

Chapter 2

Technology, Society, and Education

Abstract The relation between social and cognitive evolution and changes in mass communications technologies was explored profoundly by Marshall McLuhan. Among the various ideas that his exploration uncovered was the suggestion that technology extends human faculties. The main revolutions in history, in fact, are associated with media revolutions. This chapter examines this basic framework, with discussions on the use of print and electronic media in math education, as well as the connection between math and computer science in the classroom.

Technology is just a tool. In terms of getting the kids working together and motivating them, the teacher is the most important.

Bill Gates (b. 1955).

Introductory Remarks

One of McLuhan's most famous notions was that the media of communicating information through changing technologies have more influence on social structures and individuals than the information itself, which he expressed in the well-known phrase "the medium is the message" (McLuhan 1964: 22–23):

The medium is the message. This is merely to say that the personal and social consequences of any medium—that is, of any extension of ourselves—result from the new scale that is introduced into our affairs by each extension of ourselves, or by any new technology.

A medium "shapes and controls the scale and form of human association and action" (McLuhan 1962: 9) and is therefore perceived to constitute the message itself. McLuhan put forward four laws of what media extensions are capable of bringing about: amplification, obsolescence, reversal, and retrieval. A new technological invention will at first amplify some physical, sensory, intellectual, or other faculty (Law 1). While one area is amplified, another is lessened or rendered obsolete (Law 2). Then, when the invention is used to maximum capacity it reverses its characteristics (Law 3) which are then retrieved in another medium (Law 4). The classic example of how these laws operate is mass print technology, which was

made possible in the West by Johannes Gutenberg who invented movable type around 1455, leading to the spread of the paper book as a powerful new cognitive tool, an event that allowed information to flow more broadly than at any other previous time, given that books could be produced *en masse* cheaply and made available across the world. The book thus extended the reach of human communication and of intellectual interaction. However, the spread of print literacy entailed its own set of unexpected consequences. The act of reading by oneself, rather than engaging in oral dialogue, entrenched the preeminence of individualism and privacy, rendering group identity and oral interaction obsolete until the same book was read by masses of people, a form of indirect mutual reading that allowed for the retrieval of group identity. To this day, it is true that we sense an affinity to those who read and like the same books that we do. In a similar way, these laws can be easily applied to the Internet, which has amplified all aspects of communication and information access, rendering traditional media obsolete, but also reversing and retrieving them through convergence.

Math education has always been guided by technology, in the McLuhanian sense, from the use of papyri to paper books and now the Internet. Without the development of writing, mathematics would likely never have emerged. Writing is a powerful tool, because it allows for ideas to be imprinted on surfaces through the hands, thus preserving knowledge and extending communal memory and making continuous history possible, given that written materials can be preserved for longer periods than they can by oral transmission. Writing down mathematical ideas as in the early papyri made it possible to gradually institutionalize mathematics as a discipline, and then teach it systematically, because the written text stabilizes knowledge making it more permanent and capable of being passed on to others both across space and across time. Euclid's *Elements* is a case-in-point. Without it, we would only be left with indirect oral accounts of his ideas which would be rather imperfect, since each time they would be recounted orally they would be bound to change. But we have his text for all time and can refer to it whenever we want, allowing us to develop new ideas from it.

The current proposals by educators on how to use blogs, social media, video games really presents itself as an opportunity to extend writing via electronic media, making it much more versatile, amplifying both how we process and make knowledge. Many enthusiastic educators are even prepared to discard all previous media (as we saw in the previous chapter). But this is a symptom of technological newness, as McLuhan argued, which can dupe us into believing that all previous technologies and media are rendered obsolete. So, there is a warning here—new media cannot be extricated from the past for the reason that they are connected to the past via extension.

The term *convergence* was introduced in the mid-1990s to describe essentially the McLuhanian law of retrieval in the Digital Age. It refers to the integration and amalgamation of media, technologies, and their content through digitization. It also refers to the merging of different modes of communication into one huge connected system that McLuhan (1964) called, as we saw, the electronic Global Village. This Village is very much like a real tribal village, where a new virtual form of orality

has emerged accompanied by the retrieval of *mythos* (previous chapter). A feature of orality is that it entails real-time responses, whereas traditional written communication involves a lag in receiving responses. The same feature of orality is true of digital communications today, since an interaction is perceived as occurring in real time or in slightly delayed time. Tribal orality is called “primary;” the orality that has surfaced today through digital communication is called “secondary” (Ong 1982). So, paradoxically, even though we communicate by writing (emails, text messages, and so on), our expectations from the act of communication is identical to that of oral communication—that is, we expect an immediate or quasi-immediate response. Secondary orality is also akin to primary orality in having brought about a new form of *mythos* (as mentioned), or a reliance on belief over reason and rationality. This became apparent on the threshold of the year 2000 when the “millennium bug” was thought to be a harbinger of doom. So reliant had people become on the computer that a simple technological problem—making sure that computers could read the 00 date as 2000 and not 1900 or some other date—was interpreted in apocalyptic terms. That fear was striking evidence that computers had acquired a meaning that far exceeded their original function as machines.

A third aspect of secondary orality is that it retrieves the sense of what can be called “immediate reality.” In tribal villages, the space between interlocutors was physically real. In the Global Village, it is electronic and thus “hyperreal,” to use the term introduced by the late social critic Baudrillard (1983). Hyperreality is perceived as real and is guided by electronic media. As Benedikt (1991: 1) observed when the Internet was in its fledgling stages, in this space “the tablet becomes a page becomes a screen becomes a world, a virtual world. Everywhere and nowhere, a place where nothing is forgotten yet everything changes.”

