagnosis ranges from routine and advanced use of X-rays (for radiographs, radioscopy radiographs and computed tomography (CT)) to injection of radioisotopes—known as radiopharmaceuticals—for gamma imaging (positron emission tomography (PET), single photon emission computed tomography (SPECT), scintigraphy, etc.).

Radiopharmaceuticals are used as “tracers” administered to patients. Wherever a tracer goes in the body, it can be followed by suitable detectors which measure its characteristic radiation. It can be made into a chemical compound, that concentrates in particular organs and biological structures, allowing to determine the precise location, shape and biochemical function of the structures to be mapped out. It is thus possible to identify and perform an early diagnosis of diseases such as cancer and degenerative syndromes of the central nervous system (such as Alzheimer’s and Parkinson’s disease). The cells, in fact, are equipped with highly specific receptors for certain molecules, which are captured even if present in very low concentrations. In more recent years, the design of radiopharmaceuticals, able to bind specifically and selectively to these receptors, has allowed to study the metabolism of the cell at the molecular level and sometimes even in real time.

Positron emission tomography is an important and widely diffused method in which radioactive tracers are used in medicine [10]. When compounds containing short-lived, positron-emitting isotopes are injected into a patient, they can be followed as they concentrate in particular organs. This is possible because the positrons emitted as they decay very quickly encounter their anti-particles, atomic electrons. When this happens, the electron and positron rapidly annihilate each other and two energetic photons are emitted in precisely opposite directions. By detecting such pairs of photons, with nanosecond timing, the place in the body where the radioactive isotope has decayed can be determined quite accurately. By detecting many such pairs over a period of time, the detailed changes in concentrations of the chemicals can be mapped out. The functioning of living hearts or brains and other organs can be studied and their diseases diagnosed. Brain damage suffered in a stroke, for example, can be mapped out precisely. Positron emission tomography has also become an important method for understanding the normal function of the brain and other organs.

The best-known use of radiation for treating disease is radiotherapy, used against cancer, because it destroys diseased tissue with an efficiency greater than for healthy tissue. The radiation used for cancer treatment may come from a machine outside the body (external-beam radiation therapy), or it may come from radioactive material placed in the body near cancer cells (internal radiation therapy). In the latter case, using the above-described mechanism of the capture of radiopharmaceuticals by the cells, a highly effective approach can be the administration to the patient of compounds containing radionuclides, which radiate preferentially inside cancer cells with limited damage to the surrounding healthy tissue.

An innovative and efficient method to treat tumours is hadrontherapy [10]. It consists in irradiating tumours with protons or light nuclei (alphas, carbon ions). Compared to conventional radiotherapy, it presents two main advantages: a better spatial accuracy in targeting the tumour and a more accurate and efficient irradiation of the tumour. Indeed, while X-rays lose energy slowly and mainly exponentially as they penetrate tissues, charged particles release more energy at the end of their range in matter. This makes it possible to target a well-defined region at a specific depth in the body that can be tuned by adjusting the energy of the incident particle beam, with reduced damage to the surrounding healthy tissue.

Ionising radiation is also routinely used for sterilization of needles, surgical instruments, and dressings—especially those made of non-heat-resistant materials—because they kill bacteria and viruses.

2.11.2 Research Applications

X-ray diffraction is used in the study of molecular tridimensional structures from mineral to biological macromolecules. It is the primary method for determining the molecular conformation of biological macromolecules, particularly proteins and nucleic acids such as deoxyribonucleic acid (DNA) and ribonucleic acid (RNA). It is worth to recall that the double-helical structure of DNA was deduced from X-ray diffraction data.

Neutron diffraction is also used to study properties of molecules in solution or to help refine structures obtained by X-ray images. The methods are often viewed as complementary, as X-rays are sensitive to the spatial distributions of atomic electrons and scatter most strongly off heavy atoms, while neutrons are sensitive to the spatial location of nuclei and scatter strongly even off many light isotopes, including hydrogen and deuterium.

Electron diffraction has been used to determine some protein structures, most notably membrane proteins and viral capsids.

