

Preface

The philosophy of mathematics has had a tumultuous life, even when only the last one and half century is considered. While the foundational crisis between the late nineteenth century and the early twentieth century left philosophers with no clear indication of how concerns prompted by reflection on mathematics, its language, and its objects could be eventually solved, mathematics has maintained a central place in philosophical investigations. Led both by empiricist (when not sociologically leaned) approaches attempting at framing mathematics within an overall conception of natural knowledge, and by novel foundational perspectives striving to preserve an autonomous place for mathematical knowledge, the philosophy of mathematics has witnessed a growth of studies since the Seventies of the past century, and is today one of the liveliest and most stimulating areas of philosophical research, where disciplines as diverse as logic, history of mathematics, philosophy of language and science, epistemology and metaphysics find an impressively fertile common ground. While new and provoking positions have been developed on all sides of possible theoretical divides (platonist vs. nominalist, realist vs. anti-realist, empiricist vs. apriorist, philosophy-oriented vs. practice-oriented, and so on), a wealth of investigations has been flourishing, research groups gathering around specific proposals have been forming, and novel perspectives and conceptual tools have been emerging. The essays collected in this volume are meant to be a vivid example of this renewed philosophical milieu, and are the outcome of activities organized by one among the recently born international communities devoted to the philosophy of mathematics.

The background milieu for what would later become the Italian Network for the Philosophy of Mathematics, FilMat, was provided by recurrent meetings of, and collaborations between, national research groups (such as SELP and COGITO), promoted by several researchers in the philosophy of mathematics and logic based at various Universities in Italy (among which the University of Bologna, Scuola Normale Superiore in Pisa, San Raffaele University in Milan, the University of Padua, the University of Milan). Also through the continued encouragement of colleagues like Marco Panza at IHPST (Paris), and a renewed interest in

publications in the philosophy of mathematics by Italian publishers (among which we would like to stress the role played by Carocci Editore) participants in those groups came to realize that their network of national and international relations should be given some more stability.

The initial suggestion for the creation of the network was then prompted by a first very successful conference that was organized at Scuola Normale Superiore, Pisa, in 2012, by Gabriele Lolli and Giorgio Venturi. The aim of that conference was to gather Italian scholars in philosophy of mathematics and closely related fields, in order to bring something to the fore: despite the philosophy of mathematics is unduly underrepresented in Italian academia, the community of Italian researchers successfully involved in the discipline is lively and conspicuous. This was not only—and not so much—meant to apply to well-known scholars already based in prestigious universities all around the world, but especially to young scholars, including PhDs and post-docs, who strive to find adequate opportunities in this field in their country, and are most of the time bound to flee towards non-Italian Universities (well beyond what is a legitimate and necessary need for international exchanges and cooperation). The aim of the Pisa conference was to make all Italian researchers, at any level, be they based in Italy or abroad, feel part of a compact and collaborative community, with clear national ties while still extremely well entrenched in international scientific research, with a significant potential for fostering successful careers of young scholars. That potential was displayed in a Springer volume that selected a number of papers from that conference, edited by Gabriele Lolli, Marco Panza and Giorgio Venturi and published in the *Boston Studies for the Philosophy and History of Science* series in 2015: *From Logic to Practice. Italian Studies in the Philosophy of Mathematics*. The volume gave a nice picture of Italian research in the philosophy of mathematics, with a special focus on the integration of logical, historical, and philosophical concerns in the philosophy of mathematics and mathematical practice.

Some of the participants in that conference—Francesca Boccuni, Gabriele Lolli, Marco Panza, Matteo Plebani, Luca San Mauro, Andrea Sereni, and Giorgio Venturi—felt the urge of providing that community with a more solid and visible platform, as a way of acknowledging the reception of Italian researchers in the philosophy of mathematics, of appreciating the successful placement of Italian scholars in international universities, of integrating younger students in a nationally based and yet geographically diffused web of professional connections, and, last but not least, of promoting the undeniable interest of studies in the philosophy of mathematics in Italian academia. Given the disseminated location of the potential members, the network form seemed most appropriate.