So, what do the McLuhnian laws imply for math education? They imply that the new media do amplify how we learn at the same time that they retrieve a sense of (secondary) orality, uniting people as if in a village. The way we learn today, therefore, is both real and hyperreal, oral and literate—and all this has definite implications for education. The contemporary math classroom exists in both physical reality (as a walled-in structure with physical media such as books) and electronic hyperreality (through social media and the like). Actually, these laws have already taken place. A perusal of the relevant literature on math education, as well as an Internet search of relevant websites, shows that this is indeed the case. There are at least four main uses of new technologies in the classroom:

1. *As an ancillary tool*: using digital technologies such as social media to amplify or extend classroom teaching.
2. *As an integrative tool*: using the technologies in tandem with traditional learning materials, such as print textbooks.
3. *As a collaborative tool*: using technologies to get students to interact among themselves and others outside the classroom, such as with virtual math communities, clubs, and so on.
4. *As an embedded tool*: taking into account what students bring to the classroom in terms of technological skills as “embedded” in their lives.

There are, of course other uses of new technologies (see Martinovic 2015). The main subtext is that we can no longer turn back the clock, since students live in the Global Village and thus bring with them a new learning style to class that emanates from a daily engagement in hyperreality and is, thus, distinctly different from the kind of style that students reared in the Age of Print brought to class with them. Another difference between students of the past and of the present is the relevance of pop culture to everyday life. As discussed briefly in the previous chapter, anecdotal math can be seen as a new medium itself, connecting math to cultural content and thus to many of the modalities of the Global Village.

This chapter will look at the implications for classroom math pedagogy that derive from living in the Global Village. The specific topics include: an overall survey of the new technologies, the learning and teaching of math through traditional print media, access to the broader math learning community through online media, the connection of math to computer science, and, finally, a concise pedagogical perspective that can be gleaned from this kind of consideration.

Technology and Mathematics

As Anderson (2013: 298) warns, we should be very careful to embrace technology as an educational panacea simply because it is new and trendy, since it can itself become a source of new and unwanted problems in education if we do not understand what it is and how it blends with social life:

I question silver bullet solutions to perceived social problems, among which our public education makes an easy target. This allows me to slide from teaching and learning in the early grades right into issues of later training in the social sciences, all larded with prescriptive exposure to the quantitative. Along the way, mathematics itself has been commonly misconstrued as narrowly quantitative, even reduced to arithmetic. Technology, computers included, while frequently suggested as solutions to social and educational problems often contribute to those very problems, although technology most certainly does benefit business and industry.

In a McLuhian theoretical framework, technology is defined as any extension of human abilities. This is why it is so alluring—it amplifies everything from locomotion (the automobile) to intellection (the computer). But there is always a danger in becoming completely immersed in any new technology, as McLuhan also warned, especially mass communications technology, which enhances the reach and possibilities of the human voice. Strictly defined, communication is the exchange of messages between members of the same species. In the human species, the exchange can be interpersonal (between human beings), group-based (between some individual or media outlet and audiences), and mass-based (involving communication systems that encompass entire societies). Communication occurs by means of three main media of transmission—(1) a natural biological system (the voice, touch, etc.), (2) some material artifact that involves writing or drawing (a book, a painting, etc.), or (3) electronic technology (a radio, a television set, the Internet, etc.).

All three modes are utilized today. And all three are employed to support learning. In the ancient world, learning occurred either through the oral mode or, later, through artifactual and oral modes in tandem. Today, these are still operative in learning, but they are greatly amplified by new media. Simply put, in a classroom today we interact dialogically, we use print materials, and we use electronic media—all in the service of the learning styles of contemporary students. The natural medium (voice) still has emotional power and contributes to learning through real dialogue. It is still a powerful mode for the delivery of education. A central problem with the MOOC model, discussed in the previous chapter, is that it sees the natural mode as obsolete. Of course, one can engage with people through a screen in a MOOC course, but this removes the effect of immediacy and thus of the many important learning nuances that physical contact entails. It is strictly hyperreal. Of course, some teachers are better than others at classroom oral interaction, but this is a variable that cannot be eliminated, as the MOOC model obviously attempted to do. The artifactual mode of using print materials also continues to have great importance in math education. Reading a print book is not an antiquated or anachronistic activity. It is a useful one that adds considerably to the learning process. But over-reliance on textbooks and other print materials is what McLuhan called a “rear-view mirror” perspective of learning that stultifies it by “freezing” it in time reversing the present to the Age of Print. Finally, the electronic mode of interaction has extended many possibilities of classroom learning by connecting the classroom to the outside world where information and insights can be gleaned easily and brought to bear on classroom practices.

To reiterate, technology is the making of tools (physical and intellectual) which extend the biology and psychology of the human organism. An ax extends the ability of the human hand to break wood; the wheel expands the capacity of the human foot to cover distances; the computer amplifies the capacity for memory retention (through access and retrieval functions) and for processing computations quickly and accurately; and so on and so forth. The specific types of tools developed by a society at a certain point in time will determine how that society will evolve. Any major change in technology brings about a concomitant shift in social systems and, as a byproduct, in human consciousness. Ancient cuneiform writing, impressed indelibly into clay tablets, allowed the Sumerians to develop a great civilization; papyrus and hieroglyphics transformed Egyptian society into an advanced culture; the alphabet spurred the ancient Greeks on to make extraordinary advances in mathematics, science, technology, and the arts; the alphabet also made it possible for the Romans to develop an effective system of government, among many other things; the printing press facilitated the dissemination of knowledge broadly, paving the way for the European Renaissance, the Protestant Reformation, and the Enlightenment; radio, movies, and television brought about the rise of a global pop culture in the twentieth century; and the Internet and the World Wide Web ushered in the Global Village as the twentieth century came to a close.

As McLuhan claimed, the first great paradigm shift of human civilization was a consequence of the invention of alphabetic writing and the spread of literacy, replacing orality as the main mode of learning and of knowledge-making. Reading

and writing activate linear thinking processes in the brain, because printed ideas are laid out one at a time and can thus be connected to each other sequentially and analyzed logically in relation to each other. The orality of pre-writing societies, on the other hand, is not conducive to such precise thinking, because spoken ideas are transmitted through the emotional qualities of the human voice and are, thus, inextricable from the subject who transmits them. Literacy, however, engenders the sense that knowledge and information in books and other written media serve attainment of “objectivity.” This perception is bolstered by the fact that printed information can be easily categorized and preserved in some durable material form and then maintained in buildings (such as libraries) or on personal bookshelves, literally rendering it an “object.” Without the advent and institutionalization of book-based print literacy, the spread of knowledge throughout the world that we now hold as critical to the progress of human civilization would simply not have been possible; it would have remained a subjective act that evanesced rapidly. But orality did not disappear from human life. It has always been retrieved in some way or other (Law 3). The spoken word comes naturally; literacy does not. Through simple exposure to everyday dialogue, children develop the ability to speak with little or no effort and without any training or prompting whatsoever. Literacy, on the other hand, does not emerge through simple exposure to printed texts. It is learned through instruction, practice, and constant rehearsal.

