Chapter 2
Dramatis Personae (The Actors)

> *[Ignorance] of the principle of conservation of energy ... does not prevent inventors without background from continually putting forward perpetual motion machines... Also, such persons undoubtedly have their exact counterparts in the fields of art, finance, education, and all other departments of human activity... persons who are unwilling to take the time and to make the effort required to find what the known facts are before they become the champions of unsupported opinions—people who take sides first and look up facts afterward when the tendency to distort the facts to conform to the opinions has become well-nigh irresistible.*
>
> Robert Millikan

Abstract Clarity in physics requires precise definitions of terms, such as work and energy, that have fuzzy meanings in everyday usage. Some of the common terms and concepts (e.g. force, velocity, acceleration, momentum) are thus introduced and defined, as an introduction to the material of later chapters, where they will be extensively utilized. Since the book is devoted to readers with only a basic background of mathematics, some of the necessary mathematical tools and concepts are also briefly explained.

2.1 Definitions

R.A. Millikan is undoubtedly right about the many inventors of perpetual motion machines (and incidentally also about their counterparts in several other contexts). It may therefore seem paradoxical that all objects in the Universe are apparently in perpetual motion, whether they are extremely small particles or incredibly large celestial bodies, such as stars or galaxies. To study motion in all its aspects and manifestations, we need to begin with clear definitions. In this Chapter we provide an introduction to some of the more relevant terms and concepts of Mechanics. We will also define a basic tool of the (*Infinitesimal*) *Calculus*,[1] the *derivative*, which is the key to modern mathematics and physics, and explain its meaning (although only

[1]Modern calculus is considered to have been developed independently in the 17th century by Isaac Newton and Gottfried Leibniz.

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in a very cursory way). Derivatives are also extremely important for physicists, since they are the primary ingredients of *differential equations*, which represent the most customary way of describing the laws of nature in their universality.[2]

A list of definitions is a necessary prerequisite to science, although unfortunately it may be very tedious for the reader. However, without unambiguous definitions, the door is open to sleights of hand (sometimes even involuntary), in which one may advance a thesis by slightly changing the implicit meaning of a term during the course of an argument. A lack of clarity may also lead to distortions of understanding between participants in a dialogue, due to subjective interpretations of the terminology, so that they may reach different conclusions, and still be logically correct.

Physics is concerned with properties which, at least in principle, are measurable and as such are called *quantities*. For instance, speed, temperature and humidity can be measured, while beauty, usefulness and goodness cannot and therefore are considered *qualities*. This distinction, however, depends on the capability and purpose of the (experimental) scientist. Temperature was not unambiguously measurable before the invention of thermometers, and consequently could not be considered a quantity. Similarly, it is entirely possible, although probably not very useful, to define beauty in terms of measurable criteria, such as the length of the nose or the body mass index (i.e. a person's weight in kilograms divided by the square of height in meters), as has actually been done by some *art theoreticians*, in which case it would become a *quantity* (at least for the people embracing those criteria).

Quantities may have a constant or a variable value: accordingly they are called *constants* or *variables.* Their designation depends on the context in which they are considered: e.g. they might be constant in time but not in space, or vice versa. Let us call x, y and z the space coordinates, corresponding to three arbitrarily chosen, mutually orthogonal directions in the 3D (three-dimensional) physical space. We can introduce time t as an orthogonal fourth dimension, in order to have a more comprehensive 4D space-time. Let us also assume that a given variable f depends on (or is a function of) any n ($1 \leq n \leq 4$) of the four coordinates of the 4D space-time. Then those n coordinates will define the space of dependence of the variable f.

As an example of such a process, let us consider an orographic map of a given region, i.e. a representation on a plane of the altitude *above sea level* of a land mass by means of contour lines or of colour shades. In this case we have two coordinates (longitude x and latitude y), which yield a 2D space. This is the plane upon which the map is drawn. For each point in the map the elevation e is defined as a function of x and y: i.e., $e = e(x, y)$.

[2]In most cases the laws of physics are expressed by means of differential equations, which means that the derivatives of the relevant functions are calculated keeping all variables fixed except the one of interest in the specific term.

