

Preface

This book grew out of our desire to better understand period maps in positive characteristic, analogous to those used by Kim in his program to study rational points. We very quickly realised that what was currently sorely lacking was a robust picture of p -adic cohomology for varieties over positive characteristic local fields (i.e. local function fields), and the results here consist of our attempt to provide the foundations for such a theory.

The inspiration and model for our approach is (unsurprisingly) Berthelot's theory of rigid cohomology, and in some sense the key insights here are mostly that of making the right definitions rather than proving completely new results. The crucial observation of this book is that once these definitions are in place, much of the existing literature on rigid cohomology, its construction, and most of the proofs of its fundamental properties such as finite dimensionality and cohomological descent, can be applied in more general situations than have generally been considered to date.

As a result, much of this book will be familiar to those well versed in this literature. The concepts used, such as frames, overconvergence, dagger algebras and the overconvergent site are all transported from the 'classical' theory of rigid cohomology, and the broad outlines of many of the arguments are more or less the same as their already well understood classical counterparts. This in part accounts for the length of the work, as in essence what we have had to do is go through the key literature on rigid cohomology and reprove these fundamental results in our new situation.

The version of rigid cohomology we consider here is of course only one of many possible situations one would be interested in knowing the existence of a well behaved p -adic cohomology theory, and in one sense our results here are therefore provisional. They provide the first glimpse (beyond the classical case) of a much more general picture of rigid cohomology, which one would hope could eventually apply to higher dimensional local fields, global fields, valuation rings therein or a wealth of other situations. The desire to move more quickly to arithmetic applications of this 'new' rigid cohomology means that we have not properly explored to

what level of generality these methods can be pushed, but we hope that our working out of this one case in detail will serve as a motivation to a further expansion of the scope of rigid cohomology.

One of the important technical tools in constructing the version of rigid cohomology we present here is Huber's theory of adic spaces. While others have previously made use of alternative descriptions of rigid analytic spaces in the study of rigid cohomology, most notably in Le Stum's use of Berkovich spaces to construct the overconvergent site, our use of adic spaces is absolutely fundamental, due to the need to work with rigid analytic spaces which are not locally of finite type over a non-Archimedean ground field. Another modest hope, then, for our work, is that it will provide yet more evidence that Huber's adic spaces really are the 'correct' objects of study in non-Archimedean analytic geometry, and in particular therefore provide the 'correct' setting even for classical rigid cohomology.

Like any good 'arithmetic' cohomology theory, we fully expect that the version of rigid cohomology we present here will find manifold applications beyond our original motivation for embarking on this project. The lack of a good p -adic theory has been a glaring hole in the study of the cohomology of varieties over local function fields for some time, especially given the greater arithmetic depth and information generally observed in p -adic invariants in characteristic p (or, indeed, in mixed characteristic). By taking at least the first step towards properly filling in this gap, we hope that this book will provide a useful tool for any researchers interested in the arithmetic of varieties over local function fields.

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