

Fuzzy Decision Making: Its Pioneers and Supportive Environment

Cengiz Kahraman, Sezi Çevik Onar and Başar Öztayşi

Abstract Fuzzy decision making is the collection of single or multicriteria techniques aiming at selecting the best alternative in case of imprecise, incomplete, and vague data. This chapter reviews the fuzzy decision making literature and summarizes the review results by tabular and graphical illustrations. The classification is based on the new extensions of fuzzy sets: Intuitionistic, hesitant, and type-2 fuzzy sets. Later, the media publishing fuzzy decision making papers, journals, books, conferences, and societies are summarized. Finally, fuzzy decision making examples are given for ordinary, intuitionistic, hesitant, and type-2 fuzzy sets.

Keywords Decision-making · Ordinary fuzzy set · Intuitionistic fuzzy set · Hesitant fuzzy set · Type-2 fuzzy set

1 Introduction

Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker [16]. A decision making process can be divided into the following eight steps Baker et al. [5]:

- Step 1. Definition of the problem: This step includes the identification of root causes, limiting assumptions, system and organizational boundaries and interfaces, and any stakeholder issues
- Step 2. Determination of requirements: This step includes the constraints describing the set of the feasible solutions of the decision problem
- Step 3. Establishment of goals: This step includes the definition of the goals which are broad statements of intent and desirable programmatic values

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- Step 4. Identification of alternatives: This step includes the determination of the alternatives which offer different approaches for changing the initial condition into the desired condition
- Step 5. Definition of criteria: This step includes the determination of the criteria which will discriminate among alternatives. The criteria must be based on the goals
- Step 6. Selection of a decision making technique: This step includes the determination of the analysis technique which will solve the problem
- Step 7. Evaluation of alternatives with respect to criteria: This step includes the application of the determined technique in Step 6
- Step 8. Validation of solutions: This step includes the check of the validity of the solution obtained by the evaluation techniques

Real-world decision-making problems are usually too complex and ill-structured to be considered by classical decision making techniques. In the classical decision making techniques, human's judgments are represented as exact numbers. However, in many practical cases, the data may be imprecise, or the decision makers might be unable to assign exact numerical values to the evaluation. Since some of the evaluation criteria are subjective and qualitative in nature, it is very difficult for the decision maker to express the preferences using exact numerical values [35]. Instead, decision makers usually prefer making linguistic evaluations in the decision matrix. Besides, the conventional decision making approaches tend to be less effective in dealing with the imprecise or vague nature of the linguistic assessments [20]. The real case problems including decision making problems are complex and involve vagueness and fuzziness. This has led to the development of fuzzy set theory by Zadeh [44], as a means of representing and manipulating data that were not precise, but rather fuzzy. The theory of fuzzy logic provides a mathematical strength to capture the uncertainties associated with human cognitive process, such as thinking and reasoning [32].

The term *fuzzy decision making* has first been introduced by Bellman and Zadeh [6]. They defined the fuzzy decision making as follows: The term *decision making in fuzzy environment* means a decision making process in which the goals and/or the constraints, but not necessarily the system under control, are fuzzy in nature. This means that the goals and/or the constraints constitute classes of alternatives whose boundaries are not sharply defined [6]. Later, Baas and Kwakernaak [4] applied the most classic work on the fuzzy decision making and it was used as a benchmark for other similar fuzzy decision models. Dubois and Prade [15], Zimmermann [47, 48], Chen and Hwang [14], and Ribeiro [31] differentiated the family of fuzzy decision making methods into two main phases. The first phase is generally known as the rating process, dealing with the measurement of performance ratings or the degree of satisfaction with respect to all attributes of each alternative under fuzziness. The second phase, the ranking of alternatives that is carried out by ordering the existing alternatives according to the resulted aggregated performance ratings obtained from the first phase. Yager [40] presented some ideas on the applications of fuzzy sets to

multi-objective decision making with particular emphasis on a means of including differing degrees of importance to different objectives.

After the introduction of fuzzy decision making by these pioneers, the classical decision making techniques have been transformed to their fuzzy versions within few years. Classical TOPSIS method was transformed to fuzzy TOPSIS by Chen [13]; Classical AHP was transformed to fuzzy AHP by Van Laarhoven and Pedrycz [37], Buckley [7], and Chang [11]. Classical VIKOR was transformed to fuzzy VIKOR by Opricovic [28].

After the introduction of the extensions of fuzzy sets such as type-2 fuzzy sets, intuitionistic fuzzy sets, fuzzy multisets, and hesitant fuzzy sets, the ordinary fuzzy versions of the decision making methods have started to extend to their new versions such as type-2 fuzzy TOPSIS [12], type-2 fuzzy AHP [19], type-2 fuzzy VIKOR [42], intuitionistic fuzzy TOPSIS [2], hesitant fuzzy TOPSIS [46], etc.

The rest of this chapter is organized as follows. Section 2 includes a literature review on fuzzy decision making based on the extensions of fuzzy sets. Section 3 presents the publication media of fuzzy decision making. Section 4 gives some fuzzy decision making applications. Section 5 concludes the chapter.

2 Literature Review

In the following, a literature review on decision making based ordinary fuzzy sets, hesitant fuzzy sets, intuitionistic fuzzy sets, and type-2 fuzzy sets, respectively.

2.1 Ordinary Fuzzy Decision-Making (OFDM)

If X is a collection of objects denoted generically by x , then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\} \quad (1)$$

$\mu_{\tilde{A}}(x)$ is called the membership function which maps X to the membership space M . Its range is the subset of nonnegative real numbers whose supremum is finite [44].

The search for OFDM based on Scopus database in October 2015 yielded 2159 papers. 1281 of them are articles while 775 of them are conference papers. The rest are review, book chapters, editorial, erratum, article in press, etc.

Figure 1 presents the publication frequencies of the papers on OFDM with respect to years. As it is clearly seen from the graph, significant increases in the frequencies of OFDM papers exist after the year 2000.

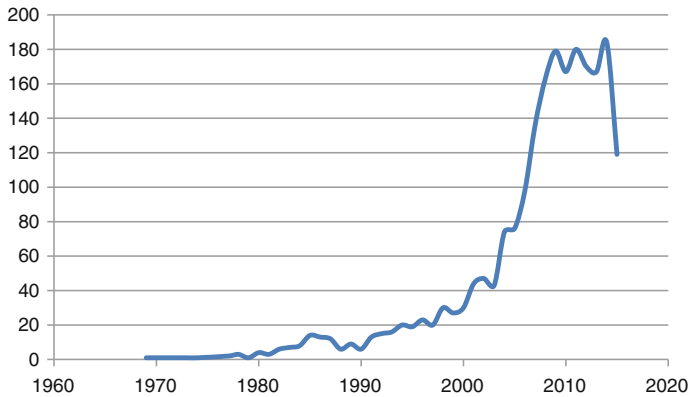


Fig. 1 Publication frequencies of papers on OFDM with respect to years

Table 1 shows the sources publishing OFDM papers and the frequencies of those papers. *Fuzzy Sets and Systems* and *Expert Systems with Applications* are by far the leading journals publishing OFDM papers.

Table 2 presents the authors most publishing OFDM papers.

Table 3 gives the affiliations of the authors publishing OFDM papers.

Figure 2 illustrates the source countries publishing OFDM papers. China is by far the first country publishing OFDM papers.

Figure 3 gives classifies the OFDM papers with respect to their subject areas. Engineering (55.1 %), computer sciences (52.3 %), mathematics (24.9 %), and decision sciences (13.7 %) are the first four areas that OFDM papers come from.

2.2 Hesitant Fuzzy Decision-Making (HFDM)

Hesitant fuzzy sets (HFSs), initially developed by Torra [34], are the extensions of regular fuzzy sets which handle the situations where a set of values are possible for the membership of a single element.

Torra [34] defines hesitant fuzzy sets (HFSs) as follow: Let X be a fixed set, a HFS on X is in terms of a function that when applied to X returns a subset of $[0, 1]$. Mathematical expression for HFS is as follows:

$$E = \{ \langle x, h_E(x) \rangle | x \in X \}, \quad (2)$$

where $h_E(x)$ is a set of some values in $[0, 1]$, denoting the possible membership degrees of the element $x \in X$ to the set E .

There are different types of HFSs that define membership degrees in various formats such as hesitant fuzzy linguistic term sets (HFLTTS), interval valued hesitant

Table 1 Sources publishing OFDM papers and the frequencies of those papers

Source	Number of documents
Fuzzy Sets and Systems	79
Expert Systems with Applications	65
IEEE International Conference on Fuzzy Systems	53
Information Sciences	30
Xi Tong Gong Cheng Yu Dian Zi Ji Shu Systems Engineering and Electronics	28
Studies in Fuzziness and Soft Computing	24
European Journal of Operational Research	20
Journal of Intelligent and Fuzzy Systems	18
Computers and Industrial Engineering	17
International Journal of Production Research	17
Annual Conference of the North American Fuzzy Information Processing Society NAFIPS	16
Applied Mathematical Modelling	16
Cybernetics and Systems	16
Fuzzy Optimization and Decision Making	16
Knowledge Based Systems	16
Proceedings of the IEEE International Conference on Systems Man and Cybernetics	16
International Journal of Advanced Manufacturing Technology	15
Kongzhi Yu Juece Control and Decision	15
Xitong Gongcheng Lilun Yu Shijian System Engineering Theory and Practice	14
Advanced Materials Research	13
Applied Mechanics and Materials	13
Computers and Mathematics with Applications	12
International Journal of Information Technology and Decision Making	12
International Journal of Intelligent Systems	12
International Journal of Uncertainty Fuzziness and Knowledge Based Systems	12
Studies in Computational Intelligence	12
Soft Computing	10

fuzzy sets (IVHFS), triangular hesitant fuzzy sets (THFS), and trapezoidal hesitant fuzzy sets (TrHFS).