In the last decades *synchrotron radiation*⁹ has become an increasingly important tool for the study of biological macromolecules and for research in the fields of art, archaeology, and the conservation of objects of cultural heritage significance, being specifically suitable for micro-non-destructive analyses.

In archaeology and geology, radioactive decay has found interesting applications to determine the age of fossils and other objects, dates of very remote events and measure very large time intervals.

A number of methods using either protons or neutrons have been developed for performing elemental analysis of samples in various fields, from environmental analyses to archaeology and history of art. Among the most important methods are Rutherford back scattering (RBS); proton induced X-ray emission (PIXE); neutron activation¹⁰ analysis (NAA); neutron radiography and prompt gamma activation analysis (PGAA).

⁹Synchrotron radiation is called the electromagnetic radiation emitted when charged particles are subject to a circular acceleration, e.g. when their trajectory is bent by a magnetic field.

¹⁰Activation, i.e. the transformation of a stable isotope of a certain element into a radioactive one, occurs when nuclear collisions due to the accelerated beam impinging on surrounding materials are such that either a proton or a neutron can be stripped off a stable isotope of the struck material. The resulting isotope can be a β^+ or β^- emitter. Activation can also occur when a (slow) neutron is captured by a nucleus. The capture process forms an excited nucleus which quickly decays to its ground state by emitting γ -rays. The newly formed isotope containing one more neutron can be unstable and will typically undergo β^- decay.

In agriculture, research involving the development of new plant species employs radioisotopes to engender genetic mutations.

2.11.3 Industrial Applications

Radioisotopes are widely used throughout industry. Often they are source of penetrating gamma rays used for examining metal structures in order to find otherwise imperceptible defects in metallic castings and welds and to highlight imperceptible possible flaws before they cause dangerous failures (for example, this technique is used in the periodic checks of ball bearings and jet turbine blades). Radiography is also used to check the flow of oil in sealed engines and the rate and way various materials wear out. Well-logging devices use a radioactive source and detection equipment to identify and record formations deep within a well hole for oil, gas, mineral, groundwater, or geological exploration.

Furthermore, radioisotopes are used: for producing new materials with specific chemical-physical characteristics, through transmutation reactions; in the domestic smoke detectors; for sorting scrap metals and analysing alloys; for the treatment of water and sewage sludge of industrial plants or hospitals; to trace and analyse pollutants; to inspect airline luggage and shipping cargo containers for hidden explosives; etc.

Engineers also use gauges containing radioactive substances to measure the thickness of paper products, fluid levels in oil and chemical tanks, and the moisture and density of soils and materials at construction sites.

The agricultural industry makes use of radiation to improve food production, preservation and packaging. Plant seeds, for example, have been exposed to radiation to bring about new and better types of plants. Besides making plants stronger, radiation can be used to control insect populations, thereby decreasing the use of dangerous pesticides. Irradiation with gamma rays is used to increase the shelf life of certain plants and in some types of pest control of foodstuffs. Irradiation of food can stop vegetables or plants from sprouting after they have been harvested. It also kills bacteria and parasites, and controls the ripening of fruits.

Radioactive material is also used in gauges that measure the thickness of eggshells to screen out thin, breakable eggs before they are packaged in egg cartons. In addition, many of our foods are packaged in polyethylene shrink-wrap that has been irradiated so that it can be heated above its usual melting point and wrapped around the foods to provide an airtight protective covering.

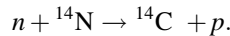
Finally, electricity produced by nuclear fission is one of the greatest uses of radiation. As of 31 October 2015, there were 439 commercial nuclear power reactors operable in 30 countries, with over 380,000 MW of total electric capacity [11, 12]. They provide over 11 % of the world's electricity as continuous, reliable base-load power, without carbon dioxide emissions (see Sect. 4.2 for more details).

2.11.4 Radioactive Dating

A radioactive nucleus has a half-life that is independent of the state of chemical combinations of the atom in which it resides. A collection of radioisotopes therefore forms an excellent clock. It is, thus, possible to establish dates which are of considerable interest in archaeology, history and geology.

