

# Efficient Data Fusion and Practical Considerations for Structural Identification

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**Abstract** This chapter represents a partial summary of several presentations given as part of the associated short course at CISM. The primary topic is on the use of data fusion techniques in structural system identification. Here the data fusion concept means the bringing together of sensor measurements from different kinds of sensors measuring different dynamic response quantities to provide a more accurate estimate of the dynamic states as well as the improved identification of model parameters. Data fusion scenarios discussed include the situation when different sensors are either collocated or not. In the last section, some separate work related to practical challenges of damping estimation is examined using operational modal analysis in the context of driving frequencies caused by traffic on multi-span bridge structures.

## 1 Introduction and Motivation

It is widely known that the estimation of dynamic displacements from acceleration measurements is fraught with significant problems due to integration amplification of low-frequency noise in the original acceleration measurement. An accurate estimate of the displacement can however be very valuable in a wide number of structural monitoring applications. Displacements (and strains) can be associated with permanent deformations which may be indicative of damage or important settlements in structural systems.

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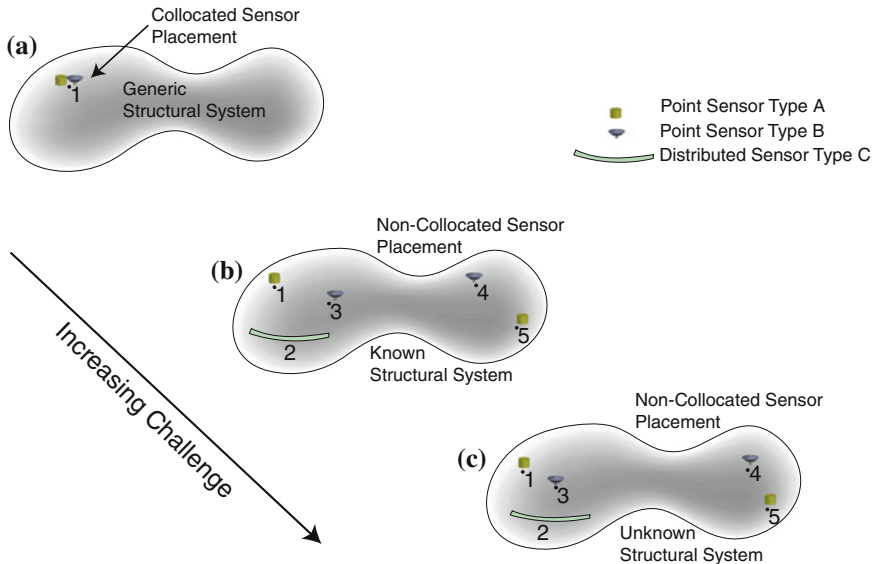
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Smyth and Wu (2007) presented a simple Kalman Filter -based technique which also included a smoothing step to provide an estimate of displacement and velocity based on a relatively noisy displacement measurement taken together (at the same location) with an acceleration measurement. This collocated fusion technique is relatively simple because the theoretical relationship between the displacement and acceleration motions is known exactly. Moreover the state-space system of equations in that case is perfectly linear.

The progression of complexity of the data fusion challenge is illustrated schematically in Fig. 1. As mentioned, the simplest case is that of the collocated displacement and acceleration sensor pair. Next, the case when different sensors are combined on a perfectly known structural system (linear or nonlinear). This second case is much like the first from the point of view that the state-space equations of motion relating the measured states with one another are perfectly known with the exception of the measurement noise. The last and most challenging category is when the sensors are non-collocated and the dynamic system is itself not perfectly known. For the most part, we will consider cases where the model form of the system is known, but its model parameters are not known. Because of the efficiency of the methodologies pursued here for this problem, it will also be possible to permit different assumed model forms to compete with one another (in parallel) to not only identify model parameters but to select the most suitable assumed model form.



**Fig. 1** The data fusion and joint state estimation problems at varying levels of complexity for a generic structural system: **a** collocated sensing, **b** non-collocated sensors, but perfectly known system, and **c** non-collocated sensors and system to be identified

### ***1.1 Redundancy Through Displacement Sensing***

As mentioned, adding some coarse displacement sensing for a dynamic system can provide valuable information on low-frequency motion which might not be easily resolved from the noise level in the accompanying acceleration. Absolute displacement is in general more expensive to measure than acceleration. Some methods used recently include differential GPS measurements of civil infrastructure in situations where low-frequency motions are expected to play an important role. Such a GPS-based measurement, while relatively noisy, has been shown to provide very useful data which can be “fused” with collocated accelerations to yield very accurate displacement and velocity estimates.