The search for HFDM based on Scopus database in October 2015 yielded 221 papers. 181 of them are articles while 25 of them are conference papers. The rest are review, editorial, erratum, and article in press.

Figure 4 presents the publication frequencies of the papers on HFDM with respect to years. As it is clearly seen from the graph, significant increases in the frequencies of HFDM papers exist after the year 2012.

Table 2 Authors most publishing OFDM papers

Authors	Number of OFDM papers he/she published
Herrera-Viedma, E.	22
Kahraman, C.	21
Yano, H.	20
Sakawa, M.	19
Kacprzyk, J.	18
Ekel, P.Y.	18
Lu, J.	17
Zhang, G.	15
Mousavi, S.M.	13
Chiclana, F.	12
Li, D.F.	12
Ruan, D.	11
Pedrycz, W.	11
Tavakkoli-Moghaddam, R.	11
Chen, S.Y.	11
Lee, H.S.	10
Liu, P.	10

Table 3 Affiliations of the authors publishing OFDM papers and frequencies

Affiliations	Number of papers
Islamic Azad University	44
University of Tehran	36
Istanbul Technical University	31
Huazhong University of Science and Technology	28
North China Electric Power University	27
Universidad de Granada	27
Galatasaray University	25
Southeast University	23
Dalian University of Technology	22
National Chiao Tung University Taiwan	21
National Taiwan Ocean University	21
National Taiwan University of Science and Technology	21
University of Technology Sydney	20

Table 4 shows the sources publishing hesitant fuzzy decision-making papers and the frequencies of those papers. *Journal of Intelligent and Fuzzy Systems*, *Information Sciences* and *Knowledge Based Systems* are by far the leading journals publishing HFDM papers.

Figure 5 presents the authors most publishing HFDM papers.

Table 5 gives the affiliations of the authors publishing HFDM papers.

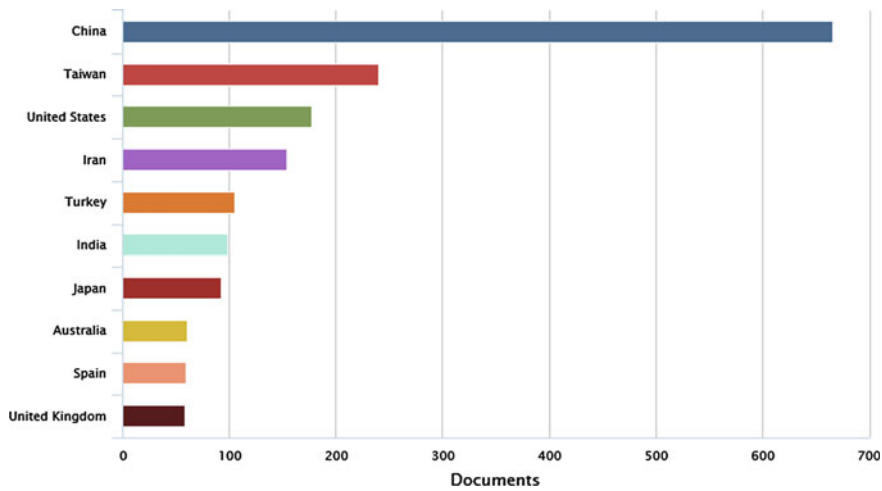


Fig. 2 Source countries publishing OFDM papers

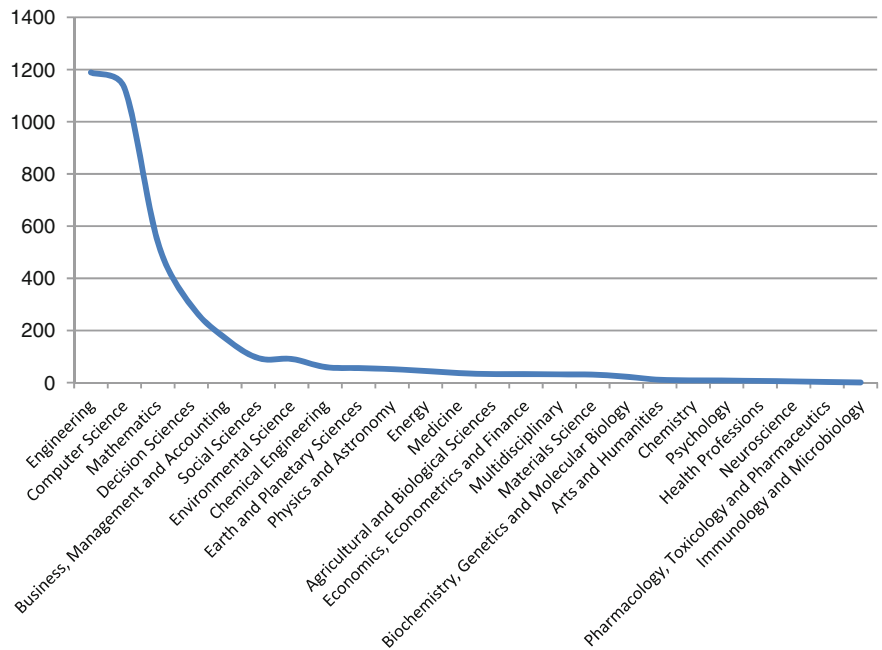


Fig. 3 OFDM papers with respect to their subject areas and frequencies

Fig. 4 Publication frequencies of the papers on HFDM with respect to years

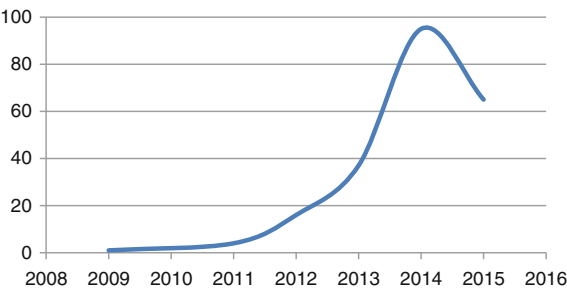


Table 4 Sources publishing HFDM papers and the frequencies of those papers

Source	Number of documents
Journal of Intelligent and Fuzzy Systems	39
Information Sciences	16
Knowledge Based Systems	14
Mathematical Problems in Engineering	8
International Journal of Intelligent Systems	8
Journal of Applied Mathematics	6
Soft Computing	5
Computers and Industrial Engineering	4
Applied Mathematical Modelling	4
Journal of Information and Computational Science	4
Journal of Convergence Information Technology	4
International Journal of Uncertainty Fuzziness and Knowledge Based Systems	4
Expert Systems with Applications	4
International Journal of Computational Intelligence Systems	4
IEEE Transactions on Fuzzy Systems	4
Kongzhi Yu Juece Control and Decision	3
Journal of Computational Information Systems	3
International Journal of Fuzzy Systems	3
International Journal of Systems Science	3
International Journal of Digital Content Technology and Its Applications	3

Figure 6 illustrates the source countries publishing HFDM papers. China is again by far the first country publishing HFDM papers.

Figure 7 gives classifies the HFDM papers with respect to their subject areas. Computer sciences (77.2 %), mathematics (55.7 %), engineering (48.9 %), and decision sciences (17.8 %) are the first four areas that HFDM papers come from.

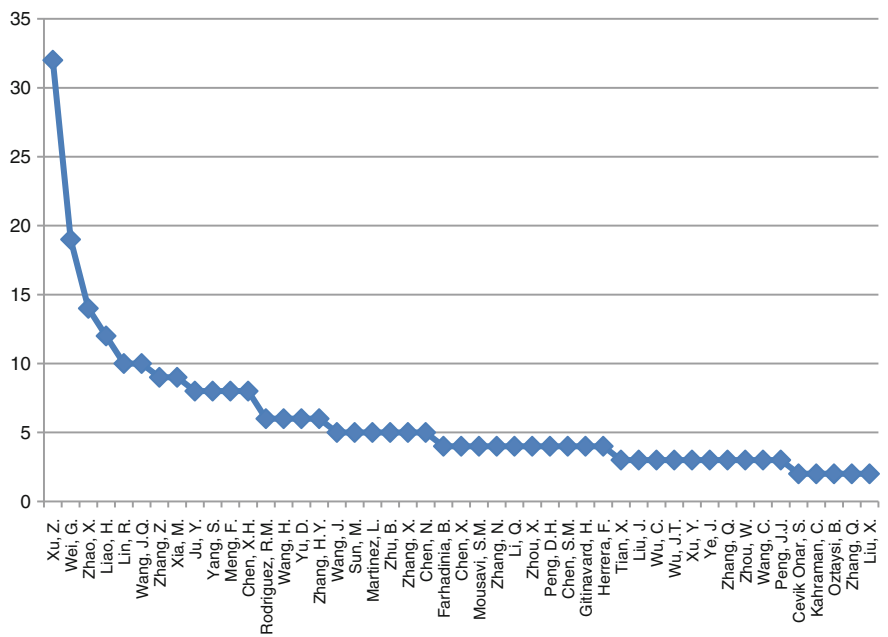


Fig. 5 Authors most publishing HFDM papers

Table 5 Affiliations of the authors publishing HFDM papers and frequencies

Affiliations	Number of papers
Central South University China	25
Southeast University	23
Chongqing University	21
Sichuan University	19
Beijing Institute of Technology	14
PLA University of Science and Technology	10
Hebei University	9
Shanghai Jiaotong University	9
Zhejiang University of Finance and Economics	8
Qingdao TechNological University	7
Southwest Jiaotong University	6
Tsinghua University	6
Universidad de Granada	6
Universidad de Jaen	5
Harbin Institute of Technology	5
National Taiwan University of Science and Technology	5
Zaozhuang University	5

(continued)

Table 5 (continued)

Affiliations	Number of papers
Iran University of Science and Technology	4
Nanjing University of Finance and Economics	4
Shahed University	4
Kunming University of Science and Technology	4
Harbin University of Science and Technology	4
Hunan University	4
Hebei University of Engineering	4
University of Manchester	3
Yangzhou University	3
Hohai University	3
Beijing Jiaotong Daxue	3
Beijing Normal University	3
Shaoxing University	3
Yunnan University	3
De Lin Institute of Technology	3
Hunan Institute of Science and Technology	3
Jining Medical University	2
Quchan Institute of Engineering and Technology	2
Quchan University of Advanced Technology	2
Hefei University of Technology	2
Zhejiang Normal University	2
Northwestern Polytechnical University	2
Central South University of Forestry and Technology	2
Beijing Forestry University	2
Anhui University of Technology	2
Universidad de Chile	2
Guangxi Normal University	2
Nanjing University of Aeronautics and Astronautics	2
Istanbul Technical University	2
South China University of Technology	2
Qufu Normal University	2

2.3 *Intuitionistic Fuzzy Decision-Making (IFDM)*

Atanassov [3] intuitionistic fuzzy sets (IFSs) include the membership value as well as the non-membership value for describing any x in X such that the sum of membership and non-membership is at most equal to 1.

