

Preface

There are two main methods for analyzing a structure: the *force method* and the *displacement method*. Some will argue the former is passé. We do not follow that argument. The force method may have lost its prominence as a structural analysis tool, but it is still conducive to the way engineers think about a structure. Forces and equilibrium are intuitive from infancy, for example, when we tried out our first solo steps. There is also the early childhood sense of triumph when an eighth wooden block is positioned correctly on a stack of seven, or the feeling of disappointment when the last cube makes the tower tumble.

This innate feeling of forces and equilibrium has remained with us. We usually think of structures in terms of forces, although in practice our computers will perform the analysis by way of displacements. This duality is reflected in this treatise.

My first book on structures was a text written by André Paduart of the Free University of Brussels (ULB). It was called M, N, T, R, δ . The title started out with the internal loads: M for bending Moment, N for Normal force, T for shear force (effort Tranchant—the text was in French), R for Reactions I guess, and lastly δ for displacements. At that time (1963) displacements were indeed a nuisance and you computed them only when you absolutely needed them. In those days the analysis method was the force method.

Things have changed. Nowadays the analysis method of choice is indisputably the displacement method, where the basic variables are the displacements of the structure. Once these have been computed the internal forces and reactions easily follow suit. The shift in paradigm was heralded by the advent of tremendous, readily available computing power. Arguably, a basic tenet in the classical days of structural analysis was never to have more than three linear equations with three unknowns at any time, no matter how complex the structure. This led to some ingenious modeling and clever assumptions in order to avoid the three-equations obstacle.

The displacement method, on the other hand, albeit deceptively simple and relatively boring, because it involves tedious matrix calculations, requires the solution of many linear equations with as many unknowns. With the finite-element method, an offshoot of the displacement method, it is not unusual to solve tens of

thousands of linear equations when, for instance, considering the analysis of an aircraft wing or a cable-stayed bridge, a task perfectly suited to computerized calculations. Moreover, the writing of the equations, what we call assembly, is also automated in the displacement method; again something computers do well.

This is also how we present these two aspects of analysis: performing an analysis by the force method is a human endeavor. It can be applied to small and medium-sized structures. It requires an understanding of the behavior of the structure. Its results, the internal loads, can be understood and related to and, as we shall see, even qualitatively predicted in some cases. The displacement method, on the other hand, is for computers. It is algorithmic and can easily be programmed, it manipulates vast arrays of numbers, and does not require too deep an understanding of the process.

This book is intended for a graduate level of instruction although undergraduate material is embedded in the text. Like most structural courses it assumes that the reader has some basic knowledge of the field. Courses in Statics and Solid Mechanics (Strength of Materials) are prerequisites, and essentials of these courses are the first topics of this book.

Part I: Preliminaries

We start with principles of equilibrium (Chap. 1) from which we need to postulate the existence of internal loads. This is followed immediately by a probably too lengthy chapter (Chap. 2) on how to compute and display these internal forces by means of the *nsm*-diagrams. We next discuss the principle of virtual work (Chap. 3) without having introduced Hooke's law. This is very important from a didactic viewpoint. It shows that the principle, in its basic form, is a mathematical concept and not really connected to a structure (a structure is something that deforms under loading). We also show that for the principle to work we need to define quantities that turn out to be deformations (Chap. 4). This way the principle is not some hat trick but simply integration by parts, a mathematical tool. In the next chapter (Chap. 5) we introduce elasticity, that is, the linear relation between the internal forces and the deformations. When external loads are applied to a structure, it deforms so as to engender internal forces which balance the applied ones. We discuss the Timoshenko beam and its relation to the Bernoulli–Euler beam used in engineering beam theory, a graduate course topic. Finally, in the last chapter of Part I (Chap. 6) we introduce a technique for calculating a displacement or a rotation at any location along a structure. The method is closely connected to virtual work and is often taught in introductory courses to structures.

Part II: The Structures

The first chapter in Part II (Chap. 7) describes the types of structures dealt with in this book: beams, plane frames and, to a lesser extent, plane trusses. Such structures are composed of line elements fully or partially connected to one another or to the ground, as described in this chapter. The following chapter (Chap. 8) is an introduction to and a comparison of the two main methods of analysis. This is done by analyzing a stylized structure, a rigid beam on several elastic supports, by both the force method and the displacement method. The last chapter of Part II (Chap. 9) is usually taught to architectural students but unfortunately less so to engineering students (See, for instance, D.L. Schodek and M. Bechthold, *Structures*, 6th edition, Pearson Prentice Hall 2008.) We consider this material extremely important. It often gives a hint of the expected results before actually performing the analysis.