In archaeology the *radiocarbon dating*, or ^{14}C -dating, is widely used for dating organic materials (bone, wood, textile fibres, seeds, wood charcoal). The method, based on the measurement of the relative abundances of the isotopes of carbon, allows the dating of materials from ages between 50,000 and 100 years ago.

Carbon, which is a chemical element essential for life, is present in all organic substances. In nature there are three carbon isotopes, two are stable (^{12}C and ^{13}C) and one is radioactive (^{14}C). The latter transforms into nitrogen (^{14}N) by β^- -decay, with a half-life of 5700 years. Consequently, it would disappear in the long run, if it were not continually replenished. The production of new ^{14}C occurs in nature, in the upper atmosphere, from the capture of neutrons by nitrogen atoms (the neutrons are produced by the cosmic rays that constantly bombard the Earth and smash atmospheric nuclei), via the specific reaction



The dynamic equilibrium that is established between production and radioactive decay keeps the concentration of ^{14}C constant in the atmosphere, where it is presently mainly bonded to oxygen in the form of carbon dioxide. The latter contains the radioactive isotope ^{14}C in the ratio of 1 atomic part in about 10^{12} of the stable isotope ^{12}C .

All living organisms (plants and animals) continually exchange carbon with the atmosphere through the processes of breathing, photosynthesis or feeding. As a result, as long as an organism is alive, all its tissues have the same concentration of ^{14}C as that found in the atmosphere. As soon as the animal or plant dies, its intake of carbon stops, and, from that time, the carbon-14 it contains begins to decrease because it decays without replenishment. Then the concentration of ^{14}C decreases regularly according to Eq. (2.9).

$$C(t) = C_0 \times e^{-\lambda \Delta t}, \quad (2.36)$$

where C_0 is the concentration of carbon-14 in the atmosphere, Δt is the time elapsed since the death of the organism, and λ is the decay constant of ^{14}C .

So by measuring the amount of ^{14}C present in the organic remains, one gets their age according to the following formula:

$$\Delta t = -\frac{1}{\lambda} \ln \frac{C}{C_0}. \quad (2.37)$$

For example, if in an organic specimen the concentration of ^{14}C amounts to a quarter of the natural level, then the specimen is of the age of two half-lives. If it is

less than a factor of 1000, then the death of the organism occurred more than ten half-lives ago.

This method allows to date the specimens with a margin of error between 2 and 5 % and up to a maximum time of about 50,000 years; for older specimens, the concentration of ^{14}C is too low to be measured with sufficient accuracy. By using special preparations that enhance the ^{14}C content, it is possible to arrive to about 75,000 years ago.

Using radionuclides of longer half-life, one can perform dating further back in time as far as to estimate the age of the solar system (4.5 billion years). For example, the potassium-argon method, based on the β^+ -decay of $^{40}\text{K}_{19}$ to $^{40}\text{Ar}_{18}$, with a half-life of 1.25 billion years, is reliable in the range between 4.5 billion years and a hundred thousand years. The system rubidium-strontium, based on the β^- -decay of $^{87}\text{Rb}_{37}$ to $^{87}\text{Sr}_{38}$, with a half-life of 48.1 billion years, is reliable in the range between 4.5 billion years and five million years. Both these methods are used in geological dating.

Problem 2.10 Total body dose. A radiological technician works 3.5 h per day at a distance $d = 5.0$ m from a ^{60}Co radioactive source, with 1.0×10^9 Bq activity. The source emits two γ -rays, with energy 1.33 and 1.17 MeV, in fast sequence. Calculate the dose absorbed by the technician per day, knowing that his/her mass is $M = 80$ kg and supposing his/her body has a front section area of 1.5 m^2 and that γ -rays release in the body about 50 % of all their energy.