The FilMat Network (www.filmatnetwork.com) soon gathered a conspicuous number of affiliations of Italian scholars worldwide, counting almost 70 members at the time this Preface is written. Others may join the network in the future, and we are confident that the number of young students and early-career scholars finding in membership to the network a way of facilitating and enhancing their scientific and professional journey in this field will raise in the long run.

Despite the nation-based nature of the network, nothing is more alien to its promoting ideas and its mission than closure or isolation with respect to an international scientific community which is becoming more and more global and connected. Quite to the contrary, we believe that keeping researchers in this field in close touch despite their diffuse geographical locations is an ideal way of intensifying scientific cooperations independently of national borders, also by building on individual existing collaborations in different countries.

The FilMat Network promotes conferences, workshops, and seminars in the philosophy of mathematics and strictly related areas, also by circulating news about activities organized by its members. One of its main aims so far has been to schedule a biennial network conference. The first official FilMat international conference—*Philosophy of Mathematics: Objectivity, Cognition and Proof*—was organized by the editors of this volume at San Raffaele University, Milan, in May 29–31, 2014. As a way of stressing both the network's nation-based original inspiration and its interest in fostering international cooperation, the conference assumed a specific format. Invited speakers were selected from each of four categories: Italian scholars based in Italy (Mario Piazza, University of Chieti-Pescara), Italian scholars based abroad (G. Aldo Antonelli, UC Davies), non-Italian philosophers (Leon Horsten, University of Bristol), and early-career invited speakers (Francesca Poggiolesi, IHPST Paris). Contributed speakers were selected by double-blind review through an international call for papers.

The conference was—or so we dare say—quite a successful event. It hosted 21 talks—with the addition of an inaugural lecture by Stewart Shapiro (Ohio State University)—and received 38 submissions from 36 international universities and research institutions from Austria, Belgium, Brazil, Bulgaria, Finland, France, Germany, Hungary, Israel, Italy, Poland, Portugal, UK, and USA. Considering joint works, submissions came from a grand total of 42 authors, of which 9 Italy-based and 6 non-Italy-based Italian nationals, and 27 international authors. Among all of them, 18 were young scholars and early-career researchers. Numbers proved the format to be successful, and it is likely to be preserved in the future. The second FilMat conference has taken place at the University of Chieti-Pescara in May 26–28, 2016, organized by Mario Piazza together with Pierluigi Graziani (University of Urbino) and Gabriele Pulcini (University of Campinas), and statistics of submissions in terms of quantity and international provenance equalled those of the first conference.

We are grateful to Springer, to the Editors of the *Boston Studies in the Philosophy and History of Science* series, and in particular to Springer's publishing editor Lucy Fleet for having accepted and supported our proposal of making a volume out of selected contributions from the FilMat 2014 conference, and to the Project Coordinator Karin De Bie, as well as to Steve O'Reilly and Gowtham Chakravarthy for their support in the production process. We also wish to thank two anonymous reviewers for their precious comments on the book proposal and final draft. Together with the collection stemmed from the Pisa conference, we believe this volume will testify the value of the kind of research in the philosophy of

mathematics that has been gathering around activities of the FilMat Network, and we hope other volumes will follow suit.

Above all, we are grateful to all the authors who submitted to this volume for their cooperation and patience in going through a double-blind review process, which has often led to significant modifications and improvements of contributions based on the valuable suggestions from a panel of about 30 international reviewers, to whom we express our warmest thanks. Given changes occurred along this process, and the final distribution of themes across contributions, the title of this book has been slightly changed from the title of the original conference where most papers were originally presented.