Part III: Flexibility

The flexibility matrix (Chap. 10) lies at the heart of the force method. The chapter starts with the Betti–Maxwell reciprocal theorems, the notion of degree of freedom, leading to the definition of influence or flexibility coefficients, which are the constituents of the flexibility matrix. Several illustrated examples close the chapter, but not before discussing why normal deformations in the presence of bending deformation are usually neglected. Redundancy is treated in the following chapter (Chap. 11). The force method is applicable to redundant structures only. Determining the degree of redundancy is therefore the first step of an analysis by the force method. We also consider redundancy in the context of safety and stiffness of structures. The next chapter is the core of the force method (Chap. 12). We learn the important notions of releasing a structure and writing the compatibility equations. Thereafter, the task is reduced to the analysis of a statically determinate structure where all the applied loads are known. The chapter closes with several illustrated problems. This is followed by a chapter dealing with the effect of heat on structures (Chap. 13). It includes prestressing and manufacturing deformations or misalignments. The chapter emphasizes that all this is applicable to redundant structures only. Determinate structures without applied forces are guaranteed to be free of internal forces.

Part IV: Stiffness

We introduce the stiffness method by way of trusses (Chap. 14). We find the analysis of trusses by the force method too tedious. The stiffness method, which is essentially a numerical method to be solved algorithmically, is ideally suited to

trusses as an analysis tool. The following chapter (Chap. 15) introduces the stiffness matrix of an element. This potent concept allows us to analyze with the same ease any structure if we know the stiffness matrices of all its elements. An intermediate chapter (Chap. 16) essentially indicates a systematic way (congruent transformations) to obtain the element stiffness matrix in oblique coordinates. The following chapter (Chap. 17) is the core of the displacement (stiffness) method. We show the meaning of the equilibrium equations which compute the unknown nodal displacements. Next, we indicate the concept of ‘assembly’, an efficient algorithm to write the equilibrium equations, essentially, the system stiffness matrix. Everything we have done until now assumed that the structure was submitted to point loads applied to the nodes. This excludes distributed forces and temperature effects (applied strain). The final chapter of Part IV (Chap. 18) addresses this subject. By means of fixed-end reactions the problem is circumvented in a most elegant manner. This concludes the analysis methods presented in this text.

Part V: Four Additional Topics

The next part of the book touches on a few aspects of structural analysis and design which complement well the subject matter of this text.

A nice theorem which may be of practical use is presented next (Chap. 19). It states that a roller support of a continuous beam is optimally positioned (from a stiffness perspective) if the slope of the beam over the support is horizontal. A simple numerical technique to find the optimal location is also indicated.

A redundant truss cannot, in general, be fully stressed. In the following chapter (Chap. 20) we show a class of redundant trusses which can be fully stressed.

The next chapter (Chap. 21) indicates a way to cast the analysis equations of a frame in the form of truss equations by means of unimodal normal, shear, and moment elements. Consequently, a unified approach to structural analysis is possible by considering any frame as a generalized truss.

In the penultimate chapter of this book (Chap. 22) we present a very elegant formula for the internal forces in trusses explicitly in terms of the stiffnesses of the bars. Unfortunately, the formula is plagued by the so-called curse of dimensionality.

Part VI: Epilogue

Finally, in the concluding chapter (Chap. 23) we summarize the essentials of all that was said by means of the simplest structure of them all: an assembly of linear springs.

One of our guidelines when writing the book was to keep everything as simple as possible so as to convey the basic ideas without unnecessary burdens. We use plane structures, straight-linear uniform elements, uniformly distributed forces, beams with lateral forces only (which precludes normal internal forces in beams). Using planar structures ignores torsional effects, which is a drawback. Students will have ample leisure to include all this when taking a course in finite elements or when performing a finite-element analysis. We have also avoided talking of energy-related concepts (Castigliano, Clapeyron, etc.). Instead, we emphasize Virtual Work. The latter has a large enough gangplank to take all these methods on board. This was also the reason for avoiding, as much as possible, analyzing trusses manually. In our view trusses are computer territory. All this made it possible to make space for understanding structural behavior which should after all be the main objective of a book on structures.

I have read and browsed through many books on structures. A few stand out. KH Gerstle's *Basic Structural Analysis*, Prentice-Hall, 1974, is probably my first choice. I have benefitted a lot from Schodek's *Structures*, mentioned earlier. Then there is, of course, SO Asplund (*Structural Mechanics: Classical and Matrix Methods*, Prentice-Hall, 1966), an amazing book. I have seen it referred to as the bible of modern structural theory. I fully agree with this description.

I am indebted to Professor Z. Hashin (TAU) and to the late Professors M.A. Brull (TAU) and A. Libai (Technion), who in their own way have been instrumental in making my career in structures possible. I am grateful to my graduate and PhD students who were an inspiration. Many thanks go to Tim Love of Cambridge University, a rare altruist, who was so helpful during a 2-months' summer stay at the Engineering Department. Thanks to Miriam Hercberg for her help in checking the manuscript and to Guy Shiber for verifying and correcting the equations, calculations, and figures.

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