Preface to the First Edition

This book grew out of lecture notes from a course given at Graz University of Technology (autumn 1990), The Institute of Technology at Linköping University (autumn 1991), and the Ecole Polytechnique in Montréal (autumn 1993). Originally conceived of as a Ph.D. course, it attempts to present a coherent and mathematically rigorous theory of deterministic global optimization. At the same time, aiming at providing a concise account of the methods and algorithms, it focuses on the main ideas and concepts, leaving aside too technical details which might affect the clarity of presentation.

Global optimization is concerned with finding global solutions to nonconvex optimization problems. Although until recently convex analysis has been developed mainly under the impulse of convex and local optimization, it has become a basic tool for global optimization. The reason is that the general mathematical structure underlying virtually every nonconvex optimization problem can be described in terms of functions representable as differences of convex functions (dc functions) and sets which are differences of convex sets (dc sets). Due to this fact, many concepts and results from convex analysis play an essential role in the investigation of important classes of global optimization problems. Since, however, convexity in nonconvex optimization problems is present only partially or in the "other" way, new concepts have to be introduced and new questions have to be answered. Therefore, a new chapter on dc functions and dc sets is added to the traditional material of convex analysis. Part I of this book is an introduction to convex analysis interpreted in this broad sense as an indispensable tool for global optimization.

Part II presents a theory of deterministic global optimization which heavily relies on the dc structure of nonconvex optimization problems. The key subproblem in this approach is to transcend an incumbent, i.e., given a solution of an optimization problem (the best so far obtained), check its global optimality, and find a better solution, if there is one. As it turns out, this subproblem can always be reduced to solving a dc inclusion of the form $x \in D \setminus C$, where D, C are two convex sets. Chapters 4–6 are devoted to general methods for solving concave and dc programs through dc inclusions of this form. These methods include successive partitioning and cutting, outer approximation and polyhedral annexation, or combination of the

concepts. The last two chapters discuss methods for exploiting special structures in global optimization. Two aspects of nonconvexity deserve particular attention: the rank of nonconvexity, i.e., roughly speaking the number of nonconvex variables, and the degree of nonconvexity, i.e., the extent to which a problem fails to be convex. Decomposition methods for handling low rank nonconvex problems are presented in Chap. 7, while nonconvex quadratic problems, i.e., problems involving only nonconvex functions which in a sense have lowest degree of nonconvexity, are discussed in the last Chap. 8.

I have made no attempt to cover all the developments to date because this would be merely impossible and unreasonable. With regret I have to omit many interesting results which do not fit in with the mainstream of the book. On the other hand, much new material is offered, even in the parts where the traditional approach is more or less standard.

I would like to express my sincere thanks to many colleagues and friends, especially R.E. Burkard (Graz Technical University), A. Migdalas and P. Värbrand (University of Linköping), and B. Jaumard and P. Hansen (University of Montréal), for many fruitful discussions we had during my enjoyable visits to their departments, and P.M. Pardalos for his encouragement in publishing this book. I am particularly grateful to P.T. Thach and F.A. Al-Khayyal for many useful remarks and suggestions and also to P.H. Dien for his efficient help during the preparation of the manuscript.

Hanoi, Vietnam Hoang Tuy
October 1997

Preface to the Second Edition

Over the 17 years since the publication of the first edition of this book, tremendous progress has been achieved in global optimization. The present revised and enlarged edition is to give an up-to-date account of the field, in response to the needs of research, teaching, and applications in the years to come.

In addition to the correction of misprints and errors, much new material is offered. In particular, several recent important advances are included: a modern approach to minimax, fixed point and equilibrium, a robust approach to optimization under nonconvex constraints, and also a thorough study of quadratic programming with a single quadratic constraint.

Moreover, three new chapters are added: monotonic optimization (Chap. 11), polynomial optimization (Chap. 12), and optimization under equilibrium constraints (Chap. 13). These topics have received increased attention in recent years due to many important engineering and economic applications.

Hopefully, as main reference in deterministic global optimization, this book will replace both its first edition and the 1996 (last) edition of the book *Global Optimization: Deterministic Approaches* by Reiner Horst and Hoang Tuy.

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