Let $X \neq \emptyset$ be a given set. An intuitionistic fuzzy set (IFS) in X is an object A given by

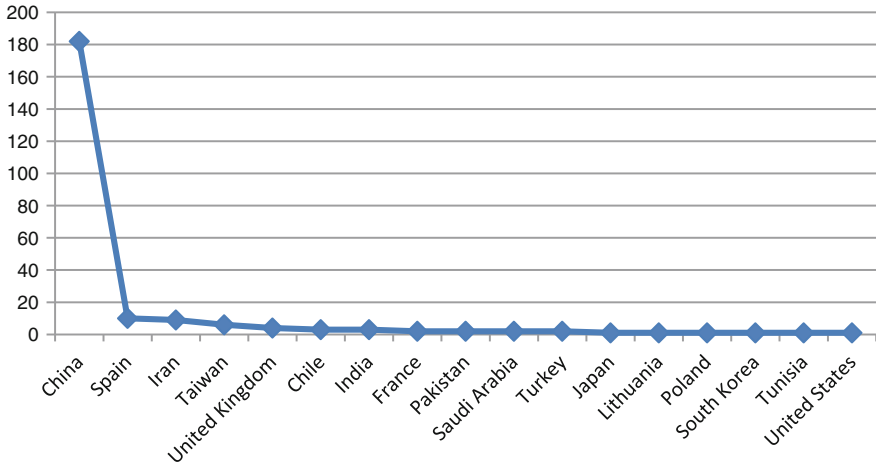


Fig. 6 Source countries publishing HFDM papers

$$\tilde{A} = \{x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x); x \in X\}, \quad (3)$$

where $\mu_{\tilde{A}}: X \rightarrow [0, 1]$ and $v_{\tilde{A}}: X \rightarrow [0, 1]$ satisfy the condition

$$0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1, \quad (4)$$

for every $x \in X$.

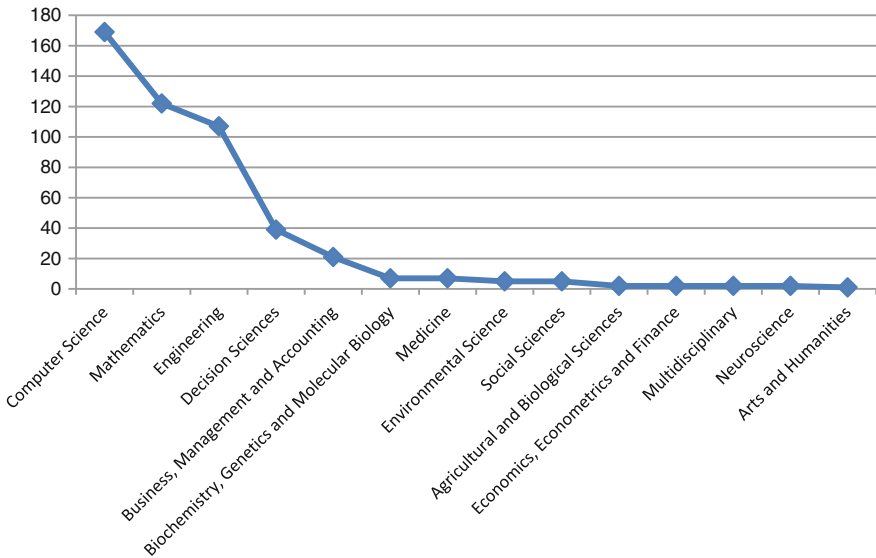
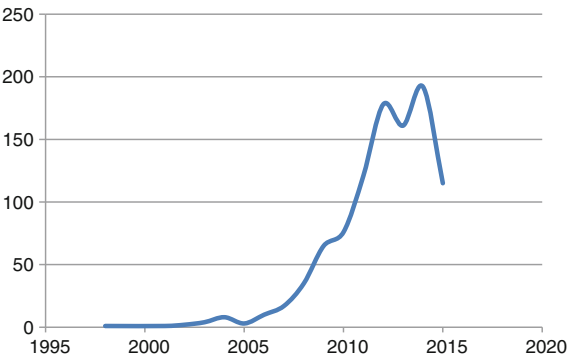


Fig. 7 HFDM papers with respect to their subject areas and frequencies

Fig. 8 Publication frequencies of the papers on IFDM with respect to years



There are different types of IFSs that define membership and nonmembership values in various formats such as interval valued intuitionistic fuzzy sets (IVIFS), triangular intuitionistic fuzzy sets (TIFS), and trapezoidal intuitionistic fuzzy sets (TrIFS).

The search for IFDM based on Scopus database in October 2015 yielded 994 papers. 719 of them are articles while 236 of them are conference papers. The rest are review, erratum, book and article in press.

Figure 8 presents the publication frequencies of the papers on IFDM with respect to years. As it is clearly seen from the graph, significant increases in the frequencies of IFDM papers exist after the year 2005.

Table 6 shows the sources publishing IFDM papers and the frequencies of those papers. *Journal of Intelligent and Fuzzy Systems*, *Expert Systems with Applications*, *Kongzhi Yu Juece Control and Decision*, and *Knowledge Based Systems*, *Applied Mathematical Modelling* are the leading journals publishing IFDM papers.

Table 6 Sources publishing IFDM papers and the frequencies of those papers

Source	Number of documents
Journal of Intelligent and Fuzzy Systems	47
Expert Systems with Applications	43
Kongzhi Yu Juece Control and Decision	33
Knowledge Based Systems	30
Applied Mathematical Modelling	27
Information Sciences	22
IEEE International Conference on Fuzzy Systems	21
International Journal of Uncertainty Fuzziness and Knowledge Based Systems	21
Advances in Information Sciences and Service Sciences	19
International Journal of Digital Content Technology and Its Applications	18

(continued)

Table 6 (continued)

Source	Number of documents
Mathematical Problems in Engineering	16
International Journal of Intelligent Systems	16
Fuzzy Optimization and Decision Making	16
IEEE Transactions on Fuzzy Systems	15
Journal of Applied Mathematics	13
Applied Mechanics and Materials	13
Journal of Computational Information Systems	12
Soft Computing	12
Journal of Convergence Information Technology	11
Xi Tong Gong Cheng Yu Dian Zi Ji Shu Systems Engineering and Electronics	11
Xitong Gongcheng Lilun Yu Shijian System Engineering Theory and Practice	11
International Journal of Computational Intelligence Systems	10
Journal of Information and Computational Science	10
International Journal of Advancements in Computing Technology	10
Communications in Computer and Information Science	10
Group Decision and Negotiation	9
Information Fusion	9
Journal of Systems Science and Systems Engineering	8
Technological and Economic Development of Economy	8

Figure 9 presents the authors most publishing IFDM papers.

Table 7 gives the affiliations of the authors publishing IFDM papers. Chinese universities again appear at the top of the table.

Figure 10 illustrates the source countries publishing IFDM papers. China is again by far the first country publishing IFDM papers.

Figure 11 gives classifies the IFDM papers with respect to their subject areas. Computer sciences (70.2 %), mathematics (40.3 %), engineering (44.4 %), and decision sciences (10.4 %) are the first four areas that IFDM papers come from.

2.4 Type-2 Fuzzy Decision-Making (T2FDM)

Zadeh [45] introduced type-2 fuzzy sets in 1975. A type-2 fuzzy set lets us incorporate uncertainty about the membership function into the fuzzy set theory. A type-2 fuzzy set $\tilde{A}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{A}}$ shown as follows [27]:

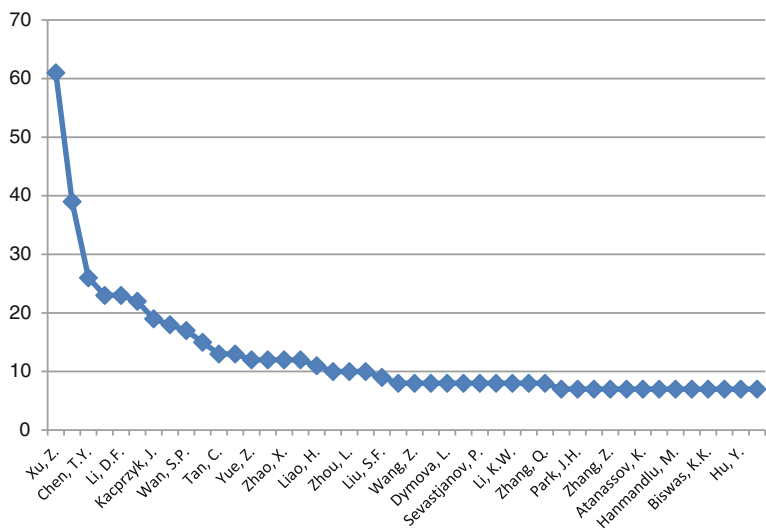


Fig. 9 Authors most publishing IFDM papers

Table 7 Affiliations of the authors publishing IFDM papers and frequencies

Affiliations	Number of papers
Central South University China	51
Chongqing University	51
Southeast University	44
PLA University of Science and Technology	32
Zhejiang University of Finance and Economics	28
Chang Gung University	25
Jiangxi University of Finance and Economics	24
Shanghai Jiaotong University	20
Systems Research Institute of the Polish Academy of Sciences	19
Hohai University	18
Sichuan University	18
Fuzhou University	18
Anhui University	17
Beijing Institute of Technology	17
Nanjing University of Aeronautics and Astronautics	14
Tsinghua University	14
Zhejiang Normal University	13
Beihang University	13
Guangdong University of Foreign Studies	12
Guangdong Ocean University	12
Hebei University	11