More generally the variable f may be a function of n other variables, such as temperature, frequency or density. We may call all of these variables *coordinates*, and define a corresponding (generalized) *nD space*, of which our familiar 3D space and the 4D space-time are but special cases. Just as x, y and z define a *point* in our 3D space, the set of values of the generalized coordinates defines a *point* in the corresponding nD *space*. As an example let us consider the cooking of spaghetti. Here the "coordinates", upon which the quality q of the cooked *pasta* depends, are the cooking time t, the relative quantity of salt s, and the air pressure p: i.e. $q = q(t, s, p)$. A point in the space (t, s, p) *will* define the ideal *parameters* to cook perfect *spaghetti al dente.*

In the following chapters we shall encounter several of the all-important constants of physics. To take just one example here, we may mention the *universal gravitational constant* G, first introduced by Newton [1]. He realized that the motion of planets and the fall of an apple to the ground, as well as the reciprocal attraction of any two massive bodies, all obey the same physical law: namely that the force between them is directly proportional to the product of the masses and inversely proportional to the square of their distance, with a constant of proportionality G. Later we will return to Newton's law, but here we wish to remark that G has a specific value,[3] which is (for what we know) universal. The Universe would be a completely different place if its value were even slightly different. Also, while we normally assume that G has never changed[4] and expect it to remain constant in the future, we have no real proof that this is the case. All we can do here, as in many other cases, is to rely on the so-called *Occam's razor*,[5] which requires us always to choose the *simplest* explanation for the available data, i.e. the most economical one in terms of the assumptions made.

Another basic concept, as we mentioned before, is that of *derivative*, which we define as the instantaneous rate of change of a given function. In other words, if the variable f changes (at a given time) by a quantity Δf in the time interval Δt, its derivative, w, is given by the ratio of these two quantities, provided that Δt is infinitesimal (i.e. arbitrarily small[6]).[7] Using a mathematical notation we write

[3] Approximately $G = 6.673 \times 10^{-11}$ N(m/kg)^2.

[4] The possible variability of fundamental physical constants is discussed in Chap. 13.

[5] Occam's razor is an epistemological principle, devised by William of Ockham (c. 1287–1347), which states that, among competing hypotheses that predict equally well, one should always select the one with the fewest assumptions. Other, more complicated solutions may ultimately provide better predictions, but, in the absence of differences in predictive ability, the fewer assumptions, the better.

[6] To be more precise, the derivative is the *limit* of the ratio $\Delta f / \Delta t$, when Δt *tends to zero.*

[7] If the function f depends not only on time (as in the example in the text) but also on other variables, such as the coordinates of the position where the function is evaluated, the derivative may be calculated as a *partial derivative* proceeding as illustrated in the text, but keeping all variables fixed with the exception of one of them. E.g., given a function $f(x, y, z)$ of x, y and z, the partial derivative with respect to y is written $w = \partial f / \partial y$.

$$w = \frac{df}{dt}$$

As an example, consider a traveller driving between Naples and Venice (never mind whether those two cities are the ones in Florida or in Italy). To predict when he/she will arrive, we require the *average velocity*. For a physicist, however, the velocity is defined as the (instantaneous) *displacement* (or shift in *position*) in an *infinitesimal* time interval, i.e. as the derivative of the *position* with respect to time. The velocity defined in this way has a value that may vary continuously, both in magnitude and direction, throughout the entire journey.

Besides the derivative, another ubiquitous tool for physics is the *integral*. Basically an integral is a sum and the symbol used for it, $\int$, is indeed a stretched *S*, as in Sum. On one side the (indefinite) integral is the inverse of the derivative. Inverting the above formula for the derivative gives:

$$f = \int w dt = \int df$$

On the other side, the sum may be understood by resorting to geometry. To help us, we refer to Fig. 2.1, where the function $w(t)$ is represented.

The product of a given value of w times the interval dt is approximately equal to the area of a strip of width dt. The sum over all wdt's approximately gives the total area under the curve between t_A and t_B. If we let dt become smaller and smaller the number of addends in the sum becomes larger and larger and the result tends to coincide with the actual area. For an infinite number of infinitesimally small addends we obtain the (definite) *integral* of w over t, between t_A and t_B.