### ***1.2 Where to from Here?***

So the question arises whether displacement measures (or other sensor information) can be used to help correct multiple distributed sensor (often accelerometer) measurements. As mentioned this is most challenging when the dynamic system being monitored by the distributed network is not perfectly known. This prompts one to consider a joint state and parameter estimate framework because the system parameters are also not known. In this review the parameters will be considered in an augmented state vector. In this framework, even if the underlying dynamic system is linear, the overall state-space equation will be nonlinear because there will be product terms of the unknown parameters with states which are also to be estimated. The following subsections, each briefly deals with each one of the questions raised and provides references for the interested reader.

## **2 Sensor Data Fusion: Integration Challenges in Kalman Filtering**

Many damage detection and system identification approaches benefit from the availability of both acceleration and displacement measurements. This is particularly true in the case of suspected nonlinear behavior and permanent deformations. In civil and mechanical structural modeling accelerometers are most often used, however displacement sensors, such as noncontact optical techniques as well as GPS-based methods for civil structures are becoming more common. It is suggested, where possible, to exploit the inherent redundancy in the sensor information and combine the collocated acceleration and displacement measurements in a manner which yields highly accurate motion data.

This circumvents problematic integration of accelerometer data that causes low-frequency noise amplification, and potentially more problematic differentiation of

displacement measurements which amplify high-frequency noise. Another common feature of displacement-based sensing is that the high-frequency resolution is limited, and often relatively low sampling rates are used. In contrast, accelerometers are often more accurate for higher frequencies and higher sampling rates are often available. The fusion of these two data types must, therefore, combine data sampled at different frequencies. In the study of Smyth and Wu (2007), a multirate Kalman filtering approach is proposed to solve the problem of different sampling rates of acceleration and displacement measurements. This approach was combined with an important smoothing step discussed briefly below.

Consider the discrete linear state-space model (discretized at intervals  $T_d$ , i.e., when acceleration measurements are obtained)

$$\mathbf{x}(k+1) = \mathbf{A}_d \mathbf{x}(k) + \mathbf{B}_d \mathbf{u}(k) + \mathbf{w}(k) \quad (1)$$

$$\mathbf{y}(k) = \mathbf{H} \mathbf{x}(k) + \mathbf{v}(k) \quad (2)$$

where the process noise  $\mathbf{w}(k)$  and measurement noise  $\mathbf{v}(k)$  are zero-mean white Gaussian processes with covariance matrices  $\mathbf{Q}_d$  and  $\mathbf{R}_d$ , respectively. Then the Kalman filter algorithm for the above system is formulated as follows:

*Time update:*

$$\hat{\mathbf{x}}(k+1|k) = \mathbf{A}_d \hat{\mathbf{x}}(k|k) + \mathbf{B}_d \mathbf{u}(k) \quad (3)$$

$$\mathbf{P}(k+1|k) = \mathbf{A}_d \mathbf{P}(k|k) \mathbf{A}_d^T + \mathbf{Q}_d \quad (4)$$

*Measurement update:*

$$\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{K}(k+1) [\mathbf{y}(k+1) - \mathbf{H} \hat{\mathbf{x}}(k+1|k)] \quad (5)$$

$$\mathbf{P}(k+1|k+1) = [\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}] \mathbf{P}(k+1|k) \quad (6)$$

where the Kalman gain  $\mathbf{K}(k+1)$  is given by

$$\mathbf{K}(k+1) = \mathbf{P}(k+1|k) \mathbf{H}^T [\mathbf{H} \mathbf{P}(k+1|k) \mathbf{H}^T + \mathbf{R}_d]^{-1} \quad (7)$$

Assume the displacement measurement sampling interval is  $T_d$ , where  $T_d/T_a = M$ ,  $M$  is an integer. Since no displacement measurements are available between the times  $kT_d$ , where  $k$  is an integer, this is equivalent to optimal filtering with arbitrarily large measurement errors, so  $\mathbf{R}_d \rightarrow 0$  and hence  $\mathbf{K} \rightarrow 0$ . Thus, only the time update is performed and the optimal estimate is

$$\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) = \mathbf{A}_d \hat{\mathbf{x}}(k|k) + \mathbf{B}_d \mathbf{u}(k) \quad (8)$$

$$\mathbf{P}(k+1|k+1) = \mathbf{P}(k+1|k) = \mathbf{A}_d \mathbf{P}(k|k) \mathbf{A}_d^T + \mathbf{Q}_d \quad (9)$$