An implicit fifth law of media, which is not formulated directly by McLuhan, can be called the “3-E” Law of Economy, Efficiency, and Effectiveness. Basically, it stipulates that when each new system emerges it is normally more economical (in form) than previous ones and also more efficient and effective to meet new needs. Consider the development of positional number systems. The invention of the decimal system, for example, made it possible to represent numbers and mathematical operations in economical ways, that is, in ways that compressed numerical information better than, say, the Roman numeral system. This new tool thus allowed mathematics to evolve more rapidly, allowing us to do a lot with very little—ten digits are all that is needed to represent numbers ad infinitum. In contrast to non-positional systems of notation, it is efficient, mirroring the efficiency of the alphabet. Indeed, after the Greeks developed their alphabet characters they used them also for numbers. This “doing a lot with very little” is known as *double articulation*, a term introduced by the French linguist Martinet (1955).

Consider a concrete example. The number “two thousand two-hundred and fifty-three” is represented in the Roman system as follows:

MMCCLIII.

Now, when compared to the decimal representation of the same number, 2,253, it is instantly evident that it is much easier to read, if we understand the positional rule used to construct it—the position of each digit in the numeral indicates its value as a power of ten. The advantages of the decimal system become even more conspicuous if we consider carrying out arithmetical operations, as, for instance, adding $2,253 + 1,337$, with Roman numerals. Here is how it would look on paper:

$$\text{MMCCLIII} + \text{MCCCXXXVII} = \text{MMMDC}.$$

The task is a daunting one, especially for anyone accustomed to using the decimal system. It is further complicated by the fact that a smaller numeral appearing before a larger one indicates that the smaller one is to be subtracted from the larger one. Clearly, it would take quite a bit of effort to carry out the addition, keeping track of all the letter-to-number values, especially when we compare it with the minimal effort expended to perform it with decimal numerals. The latter is, in other words, more effective.

The positional system amplifies our understanding of number because it allows us to eliminate a lot of symbolic material and concentrate on the number concept itself. The 3-E Law thus implies that economy and efficiency allow for a more direct contact with the ideas below the symbols. For a place-value notation system to work it requires the use of a new kind of “dummy” symbol, or place holder—a symbol showing that a certain place is to be left “empty” (without value). That symbol is, as we now know, 0. The 0 makes it possible to differentiate between numbers such as “eleven” (=11), “one hundred and one” (=101), and “one thousand and one” (=1001) without the use of additional symbols, positional rules, and so on. The 0 symbol in a numeral tells us, in a word, that the position is void or empty. It was probably first conceptualized by the Babylonians, who left a blank space for it, but banned by the Greeks (of all people). The Chinese also left an empty space on their counting boards. There is archeological evidence that the Mayans had a symbol for zero by about 250 CE and that the Hindus, who developed the symbol we use today, had devised it by the late 800s. The word *zero* derives from *ziphirum*, a Latinized form of the Arabic word *sifr* which, in turn, is a translation of the Hindu word *sunya* (void or empty). In 976 CE the Persian mathematician al-Khwarizmi defined it simply as a place-holder, remarking that if no number appears in the place of tens, then a little circle should be used to keep the rows.

The zero symbol changed everything. The decimal system reached Europe first in 1000 CE through the efforts of Pope Sylvester II. But it hardly got noticed at the time. It was reintroduced in a much more practical way to medieval Europeans a few centuries later, as is well known, by Fibonacci. With the publication in 1202 of his *Liber Abaci*, Fibonacci succeeded in convincing his fellow Europeans that the decimal system was far superior to the Roman one (Devlin 2011a). But Fibonacci realized that a symbol for “nothingness” would bring about philosophical objections. So he started off his book reassuring readers that zero was only a sign that allowed for all numbers to be written (cited in Posamentier and Lehmann 2007: 11):

The nine Indian figures are: 9 8 7 6 5 4 3 2 1. With these nine figures, and with the sign 0, which the Arabs call zephyr, any number whatsoever is written.

Now, the number of new mathematical ideas that have come from the adoption of zero is enormous. As an element in the new “toolkit” of positional numbers it has allowed mathematics to evolve in ways that would have been impossible without it. It is a virtual law of mathematical discovery that any change to its notational and symbolic toolkits, will present further ideas and suggest new paths of discovery.

Another example is negative numbers. Negative numbers were known in antiquity. The concept probably surfaced first in China, where it is found in a 250 BCE text titled *Chui-chang swan-shu* (The Nine Chapters). During the seventh century negative numbers are found in the bookkeeping practices and astronomical calculations of the Hindus. It was not until the sixteenth century that such numbers surfaced in Europe, appearing in Girolamo Cardano's works. It is not coincidental that the term *negative* comes from the Latin *negare* ("to deny"), perhaps because the existence of such numbers had been denied for so long, or because it implied the "denial" of the positive, so to speak. On the one hand, nothing new was accomplished by introducing the concept of negative number. It was already part of accounting practices, such as showing loss on a ledger. But, coupled with the use of zero the new negative numbers suggested a broader number system and thus became included in the old—the set of integers. This shows how the law of retrieval works—a new invention makes the old system obsolete (the positive integers by themselves), but also retrieves it and amplifies it considerably. The natural numbers were thus enlarged to include negative numbers, and the system became a new powerful cognitive tool guiding discoveries that would have literally been unthinkable before.