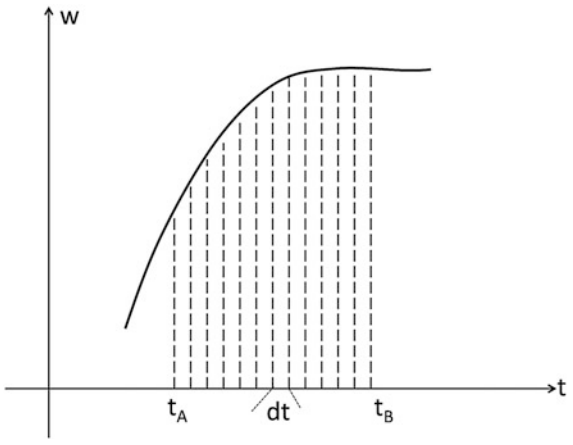


Fig. 2.1 The area underneath the curve $w(t)$ between t_A and t_B is the definite integral of the function $w(t)$ between the two values of the independent variable t

2.2 The Laws of Physics

Likewise, the *acceleration* is defined as the change of velocity in an infinitesimal time interval, i.e. as the (first) derivative of the velocity or the second derivative of the position with respect to time. As we shall see later, an acceleration is always the consequence of an *applied force*; i.e., without an applied force, an object remains forever in its original *state of motion*, whether not moving or moving with constant velocity. This *first law of Mechanics* is counterintuitive, and in fact was clearly stated only about 400 years ago by Galilei [2] as the *principle of inertia*. Since we are continuously surrounded by *fields of force* (mostly gravity), we never experience such an unchanging state of motion.

Even though most students of basic physics may not realize it, the introduction of the concept of force through *the second law of mechanics*, i.e. the famous $\boldsymbol{F} = m\boldsymbol{a}$ (see Chap. 4), arguably one of only two physics formulae known by most non-scientists,[8] requires a non-trivial trick, namely the definition of two new entities (force and mass) by means of a single formula. Of course, once we accept that an acceleration requires a driving force and is proportional to it, the *mass* may be simply defined as the constant of proportionality. However, the mass is also the *source* of gravity. The correspondence of the two interpretations into a single quantity is one of the pillars of Einstein's *Theory of Relativity* (see Chap. 7).

For a complete specification, to define the displacement as the distance an object has moved in a given time interval is not sufficient since the direction in which the movement took place must also be specified. To this end, *vectors* are needed, which are mathematical entities defined by a number (their magnitude in the adopted *units*) and a *direction*. For example, a displacement might be specified as a movement of ten meters in the direction south-to-north. Since our goal in this book is not the application of physics, but rather the illustration of its basic concepts and developments, we will not engage here in the details of vector calculus, which can be found in any basic textbook,[9] but just limit ourselves in the following to providing a few basic examples.

In a *Cartesian*[10] plane, one can define a *position vector* $\boldsymbol{r}$ (by convention, vectors are denoted with bold type) as the *arrow* going from the origin of the coordinate system to the point identifying the current position. Then if one moves from a point P_1 to another P_2, the corresponding displacement $\boldsymbol{l}$ is given by the arrow joining the two points P_1 and P_2, and the average velocity $\boldsymbol{v}_{av}$ by the ratio $\boldsymbol{l}/t$, where t is the time required for the move. Time is a scalar, and is specified by only one number,

[8] The other being Einstein's $E = mc^2$ (see Chap. 7).

[9] E.g. D. Halliday, R. Resnick, J. Walker: Fundamentals of Physics, or Young, Freedman and Lewis Ford: *University Physics with Modern Physics*, or Douglas C. Giancoli: *Physics for Scientists and Engineers with Modern Physics*, as well as many others.

[10] A Cartesian coordinate system is a coordinate system that specifies each point uniquely in a plane by a pair of numerical coordinates, which are the signed distances from the point to two fixed perpendicular axes.

e.g. 2 s or three hours. More precisely, the instantaneous velocity $\boldsymbol{v}$ is given by the derivative of $\boldsymbol{l}$ with respect to t, for which the symbol $d\boldsymbol{l}/dt$ is used. And the acceleration $\boldsymbol{a}$ is defined as the derivative of $\boldsymbol{v}$ with respect to t (i.e. $d\boldsymbol{v}/dt$).

2.3 The Variables of Mechanics

Translations and *rotations* are defined as displacements along a straight line or around a given circle (i.e. at a constant distance from a given point), respectively. A basic theorem of Mechanics states that every infinitesimal displacement can be decomposed into the *sum* of an infinitesimal translation plus an infinitesimal rotation. As a consequence one can study even the most intricate trajectories, by analysing their two components (translational and rotational) and considering them separately, which obviously simplifies the treatment enormously.