When displacement measurements are available at times  $kT_d$ , both the time update and measurement update should be performed. It is important to note that so far, as presented, this does not exploit the possible future correction in displacement measurement as each displacement sample becomes available. Therefore, displacement estimates can drift within the large interval  $T_d$ . Thus some smoothing (using the Rauch–Tung–Striebel algorithm), albeit a noncausal procedure, is beneficial. Through trials with simulated data the procedure’s effectiveness was shown to be quite robust at a variety of noise levels and relative sample rates for this practical problem.

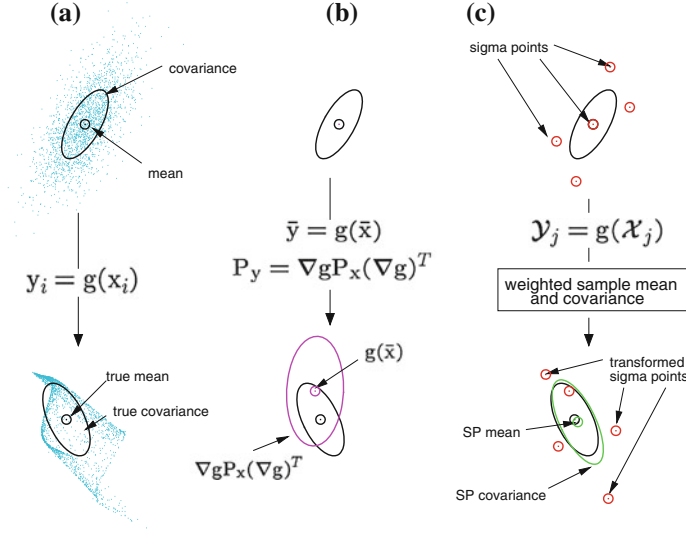
### 3 Sensor Data Fusion: Nonlinear Systems

#### 3.1 Theory and Computational Tools

There are several ways to go about solving the state estimation problem in the context of nonlinear systems. The extended Kalman filter (EKF) has been the standard Bayesian state-estimation algorithm for nonlinear systems for the last 30 years and has been applied over a large number of civil engineering applications. Despite its wide use, the EKF is only reliable for systems that are almost linear on the time scale of the updating intervals. The main concept of the EKF is the propagation of a Gaussian random variable (GRV), which approximates the state through the first-order linearization of the state transition and observation matrices of the nonlinear system, through Taylor series expansion. Therefore, the degree of accuracy of the EKF relies on the validity of the linear approximation and is not suitable for highly non-Gaussian conditional probability density functions (PDFs) due to the fact that it only updates the first two moments.

The unscented Kalman filter (UKF), on the other hand, does not require the calculation of Jacobians (in order to linearize the state equations). Instead, the state is again approximated by a GRV, which is now represented by a set of deterministically chosen points (sigma points). These sample points completely capture the true mean and covariance of the GRV and when propagated through the actual nonlinear system they capture the posterior mean and covariance accurately to the second order for any nonlinearity (third order for Gaussian inputs). The UKF appears to be superior to the EKF especially for higher order nonlinearities as it is often encountered in civil engineering problems. Detailed derivation of the UKF algorithm for implementation in civil engineering applications can be found in Wu and Smyth (2007). A schematic comparison of the EKF and UKF approaches is illustrated in Fig. 2

The sequential Monte Carlo methods or particle filters (PFs) can deal with nonlinear systems with non-Gaussian posterior probability of the state, where it is often desirable to propagate the conditional PDF itself. The concept of the method is that



**Fig. 2** Propagation of mean and variance of the GRV (from Van Der Merwe et al. 2004): **a** sampling, **b** linearization (EKF), and **c** sigma point approach (UKF). Be aware of the notation being quite different to what is used in this text

the approximation of the posterior probability of the state is done through the generation of a large number of samples (weighted particles), using Monte Carlo methods. PFs are essentially an extension to point-mass filters with the difference that the particles are no longer uniformly distributed over the state but instead are concentrated in regions of high probability. The basic drawback is the fact that depending on the problem a large number of samples may be required, thus making the PF analysis computationally expensive.