(continued)

Table 7 (continued)

Affiliations	Number of papers
Xiamen University	11
Harbin Institute of Technology	11
North China Electric Power University	11
Dalian University of Technology	10
Tianjin University	10
PLA Dalian Naval Academy	10
Zaozhuang University	10

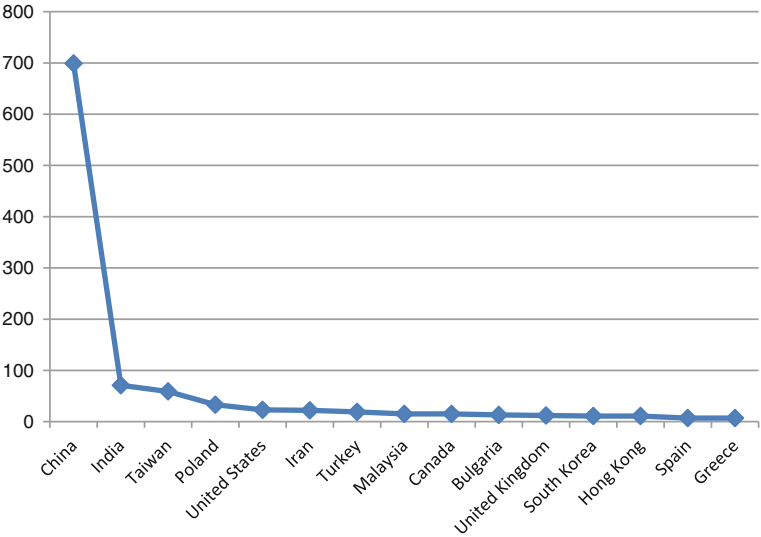


Fig. 10 Source countries publishing IFDM papers

$$\tilde{A} = \left\{ \left((x,u), \mu_{\tilde{A}}(x,u) \right) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{A}}(x,u) \leq 1 \right\} \tag{5}$$

where J_x denotes an interval $[0, 1]$. The type-2 fuzzy set $\tilde{A}$ also can be represented as follows [27]:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u) / (x,u), \tag{6}$$

where $J_x \subseteq [0, 1]$ and $\int \int$ denote union over all admissible x and u .

The search for T2FDM based on Scopus database in October 2015 yielded 229 papers. 124 of them are articles while 95 of them are conference papers. The rest are conference review, erratum, book chapter, book and article in press.

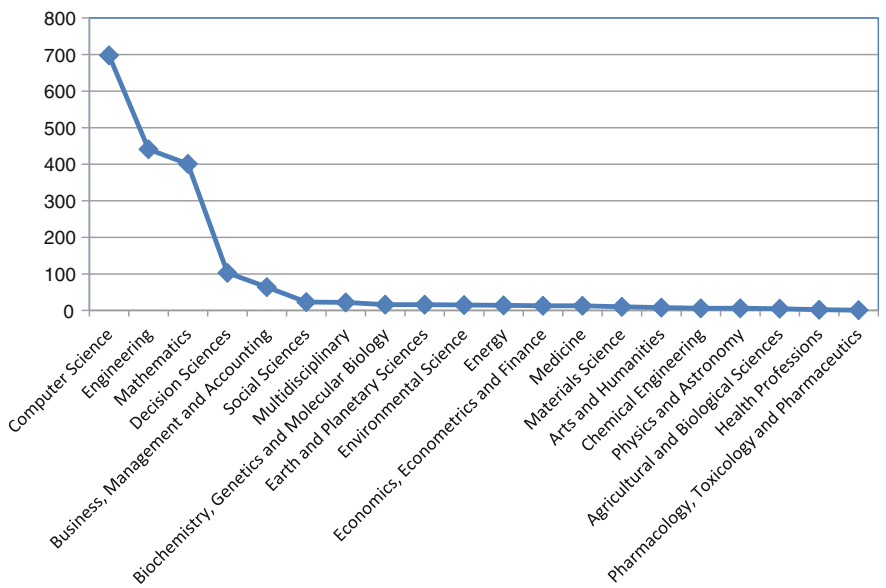


Fig. 11 IFDM papers with respect to their subject areas and frequencies

Figure 12 presents the publication frequencies of the papers on T2FDM with respect to years. As it is clearly seen from the graph, significant increases in the frequencies of T2FDM papers exist after the year 2007.

Table 8 shows the sources publishing T2FDM papers and the frequencies of those papers. *IEEE International Conference on Fuzzy Systems, Expert Systems with Applications, Information Sciences, Knowledge Based Systems, Soft Computing*, and *IEEE Transactions on Fuzzy Systems* are the leading journals publishing T2FDM papers.

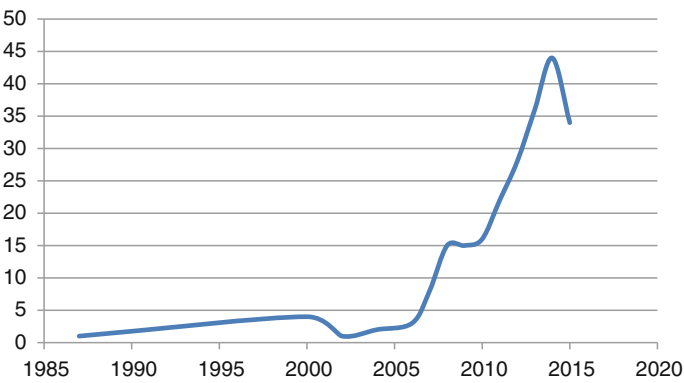


Fig. 12 Publication frequencies of the papers on T2FDM with respect to years

Table 8 Sources publishing T2FDM papers and the frequencies of those papers

Source	Number of documents
IEEE International Conference on Fuzzy Systems	21
Expert Systems with Applications	8
Information Sciences	7
Knowledge Based Systems	7
Soft Computing	7
IEEE Transactions on Fuzzy Systems	6
Annual Conference of the North American Fuzzy Information Processing Society NAFIPS	4
Computers and Industrial Engineering	3
International Journal of Computational Intelligence Systems	3
Studies in Computational Intelligence	3
Studies in Fuzziness and Soft Computing	3
Frontiers of Computer Science	2
International Journal of Electrical Power and Energy Systems	2
IEEE Transactions on Systems Man and Cybernetics Part B Cybernetics	2
International Journal of Information Technology and Decision Making	2
International Journal of Uncertainty Fuzziness and Knowledge Based Systems	2
Journal of Applied Mathematics	2
Journal of Industrial and Production Engineering	2
Journal of the Chinese Institute of Industrial Engineers	2
Advances in Soft Computing	2
Aip Conference Proceedings	2
International Journal of Advanced Manufacturing Technology	2
Engineering Applications of Artificial Intelligence	2
Communications in Computer and Information Science	2
Applied Mathematical Modelling	1
Applied Mathematical Sciences	1
Applied Mechanics and Materials	1
Arpn Journal of Engineering and Applied Sciences	1
Computers in Industry	1

Figure 13 presents the authors most publishing T2FDM papers.

Table 9 gives the affiliations of the authors publishing T2FDM papers.

Figure 14 illustrates the source countries publishing T2FDM papers. China and Taiwan are by far the first two countries publishing T2FDM papers.

Figure 15 gives classifies the T2FDM papers with respect to their subject areas. Computer sciences (78.7 %), engineering (41.7 %), mathematics (34.3 %), and decision sciences (8.3 %) are the first four areas that T2FDM papers come from.

Table 9 Affiliations of the authors publishing T2FDM papers and frequencies

Affiliations	Number of papers
Chang Gung University	14
Universiti Malaysia Terengganu	13
National Taiwan University of Science and Technology	12
University of Essex	9
Istanbul Technical University	9
University of Southern California	9
Hebei University	7
Tijuana Institute of Technology	6
Yildiz Technical University	6
Southeast University	5
Universidad Autonoma de Baja California, Campus Tijuana	5
Central South University China	5
De Montfort University	5
Jinwen University of Science and Technology	5
Amirkabir University of Technology	4
Politechnika Czesochowska	4
Deakin University	4
Hohai University	3
Selcuk University	3