Solution: The total energy of the two γ -rays released in each decay is $(1.33 + 1.17) \text{ MeV} = 2.50 \text{ MeV}$. Then the total energy per second released by the radioactive source is

$$1.0 \times 10^9 \times 2.50 = 2.5 \times 10^9 \text{ MeV/s.}$$

This energy is uniformly released in the space around the source. Then, taking into account that the technician works at a distance of 5.0 m from the source, the fraction of the energy intercepted by his body is equal to the ratio of the frontal section area of his body divided by the surface of the sphere of radius d :

$$\frac{1.5}{4\pi d^2} = \frac{1.5}{4 \times 3.14 \times 5^2} = 4.8 \times 10^{-3}.$$

Recalling that γ -rays release only 50 % of all their energy in the technician's body, the energy deposited in the body is:

$$E = \frac{1}{2} 4.8 \times 10^{-3} \times 2.5 \times 10^9 \times 1.6 \times 10^{-13} = 9.6 \times 10^{-7} \text{ J/s.}$$

Recalling that $1 \text{ Gy} = 1 \text{ J/kg}$, the dose absorbed by the technician's body per unit time is

$$9.6 \times 10^{-7} / 80 = 1.2 \times 10^{-8} \text{ Gy/s.}$$

Then, in 3.5 h (=12,600 s), the technician absorbs a dose of

$$12,600 \times 1.2 \times 10^{-8} = 1.5 \times 10^{-4} \text{ Gy.}$$

Recalling that the radiation weighting factor of γ -rays is $w_R = 1$, the equivalent dose is 0.15 mSv.

Problem 2.11: Initial and residual activity. A laboratory has $1.49 \mu\text{g}$ of $^{13}\text{N}_7$ with half-life 600 s. Calculate the initial activity of the specimen and its residual activity after 1 h.

Solution: One mole of the radioactive specimen (13 g) contains 6.02×10^{23} nuclei $^{13}\text{N}_7$. Therefore, the number of nuclei $^{13}\text{N}_7$ initially present in the specimen is:

$$N_0 = \frac{1.49 \times 10^{-6}}{13} 6.02 \times 10^{23} = 6.90 \times 10^{16} \text{ nuclei.}$$

Using Eq. (2.10) one has:

$$\lambda = \frac{0.693}{\tau_{1/2}} = \frac{0.693}{600 \text{ s}} = 1.16 \times 10^{-3} \text{ s}^{-1}.$$

Then, using Eq. (2.12), the activity at the time $t = 0$ is:

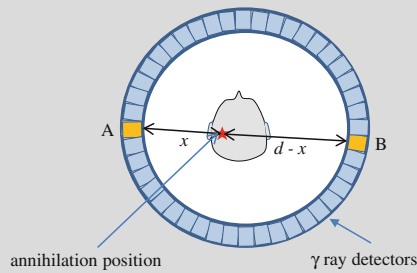
$$\left(\frac{dN}{dt} \right)_0 = \lambda N_0 = (1.16 \times 10^{-3} \text{ s}^{-1})(6.90 \times 10^{16}) = 8.00 \times 10^{13} \text{ Bq,}$$

and the residual activity after 1 h is:

$$\begin{aligned} \frac{dN}{dt} &= \left(\frac{dN}{dt} \right)_0 e^{-\lambda t} = (8.00 \times 10^{13} \text{ s}^{-1}) e^{-(1.16 \times 10^{-3} \text{ s}^{-1})(3600 \text{ s})} \\ &= 1.23 \times 10^{12} \text{ Bq.} \end{aligned}$$

Problem 2.12: Time-of-flight PET. Positron emission tomography measures the two annihilation photons that are produced back-to-back after positron emission from a radionuclide-tagged tracer molecule, which is chosen to mark a specific function in the body on a biochemistry level. The photons are measured by an annular array of suitable detectors placed around the body of the patient, as shown in the figure.

Consider an annihilation event where the two emitted photons are detected by the detectors A and B positioned on opposite sides of the patient. From the time difference, Δt , of the arrival of the two photons at the detectors A and B (*time-of-flight difference*) it is possible to reconstruct the annihilation position (red star in the figure). Calculate this position on the line connecting the two detectors A and B, knowing that they are at a distance d from each other.