In sum, technology is an extension—physical (the wheel extends the foot), intellectual (the alphabet extends the ability to record knowledge), symbolic (mathematics extends the ability to count), printing press (extends the use of writing), and so on. And as each age changes its technologies, so too does it change its modes of knowledge-making, its understanding of the world, its transmission patterns, and thus leads to new forms of consciousness. In the Age of Print, there was no question that the book was the main tool for conducting classroom learning. It was a synchronic one for the era—it was a tool that teachers and learners knew how to use efficiently and effectively. As the Age of Print gave way to the Electronic Age, now the Digital Age, new tools for knowledge-making have also changed, but the tools of print have not disappeared, they are retrieved in various ways.

The Gutenberg Galaxy

From ancient times, math education was based on two basic media—an oral dialogue between the teacher and the student and a text, which presented topics and relevant illustrations, such as the *Ahmes Papyrus* in Egypt, Euclid's *Elements*, Recorde's books, and so on. By the time of the reformers, the print text became even more critical as a teaching and learning tool, given that it was organized in a sequential fashion, since the topics were laid out to mirror the purported increasing complexity of the subject matter and the psychology of the learner. By the early years of the twentieth century, the textbook became the key medium (and thus tool) in the delivery of math education, increasingly bolstered with pedagogical features such as visual illustrations and practical applications sections. The print textbook became the hub on which pedagogy turned because, as McLuhan pointed out, from the early 1500s to virtually the present day, the world was immersed in the "Gutenberg

Galaxy,” a synonym for the Age of Print, named after Johannes Gutenberg (mentioned above). This is a universe where print guides and structures social activities, knowledge-making, and influences all aspects of human interaction.

Writing was the first artifactual technology. Ancient writing systems were a mixture of pictographs, ideographs, and some proto-alphabetic characters called phonographs. The early civilizations were built on writing systems and written texts were preserved, for example, on walls, on tablets, on papyri, and other materials. This led to the concept of the library. One of the first libraries was the one founded in ancient Alexandria; it preserved books and was a school as well, where students went to gain knowledge. Euclid was one of them. With the invention of the alphabet in ancient Phoenicia around 1000 BCE, the new mode of writing altered the way people transmitted and recorded knowledge in any part of the world where it was adopted. Alphabets reflect the 3-E Law (above)—they are economical (a small set of characters can be used to make words ad infinitum), efficient (phonetic writing takes up much less space than pictographic writing), and effective (they allow for ideas to be laid out in sequence and organized on a page). Writing has been the basis for recording, spreading, and preserving knowledge ever since humans left the ancient tribes to establish the first civilizations. It engendered the first true cognitive paradigm shift in human history.

Until the late 1400s, all paper materials were written by hand, known as manuscripts. Copyists called scribes, many of whom were monks, made duplicates of manuscripts. But they were very expensive, because the scribes decorated them with pictures and designs. They were also rare and generally inaccessible. Knowledge was still in the hands of a privileged few. All that changed with the advent of the printing press and mass paper-based typesetting technologies. Although it was a Chinese printer named Bi Sheng who had invented movable type in the 1000s, it was not until 1447 that the German printer Johannes Gutenberg developed movable metal type technology, leading to the first printing press capable of producing numerous copies of paper documents quickly and cheaply.

The event was truly revolutionary. As books and paper materials became available in massive quantities, printing shops sprung up all over, publishing books, newspapers, pamphlets, and many other kinds of print documents inexpensively. Publishing became a major business shaping trends of all kinds, as more people gained literacy. The latter became a necessary skill in the ever-changing workplace, leading eventually to the view of education as necessary for everyone, and to the 3-R philosophy mentioned in the previous chapter. With more and more people able to read and write, ideas spread more broadly than ever before. Revolutions of a religious, political, social, and scientific nature were inspired indirectly by people literally “reading each other.” Books could be sent all over the world, and ideas started crossing political borders, increasingly uniting the world and leading to standardized ways of doing things in the scientific and business domains. In a phrase, the invention of the printing press was the technological event that paved the way for the establishment of a global civilization.

With the subsequent advent of the Enlightenment and then the Industrial Age, print literacy became even more crucial. Treatises, textbooks, and the like became central in mathematics and math education. The publishing business surged in the

twentieth century, as print materials became even more inexpensive and available *en masse*. New types of print materials, such as comic books and pulp fiction magazines, emerged, leading to an ever-burgeoning pop culture shaped in part by such materials. These also quickly jumped from print to electronic platforms such as radio and cinema. The twentieth century thus saw the beginning of the weakening of the Gutenberg Galaxy, as writing started to be possible on different surfaces and print texts to be produced electronically. By the 1980s, when photocopying made duplication easier and more rapid and desktop publishing (the design and production of publications of all kinds using microcomputers with graphics capabilities) became widespread, the traditional typesetting technologies started to become obsolete. Sophisticated word-processing software emerged to produce all kinds of print materials, which could be transmitted instantly via computer communication systems to other locations for editing, redesigning, printing, and distributing. As the Internet became a daily reality by the mid-1990s, the Gutenberg Galaxy had come to an end, and along with it a new paradigm shift occurred ushering in the Digital Age. The children born and raised in this age thus bring different expectations of the classroom than those born in the Age of Print. But the law of retrieval implies that print has not disappeared; it continues to play significant roles in education.

Math Education via Print

In an age of convergence the print textbook is still around—as the law of retrieval claims, a previous medium is eventually retrieved for its usefulness in the context of emerging new media. This law allows us to argue against three myths that have crystallized in the post-Gutenberg Galaxy:

1. the printed textbook is irrelevant, given all the online resources available today and access to all kinds of written texts on the web;
2. students no longer read; they only extract from a text what they need;
3. textbooks are too constraining since their content is fixed, literally, on paper, and can only be updated by new editions which take time and money.

The first myth can easily be dismissed, since most math classrooms and curricula still envision a role for textbooks (to varying degrees). The textbook is a script that gives the outline and sequence of events that can be adapted, generally, to various learning situations and teaching styles. Without it, there would have to be an ongoing negotiation of content and exercise material between teacher and learner. This works well for individualized or private lessons, as is done by many private businesses who offer math training to individuals outside the school system. But even those businesses develop their own print materials for teaching learners privately.