Consider the discrete nonlinear state-space model

$$\mathbf{x}_{k+1} = \mathbf{F}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k) \quad (10)$$

$$\mathbf{y}_k = \mathbf{H}(\mathbf{x}_k, \mathbf{v}_k) \quad (11)$$

The inference problem for the above discrete nonlinear state-space model is that of recursively estimating the state vector  $\mathbf{x}_k$  considering all measurements up to  $\mathbf{y}_k$ —more precisely the posterior density of the state vector  $p(\mathbf{x}_k | \mathbf{y}_{1:k})$

$$p(\mathbf{x}_k | \mathbf{y}_{1:k}) = p(\mathbf{x}_k | \mathbf{y}_k, \mathbf{y}_{1:k-1}) = \frac{\overbrace{p(\mathbf{y}_k | \mathbf{x}_k, \mathbf{y}_{1:k-1})}^{\substack{\text{conditional} \\ \text{independence} \\ = p(\mathbf{y}_k | \mathbf{x}_k)}} \overbrace{p(\mathbf{x}_k | \mathbf{y}_{1:k-1})}^{\text{prediction}}}{p(\mathbf{y}_k | \mathbf{y}_{1:k-1})} \quad (12)$$

where  $p(\mathbf{x}_k|\mathbf{y}_{1:k-1})$  is given by the Chapman–Kolmogorov equation

$$p(\mathbf{x}_k|\mathbf{y}_{1:k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) \underbrace{p(\mathbf{x}_{k-1}|\mathbf{y}_{1:k-1})}_{\text{previous step}} d\mathbf{x}_{k-1} \quad (13)$$

The methods mentioned at the beginning of this section essentially give a different approximation to the posterior PDF given by Eq. (12). Once this approximation is obtained, then an estimate of the state can be determined using different criteria; the one that minimizes the mean squared-error  $\|\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\|^2$  is given by the conditional expectation

$$\hat{\mathbf{x}}_{k|k} = \mathbb{E}_{\mathbf{x}_k} [\mathbf{x}_k|\mathbf{y}_{1:k}] = \int \mathbf{x}_k p(\mathbf{x}_k|\mathbf{y}_{1:k}) d\mathbf{x}_k \quad (14)$$

### 3.2 Joint State and Parameter Identification

To perform joint state and parameter estimation using a nonlinear filter, the state vector is augmented to comprise the original state vector  $\mathbf{x}_k$  and the (structural) parameter vector  $\boldsymbol{\theta}_k$  to be estimated

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{x}_k \\ \boldsymbol{\theta}_k \end{bmatrix} \quad (15)$$

The UKF for nonlinear structural identification is favored (as opposed to the EKF or the PF), and this choice is justified by the findings of Wu and Smyth (2007) and Chatzi and Smyth (2009), which will be discussed in a later section. Since parameters (time-invariant variables) and original states (time-variant variables) interact, even for linear dynamic systems, this joint approach can result in nonlinear dynamics over the augmented hidden states. This approach directly models uncertainties and correlations between parameters and states (since both are treated completely symmetrically) and can be thought of as iteratively implementing a Gaussian approximation to the recursive Bayes' rule computations for the joint posterior  $p(\mathbf{z}_k, \boldsymbol{\theta}_k|\mathbf{y}_{1:k})$ . Alternatively, there is the dual estimation approach, where two interacting, but distinct, filters operate simultaneously. One computes a Gaussian approximation of the state posterior given a parameter estimate and observations  $p(\mathbf{z}_k|\hat{\boldsymbol{\theta}}_k, \mathbf{y}_{1:k})$ , while the other computes a Gaussian approximation of the parameter posterior given the estimated states  $p(\boldsymbol{\theta}_k|\hat{\mathbf{z}}_k, \mathbf{y}_{1:k})$ . The two UKFs interact by each feeding its estimate (i.e., the posterior means  $\hat{\mathbf{z}}_k$  and  $\hat{\boldsymbol{\theta}}_k$ ) into the other. Several applications of the joint approach are referenced in the following sections.

### ***3.3 Online Identification of Degrading and Pinching Hysteretic Systems***

The modeling and identification of nonlinear hysteretic systems is a problem often encountered in the engineering mechanics field. Nonlinear hysteretic behavior is commonly seen in civil and mechanical structures subjected to severe cyclic loadings such as earthquakes, wind, or sea waves, and in aerospace structures incorporating joints. Due to the memory aspect of hysteresis in which the restoring force depends not only on the instantaneous displacement, but also on the past history of displacement, the nonlinear force cannot be expressed in the form of an algebraic function involving the instantaneous values of the state variables of the system. Because of the importance in structural response prediction, structural control, and damage detection and health monitoring, considerable effort has been devoted by numerous investigators to the development of models of hysteretic restoring forces and techniques to identify such systems.