Solution: Calling x and $(d - x)$ the distance from the annihilation position and the detectors A and B, respectively; c the speed of the light; and t_A and t_B the times of arrival of the two photons at the detectors A and B, respectively, one can write

$$\begin{aligned} ct_A &= x, \\ ct_B &= d - x, \end{aligned}$$

Subtracting the first equation from the second and calling $\Delta t = (t_B - t_A)$, one gets:

$$c\Delta t = d - 2x,$$

from which it follows that the annihilation occurs at a distance x from detector A:

$$x = \frac{d - c\Delta t}{2}.$$

Time-of-flight PET imaging was investigated in the past, but has never been adopted because of limitations in the available instrumentation. The recent development of scintillators suitable for time-of-flight PET combined with advances in timing resolution and time stability of detector electronics have led to a resurgence of interest in time-of-flight PET scanners, with commercial models being introduced. The advantage of time-of-flight PET, however, remains to be evaluated over the next few years.

Problem 2.13: Radioactive dating. An antique wood relic contains only 6 % of ^{14}C nuclei compared to a specimen of fresh wood. How old is the relic?

Solution: Calling N_0 and N the number of nuclei ^{14}C respectively present in the specimen of fresh wood and in the relic, and T the age of the relic, and using Eq. (2.9), we have:

$$\frac{N}{N_0} = e^{-\lambda T},$$

from which, by taking the logarithm of the first and second member, one gets:

$$\ln \frac{N}{N_0} = -\lambda T,$$

which, using Eq. (2.10) and recalling that the half-life of ^{14}C is $\tau_{1/2} = 5700$ a (see Table 2.2), gives:

$$T = -\frac{1}{\lambda} \ln \frac{N}{N_0} = -\frac{\tau_{1/2}}{0.693} \ln \frac{N}{N_0} = \frac{5700}{0.693} \ln(0.06) \approx 2.3 \times 10^4 \text{ a.}$$

Problem 2.14: Radioactive dating. A sedimentary rock, which includes fossils of prehistoric animals, contains nuclei strontium $^{87}\text{Sr}_{38}$ and rubidium $^{87}\text{Rb}_{37}$ in the atomic ratio 0.016. Calculate the age T of the fossils assuming the total absence of $^{87}\text{Sr}_{38}$ at the time of formation of the rock (the half-life of Rb is 4.81×10^{10} a).

Solution: Calling N_0 the number of initial nuclei Rb, we know that, in the time T , $0.016 \times N_0$ Rb nuclei have undergone β^- decays and formed as many nuclei Sr. Therefore, according to Eqs. (2.9) and (2.10) the age of the fossils is:

$$T = -\frac{\tau_{1/2}}{\ln(2)} \ln(1 - 0.016) = \frac{4.81 \times 10^{10}}{0.693} \times 1.613 \times 10^{-2} = 1.12 \times 10^9 \text{ a.}$$

Problems

- 2.1 Polonium-210 is an alpha emitter that decays directly to its stable daughter isotope, lead-206, with a half-life of 138.4 days. Evaluate the energy available for this α -decay, knowing that the atomic mass of ^{210}Po is 209.982874 u and that of ^{206}Pb is 205.974465 u.
[Ans.: 5.40 MeV]
- 2.2 Calculate the amount of energy released in the alpha decay of 1 mol of plutonium-239 into uranium-235. (The masses of Pu-239, U-235 and the

alpha particle are 239.0521634 u, 235.0439299 u, and 4.002603 u, respectively.)

[Ans.: 5.1×10^{11} J]

- 2.3 The kinetic energy of the α -particle from the decay of ^{226}Ra ($Z = 88$, atomic mass 226.0254098 u) is 4.78 MeV and the recoil energy of the daughter nucleus, ^{222}Rn ($Z = 86$), is 0.09 MeV. Calculate the atomic mass of the ^{222}Rn nucleus.

[Ans.: 222.0175785 u]

- 2.4 Tritium (^3H), the heaviest isotope of hydrogen, is radioactive and β^- decays, with half-life of 12.3 years, into a helium isotope ^3He . Calculate the maximum value of the energy of the electron emitted in the decay, knowing that the atomic masses of the two isotopes are $M[^3\text{H}] = 3.01604928178$ u and $M[^3\text{He}] = 3.01602932243$ u.