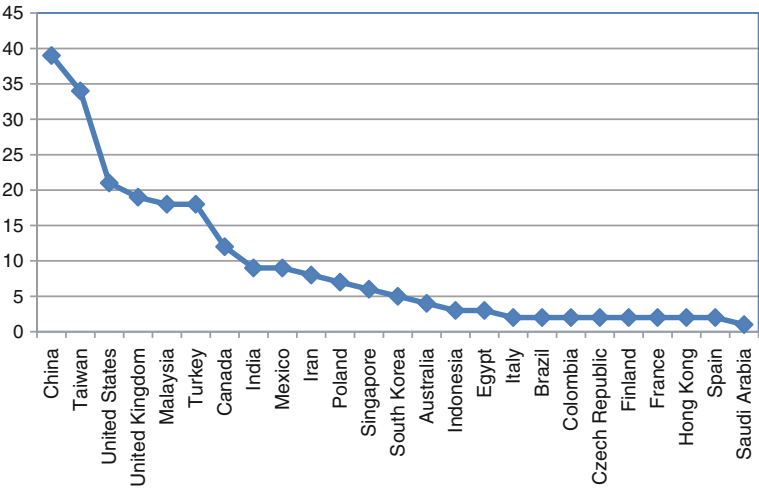


Fig. 14 Source countries publishing T2FDM papers

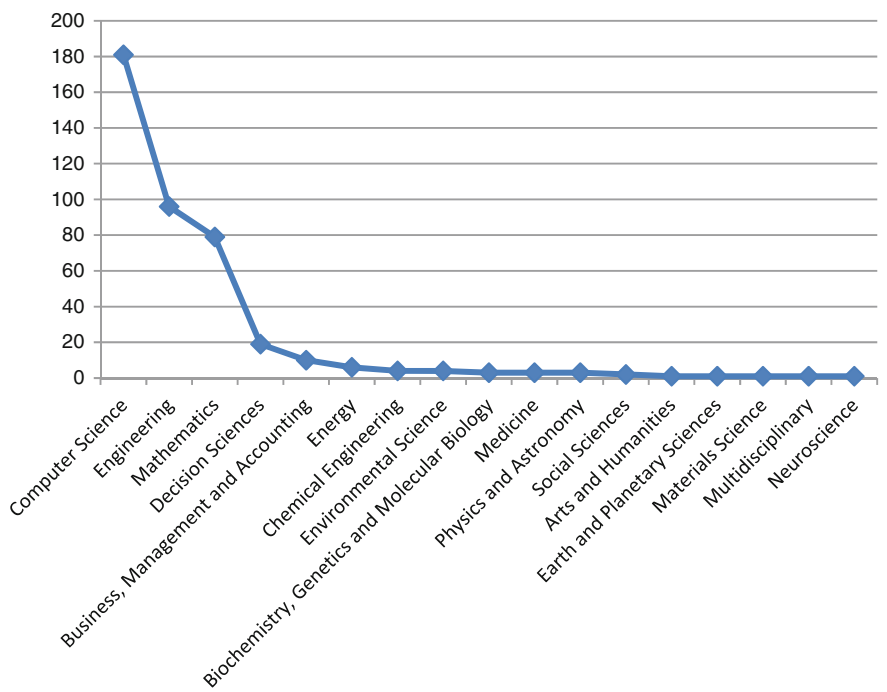


Fig. 15 T2FDM papers with respect to their subject areas and frequencies

Electric vehicles have been popular all around the world in recent years. Since one of the drawbacks of electric vehicles is the charge capacity of cars, locating charging stations has become a very important problem. Assume there are four possible locations and five criteria to be used for decision making. The criteria can be listed as follows:

- Destruction degree on vegetation and water (C1)
- Construction cost (C2)
- Annual operation and maintenance cost (C3)
- Traffic convenience (C4)
- Service capability (C5).

4.1 Charging Station Location Selection Using Ordinary Fuzzy TOPSIS

In Fuzzy TOPSIS approach the decision maker assigns importance weight for each criterion and score of each alternative with respect to the criteria. For the given problem the weights and scores are determined as shown in Table 13 [13].

Table 10 Journals publishing fuzzy decision making papers

Journal	Publisher	Number of fuzzy papers published
Fuzzy Sets and Systems	Elsevier	402
Expert Systems with Applications	Elsevier	381
IEEE International Conference on Fuzzy Systems	IEEE	314
Journal of Intelligent and Fuzzy Systems	IOS Press	177
Information Sciences	Elsevier	160
European Journal of Operational Research	Elsevier	130
Knowledge Based Systems	Elsevier	98
Applied Mathematical Modelling	Elsevier	83
Fuzzy Optimization and Decision Making	Springer	83
Computers and Industrial Engineering	Elsevier	74
International Journal of Uncertainty Fuzziness and Knowledge Based Systems	World Scientific	74
International Journal of Production Research	Taylor and Francis	71
International Journal of Advanced Manufacturing Technology	Springer	71
IEEE Transactions on Fuzzy Systems	IEEE	70
Soft Computing	Springer	64
International Journal of Intelligent Systems	John Wiley and Sons	58
Computers and Mathematics with Applications	Pergamon	52
Mathematical Problems in Engineering	Hindawi	51
International Journal of Computational Intelligence Systems	Atlantis	38
International Journal of Production Economics	Elsevier	38
Journal of Applied Mathematics	Hindawi	35
International Journal of Fuzzy Systems	Chinese Fuzzy Systems Association	35
Journal of Multiple Valued Logic and Soft Computing	Oldcity	34
International Journal of Approximate Reasoning	Elsevier	31
International Journal of Information Technology and Decision Making	World Scientific	30
International Journal of General Systems	Taylor and Francis	28
Engineering Applications of Artificial Intelligence	Elsevier	28
Group Decision and Negotiation	Springer	28
Technological and Economic Development of Economy	Taylor and Francis	27

Table 11 Fuzzy decision making books

Book	Editor and publisher	Tools and techniques
Fuzzy sets, decision making, and expert systems	Zimmermann [47, 48], Springer	Individual decision making in fuzzy environments, multi-person decision making in fuzzy environments, multi-criteria decision making in ill-structured situations
Combining fuzzy imprecision with probabilistic uncertainty in decision making	Kacprzyk and Fedrizzi [17], Springer	Decision making in a probabilistic fuzzy environment, fuzzy statistical decision making, decision evaluation methods under uncertainty and imprecision
Multiperson decision making models using fuzzy sets and possibility theory	Kacprzyk and Fedrizzi [18], Springer	Multiperson decision making models using fuzzy sets and possibility theory
Fuzzy multiple objective decision making: methods and applications	Lai and Hwang [22], Springer	Multiple objective decision making, fuzzy multiple objective decision making, possibilistic multiple objective decision making
Fuzzy sets and fuzzy Decision-Making	Li and Yen [24], CRC Press	Multifactorial decision-making, additive standard multifactorial functions
Fuzzy logic for planning and decision making	Loostma [25], Springer	Fuzzy logic in operations research, fuzzy PERT, fuzzy SMART, fuzzy multi-objective optimization, fuzzy AHP
Fuzzy and multi-level decision making: an interactive computational approach	Lee and Shih [23], Springer	Fuzzy interactive multi-level decision making, possibility programming, aggregation of fuzzy systems in multi-level decisions
Dynamical aspects in fuzzy decision making	Yoshida [43], Physica-Verlag	Fuzzy dynamic programming, fuzzy portfolio model, interactive fuzzy programming, fuzzy optimization
Fuzzy reasoning in decision making and optimization	Carlsson and Fullér [9], Physica-Verlag	Fuzzy multicriteria decision making, fuzzy reasoning and decision making
Fuzzy decision making in modeling and control	Sousa and Kaymak [33], World Scientific	Model-based control with fuzzy decision functions, fuzzy decision functions, Fuzzy aggregated membership control
Multi-objective group decision making: methods, software and applications with fuzzy set techniques	Lu et al. [26], Imperial College Press	Fuzzy multi-objective decision making, fuzzy multi-objective DSS, fuzzy group decision making

(continued)

Table 11 (continued)

Book	Editor and publisher	Tools and techniques
Fuzzy sets and their extensions: representation, aggregation and models intelligent systems from decision making to data mining	Bustince et al. [8], Springer	Aggregation operators, voting systems, linguistic decision based model, group decision making
Fuzzy multi-criteria decision making: theory and applications with recent developments	Kahraman [21], Springer	Fuzzy AHP, fuzzy outranking methods, fuzzy TOPSIS, fuzzy multi-objective decision making, fuzzy geometric programming
Fuzzy multicriteria decision-making: models, methods and applications	Pedrycz et al. [29], John Wiley & Sons	Discrete models of multicriteria decision-making, fuzzy group decision making models
Fuzzy-like multiple objective decision making	Xu and Zhou [38], Springer	Fuzzy random multiple objective decision making, fuzzy-like multiple objective decision making, bifuzzy multiple objective decision making
Decision making in the manufacturing environment using graph theory and fuzzy multiple attribute decision making methods, vol. 2	Rao [30], Springer	Fuzzy ELECTRE, fuzzy PROMETHEE, fuzzy COPRAS, fuzzy VIKOR, fuzzy AHP
Intuitionistic fuzzy information aggregation: theory and applications	Xu and Cai [39], Springer	Intuitionistic preference relations, dynamic intuitionistic fuzzy multi-attribute decision making, nonlinear optimization models for multi-attribute group decision making
Fuzzy multiple objective decision making	Tzeng and Huang [36], Chapman and Hall/CRC	multi-objective evolutionary algorithms, network data envelopment analysis, fuzzy goal programming, multi-level multi-objective programming
Decision theory with imperfect information	Aliev and Huseynov [1], World Scientific	Hierarchical models for decision making, behavioral decision making with combined states, decision making under unprecisiated imperfect information

The scale used in this approach is given in Table 14.

The fuzzy values given in Table 14 are used to transform the linguistic evaluations into fuzzy numbers as given in Table 15.

The scores of the alternatives are then normalized to find the normalized scores (Table 16).