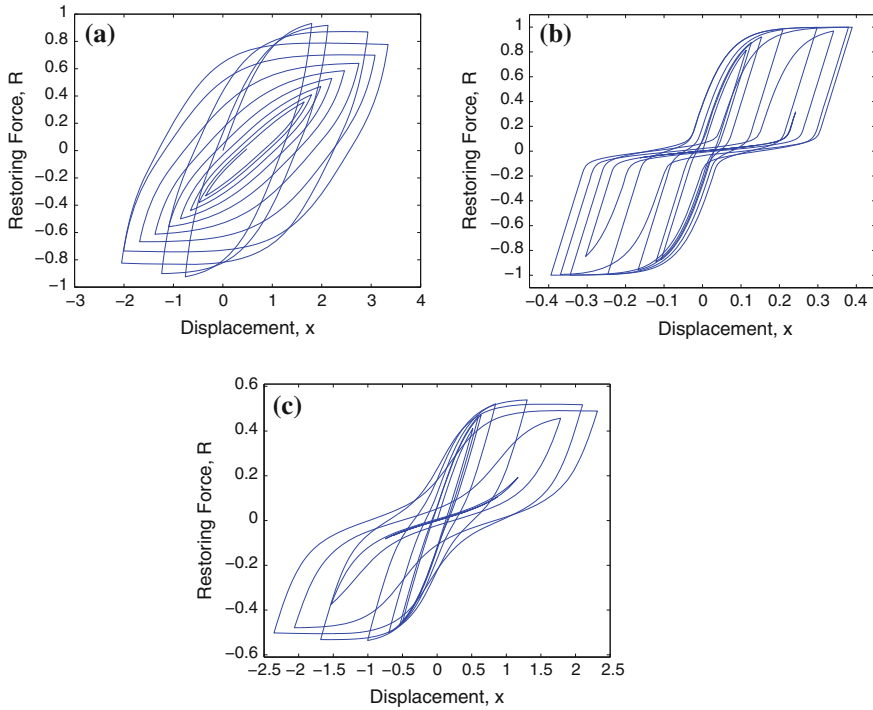
Various hysteretic models have been proposed in the past few decades. One of the most popular models is the differential Bouc-Wen model (1976) which can capture many commonly observed types of hysteretic behavior. Based on this classical Bouc-Wen model, several further extensions have been made and thus provide more capabilities to capture hysteresis which include degradation and pinching. Baber and Wen (1981) introduced degradation shape functions which allow the model take into account strength deterioration and stiffness degradation. Baber and Noori (1985) further proposed a generalized hysteretic model to incorporate pinching, a sudden loss of stiffness associated with opening and closing of cracks, commonly observed in concrete and masonry structural systems. The plots of the hysteretic restoring force against the corresponding displacement for three different types of hysteretic response are shown in Fig. 3.

Wu and Smyth (2008) apply the UKF, which is capable of handling any functional nonlinearity, to the online parametric system identification of hysteretic differential models with degradation and pinching. It has been shown through the simulation studies that with only the measurements of acceleration response and the earthquake ground acceleration the UKF is capable of tracking online system states and parameters of the complicated hysteretic systems accurately.

### ***3.4 Online Identification with Model Uncertainty***

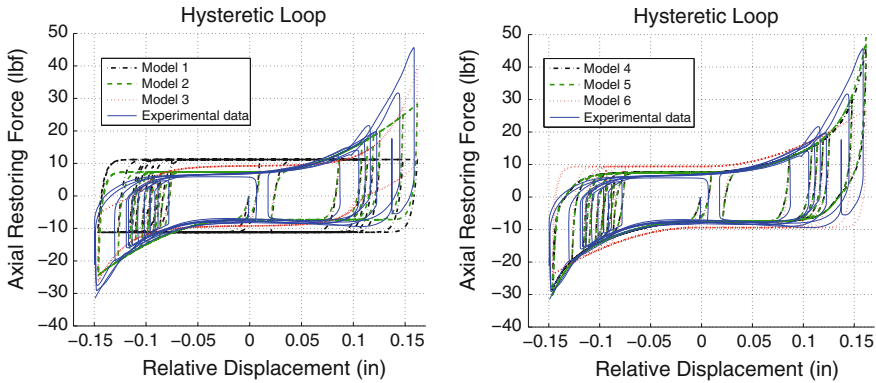
Chatzi et al. (2010) present a methodology for the online identification of nonlinear hysteretic systems where not only the parameters of the system are unknown, but also the nature of the analytical model describing the system is not clearly established. They employ the UKF method in order to investigate the effects of model complexity and parametrization. The latter can be especially challenging in the case of realistic applications involving limited information availability. The state-space formulation