Furthermore, between the two types of movement (translation and rotation) there exists a very useful symmetry. This is our first instance of the abstract *beauty* of physics, which was hinted at in Chap. 1. In order to stress the symmetry we summarize all of the basic variables of Mechanics (the *actors of our game*) in the two columns of Fig. 2.2. Of course students of Engineering in their first course of Physics require a deeper understanding of these variables, and of their applications. For us it is enough here to list the definitions in Fig. 2.2, and provide more details later as required. In the next paragraph, however, we wish to discuss in more detail the rotational equivalent of the basic translational law $\boldsymbol{F} = m\boldsymbol{a}$.

As an example, let us assume that we wish to close a heavy door. Obviously we must apply a force, but where and how to apply it is also important. In fact, let us assume that we pull or push with a force perpendicular to the door itself. If we apply such a force on the side of the door where the hinges lay, we achieve nothing. The easiest way to close the door is clearly to push it on the other side (where the handles are). Why? Because in a rotation the relevant quantity is not the force, but the *torque*, which in the case of a force perpendicular to the door is given, in magnitude, by the product of the force times its *lever arm*. The latter, in this case, is the distance between the point where the force is applied and the side of the door with hinges.

If we substitute the force with the torque and the linear acceleration with the angular one, we obtain the second law of Mechanics for rotational motion, i.e. *the torque is proportional to the angular acceleration.* However, the constant of proportionality will no longer be the mass, but another quantity called the *moment of inertia* (usually denoted with the letter I). For a single point-like body of mass m, I is given by mr^2, where r is the distance of the body from the axis of rotation (i.e. the straight line around which the body rotates). In the case of the door, the axis of rotation is the line of the hinges. The body's *angular velocity* is correspondingly defined as the instantaneous rate of change of its *rotational displacement* (the angle by which it rotates), and its *angular acceleration* as the derivative of its angular velocity with respect to time.

Translational Mechanics	Rotational Mechanics
linear coordinate l	angular coordinate θ
linear velocity $v = \frac{\mathrm{d}l}{\mathrm{d}t}$	angular velocity $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$
linear acceleration $a = \frac{\mathrm{d}v}{\mathrm{d}t}$	angular acceleration $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$
mass m	moment of inertia I
linear momentum $\mathbf{p} = m\mathbf{v}$	angular momentum $\mathbf{L} = I\boldsymbol{\omega}$
force $\mathbf{F}$	torque $\boldsymbol{\tau}$
$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$, $\mathbf{F} = m\mathbf{a}$	$\boldsymbol{\tau} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}$, $\boldsymbol{\tau} = I\frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t}$
if $\mathbf{F} = 0$, $\mathbf{v} = \text{const}$	if $\boldsymbol{\tau} = 0$, $\boldsymbol{\omega} = \text{const}$
work $W = F\,\Delta l$	work $W = \tau\,\Delta\theta$
power $P = \frac{\mathrm{d}W}{\mathrm{d}t}$	power $P = \frac{\mathrm{d}W}{\mathrm{d}t}$
kinetic energy $K = \frac{1}{2}mv^2$	kinetic energy $K = \frac{1}{2}I\omega^2$

Fig. 2.2 Some of the main definitions and laws of Mechanics. The symmetrical correspondence between Translational and Rotational Mechanics may be very helpful, both to understand their meanings and for applications. For the sake of simplicity, we have in some cases limited ourselves to rectilinear or, respectively, circular motion. The generalization to all kinds of trajectories, using vectorial notations, is straightforward. A comprehensive discussion of the laws of Mechanics will be included in Chap. 3

2.4 Conservation Laws

A very important variable of translational Mechanics is the *linear momentum* $\boldsymbol{p}$, which for an individual body is defined as the product of its mass m times its velocity $\boldsymbol{v}$. It can be easily seen[11] that, if m is constant, the time derivative of $\boldsymbol{p}$ is the applied force, i.e. $\boldsymbol{F} = d\boldsymbol{p}/\mathrm{d}t$. In fact, in Classical Physics, the latter is an alternative formulation to $\boldsymbol{F} = m\boldsymbol{a}$. We will see in Chap. 7 that, in the T*heory of Relativity,* the two formulations do not coincide in our customary 3D space because the mass is no longer constant, and only $\boldsymbol{F} = d\boldsymbol{p}/\mathrm{d}t$ is correct. What happens if no force is applied? In this case $d\boldsymbol{p}/\mathrm{d}t = 0$ and $\boldsymbol{p}$ is constant. The derivative of a constant vanishes, since by definition a constant does not change, hence the rate of change of the linear momentum is zero.

