

# Preface

This is a book discussing the Bogoliubov-de Gennes (BdG) method and its major applications to superconductors. It was originally formulated by De Gennes in his book *Superconductivity of Metals and Alloys* and is based on the BCS theory of superconductivity, i.e., Cooper pair formation due to an effective pairing interaction around the Fermi energy. It consists essentially of a coupled set of Schrödinger equations, enabling a description of superconductivity with much information on both quasiparticle and superconducting order parameter properties. The method becomes particularly useful by providing insight into quasiparticle properties down to the atomic scale, which can be detected by such local probes as scanning tunneling microscopy. The understanding of quasiparticle properties is important in uncovering the mechanism for superconductivity. Therefore, the BdG formalism serves as a complementary approach to the Ginzburg–Landau theory, which is a choice to describe the spatial variations of superconducting order parameter slowly varying at a larger length scale.

The BdG theory was initially applied to understand the variation of superconducting order parameter in layered structures and quasiparticle states around a single vortex core in conventional *s*-wave superconductors. At that time, the BdG equations were solved analytically for some special cases. With significant advance in computer power, the BdG method is now widely used to address the local electronic structure in superconductors in much more complicated situations. Its application has recently been expanded to unconventional superconductors like high-temperature cuprates and topological superconductors. Due to its simplicity, the BdG formalism can be easily accessed by undergraduate and graduate students with the knowledge of quantum mechanics, who want to learn some interesting superconducting phenomena by solving a generalized set of Schrödinger equations through computer simulations. Throughout the book, we use the synonyms, BdG method, BdG formalism, and BdG theory, interchangeably with the same meaning.

The book is organized into two parts. The first part is on the formalism itself and consists of two chapters. The second part of the book, Chaps. 3–7, covers important applications of the BdG method. Chapter 1 is on the derivation of the BdG equations in the continuum model. Chapter 2 gives an alternative derivation of the equations in

the tight-binding model, based on which a more realistic description of the normal-state band structure can be incorporated. The structure of the equations and its connection with the Abrikosov–Gorkov Green function method, as well as solutions in the uniform case, are discussed in these first two chapters. Chapter 3 deals with the local electronic structure around a single impurity in a superconductor. The Yu-Shiba-Rusinov state around a magnetic impurity and Majorana mode from a coupling to a spin chain in *s*-wave superconductors and impurity resonance states in *d*-wave superconductors are discussed. Chapter 4 treats the influence on macroscopic properties by disorder effects in both *s*-wave and *d*-wave superconductors. The localization of impurity-induced resonance states in a *d*-wave superconductor is also discussed by using the one-parameter scaling analysis within the BdG formalism. Chapter 5 deals with the magnetic field effects on the local electronic structure of superconductors. The quasiparticle states around a single vortex core and in the mixed-state of conventional and unconventional superconductors are considered. The effect from a competing order on the quasiparticle states in a *d*-wave vortex core is explored. In addition, the Fulde–Ferrell–Larkin–Ovchinnikov state due to the spin Zeeman interaction of a magnetic field is also considered. In Chap. 6, I discuss the transport properties of junctions formed with a superconductor. The Andreev reflection on the differential conductance and its response to spin polarization and topological superconductivity are explored. Chapter 7 covers the topological effects with a focus on the periodicity of supercurrent in multiply connected geometries at mesoscale and quantum size effects on the superconducting properties in nanoscale superconductors. I would like to comment that the topics chosen in the chapters on the applications of the BdG equations have a direct relevance to the present frontiers of research in such emerging fields as iron-based superconductors, topological superconductors, and cold atoms.

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