# Preface

This volume is a compilation of research and survey papers in number theory, written by members of the *Women in Numbers* (WIN) network, principally by the collaborative research groups formed at *Women in Numbers 3*, a conference at the Banff International Research Station in Banff, Alberta, on April 21–25, 2014.

The WIN conference series began in 2008, with the aim of strengthening the research careers of female number theorists. The series introduced a novel research-mentorship model: women at all career stages, from graduate students to senior members of the community, joined forces to work in focused research groups on cutting-edge projects designed and led by experienced researchers. This model had tremendous success, branching out not only to *WINE (Women in Numbers Europe)* but also to *Algebraic Combinatorixx*, *WIT (Women in Topology)*, and others. The Association for Women in Mathematics (AWM), funded by the National Science Foundation, is now supporting this research-mentorship model under the umbrella of the *Research Collaboration Conferences for Women* initiative.

The goals for *Women In Numbers 3* were to establish ambitious new collaborations between women in number theory, to train junior participants about topics of current importance, and to continue to build a vibrant community of women in number theory. The majority of the week was devoted to research activities. Before the conference, the participants were organized into nine project groups by research interest and asked to learn background for their project topics. This led to more productive on-site research conversations and the groups were able to share preliminary results on the last day. The workshop also included a lecture series about arithmetic of curves, including elliptic curves, modular curves, and Shimura curves.

Forty-two women attended the WIN3 workshop, which was organized by the last three editors of this volume. This included 15 senior and mid-level faculty, 15 junior faculty and postdocs, and 12 graduate students. This volume is the fourth proceedings to come out of the WIN conference series. It is also the first in the series published by Springer for AWM.

The editors invited WIN3 research groups and members of the larger WIN3 community to submit articles in 2014. After a thorough referee process by external experts, we accepted 10 papers for the volume. One interesting attribute of the

collection is the interplay between deep theory and intricate computation. The papers span a wide range of research areas: arithmetic geometry, analytic number theory, algebraic number theory, and applications to coding and cryptography. In this preface, we point out a few connections between the papers.

A major theme of the volume is the study of rational points on varieties via cohomological methods. Three papers on this theme are about rational points over number fields. The paper *Insufficiency of the Brauer-Manin obstruction for rational points on Enriques surfaces* (Balestrieri et al.) is about the failure of the Hasse principle for surfaces. In the paper *Shadow lines in the arithmetic of elliptic curves* (Balakrishnan et al.), the authors use information about analytic ranks and Tate-Shafarevich groups to develop an algorithm for computing anticyclotomic $p$-adic heights and shadow lines cast by rational points on elliptic curves over imaginary quadratic fields. In the paper *Galois action on the homology of Fermat curves* (Davis et al.), the authors use topology and the étale fundamental group to study obstructions for points on Fermat curves defined over cyclotomic fields.

The paper *Zeta functions of a class of Artin-Schreier curves with many automorphisms over finite fields* (Bouw et al.) is a bridge between several of the disparate topics. It fits in the vein of studying rational points via cohomological methods, because the $\ell$-adic cohomology provides information about points on curves defined over finite fields. It connects to the topic of applications to coding theory and cryptography, because the class of Artin-Schreier curves produces large families of supersingular curves useful for error-correcting codes. Similarly, the paper *Hypergeometric series, truncated hypergeometric series, and Gaussian hypergeometric functions* (Deines et al.) draws together several topics. The hypergeometric varieties are higher-dimensional analogues of Legendre curves and the authors obtain information about the number of points on these varieties defined over finite fields. This paper also connects to the more analytic papers in the volume.

There are two other papers with an analytic and geometric focus. The paper *A generalization of S. Zhang's local Gross-Zagier formula for* $\mathrm{GL}_2$ (Maurischat) is about Hecke operators and contains a fundamental lemma for some relative trace formulae. The paper *p-adic q-expansion principles on unitary Shimura varieties* (Caraiani et al.) has results about vanishing theorems for $p$-adic automorphic forms on unitary groups of arbitrary signature.

The final three papers are about applications of algebraic number theory. The paper *Kneser-Hecke-operators for codes over finite chain rings* (Feaver et al.) is about theta series for lattices for codes over finite fields and an analogue for Hecke operators in this context. In *Ring-LWE cryptography for the number theorist* (Elias et al.), the authors give a survey about attacks on the ring and polynomial learning with errors problems and discuss connections with open problems about algebraic number fields. Finally, the volume ends with a survey about arithmetic statistics in algebraic number theory, *Asymptotics for number fields and class groups* (Wood). This survey is an extended version of Wood's lecture notes for the Arizona Winter School in 2014, on the topic of counting number fields and the distribution of class groups.

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## Workshop Website

http://www.birs.ca/events/2014/5-day-workshops/14w5009

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