Table 12 Conferences accepting fuzzy decision making papers

Conference	Organizing institutes	Number of decision making papers presented
IEEE International Conference on Fuzzy Systems (FUZZIEEE)	IEEE	423
Annual Conference of the North American Fuzzy Information Processing Society (NAFIPS)	NAFIPS	136
IEEE International Conference on Systems Man and Cybernetics	IEEE	91
Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT)	EUSFLAT	82
International Federation of Automatic Control Conference (IFAC)	IFAC	68
World Congress of the International Fuzzy Systems Association (IFSA)	IFSA	61
International Conference on Machine Learning and Cybernetics (ICMLC)	ICMLC	44
World Congress on Intelligent Control and Automation (WCICA)	WCICA	43
Computers and Industrial Engineering (CIE) Conference	CIE	34
Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU)	IPMU	33
International Federation for Information Processing (IFIP) Advances in Information and Communication Technology	IFIP	25
Joint Conference on Information Sciences	INFORMS	24
Fuzzy Logic in Nuclear Science (FLINS)	FLINS	22
International Joint Conference on Neural Networks, International Neural Network Society (INNS)	INNS	20
Industrial Electronics Conference	IEEE	15
Canadian Conference on Electrical and Computer Engineering	IEEE	14
ASME Design Engineering Technical Conference	ASME	13
IEEE International Symposium on Multiple-Valued Logic	IEEE	13
Annual Meeting of the Decision Sciences Institute (DSI)	DSI	12
IEEE Conference on Decision and Control	IEEE	12
IEEE International Symposium on Intelligent Control	IEEE	12
International IEEE Conference Intelligent Systems	IEEE	12
IEEE International Conference on Neural Networks Conference	IEEE	11

(continued)

Table 12 (continued)

Conference	Organizing institutes	Number of decision making papers presented
IEEE International Engineering Management Conference	IEEE	11
IEEE International Conference on Intelligent Robots and Systems	IEEE	10
IEEE Conference on Intelligent Transportation Systems Proceedings ITSC	IEEE	9
International Conference on Intelligent Systems and Knowledge Engineering (ISKE)	ISKE	9
Society of Instrument and Control Engineers (SICE) Annual Conference	SICE	9
Association for Computing Machinery (ACM) Symposium on Applied Computing	ACM	8
IEEE International Conference on Industrial Technology	IEEE	8

Table 13 Importance of each criterion and scores of each alternative with respect to the criteria

Criteria	Importance	Scores			
		A1	A2	A3	A4
C1	M	MG	MG	MP	G
C2	VH	MG	P	MG	G
C3	VH	G	P	VG	MP
C4	M	F	MP	MP	G
C5	H	F	MG	F	MG

Table 14 Linguistic scales used for fuzzy TOPSIS

Linguistic variables for importance	Fuzzy values	Linguistic variables for evaluations	Fuzzy values
Very low (VL)	(0, 0, 0.1)	Very poor (VP)	(0, 0, 1)
Low (L)	(0, 0.1, 0.3)	Poor (P)	(0, 1, 3)
Medium low (ML)	(0.1, 0.3, 0.5)	Medium poor (MP)	(1, 3, 5)
Medium (M)	(0.3, 0.5, 0.7)	Fair (F)	(3, 5, 7)
Medium high (MH)	(0.5, 0.7, 0.9)	Medium good (MG)	(5, 7, 9)
High (H)	(0.7, 0.9, 1)	Good (G)	(7, 9, 10)
Very high (VH)	(0.9, 1, 1)	Very good (VG)	(9, 10)

Table 15 Triangular fuzzy importance and scores

Criteria	Importance	Scores			
		A1	A2	A3	A4
C1	(0.3, 0.5, 0.7)	(7, 9, 10)	(1, 3, 5)	(7, 9, 10)	(3, 5, 7)
C2	(0.9, 1, 1)	(5, 7, 9)	(0,1,3)	(5, 7, 9)	(7, 9, 10)
C3	(0.9, 1, 1)	(7, 9, 10)	(3, 5, 7)	(9, 10)	(5, 7, 9)
C4	(0.3, 0.5, 0.7)	(7, 9, 10)	(5, 7, 9)	(7, 9, 10)	(7, 9, 10)
C5	(0.7, 0.9, 1)	(3, 5, 7)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)

Table 16 Normalized scores

	A1	A2	A3	A4
C1	(0.7, 0.9, 1)	(0.1, 0.3, 0.5)	(0.7, 0.9, 1)	(0.3, 0.5, 0.7)
C2	(0.5, 0.7, 0.9)	(0, 0.1, 0.3)	(0.5, 0.7, 0.9)	(0.7, 0.9, 1)
C3	(0.7, 0.9, 1)	(0.3, 0.5, 0.7)	(0.9, 1, 1)	(0.5, 0.7, 0.9)
C4	(0.7, 0.9, 1)	(0.5, 0.7, 0.9)	(0.7, 0.9, 1)	(0.7, 0.9, 1)
C5	(0.33, 0.56, 0.78)	(0.56, 0.78, 1)	(0.33, 0.56, 0.78)	(0.33, 0.56, 0.78)

Then the fuzzy importance weights are multiplied by the normalized scores in order to find the weighted normalized scores as in Table 17.

Using the values given in Table 17, the positive ideal solution (PIS) and negative ideal solution (NIS) are determined. Using these values, the distances to PIS and NIS and closeness coefficients are calculated as shown in Table 18.

Table 17 Weighted normalized scores

	A1	A2	A3	A4
C1	(0.21, 0.45, 0.7)	(0.03, 0.15, 0.35)	(0.21, 0.45, 0.7)	(0.09, 0.25, 0.49)
C2	(0.45, 0.7, 0.9)	(0, 0.1, 0.3)	(0.45, 0.7, 0.9)	(0.63, 0.9, 1)
C3	(0.63, 0.9, 1)	(0.27, 0.5, 0.7)	(0.81, 1, 1)	(0.45, 0.7, 0.9)
C4	(0.21, 0.45, 0.7)	(0.15, 0.35, 0.63)	(0.21, 0.45, 0.7)	(0.21, 0.45, 0.7)
C5	(0.23, 0.5, 0.78)	(0.39, 0.7,1)	(0.23, 0.5, 0.78)	(0.23, 0.5, 0.78)

Table 18 Distance measures and closeness coefficients

	d^-	d^+	CC_i	Ranking
A1	3.108	2.297	0.575	2
A2	2.089	3.295	0.387	4
A3	3.191	2.185	0.593	1
A4	2.934	2.457	0.544	3

4.2 Charging Station Location Selection Using HFLTS

When the same problem is handled using HFLTS, the input data contain linguistic operators such as lower than, between, higher than etc. For this problem, the pairwise comparison matrices are determined as given in Tables 19, 20, 21, 22, 23 and 24 [41].

Table 19 Pairwise comparison of the criteria using HFLTS

	C1	C2	C3	C4	C5
C1	–	Lower than very low	Is low	Is medium	Between low and medium
C2	Between high and absolute	–	Is high	Greater than high	Between medium and high
C3	Is vh	Is low	–	Between medium and high	Is high
C4	Is medium	Lower than low	Between medium and low	–	Is medium
C5	Between low and high	At most high	Is low	Is medium	–

Table 20 Pairwise comparison of the alternatives with respect to C1

	A1	A2	A3	A4
A1	–	Is medium	Between medium and vh	Greater than high
A2	Is medium	–	Between low and medium	Between high and vh
A3	Between very low and medium	Between medium and high	–	At most high
A4	Lower than low	Between low and very low	At most low	–

Table 21 Pairwise comparison of the alternatives with respect to C2

	A1	A2	A3	A4
A1	–	Between low and medium	Between low and medium	Is low
A2	Between very low and low	–	Between very low and low	Lower than very low
A3	Between medium and high	Between high and vh	–	Is low
A4	Is high	Greater than vh	Is medium	–

Table 22 Pairwise comparison of the alternatives with respect to C3

	A1	A2	A3	A4
A1	–	Between high and vh	Between low and medium	Greater than vh
A2	Between very low and low	–	At least vh	Is high
A3	Between medium and high	At most very low	–	Is absolute
A4	Lower than very low	Is low	Is none	–

Table 23 Pairwise comparison of the alternatives with respect to C4

	A1	A2	A3	A4
A1	–	Between high and vh	Is high	Between medium and low
A2	Between very low and low	–	Is medium	Is very low
A3	Is low	Is medium	–	Between very low and low
A4	Between medium and high	Is vh	Between high and vh	–

Table 24 Pairwise comparison of the alternatives with respect to C5

	A1	A2	A3	A4
A1	–	Between low and medium	Is medium	Is medium
A2	Between medium and high	–	Between medium and vh	Between medium and high
A3	Is medium	Between very low and medium	–	Between low and medium
A4	Is medium	Between low and medium	Between medium and high	–

Due to the page constraints only sample calculations are provided here. Using Table 19, the pessimistic and optimistic preference matrices are provided as given in Table 25.

Using the scale given in Table 26, the linguistic intervals are converted to numerical intervals.

In order to calculate the linguistic intervals given in Table T9, the mean value of each row in pessimistic and optimistic preference matrices is calculated. For example, in the interval value $[(1, -0.25), (1, -0.0)]$, the lower bound is determined

Table 25 Pessimistic and optimistic preference matrices

Pessimistic collective preference					
	C1	C2	C3	C4	C5
C1	–	(n, –0.0)	(l, –0.0)	(m, –0.0)	(l, –0.0)
C2	(h, –0.0)	–	(h, –0.0)	(vh, –0.0)	(m, –0.0)
C3	(vh, –0.0)	(l, –0.0)	–	(m, –0.0)	(h, –0.0)
C4	(m, –0.0)	(n, –0.0)	(l, –0.0)	–	(m, –0.0)
C5	(m, –0.33)	(n, –0.0)	(l, –0.0)	(m, –0.0)	–
Optimistic collective preference					
	C1	C2	C3	C4	C5
C1	–	(n, –0.0)	(l, –0.0)	(m, –0.0)	(m, –0.0)
C2	(a, –0.0)	–	(h, –0.0)	(a, –0.0)	(h, –0.0)
C3	(vh, –0.0)	(l, –0.0)	–	(h, –0.0)	(h, –0.0)
C4	(m, –0.0)	(vl, –0.0)	(m, –0.0)	–	(m, –0.0)
C5	(h, –0.0)	(h, –0.0)	(l, –0.0)	(m, –0.0)	–

Table 26 The scale for the linguistic terms

Ni	vli	li	mi	hi	vhi	ai
0	1	2	3	4	5	6

by calculating the mean values of (n, –0.0), (l, –0.0), (m, –0.0), and (l, –0.0). In a similar way, the upper bound is the mean of the first line of optimistic preference matrix which is (n, –0.0), (l, –0.0), (m, –0.0), and (m, –0.0). Table 27 shows the interval utilities, midpoint values and weights of each criterion.