[11] $\boldsymbol{F} = m\boldsymbol{a} = m\ d\boldsymbol{v}/dt =$ (if m is constant) $d(m\boldsymbol{v})/\mathrm{d}t = d\boldsymbol{p}/\mathrm{d}t$.

What we have just discovered is our first example of a conservation law, the law of conservation of linear momentum. A second example follows immediately, thanks to the above mentioned correspondence between translational and rotational motion. Since the torque, moment of inertia and angular velocity correspond to the force, mass and linear velocity, respectively, the angular momentum is conserved if no torque is present.

Why are conservation laws so important? Because they can immediately explain many *effects* that are easily observed. For instance, if a boat is stationary in water, since no external force is applied its linear momentum is conserved. Then, if a passenger moves forwards, the boat *must* float backwards, so that the total $\boldsymbol{p}$ remains zero. Likewise, if spinning dancers wish to increase their angular velocity (i.e. rotate faster), all they have to do is bring their arms and legs as close as possible to their rotational axes to decrease their moment of inertia. As their angular momentum (see Fig. 2.2) must remain constant, since no external torque is present, their angular velocity increases to compensate for the decrease in their moment of inertia.

2.5 Work and Energy

There is a third conservation law, *the law of conservation of energy*, which is extremely important throughout all of Physics. It is a modern version of the famous principle "*Nothing is lost, nothing is created, everything is transformed*" of Antoine Lavoisier,[12] which in another context is known as the *First Law of Thermodynamics* (see Chap. 5). Note the word "*Law*", which implies that it cannot be proved (like most of Physics and in sharp contrast with Mathematics) and might even turn out to be incorrect in as yet unexplored realms of Physics. In fact, its original formulation had to be extended (to include energy arising from matter) when phenomena, such as the annihilation between a particle and its corresponding antiparticle, were discovered.

In daily life the word *energy* can assume different meanings. In classical Physics, energy has an unambiguous definition as the capacity for doing *mechanical work* and overcoming *resistance*. Let us therefore start with the definition of (mechanical) work W. If a constant force pulls a body along a straight line for a distance l, then the work performed by the force is defined as the product of the two *moduli* $\boldsymbol{F}$ and $\boldsymbol{l}$. More generally, since the directions of the two vectors $\boldsymbol{F}$ and $\boldsymbol{l}$ may not coincide, the work is given by the product of F, l and the cosine of the angle between the directions of $\boldsymbol{F}$ and $\boldsymbol{l}$. If the two vectors have the same direction, the angle is zero,

[12]Incidentally Lavoisier was guillotined during the French revolution, but not because of his sayings. Among his accusers there was an amateur chemist, Jean Paul Marat, whom Lavoisier had previously rejected as an associate to the Academy of Sciences.

the cosine is 1 and $W = Fl$; if they have opposite directions, the cosine is -1[13] and $W = -Fl$; if they are orthogonal (perpendicular to each other), the cosine is zero and $W = 0$.

A negative value for the work means that the force is resistive and the displacement takes place *in spite* of the force. A vanishing work means that the force is irrelevant to the motion, as when an object is moved across a horizontal floor. In this case gravity, being perpendicular to the floor, performs no mechanical work by itself. It may, however, have other important effects, such as increasing the friction between the object and the floor and consequently increasing our work, if we wish to push the object along.

Of course, such a definition of work does not always coincide with our common perception of the term. In fact, according to this definition, the work performed by a teacher is extremely small, since it is limited only to writing on the chalkboard, plus a little neuronal activity (which involves a negligible amount of work). Even worse, if we ask a porter to carry a heavy suitcase up ten flights of stairs and then we change our mind and ask him to bring it back down to the ground floor, we would owe him nothing for his labour, since the two *works*, up and down the stairs, are equal and opposite (from the viewpoint of the gravitational field) and their sum adds up to zero.