We also would like to thankfully acknowledge the support of the PRIN Italian National Project *Realism and Objectivity* (national coordinator: Pasquale Frasca, Basilicata University), and in particular to the San Raffaele research unit *Cognitive Sciences and Scientific Objectivity* (unit coordinator: Claudia Bianchi, San Raffaele University).¹

Of all authors who directly or indirectly supported our project—by participating in the conference or submitting their contribution to the volume—this collection is dedicated to G. Aldo Antonelli. Aldo prematurely and unexpectedly passed away on October 11, 2015. He was the nicest person and an outstanding philosopher of mathematics and logic. His death was an immense loss to the scientific community as a whole. Aldo was extremely supportive of the network project since we first invited him as a network member and as a speaker to the FilMat conference in Milan, and maintained his unshaken support for this volume. When he passed away, he was just about to send us a revised version of his paper. Reviewers suggested just minor revisions, and we then decided to publish the paper as it was. In their comments reviewers clearly manifested sincere appreciation for Aldo's paper. Sean Walsh emphasized that “the paper was eloquently composed and a joy to read.” Roy Cook stressed that “the paper is excellent,” and accompanied his report with the following confidential comment:

Of course, about halfway through the paper I also became pretty confident of the identity of the author, and if I am right, then my positive report is not surprising: the person who I suspect wrote the paper usually produces excellent work that rarely needs significant modification or revision! (Of course, I could be wrong about who the author is, in which

¹Even though this volume is a self-standing enterprise, the FilMat conference that made it possible received financial support from various sources. Besides the PRIN project, we take the opportunity to thank again the Ph.D. Program in *Cognitive Neurosciences and Philosophy of Mind* (San Raffaele University/NeTS at IUSS Pavia); the Ph.D. Program in *Philosophy and Sciences of the Mind* (San Raffaele University); SELP (Seminario di Logica Permanente). The conference was held under the auspices of AILA (Italian Association for Logic and its Applications), SIFA (Italian Society for Analytic Philosophy), SILFS (Italian Society for Logic and Philosophy of Science), and in collaboration with COGITO Research Centre (Bologna) and CRESA Research Centre (San Raffaele). Special thanks go to the then Dean of the Faculty of Philosophy at San Raffaele University, prof. Michele Di Francesco, for his support in promoting this and many other activities in the philosophy of mathematics.

case the moral is that the actual author of the paper commendably met the very high standards that are typical for the person who I have in mind).

We are thankful to them for their permission to disclose their names and report parts of their comments as a way of witnessing once more the excellent quality of Aldo's research. We are all the more grateful to Aldo's partner Elaine Landry and his son Federico Antonelli for having made the publication of his paper possible. Aldo was the clearest representative of the kind of scholars the network is thought for, and we are proud to have been given the chance of meeting him and collaborating with him. He was organizing a workshop on *Ontological Commitment in Mathematics* together with Marco Panza, to be held at IHPST in Paris, where he would have presented the paper published in this volume. The workshop was turned into an event *in memoriam of Aldo Antonelli* and took place in Paris on December 14–15, 2015. Andrew Arana, in collaboration with Curtis Franks, delivered a memoir of Aldo's life and work, which they kindly gave us permission to include in this collection. As a way of homaging Aldo's work, together with his partner and his colleagues and friends Robert May and Marco Panza, we decided to include also a discussion note summarizing and systematizing the discussion that took place after Aldo's paper was read at the Paris workshop, which Robert and Marco kindly agreed to edit. We are confident that this discussion will do nothing more than stressing once more how stimulating and thought-provoking Aldo's work in the philosophy of mathematics can be.

If something has to be witnessed by the papers included in this collection, it is the variety of philosophical concerns that may be prompted by current reflection on mathematics. It goes without saying that what is offered here is a necessarily partial picture—as the vast production of papers and books in this field in recent years, supported by the creation of dedicated networks and research groups, testifies. As Stewart Shapiro emphasizes through the consideration of three case studies, there is a variety of stimulating ways in which mathematics and philosophy can reciprocally contribute to an improved understanding of their respective fields. Of these interactions, and more generally of the philosophical concerns that mathematics raises, three are the main areas on which the papers collected here focus, briefly codified in the three key notions in the title: *Objectivity, Realism, and Proof*.