The second myth can also be easily rejected. It is simply not true that young people read less than previous generations of students. Everyone reads books to get information or for recreation (novels and trade books). With the arrival of cinema and radio in the early part of the twentieth century, reading for recreation converged

with viewing and listening. Although the new electronic media also ventured into the fields of information and knowledge (as in newscasting and documentaries), by and large these were areas that were still felt to be best suited to print texts. What the new media brought about, however, was a new genre of print, called pop math, which will be discussed below—a genre that was inspired by, and modeled after, the entertainment styles of radio, cinema, and later television.

The third myth—that textbooks are constraining—has always been true. A text is just that—something to be used and adapted according to situation. The variability lies in the situation itself, not in the subject matter as “contained” in, and “constrained” by, a print textbook. This is why in the Age of Print, textbook publishing became a major industry, as publishers strived to put out “different” textbooks for every teaching need and expectation. This is, by and large, still the case.

Another myth is that print materials can easily be transformed into online texts, which are more broadly accessible, free, and just as feasible. Recall that one of the reasons for the lack of success of MOOCS (previous chapter) was that, for many students, the MOOC format was really nothing more than “an online textbook” with hypertextual capacities. I myself conducted an anecdotal survey in one of my larger classes at the University of Toronto. The university offered my lectures in online form (they were taped and then put on an electronic blackboard). Students thus could easily opt out of attending the class and avoid buying the textbook, since my lectures followed its contents exactly. I noticed two patterns during the experimental course—(1) most of the students still came to class and, (2) after distributing a questionnaire at the end of the course, most of the students who filled it out (over 80 %) said that they bought the textbook and found it to be as useful as, if not more so, than the lectures. The two seemed to complement each other. Moreover, a few commented that reading the textbook was easier than following the lectures, because they could go back and forth and underline parts and thus study more systematically. This was much easier than going back and forth on the video, which often contained interruptions and digressions (joking by the instructor, immediate responses to tangential questions, and so on).

Needless to say, this was an unscientific and largely anecdotal survey, but there is no reason to believe, given also the MOOC failure, that it is not typical of how students perceive the classroom and the textbook used in a course. In the Digital Age, previous media do not disappear; they are retrieved and given a different focus. The same is true for all previous forms of human endeavor. We do not eliminate previous forms of poetry, music, and even science because new ones have emerged. Rather, they converge with the present ones.

Pop Math

The convergence of print with entertainment media led to mass market books that were intended for everyone to read and enjoy, bringing out that math can be enjoyed by anyone, not just studied formally at school. This was probably a

response to the growth of electronic forms of exposition of content of all kinds on radio, in cinema, and on TV. I know of no books, such as the “Math for Dummies” series of books, that were published before the electronic era. This genre continues to have a niche market that is very lucrative. As such, it is a source for extending the math classroom. For example, books written by Irving Adler and Isaac Asimov in previous decades, and contemporary ones by Keith Devlin (among many others) have captured a wide audience for mathematics, portraying it as something enjoyable in addition to being important. And, of course, there is a veritable slew of “self-study” math manuals that inundate the book market every year.

This type of publication constitutes a “pop math” genre that rivals other pop print genres (Nuessel 2013). Like anecdotal math, which focuses on aspects of the uses of math in society, pop math is part entertainment, part serious intellectual engagement. There are four main pop math genres:

1. books presenting math to a general audience in an easy-to-read fashion
2. biographical novels about mathematicians
3. collections of math puzzles for challenge and entertainment
4. fictional novels based on math in some way.

One of the classic books in category (1) is the 1940 one by James Kasner and John Newman, *Mathematics and the Imagination*. The authors show, in clearly understandable language, how mathematics is tied to imaginative thought. We come away grasping intuitively that mathematics is both a logical mode of thinking and an art, allowing us to investigate reality like any other tool of the imagination. Another classic book in this genre is the one by Reuben Hersh, *What Is Mathematics, Really?* (1998). Hersh presents an endearing account of math as a human activity and a social phenomenon, understandable as a product of history, not as a product of pure logic. Similar books in this subgenre are those by Ian Stewart, Keith Devlin, Joseph Mazur, Alex Bellos, Simon Singh, Richard Courant, William Dunham, Amir D. Aczel, Paul Hoffman, Mario Livio, Eli Maor, among many others. These are the books that make it to “Goodreads” and other such sites, and their authors are as popular as those who write popular trade books.

Such books are not without constructive classroom value. A perusal of teacher websites shows that many of these are found on the reading lists of courses, recommended for further study or for “better understanding.” The point is that in the Electronic and Digital Ages, where pop culture reigns supreme, mathematics has found a new medium for its dissemination and enjoyment. This type of book would have been considered an “insult” to math educators in the past, when pop culture was not as widespread as it is today. The pop math book has not rendered the technical book obsolete; on the contrary it is a conduit into the world of serious math, retrieving it more effectively and efficiently for a new generation.

Genre (2) above—books written about the lives and achievements of famous mathematicians—are now also part of pop culture. The best known one in recent times is Silvia Nasar’s, *A Beautiful Mind* (2011), which was turned into a major movie. It is based on the true story of mathematical genius John Nash, who at the age of thirty slipped into madness, but who, thanks to the loyalty of a woman and the math

community, eventually won a Nobel Prize for game theory. In the same category is the 1992 book by Robert Kanigel, *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, which recounts the amazing story of an unschooled Indian clerk, Srinivasa Ramanujan, who wrote a famous letter in 1913 to G.H. Hardy about some ideas he had in the field of number theory. From this, a marvelous collaboration started leading to many new explorations in advanced mathematics. The narrative has a tragic nuance to it, since, according to the author, Ramanujan's creative intensity eventually took its toll, leading to his death at the age of thirty-two and leaving behind a legacy that is still being plumbed for its many insights to this day.