After linguistic intervals are calculated, they are transformed to interval utilities. By getting the mean of the upper and lower bounds of the interval, the midpoints are calculated. Finally, the midpoint values are normalized and the weights of each criterion are determined.

Table 27 Interval utilities, midpoint values and weights of each criterion

Criteria	Linguistic intervals	Interval utilities	Midpoints	Weights
C1	[(l, –0.25), (l, –0.0)]	[1.75, 2]	1.87	0.13
C2	[(h, –0.0), (vh, –0.0)]	[4, 5]	4.5	0.30
C3	[(h, –0.50), (h, –0.25)]	[3.5, 3.75]	3.62	0.24
C4	[(l, –0.0), (l, 0.50)]	[2, 2.5]	2.25	0.15
C5	[(l, –0.8), (m, 0.25)]	[1.92, 3.25]	2.58	0.17

Table 28 Linguistic intervals and weights of the criteria

	Wrt C1	Wrt C2	Wrt C3	Wrt C4	Wrt C5	Weighted score	Rank
Weights	0.13	0.30	0.24	0.15	0.17		
A1	0.38	0.24	0.36	0.31	0.24	0.297	1
A2	0.30	0.09	0.31	0.15	0.31	0.216	4
A3	0.23	0.29	0.28	0.18	0.21	0.249	2
A4	0.09	0.38	0.06	0.36	0.25	0.238	3

The process is repeated for the other matrices and the weights of the alternatives with respect each criterion are determined as shown in Table 28. In order to find the ranking of the alternatives, the weighted scores are calculated as in Table 28.

4.3 Charging Station Location Selection Using Pairwise Linguistic Evaluations Using IVIFS

This approach is based on pairwise linguistic comparison of the criteria and alternatives. As the evaluations are made using linguistic terms, they are transformed to IVIFS and the steps of the methodology are applied. The details of the methodology we apply can be obtained from Onar et al. [10]. The linguistic terms and related IVIFS numbers are listed in Table 29.

In accordance with the problem, the pairwise comparison matrices are formed and filled by the decision maker. Table 30 shows the pairwise comparison of the criteria and Table 31 shows the pairwise comparison of each alternative with respect to the criteria.

Due to the page constraints, we only give the details of the calculations for criteria weights. Table 32 gives the pairwise comparisons of the criteria expressed as IVIFS.

Table 29 Linguistic scale and its corresponding IVIFS

Linguistic terms		Membership
Absolutely low (AL)	AL	([0.10, 0.25], [0.65, 0.75])
Very low (VL)	VL	([0.15, 0.30], [0.60, 0.70])
Low (L)	L	([0.20, 0.35], [0.55, 0.65])
Medium low (ML)	ML	([0.25, 0.4], [0.50, 0.60])
Equal (E)	E	([0.45, 0.55], [0.30, 0.45])
Medium high (MH)	MH	([0.50, 0.60], [0.25, 0.40])
High (H)	H	([0.55, 0.65], [0.20, 0.35])
Very high (VH)	VH	([0.60, 0.70], [0.15, 0.30])
Absolutely high (AH)	AH	([0.65, 0.75], [0.10, 0.25])
Exactly equal	EE	([0.5, 0.5], [0.5, 0.5])

Table 30 Pairwise comparison of the criteria

Wrt goal	C1	C2	C3	C4	C5
C1	EE	AL	L	EE	ML
C2		EE	H	VH	H
C3			EE	H	H
C4				EE	EE
C5					EE

Table 31 Pairwise comparison of the alternatives with respect to the criteria

Wrt C1	A1	A2	A3	A4
A1	EE	E	VH	ML
A2		EE	VH	ML
A3			EE	AL
A4				EE
Wrt C2	A1	A2	A3	A4
A1	EE	H	EE	ML
A2		EE	VL	L
A3			EE	MH
A4				EE
Wrt C3	A1	A2	A3	A4
A1	EE	VH	ML	AH
A2		EE	AL	ML
A3			EE	AH
A4				EE
Wrt C4	A1	A2	A3	A4
A1	EE	MH	MH	ML
A2		EE	EE	VL
A3			EE	MH
A4				EE
Wrt C5	A1	A2	A3	A4
A1	EE	ML	EE	ML
A2		EE	MH	EE
A3			EE	MH
A4				EE

According to the steps given in Onar et al. [10], the scores of the IVIFS values are calculated using score function, as shown in Table 33.

Then, the interval multiplicative matrix $\tilde{A} = (\tilde{a}_{ij})_{n \times n} = \left[10^{\left(\mu_{g_{ij}}^- - v_{g_{ij}}^+ \right)}, 10^{\left(\mu_{g_{ij}}^+ - v_{g_{ij}}^- \right)} \right]$ is calculated as given in Table 34.

Table 32 Pairwise comparisons of the criteria expressed as IVIFS

	C1	C2	C3	C4	C5
C1	$\langle [0, 5, 0, 5], [0, 5, 0, 5] \rangle$	$\langle [0, 1, 0, 25], [0, 65, 0, 75] \rangle$	$\langle [0, 2, 0, 35], [0, 55, 0, 65] \rangle$	$\langle [0, 5, 0, 5], [0, 5, 0, 5] \rangle$	$\langle [0, 25, 0, 4], [0, 5, 0, 6] \rangle$
C2	$\langle [0, 65, 0, 75], [0, 1, 0, 25] \rangle$	$\langle [0, 5, 0, 5], [0, 5, 0, 5] \rangle$	$\langle [0, 55, 0, 65], [0, 2, 0, 35] \rangle$	$\langle [0, 6, 0, 7], [0, 15, 0, 3] \rangle$	$\langle [0, 55, 0, 65], [0, 2, 0, 35] \rangle$
C3	$\langle [0, 55, 0, 65], [0, 2, 0, 35] \rangle$	$\langle [0, 2, 0, 35], [0, 55, 0, 65] \rangle$	$\langle [0, 5, 0, 5], [0, 5, 0, 5] \rangle$	$\langle [0, 55, 0, 65], [0, 2, 0, 35] \rangle$	$\langle [0, 55, 0, 65], [0, 2, 0, 35] \rangle$
C4	$\langle [0, 5, 0, 5], [0, 5, 0, 5] \rangle$	$\langle [0, 15, 0, 3], [0, 6, 0, 7] \rangle$	$\langle [0, 2, 0, 35], [0, 55, 0, 65] \rangle$	$\langle [0, 5, 0, 5], [0, 5, 0, 5] \rangle$	$\langle [0, 5, 0, 5], [0, 5, 0, 5] \rangle$
C5	$\langle [0, 5, 0, 6], [0, 25, 0, 4] \rangle$	$\langle [0, 2, 0, 35], [0, 55, 0, 65] \rangle$	$\langle [0, 2, 0, 35], [0, 55, 0, 65] \rangle$	$\langle [0, 5, 0, 5], [0, 5, 0, 5] \rangle$	$\langle [0, 5, 0, 5], [0, 5, 0, 5] \rangle$

Table 33 Score judgement matrix

	C1	C2	C3	C4	C5
C1	[0, 0]	[-0.65, -0.4]	[-0.45, -0.2]	[0, 0]	[-0.35, -0.1]
C2	[0.4, 0.65]	[0, 0]	[0.2, 0.45]	[0.3, 0.55]	[0.2, 0.45]
C3	[0.2, 0.45]	[-0.45, -0.2]	[0, 0]	[0.2, 0.45]	[0.2, 0.45]
C4	[0, 0]	[-0.55, -0.3]	[-0.45, -0.2]	[0, 0]	[0, 0]
C5	[0.1, 0.35]	[-0.45, -0.2]	[-0.45, -0.2]	[0, 0]	[0, 0]

Table 34 Interval multiplicative matrix

	C1	C2	C3	C4	C5
C1	[1, 1]	[0.22, 0.4]	[0.35, 0.63]	[1, 1]	[0.45, 0.79]
C2	[2.51, 4.47]	[1, 1]	[1.58, 2.82]	[2, 3.55]	[1.58, 2.82]
C3	[1.58, 2.82]	[0.35, 0.63]	[1, 1]	[1.58, 2.82]	[1.58, 2.82]
C4	[1, 1]	[0.28, 0.5]	[0.35, 0.63]	[1, 1]	[1, 1]
C5	[1.26, 2.24]	[0.35, 0.63]	[0.35, 0.63]	[1, 1]	[1, 1]

In the next step, the priority vector of each criterion is calculated by using interval multiplicative matrix as in Table 35.

Possibility degree matrix is formed as given in Table Y8 and the weights of the criteria are determined.

The same steps are applied to the pairwise comparison matrices given in Table 36 and weights of the alternatives with respect to the criteria are determined. In order to find the overall rankings the weighted score of each alternative is calculated as in Table 37.