**Fig. 3** Example of hysteresis loops generated by **a** the differential model with degradation only, **b** the differential model with pinching only, and **c** the generalized differential model

incorporates a Bouc–Wen type hysteretic model properly modified with additional polynomial or exponential-type nonlinear terms that are properly weighted throughout the identification procedure. The parameters associated with the candidate models might be subjected to constraints that can affect the stability of the estimation process when violated. In addition, a two-fold criterion based on the smoothness of the parameter prediction and the accuracy of the estimation is introduced in order to investigate the required model complexity as well as to potentially rule out ineffective terms during the identification procedure (online). The case of more severe nonlinearities is explored that call for the expansion of the hysteretic models commonly used in literature. The method is validated through the identification of the highly nonlinear hysteretic behavior produced by an experimental setup involving displacement and restoring force measurements. This nontypical hysteretic behavior recorded during the experiment along with the validated results is illustrated in Fig. 4.



**Fig. 4** Model evaluation using the final parameter estimates. The displacement axial restoring force time histories are reproduced and the results are plotted against the experimental data set hysteretic loop (solid blue line)

### 3.5 Comparison of Online Bayesian Estimators

The well-known EKF is often used to deal with nonlinear system identification in many civil engineering applications. In spite of that, applying an EKF to highly nonlinear structural systems is not a trivial task, particularly for those subjected to severe loading. The EKF is based upon the principle of linearizing the nonlinear state transition function and observation function with Taylor series expansions. The derivation of the Jacobian matrices and the linearization approximations to the nonlinear functions can lead to implementation difficulties. The linearization process can also introduce large errors which may lead to poor performance and estimation divergence of the filter for highly nonlinear problems. The estimates may converge to incorrect values or diverge if the initial guesses of the unknown parameters are outside the region of convergence. Unlike the EKF, the UKF does not approximate nonlinear functions of the system and measurement equations. Instead, it approximates the posterior probability density by a Gaussian density, which is represented by a set of deterministically chosen sample points. When sample points are propagated through a nonlinear transform, they capture the true mean and covariance up to the second order for any nonlinearity. The UKF operates on the premise that it is easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function. In the study of Wu and Smyth (2007), the EKF and UKF are compared and applied for nonlinear structural system identification, in particular to linear, nonlinear elastic, and nonlinear hysteretic systems. Simulation results show that the UKF produces better state estimation and parameter identification than the EKF and is also more robust to measurement noise levels.

The use of heterogeneous, non-collocated measurements for nonlinear structural system identification is explored in Chatzi and Smyth (2009). Two techniques are examined; the UKF, and the PF. In particular, two particle filter techniques are con-

sidered and their properties and implementation issues are discussed; the generic PF (or bootstrap filter), and the Gaussian mixture sigma point particle filter (GMSPPF). The methods are compared and their efficiency is evaluated through the example of a three degree-of-freedom system, involving a Bouc–Wen hysteretic component, where the availability of displacement and acceleration measurements for different degrees of freedom is assumed. For the example considered, the UKF and GMSPPF techniques are the most efficient ones when performing a validation comparison using the final identified parameters, with the GMSPPF method proving to be the most accurate one especially when it comes to the estimation of time-invariant model parameters. The performance of the generic PF method, which is the less accurate than the two aforementioned ones, can be improved through the addition of some artificial process noise, corresponding to the time-invariant model parameters, as this helps to overcome the sample depletion problem. In fact, the latter leads to improved generic PF estimates even when using a lesser number of particles.

## 4 Sensor Data Fusion: Observability Issues

A more advanced but very relevant topic is briefly introduced here. The question of a priori observability of a dynamic system, that is, whether the states of a system can be identified given a particular set of measured quantities, is of utmost importance in multiple disciplines including engineering. More often than not, some of the parameters of the system need to be identified, and thus the issue of identifiability, that is, whether the measurements result in unique or finite solutions for the values of the parameters, is of interest. Identifiability arises in conjunction with the question of observability, when the notion of states may be augmented to include both the actual state variables of the dynamic system and its parameters (joint estimation), as implemented in all previous numerical applications referenced. This results in the formulation of a nonlinear augmented system even though the dynamic equations of motion of the original system might be linear.