Genre (3) above—recreational or puzzle mathematics—consists of collections of ingenious puzzles, by classic puzzle makers such as Lucas and Edouard (1882), Carroll (1886), Loyd (1914, 1952, 1959), Dudeney (1919, 1958, 1967), Heath (1953), Gardner (1956, 1979, 1982, 1987, 1997, 2001), Ball (1972), Smullyan (1978, 1997), to many others today, such as Averbach and Chein (1980), Wells (1992), Olivastro (1993), Delft and Botermans (1995), and Moscovich (2001). Puzzle math constitutes a substantial niche in mass market publishing, having obvious educational value. Although the use of problems to explore math can be traced right back to antiquity, collections of puzzles as a pop math genre is essentially a modern-day phenomenon. Sudoku, for instance, is now a main staple in math courses intended to impart notions of logic. In their interesting book, *Taking Sudoku Seriously* (2011), Jason Rosenhouse and Laura Taalman show how the inner logic of Sudoku mirrors inherent processes of mathematical cognition.

The periodical format of print has also ventured into these three areas as well. Journals such as *The American Mathematical Monthly*, *The Mathematics Gazette*, *Mathematics Magazine*, *Notices of the AMS*, alongside popular science magazines such as *New Scientist*, *Scientific American*, and *Plus Magazine*, which cover many of the same genres discussed above, alongside puzzle and games periodicals, such as *The Journal of Recreational Mathematics*, *Recreational Mathematics Magazine*, *Eureka*, and *Games* have captured many of the same niche markets of the books.

Genre (4) above—fictional or semi-fictional narratives that deal with mathematical topics (Mann 2010; Sklar and Sklar 2012)—also have great mass appeal and educational value. After Lewis Carroll's *Alice in Wonderland* and *Through the Looking-Glass*, whose mathematical structure has been studied in depth by puzzle math writers, including Martin Gardner in his introductions to re-publications of Carroll's novels (in 2000, 2006) and Devlin (2010), perhaps the most famous book in this category is the novella, *Flatland: A Romance of Many Dimensions*, written by the preacher and literary critic Edwin A. Abbott in 1884. The characters of the novel are geometrical figures living in a two-dimensional universe called Flatland. They see each other edge-on, and thus as dots or lines, even though, from the vantage point of an observer in three-dimensional space looking down upon them from above, they are really lines, circles, squares, triangles, etc. Derivatives of the novel include, *The Dot and the Line: A Romance in Lower Mathematics* (Juster 1963), *Sphereland: A Fantasy about Curved Spaces and an Expanded Universe* (Burger 1965), *The Planiverse: Computer Contact with a Two-Dimensional World* (Dewdney 1984), *Flutterland: Like Flatland, Only More So* (Stewart 2001), and

Spaceland: A Novel of the Fourth Dimension (Rucker 2002). Stewart's *Flutterland* basically updates the geometry that was known to Abbott during his times, including notions such as fractional dimensions, isolated points, topology, and hyperbolic geometry.

There are various film versions of *Flatland*, including *Flatland* (1965) narrated by Dudley Moore and Roddy Maude-Roxby, the short *Flatland* (1982) directed by mathematician Michele Emmer, and *Flatland: The Movie* (2007), an animated film with the voices of Martin Sheen, Kristen Bell, Michael York, and Tony Hale, and *Flatland 2: Sphereland* (2012), also starring Martin Sheen and Kristen Bell. Various television programs have alluded to the novel, including *The Outer Limits* (October 3, 1964) and *The Big Bang Theory* (January 11, 2010). Two video games are also based on the novel—*The Flatland Role Playing Game* by Marcus Rowland and *Miegakure*.

Other well-known books in genre category (4) include the following:

1. *The Man Who Counted* (by Júlio César de Mello e Souza 1949), published originally in Brazil (*O Homem que Calculava*), is a novel based on recreational mathematics. It was written in the style of the *Arabian Nights*, telling the story of a math wizard who travels to Baghdad, meeting a companion along the way. Their adventures provide the narrative context for discussing mathematical ideas.
2. *Fantasia Mathematica* (compiled in 1958 by Clifton Fadiman) is an anthology of stories, poetry, and other kinds of writing on mathematical topics. A companion volume, *The Mathematical Magpie* (1962), contains writings by famous writers, including Robert Heinlein and Aldous Huxley.
3. *Mathenauts: Tales of Mathematical Wonder* (compiled in 1987 by Rudy Rucker) is an anthology of science fiction stories that revolve thematically around mathematics.
4. *Uncle Petros and Goldbach's Conjecture* (by Doxiadis 1992) is a novel about a young man's relation with his reclusive uncle, who sought to prove Goldbach's famous conjecture. The novel highlights mathematical problems and the history of mathematics.
5. *Math Girls* (by Yuki 2007), followed by *Math Girls: Fermat's Last Theorem* (2008), *Math Girls: Gödel's Incompleteness Theorems* (2009), and *Math Girls: Random Selection Algorithms* (2011) all deal with important math theorems in fictional ways.

To summarize, it is useful to refer to Mann's (2010: 58) typology of pop math in fiction, which is as follows:

1. Fiction by mathematicians
2. Fiction using mathematical structures
3. Fiction expounding mathematics
4. Other (such as fiction by Jorge Luis Borges)
5. Fiction about real mathematicians

6. Fiction about doing mathematics
7. Fiction about mathematical ideas
8. Fiction with mathematicians as characters

The myth that print culture has no more relevance to math education is easily debunked by looking at the many types of books and periodicals that are both popular and highly educational, alongside the traditional classroom textbooks (some of which now have incorporated elements of pop math into their styles, such as the use of cartoons and anecdotes). In other words, “math in print” is still a viable educational medium (Danesi 2013a). In fact, it has a new life through new genres and subgenres of math writing.

In sum, including the actual technical writings of mathematicians, ancient to contemporary, the communication of mathematical ideas via the print medium can be represented diagrammatically as follows (Fig. 2.1).

We certainly do not live (at least exclusively) in the Gutenberg Galaxy today. But it has left its legacy, cognitively and socially. We are still attracted to paper books—there is something unique about holding a paper book in one’s hand and reading about math or studying it by turning its pages. As Flapan (2013) has observed, using a textbook, for example, may work in some contexts, not in others, yet it cannot be discarded completely. There are many more options, of course, but print is still a useful one as a learning tool. Ultimately, Flapan suggests that the teacher is the one who can best decide what to do. There are no full-proof methods or media that will work universally. Flapan’s advice is sage indeed (Flapan 2013: 148):

I won’t tell anyone that they should teach like me, just as I won’t tell anyone that they should parent like me. You have to care about and respect your students (and your kids), to be clear in your lectures, and stay excited about the material. Beyond this, you should use you own experience and self-awareness to tailor your teaching style to your strength and interests. As long as you do these things, there is no right or wrong way to teach.