How a pivotal area of our rational life can be granted the objectivity it deserves is a classical problem in the philosophy of mathematics, which becomes extremely pressing when the shadowy nature of its objects is considered and their connection with the concrete, empirically accessible world is investigated. Essays in Part I (*The Ways of Mathematical Objectivity: Semantics and Knowledge*) are all, to different extents, bearing on these issues. Fregean and neo-Fregean philosophy of mathematics attempted to assuage similar concerns by appropriate semantic analysis of mathematical discourse, but shared solutions are a long way off. Aldo Antonelli (as also shown in the Discussion Note of his contribution edited by Robert May & Marco Panza) and Robert Knowles both confront semantic issues concerning mathematical discourse in a Fregean framework. On a different but related note, mathematics can at the same time be thought of as being constituted by *a priori*

truths and as both biologically grounded and entrenched in practice and applications. Markus Pantsar and Marina Imocrante investigate in various ways how the alleged *a priori* character of pure mathematics can be integrated with either an empiricist framework informed by cognitive sciences or an epistemology of mathematics especially focused on applications and actual practice.

Objectivity seems assured when mathematics is considered as a discourse about a well-defined realm of objects, which mathematical theories are supposed to describe. However, the nature of different mathematical objects and the structure of the mathematical universe come in a variety of shapes. Essays in Part II (*Realism in a World of Sets: From Classes to the Hyperuniverse*) focus on these issues, with a particular attention to that essential domain of mathematical objects which sets are. Leon Horsten, Brice Halimi, and Gianluigi Oliveri discuss different approaches to the nature of sets, by investigating respectively the import of conceptions of the infinite on a characterization of classes, the relationship between sets and categories, and the conception of set theory as a science of structures rather than individual objects. The remaining essays in this Part, on the other hand, are concerned with finding an adequate picture of the set-theoretic universe, by connecting a realist picture with a pluralist conception of the set-theoretic domains. Claudio Ternullo & Sy-David Friedman, Neil Barton, and Giorgio Venturi all explore, through different approaches, a conception of the set-theoretical universe today known as multiverse, respectively by relating it to the so-called Hyperuniverse program, by investigating the extent to which relativism may be acceptable in a conception of the set-theoretical domain, and by considering how techniques like forcing may support one or another realist view of such domain.

Both the problems of objectivity and realism need to face an undeniable fact: even when it is understood as aiming at a faithful description of an independent realm of mathematical objects, mathematics is a human activity, where the goal of attaining truth is pursued through symbolic languages by regimentation and clarification of more or less informal notions in appropriate formal systems. Essays in Part III (*The Logic Behind Mathematics: Proof, Truth, and Formal Analysis*) offer new perspectives on some classical issues in this vicinity. Contributions by Mario Piazza & Gabriele Pulcini, and Carlo Nicolai, both deal with specific issues concerning truth in formal theories, either as related to our access to the truth of Gödel's sentence \mathcal{G} , or as related to the relationship between axiomatic truth theories and comprehension axioms. In the last three essays, the interplay between some central notions in mathematics and metaphysics (including the metaphysics of mathematics) and their proper formalization is explored by Francesca Poggiolesi, Massimiliano Carrara & Enrico Martino & Matteo Plebani, and Samantha Pollock, either by investigating how the proper logic underlying the epistemic and metaphysical notion of grounding should be made precise, or by suggesting that a primitive notion of finiteness may be essential to singling out the standard model of

arithmetic, or finally by exploring how informal beliefs may be involved in the appreciation of technical results such as categoricity theorems.

Through their diverse approaches and focus, the essays in this volume collectively prove once more how rich and stimulating mathematics can be for philosophy on its semantic, epistemic, and ontological aspects. They offer novel perspectives on vexed theoretical issues and promise to deepen our understanding of such a fascinating part of human thought like mathematics is. We are confident that they will stimulate further discussion and will greatly contribute to current debates.

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Objectivity, Realism, and Proof

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