In the work of Chatzis et al. (2014), three methods for the observability and identifiability of nonlinear dynamic systems are considered. More specifically, for a system whose state and measurement equations are analytic, the geometric Observability Rank Condition, which is based on Lie derivatives may be used. If the equations are rational, algebraic methods are also available. These include the algebraic observability methods and the algebraic identifiability algorithms which determine the finiteness or uniqueness of the solutions for the parameters. The aforementioned methods are used to study the observability and identifiability of suitable problems in civil engineering and highlight the connections between them and the corresponding concepts.

## 5 Practical Considerations

In this last section, a distinctly different topic is reviewed. While still in the general theme of structural identification, this relates to challenges identified in the context of structural identification of bridges from ambient vibration responses while under traffic loading. It is widely known that highly accurate damping estimation is particularly challenging in structural identification, but in the context of traffic loading there are some additional potential challenges. These are outlined in this last section.

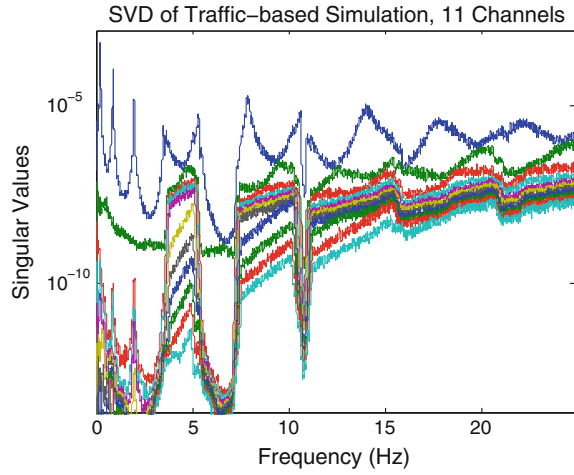
### 5.1 *Damping Estimation*

Brewick and Smyth (2013) conducted simulations of estimating the modal damping on a bridge from hour-long records of traffic loading by combining physics-based finite element modeling and signal processing. The finite element method was used to model a bridge-type structure consisting of a series of stringer beams resting atop a larger girder. The traffic loads were separated into trains and cars, with the trains modeled as partially distributed moving masses traveling along the girder and the cars modeled as point loads moving along the stringers. Vertical acceleration responses were recorded at eleven equally spaced locations along the bridge span. From these acceleration time histories, different operational modal analysis (OMA) techniques were used to find estimates for the modal coefficients of damping. The results demonstrated that a quasi-periodic component in the traffic loading introduced significant error to the damping estimates. This error could be observed in the distortion of the peaks for the power spectral densities (PSD) generated from the responses to the traffic simulations as may be seen in Fig. 5. The main OMA technique explored for the damping estimates was Enhanced Frequency Domain Decomposition (EFDD), but it could not compensate or correct for the alterations to the PSD. Other techniques such as the Stochastic Subspace Identification (SSID) method and curve-fitting frequency domain analysis were evaluated, but they produced comparable damping ratio estimates to EFDD and similarly resulted in large errors for the distorted modes. The influence of the quasi-periodic loading was perceptible, which means that the nature of traffic loads may result in damping estimates that are considerably inaccurate no matter which OMA technique is chosen.

### 5.2 *Effect of Traffic on Damping Estimation*

Accurate estimation of the damping in a structure has remained an important but challenging problem for the structural engineering community. The relative difficulty of damping estimation can be compounded when the excitation is not uniformly broadband or ambient in nature, such as when car traffic or large trains travel over a