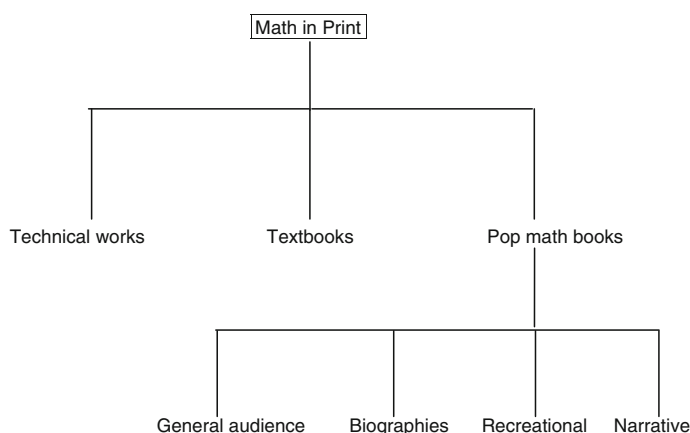


Fig. 2.1 Math in print

The Digital Galaxy

Pop math could not have emerged and thrived in previous eras, which had rigidly separated entertainment from academia. Pop math could only thrive in the Digital Galaxy, where the line between engagement and entertainment is a blurry one indeed. But this may be an overstatement. There is solid evidence that mathematics also had recreational functions in the ancient world (Olivaastro 1993). Legend has it that Archimedes devised his famous Cattle Problem to take revenge on one of his adversaries, whom he was trying to dumbfound with his mathematical prowess. Nevertheless, the Cattle Problem stimulated the development of notational standards in mathematics. Cognizant of its potential impact on mathematical method, Archimedes dedicated it to his friend, the great Alexandrian astronomer Eratosthenes.

The lesson here is that recreation and knowledge may be more intertwined than we might think. The list of puzzles that have led to some mathematical discovery, or have been instrumental in the establishment of a new branch of mathematics, is a long and impressive one (Danesi 2002). Archimedes' problem sensitized the ancient mathematicians to the need of a useful abstract notation; Fibonacci's Rabbit Puzzle led to the discovery of many patterns hidden among the integers; Euler's Königsberg's Bridges Puzzle led to the development of graph theory, combinatorics, and topology; and the list could go on and on. As Kasner and Newman (1940: 156) emphasize, the "theory of equations, of probability, the infinitesimal calculus, the theory of point sets, of topology, all have grown out of problems first expressed in puzzle form." It is therefore somewhat surprising to find that a purely *recreational* approach to math education took so long to develop, given the importance of puzzles to the evolution of the discipline. Although individuals such as Alcuin, Fibonacci, Tartaglia, and Cardano wrote mathematical puzzles for various practical and pedagogical reasons in their eras, it was not until 1612 that recreational mathematics emerged as a kind of semi-autonomous branch of mathematics. In that year, the French poet and scholar Claude-Gaspar Bachet de Mézirac published the first comprehensive collection of mathematical puzzles, a book titled *Amusing and Delightful Number Problems*. Only since then do puzzles appear in math classrooms, albeit in a limited way, usually as "extra" or "creative" forms of learning. With the advent of pop math, the situation has changed radically.

Even a superficial perusal of websites dealing with math education will reveal that there is now considerable about the possibilities afforded by the Internet for extending and enriching the classroom via media such as pop math. Moreover, one can find anything that is in any textbook or other print material on the Web. The Web is a new "tool" in the McLuhanian sense, extending various faculties that were constrained by the print medium, such as hypertextuality. There is absolutely no doubt that this has brought about a second major paradigm shift in learning and education.

The advent of the Digital Galaxy is, relatively speaking, a recent phenomenon. As computer technology improved steadily after the Second World War, smaller and cheaper computers could be built for all kinds of purposes. By the late 1970s, it became economically feasible to manufacture personal computers (PCs) for mass consumption. The first PCs were mainly word processors; that is, they simply added computer-based capacities to the typewriter in order to make writing and changing printed text significantly easier and more sophisticated. The first microcomputers had the power of older, larger machines, but could fit onto a desktop. This was accomplished because of new miniaturization technologies that allowed manufacturers to compress the memory and processing power of thousands of circuits onto tiny chips of materials called semiconductors.

The breakthrough that truly ushered in the Digital Galaxy occurred in the early 1990s with the arrival of the World Wide Web (WWW), developed by Tim Berners-Lee, a British computer scientist at the European Center for Nuclear Research (CERN). In the history of mass communications, no other device has made it possible for so many people to interact with each other, irrespective of the distance between them and of the time, on a regular basis. Moreover, in the Digital Galaxy, it is no longer accurate to talk about competing media. Advances in digital technologies and in telecommunications networks have led, as mentioned, to a convergence of all communications systems. This has led, in turn, to the emergence of new lifestyles and careers, to the creation of new institutions, and to radical changes in all domains of society. Today, the WWW contains countless documents, databases, bulletin boards, and electronic publications, such as newspapers, books, and magazines in all media forms (print, visual, etc.). Unlike print texts, Internet pages can be updated constantly and, thus, are never out of date. And this is what makes them particularly attractive over print.

For education, the WWW is a huge information-search tool and an interactive space for exchanging and conducting research. In the case of mathematics, the number of websites presenting solutions to classic problems, introducing complex concepts, and so on, is gargantuan. Some have become popular across the educational landscape. One of these is Math Forum, which hosts various pedagogically-focused websites (*Problems of the Week*, *Ask Dr. Math*, *Teacher2Teacher*, *Internet Resource Collection*, and so on). Given the enormous potential of this new tool, math educators have started compiling lists of websites thematically for a more guided and informed search. Math has literally started going online.