[Ans.: 18.6 keV]

- 2.5 A sample of radioactive material has a half-life of 12 min. If there is 8.0 g of the material at the beginning of an experiment, how much will be left after 1 h has passed?

[Ans.: 0.25 g]

- 2.6 What is the half-life of a radioactive substance whose activity decays to 1/8 of the original value in 150 s?

[Ans.: 50 s]

- 2.7 What percentage of the original nuclei of a sample of radioactive substance are left after: (a) 3 half-lives; (b) 7 half-lives; (c) 10 half-lives; (d) n half-lives.

[Ans.: (a) 12.5 %, (b) 0.78 %, (c) 0.098 %, (d) $100(1/2)^n$]

- 2.8 A chemical separation of the lead isotope ^{214}Pb from its radioactive parent is completed at 10:00 a.m. This lead isotope decays with half-life of about 27 min. At 11:00 a.m. there are 8.0×10^6 nuclei of ^{214}Pb remaining. Calculate how many of these nuclei will remain at: (a) 11:27 a.m. and (b) 11:54 a.m.

[Ans.: (a) 4.0×10^6 nuclei; (b) 2.0×10^6 nuclei]

- 2.9 A radioactive material contains 10^{12} radioactive nuclei with half-life of 1 h. Calculate how many of these nuclei decay in 1 s.

[Ans.: 1.93×10^8]

- 2.10 A radioactive antimony isotope ^{124}Sb (half-life 60 days) has an initial activity of 3.7×10^6 Bq. What is its activity after 1 year?

[Ans.: 5.5×10^4 Bq]

- 2.11 A radioactive isotope has an activity of 3.5×10^8 Bq. After 3.90 h, the activity is 2.3×10^8 Bq. What is the half-life of the isotope? What is the activity after an additional 3.90 h?

[Ans.: 6.4 h; 1.5×10^8 Bq]

- 2.12 A sample of a particular β -emitter is placed near a Geiger counter (a detector of β -rays), which registers 480 counts per minute. Twelve hours later the detector counts at a rate of 30 counts per minute. What is the half-life of the material?
[Ans.: 3 h]
- 2.13 In the year 2003, a radioactive sample was observed to decay at a rate of 800 counts per minute. In 2015, that same sample was observed to decay with a rate of 100 counts per minute. What is the half-life of the material in the sample?
[Ans.: 4 a]
- 2.14 Calculate the activity of 1 g of the isotope ^{226}Ra , knowing that its half-life is 1600 a. What will be the activity of the sample after 3200 years?
[Ans.: 3.7×10^{10} Bq; 9.2×10^9 Bq]
- 2.15 A radiation detector, placed near a sample of the radioactive isotope ^{131}I (half-life 8 days), measures an activity of 32,000 Bq. What was the activity of the sample 40 days before the measurement? How many days must elapse before the activity reduces to 100 Bq?
[Ans.: 1,024,000 Bq; 66.6 days]
- 2.16 A sample of charcoal from an archaeological site contains one-sixteenth of the carbon-14 expected in a fresh sample (the half-life of ^{14}C is 5700 a). Find the age of the charcoal sample.
[Ans.: 22,800 years]
- 2.17 In the bones of a person there are about 175 g of potassium. Calculate the activity of that person, knowing that about 0.01 % of the naturally occurring potassium is the unstable isotope ^{40}K (half-life 1.26×10^9 a).
[Ans.: ≈ 4590 Bq].
- 2.18 A chemical analysis of a bone has shown the presence of 300 g di carbon. A measurement of the activity of ^{14}C (half-life 5700 a) has given the value of 10 Bq. Calculate the age of the bone knowing that in a living material there is one ^{14}C atom for every 10^{12} ^{12}C atoms.
[Ans.: 14,459 years]

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Energy from Nuclear Fission

An Introduction

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2016, XV, 278 p. 55 illus., Hardcover

ISBN: 978-3-319-30649-0