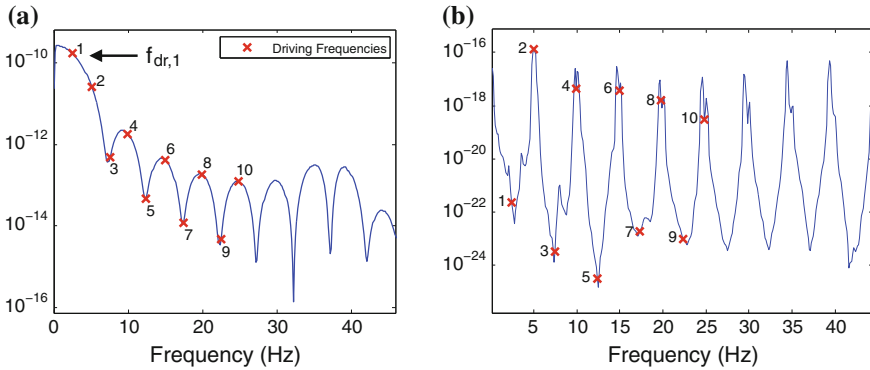
**Fig. 5** The FDD method produced a singular value decomposition (SVD) of the cross-power spectral density matrix (CPSD) of the recorded acceleration responses. The traffic simulations resulted in noticeable distortion



bridge. In the study of Brewick and Smyth (2014a) a bridge model that consisted of a series of simply supported (SS) stringers resting atop a larger girder was constructed using finite elements, and several simulations were conducted in which cars and trains crossed the bridge model. The presence of the cars and trains led to the appearance of driving frequencies  $f_{dr}$  in the response. Driving frequencies are inherent to moving loads and proportional to the velocity  $V$  of the moving loads and the length  $L$  of the beam or bridge being crossed as shown in Eq. (16), where  $n$  is the driving frequency multiple.

$$f_{dr,n} = n \frac{V}{2L} \quad (16)$$

As the moving loads traveled across the SS stringers on the bridge model, they would produce a pulse-like responses. The resulting power spectral density (PSD) of the stringers showed that significant power was concentrated at the first driving frequency and its even multiples. The vertical component of the moving load and the SS stringer responses were transferred to the girder at each support, and, owing to the continuity within the girder, each of its nodes experienced all of the car crossings. The repeated nature of the car crossings led to a PSD of the girder responses that contained peaks at the even multiples of the driving frequency. Sample PSDs of the SS stringer and girder responses are shown in Fig. 6. Over the course of a full simulation, the velocities of different cars produced peaks at slightly different driving frequencies that merged together to form shelves of elevated power. These shelves were repeated at even multiples of the driving frequencies, causing sustained regions of distortion in the frequency response spectra of the girder. Attempts were made to identify the modal damping ratios from the bridge acceleration responses using the frequency domain decomposition (FDD) and blind source separation (BSS) methods, but the driving frequencies interfered with the estimates. The regions of distortion compromised the spectra for the frequency-based methods and altered the estimated modal responses



**Fig. 6** PSDs of the response from the **a** SS stringer and **b** supporting girder due to a single car crossing

recovered from the BSS method, creating problems with identification in the time domain. The driving frequencies generated by the car and train traffic on a bridge negatively impacted both the reliability and accuracy of the damping estimates found using various operational modal analysis (OMA) techniques.

### 5.3 *Blind Source Separation for Damping Estimation*

A modified version of the blind source separation (BSS) based second-order blind identification (SOBI) method was used by Brewick and Smyth (2014b) to perform modal damping identification on a model bridge structure under varying loading conditions. The same bridge model as in Brewick and Smyth (2013) was used. The model was subjected to two different types of excitation: ambient noise and traffic loading simulated with moving loads for cars and partially distributed moving masses for trains. Acceleration responses were recorded during the simulations and treated as the mixed output signals for the BSS algorithm. The goal of the BSS algorithm is to estimate the “mixing matrix,” which in the case of structural dynamics is the same as the modal transformation matrix, and “de-mix” the outputs back into their original sources, modal responses in this case. The modified SOBI method used a windowing technique to maximize the amount of information used for blind identification from the recorded accelerations. The modified SOBI method successfully separated the individual modal responses and found the mode shapes for both types of excitation with strong accuracy. However, the power spectral densities (PSDs) of the recovered modal responses showed signs of distortion for the traffic simulations. The distortion had an adverse effect on the damping ratio estimates for some of the modes and no correlation could be found between the accuracy of the damping estimates and the accuracy of the recovered mode shapes. The responses and their PSDs were compared

to real-world collected data and patterns of similar distortion were observed, implying that this issue likely affects real-world estimates.

**Acknowledgments** This study was supported in part by the National Science Foundation under Award CMMI-1100321.

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Identification Methods for Structural Health Monitoring

Chatzi, E.; Papadimitriou, C. (Eds.)

2016, IX, 170 p. 53 illus., 39 illus. in color., Hardcover

ISBN: 978-3-319-32075-5