Let us now assume that we accelerate a car up to a certain velocity (say $\boldsymbol{v}_f$) and then take our foot off the gas pedal. The car keeps moving along the direction of $\boldsymbol{v}_f$, although its speed decreases due to friction and air resistance. We can explain this effect by saying that the work done by the motor to accelerate the car up to the velocity $\boldsymbol{v}_f$ has been transformed into *kinetic energy* (energy of motion). The kinetic energy has the capacity for doing new work overcoming the friction and air resistance for an additional distance. Similarly, if an object sits on the edge of a table, it is said to possess a *potential energy*, since, by pushing it over the edge, it falls down, enabling gravity to perform work. During the fall the object gains speed and its potential energy is transformed into an equal amount of kinetic energy.

More generally, in all physical processes the *type* of energy (potential, kinetic, heat, electromagnetic, etc.) can be indefinitely transformed, but the total amount of energy does not change. Let us consider another example: a child drops a ball from a given height h, where the ball has a potential energy mgh (m is the mass of the ball and g is the value of the gravitational acceleration on the surface of earth[14]). As the ball falls, the potential energy becomes kinetic energy, which in turn, upon hitting the floor, becomes *elastic energy* with the ball being deformed (slightly flattened). Almost instantaneously the elastic energy becomes kinetic energy again and the ball bounces back up to the original height h. At this point its speed (and kinetic energy) vanishes, but the ball has regained the original potential energy, and the process goes on indefinitely.

[13] As the vectors are in opposite directions, the angle between them is 180°, which has a cosine of −1.

[14] Approximately $g = 9.81\ ms^{-2}$.

Or does it? We all know that, even with the most elastic ball and floor, the height h at which the ball comes to rest decreases at every bounce, because each time a tiny amount of energy is lost as *heat* (another form of energy). A similar transformation of kinetic energy into heat can be noticed if one strikes an anvil with a hammer: after a few blows the anvil becomes warm to the touch. Again, no energy is lost: the kinetic energy of the hammer disappears, but an equal amount of heat energy is created.

However, in the process something important and *irreversible* occurs. Although the mechanism of transformation between potential and kinetic energy can go on repeatedly, the heat of the anvil can hardly be collected and reused (e.g. to yield again kinetic energy). In other words, heat, being energy, can still produce work but at a lower rate of *efficiency*. In the real world, most (or maybe all) natural processes are irreversible and lead to the gradual transformation of useable energy into less efficient forms of energy. This (so far) never falsified observation is called the *Second Law of Thermodynamics* (see Chap. 5). It can be summarized by paraphrasing George Orwell:[15]

All energies are equal, but some energies are more equal than others.

When it was formulated,[16] the Second Law of Thermodynamics held mostly a practical interest for its application to heat engines. Nowadays, however, due to a later formulation binding it to the concept of an ever increasing *entropy* (see Chap. 5), the Second Law has acquired an almost philosophical dimension. Because of the reinterpretation of the flow of time following Einstein's Theory of Relativity, it has been proposed to use the Second Law for the definition of the arrow of time, as we shall see in Chap. 7.

2.6 Taylor Expansions

To conclude this Chapter, we wish to mention a simple mathematical tool which we will need repeatedly in the following chapters, i.e. *power series expansions*. They can be of great help when dealing with problems for which it is not easy, or even possible, to find solutions in terms of elementary analytical functions. In fact, under some general conditions, which we are not discussing here, they may allow us to find suitable approximations to the exact solution. To explain how they work without using full mathematical rigor, we take advantage of geometric representations.

Let us consider a function $f(x)$ in the proximity of a given point P and draw the straight line tangent to the corresponding curve in P: see Fig. 2.3. The magnified

[15]From *Animal Farm* by George Orwell (1947) "All animals are equal, but some animals are more equal than others.".

[16]By Sadi Carnot (1824) and Rudolf Clausius (1850), independently. It can be easily proved that the two formulations are equivalent.

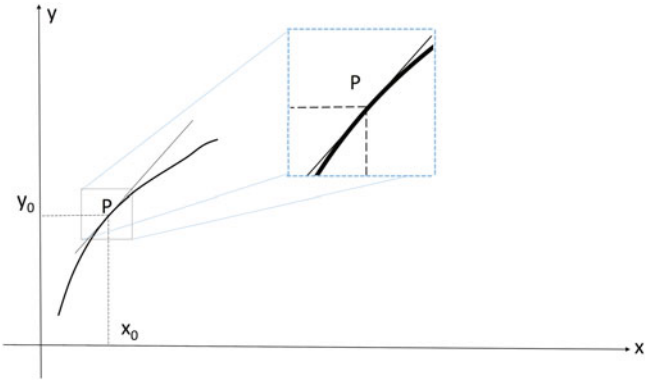


Fig. 2.3 In the vicinity of a point P a curve may be approximated by the tangent in that point