Table 35 Priority vectors

Criteria	Priority vector
C1	[0.079, 0.15]
C2	[0.227, 0.576]
C3	[0.16, 0.397]
C4	[0.095, 0.163]
C5	[0.104, 0.216]

Table 36 Possibility degree matrix and weights of the criteria

	C1	C2	C3	C4	C5	Weight
C1	0.50	0.00	0.00	0.40	0.25	0.133
C2	1.00	0.50	0.71	1.00	1.00	0.286
C3	1.00	0.29	0.50	0.99	0.84	0.256
C4	0.60	0.00	0.01	0.50	0.33	0.147
C5	0.75	0.00	0.16	0.67	0.50	0.179

Table 37 Weighted scores of the alternatives

Alternatives	wrt C1	wrt C2	wrt C3	wrt C4	wrt C5	Weighted Score	Ranking
Weights	0.13	0.29	0.26	0.15	0.18		
A1	0.28	0.26	0.32	0.28	0.17	0.266	3
A2	0.28	0.13	0.15	0.15	0.32	0.190	4
A3	0.13	0.32	0.34	0.25	0.26	0.277	1
A4	0.32	0.29	0.19	0.32	0.26	0.268	2

4.4 Charging Station Location Selection Using Interval Type-2 Fuzzy AHP

In this section, the same charging station location selection problem is solved by Interval Type-2 Fuzzy AHP method proposed by Kahraman et al. [19]. The linguistic scale used in this method is given in Table 38.

The evaluations given in Sect. 4.3 can be converted to the scale given in Table 38. The pairwise comparison matrix for the criteria is given in Table 39. The other comparison matrices are not given because of space constraints.

According to the steps provided by Kahraman et al. [19], the geometric mean for each row in Table 39 is calculated. The calculated mean values are given in Table 40.

Fuzzy weight of each criterion is determined by dividing the means given in Table 40 to the sum of all means. Interval Type-2 fuzzy weights of each criterion is given in Table 41. These values are later defuzzified and normalized in order to find the final crisp weights of the criteria.

Table 38 Linguistic scale used in interval type-2 fuzzy method

Linguistic variables	Trapezoidal interval type-2 fuzzy sets
Absolute (A)	[(7, 8, 9, 9; 1,1), (7.2, 8.2, 8.8, 8.9; 0.8, 0.8)]
Very strong (VS)	[(5, 6, 8, 9; 1, 1), (5.2, 6.2, 7.8, 8.8; 0.8, 0.8)]
Fairly strong (FS)	[(3, 4, 6, 7; 1, 1), (3.2, 4.2, 5.8, 6.8; 0.8, 0.8)]
Weak (W)	[(1, 2, 4, 5; 1, 1), (1.2, 2.2, 3.8, 4.8; 0.8, 0.8)]
Equal (E)	[(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]
If a factor i has one of the above linguistic variables assigned to it when compared with factor j. The j has the reciprocal value when compared with i	Reciprocals of above

Table 39 Pairwise comparisons expressed as type-2 fuzzy sets

	C1	C2	C3	C4	C5
C1	[(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]	[(0.14, 0.16, 0.25, 0.33; 1, 1), (0.14, 0.17, 0.23, 0.31; 0.8, 0.8)]	[(0.2, 0.25, 0.5, 1; 1, 1), (0.20, 0.26, 0.45, 0.83; 0.8, 0.8)]	[(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]	[(0.2, 0.25, 0.5, 1; 1, 1), (0.20, 0.26, 0.45, 0.83; 0.8, 0.8)]
C2	[(3, 4, 6, 7; 1, 1), (3.2, 4.2, 5.8, 6.8; 0.8, 0.8)]	[(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]	[(3, 4, 6, 7; 1, 1), (3.2, 4.2, 5.8, 6.8; 0.8, 0.8)]	[(3, 4, 6, 7; 1, 1), (3.2, 4.2, 5.8, 6.8; 0.8, 0.8)]	[(1, 2, 4, 5; 1, 1), (1.2, 2.2, 3.8, 4.8; 0.8, 0.8)]
C3	[(1, 2, 4, 5; 1, 1), (1.2, 2.2, 3.8, 4.8; 0.8, 0.8)]	[(0.14, 0.16, 0.25, 0.33; 1, 1), (0.14, 0.17, 0.23, 0.31; 0.8, 0.8)]	[(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]	[(1, 2, 4, 5; 1, 1), (1.2, 2.2, 3.8, 4.8; 0.8, 0.8)]	[(1, 2, 4, 5; 1, 1), (1.2, 2.2, 3.8, 4.8; 0.8, 0.8)]
C4	[(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]	[(0.14, 0.16, 0.25, 0.33; 1, 1), (0.14, 0.17, 0.23, 0.31; 0.8, 0.8)]	[(0.2, 0.25, 0.5, 1; 1, 1), (0.20, 0.26, 0.45, 0.83; 0.8, 0.8)]	[(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]	[(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]
C5	[(1, 2, 4, 5; 1, 1), (1.2, 2.2, 3.8, 4.8; 0.8, 0.8)]	[(0.2, 0.25, 0.5, 1; 1, 1), (0.20, 0.26, 0.45, 0.83; 0.8, 0.8)]	[(0.2, 0.25, 0.5, 1; 1, 1), (0.20, 0.26, 0.45, 0.83; 0.8, 0.8)]	[(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]	[(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]

The same steps are applied to the other matrices for comparing the alternatives with respect to each criterion. The resulting weights of the alternatives are given in Table 42. In order to rank the alternatives, the weighted scores are calculated.

Table 40 Geometric mean for each criterion row

Criteria	Means
C1	[(0.35, 0.40, 0.57, 0.80; 1, 1), (0.36, 0.41, 0.54, 0.73; 0.8, 0.8)]
C2	[(1.93, 2.63, 3.86, 4.43; 1, 1), (2.08, 2.76, 3.74, 4.32; 0.8, 0.8)]
C3	[(0.67, 1.05, 1.74, 2.10; 1, 1), (0.76, 1.12, 1.67, 2.03; 0.8, 0.8)]
C4	[(0.49, 0.52, 0.65, 0.80; 1, 1), (0.49, 0.53, 0.64, 0.76; 0.8, 0.8)]
C5	[(0.52, 0.65, 1, 1.37; 1, 1), (0.55, 0.68, 0.95, 1.27; 0.8, 0.8)]
Total	[(3.98, 5.28, 7.84, 9.52; 1, 1), (4.26, 5.53, 7.56, 9.12; 0.8, 0.8)]

Table 41 Fuzzy, defuzzified and normalized weights of the criteria

	Interval type-2 fuzzy weights	Defuzzified weights	Normalized weights
C1	[(0.037, 0.051, 0.10, 0.20; 1, 1), (0.039, 0.054, 0.098, 0.17; 0.8, 0.8)]	0.0918	0.079
C2	[(0.20, 0.33, 0.73, 1.11; 1, 1), (0.22, 0.36, 0.67, 1.01; 0.8, 0.8)]	0.5577	0.482
C3	[(0.07, 0.13, 0.32, 0.52; 1, 1), (0.083, 0.14, 0.30, 0.47; 0.8, 0.8)]	0.2482	0.215
C4	[(0.051, 0.067, 0.12, 0.20; 1, 1), (0.054, 0.07, 0.11, 0.17; 0.8, 0.8)]	0.1036	0.090
C5	[(0.055, 0.084, 0.18, 0.34; 1, 1), (0.060, 0.0907, 0.17, 0.29; 0.8, 0.8)]	0.1555	0.134

Table 42 Weights and ranking of the alternatives using Interval Type-2 Fuzzy sets

	wrt C1	wrt C2	wrt C3	wrt C4	wrt C5	Weighted score	Ranking
Weights	0.08	0.48	0.21	0.09	0.13		
A1	0.23	0.23	0.34	0.31	0.15	0.249	3
A2	0.23	0.06	0.04	0.10	0.38	0.115	4
A3	0.04	0.40	0.55	0.22	0.24	0.366	1
A4	0.51	0.31	0.07	0.37	0.24	0.271	2

4.5 Comparison of the Four Fuzzy Methods

Table 43 shows the rankings of alternatives with respect to four methods given in Sects. 4.1–4.4. The alternative A3 is the best alternative with respect to the other three methods except HFLTS based method. The IVIFS based and Type-2 Fuzzy based methods yield the same rankings since these two methods use pairwise comparison among criteria and alternatives. The alternative A2 is the worst alternative with respect to all methods.

Since each method is based on a different definition of fuzzy sets, the observation of differences in the rankings is quite expected. The methods based on the

Table 43 Rankings of alternatives with respect to four methods

Methods	Alternative ranking
Location selection using ordinary fuzzy TOPSIS	$A3 > A1 > A4 > A2$
Location selection using HFLTS	$A1 > A3 > A4 > A2$
Location selection using pairwise evaluations based IVIFS	$A3 > A4 > A1 > A2$
Location selection using interval type-2 fuzzy AHP	$A3 > A4 > A1 > A2$

extensions of ordinary fuzzy sets should be considered as more reliable since they enable a better and detailed representation of the membership functions.

5 Conclusion

Decision making is an art and a science aiming at maximizing the expected value of our decision. Almost all of real world decision making problems emerge under risk and uncertainty conditions. If sufficient data exist to be able to obtain the probability distribution of decision making parameters, probability based risk models should be preferred. If uncertainty conditions exist without sufficient data, or if the past data are not reliable, or if the expert evaluations in the past are linguistic rather than exact numerical values, then the classical risk and uncertainty models cannot process these types of data. The fuzzy set theory can capture these uncertainties since it can work with vague, incomplete and imprecise data.

Fuzzy decision making has covered ground since Bellman and Zadeh [6] first work. This chapter showed which researchers, universities, and countries most contributed to it in this period. Besides, we examined the position of decision making within the extensions of ordinary fuzzy sets. Even the extensions of ordinary fuzzy sets can be divided into interval-valued fuzzy sets, hesitant fuzzy sets, intuitionistic fuzzy sets, type-2 fuzzy sets, fuzzy multisets, and stationary fuzzy sets, we illustrated its position in hesitant fuzzy sets, intuitionistic fuzzy sets, and type-2 fuzzy sets since these extensions have been much more used with respect to the other extensions.

For further research, this work can be expanded by examining the position of fuzzy decision making in the other extensions of fuzzy sets.

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