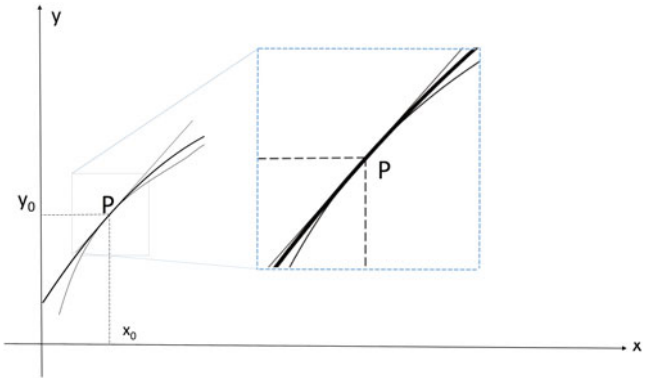


Fig. 2.4 The parabola passing through P and having the same tangent as the original curve is locally a better approximation than the tangent itself

box shows that near to P the difference between the straight line $y(x) = y(x_0) + k(x - x_0)$ where k is a constant, and the curve *f(x)*, is small: the closer to P, the smaller the difference.

An even better approximation may be obtained with a *parabola* passing through P and having there the same tangent as the curve *f(x)*: see Fig. 2.4. The equation of the parabola is also simple since the change in the y value with respect to the tangent is proportional to $(x - x_0)^2$:

$$y(x) = y(x_0) + k(x - x_0) + h(x - x_0)^2,$$

where h is another constant.

We may continue in the same way adding a term proportional to $(x - x_0)^3$, then to $(x - x_0)^4$ and so on. Under the appropriate conditions, each new term will be

smaller than the previous ones, yielding a better approximation. It can be proved that an infinite series built in this way is fully equivalent to the function $f(x)$. It is called the *Taylor (power) series*; when the reference point is the origin ($x_0 = 0$) its name is the *Maclaurin series*.

As a very elementary application of these series, let us assume that we wish to calculate the ratio 1/0.99. A simple example of the Taylor expansion is provided by the formula:

$$(1 + x)^n = 1 + nx + n(n-1)x^2/2! + n(n-1)(n-2)x^3/3! + ...$$

where the symbol "!" means *factorial*, e.g. $3! = 3 * 2 * 1 = 6$, $4! = 4 * 3 * 2 * 1 = 24$, etc. This expansion is valid for any value, positive or negative, of the exponent n if the absolute value of x is smaller than 1. If we limit ourselves to the first two terms in the series and assume $n = -1$ and $x = -0.01$, we obtain:

$$1/0.99 = (1-0.01)^{-1} = 1 + (-1) * (-0.01) = 1.01$$

Keeping three terms in the expansion, we obtain:

$$1/0.99 = 1.01 + (-1) * (-2) * (-0.01)^2/2 = 1.0101$$

Both 1.01 and 1.0101 represent increasingly better approximations to the value of 1/0.99, whose exact value is 1.01010101 …

The use of the Taylor series to provide successively better approximations to the solution of a problem in physics is a common practice, and in the course of our progress through this book, we will encounter further examples. Quite often the simple first order approximation, where only the first two terms are kept, is accurate enough for the purpose of comparing a theoretical prediction with an experimental measurement. The resultant theoretical estimate may not be 100 % accurate—in fact it may not even be possible to solve the theoretical equations to obtain the precise solution—but then experimental measurements also have errors.[17] If the errors in the Taylor expansion are no worse than the experimental errors, the theory can be meaningfully compared with experiment.

In this chapter we have introduced a few of the important quantities, principles and ideas that underlie physics. Many of them will be revisited in later Chapters. Some terms, as we have seen, have a somewhat different and more specific meaning in physics than in the everyday vernacular. However, physics as a science relies not just on the generation of new ideas and concepts, but also on the rigorous testing of these ideas with controlled and precise measurements and observations. The design and carrying out of such experiments is the task of the experimental physicist. In the next Chapter, we will provide for the lay reader a glimpse into the domain of experimental physics and the methodology of measurement.

[17]We discuss measurement errors and their sources in Chap. 3.

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