Math Online

As mentioned, there are four main uses of technology in education—as an ancillary, integrative, collaborative, and embedded tool. Above all else, it allows for instant access to resources and individuals at a distance and, because of hypertext, allows for links between resources and individuals. It also permits updating and editing on

a constant basis. Among the extensive and expansive features that the Internet offers over print, the following stand out:

1. Many, if not all, the print materials mentioned above are available online.
2. Teachers and students can now interact with others, including professional mathematicians, amateur mathematicians, historians of mathematics, other teachers, educators, other students, and so forth.
3. The Internet provides locales for diverse learning needs. YouTube offers lessons on specific math topics, various social media provide information on math problems, and the number of math communities that have cropped up online are innumerable.
4. Students today have grown up in the Digital Galaxy—they are called, in fact, “digital natives.” Awareness of how to retrieve information, enact specific communicative events, and so on are part of their digital competence.
5. The last point leads to a summary of the educational enhancements that the online world brings to math education: (a) It provides for ongoing novelty (in every sense). In print culture, one had to await for publication and then distribution. (b) It is resource-rich and easily accessible. In print culture, one had to physically go and seek out resources, such as going in person to a library to retrieve materials or make an appointment with someone, such as a teacher, to get information or clarification. The Internet provides these resources in an immediate way. (c) Those reared in the Digital Galaxy are embedded in it and thus know how to navigate its resources easily. The print book limits the navigation process to what is on the page; the screen is more expansive and interactive via hypertext and other linkage modalities.

Given these advantages, one must always keep in mind the implications of McLuhan’s four laws, which suggest that specific new challenges need to be taken into account when new media become dominant. One of these is what can be called the “quality filtering” challenge. In published print materials, we can be fairly certain that quality control has taken place, since publishers review, examine, approve, and then edit materials such as textbooks. Outside of professionally-run websites, such as the one by math journals and organizations like the NCTM, one can never be sure that the information contained on a website is accurate or, if not, if it may even be misleading. So, to “read” the Internet one needs a new kind of literacy, typically called “media literacy,” which extends to the reading of all kinds of media texts. It is defined simply as the ability to analyze, evaluate, deconstruct, and filter texts in a wide variety of media modes, genres, and formats.

Essentially, this means understanding that media texts are made by individuals or groups, who may be working to promote a particular viewpoint and might even have some hidden motive. While this was also true in print culture, the lack of the traditional quality filters increases the risk of subjective aims overriding more objective ones. Also, many sites are well written in presenting math concepts and explaining math problems, but many others are confusing and potentially detrimental as ancillary educational tools. They require a lot of editing in grammar and spelling, not to mention style of presentation. Another frustrating aspect is finding a

reference in the body of a text that is not listed in any reference or bibliography section. Also, many sites offer a highly simplified view of a topic, leading to the idea that “math is *only* fun,” thus defeating one of the classic aims of education—to instill mastery through practice and continued study.

A comparison between the learning styles of those reared in the Gutenberg Galaxy and those in the Digital Galaxy is worth considering here, since it ultimately will shape the future course of the math classroom. In the Gutenberg Galaxy the main aspects of learning style in the classroom were the following:

1. *A sense of seriousness*: It was assumed that the learning in a classroom and the materials used were serious; topics such as anecdotal or pop math would have been considered as materials to be accessed outside of the classroom.
2. *Linearity*: Reading information in print on a page means starting at the top (or bottom) and then moving the eye either up or down (depending on writing system) and from left to right or right to left.
3. *Unimodality* (in contrast to multimodality): reading print involves a single task, although illustrations, figures, and diagrams were also used as supplementary devices to the written text.
4. *Privacy*: reading is done in private and thus encourages individualism rather than collaboration.

Thinking patterns are deeply rooted in the configuration of a writing system. A number of scholars have studied the impact of writing on these patterns, suggesting that each writing system shapes the thought of its users (see Logan 1986). Linear writing promotes the view of thought as logically structured, detachable from experience, and highly abstract. On the other hand, oral and pictographic cultures appear to have a differential thinking style, which is analogical, concrete, and holistic.

In the Digital Galaxy some of the elements of pictographic-oral culture have been retrieved in terms of a secondary orality (as mentioned). Learning styles have also been amplified and diversified significantly. The main aspects are:

1. *A sense of play*: The ability to experiment with online texts creatively. The interaction with the printed page often discourages this, although by reading one can certainly experiment with the ideas in an imaginative fashion from the page; in digital environments, one is constantly encouraged to do so by the varying nature of the digital page.
2. *Improvisation*: There are many opportunities for direct improvisation in online contexts, whereas in print texts the improvisation occurs outside of the texts.
3. *Simulation*: Virtual worlds are seen as simulative of real worlds (hyperreality).
4. *Appropriation*: This involves knowing how to extract, use, remix, and amalgamate the content of different media.
5. *Negotiation*: This entails knowing how to access and traverse diverse virtual domains, discriminating and grasping multiple perspectives and norms.
6. *Multitasking*: This involves handling various tasks at the same time, including using different media in tandem to access information.

7. *Networking*: This implies knowing how to make contact with the denizens of the Internet and participating productively in various virtual communities.
8. *Transmedia navigation*: This entails knowing how to follow the flow of ideas, events, and information across multiple media sites.
9. *Judgment*: This means developing media literacy, or the ability to discern what is legitimate or not.
10. *Distributed cognition*: This involves interacting meaningfully with digital tools across the knowledge spectrum.
11. *Collective Intelligence*: This involves understanding how to pool knowledge and collaborate with Internet denizens towards common objectives.

While all of these aspects of learning existed somewhat in the Gutenberg Galaxy, the revolution in connective technology has brought about a new sense that learning can no longer be constrained to the printed page. Pedagogically, the Digital Galaxy can be seen as one huge ongoing and self-constructing textbook that involves, therefore specific questions for teachers: (1) What topics (based on specific websites) are relevant? (2) How can they be sequenced that makes sense to the specific situation? (3) How can the conceptual and the problem-solving sites be integrated with print materials? (4) How can assignments and homework be envisioned via the Web? (5) How can students be tested in the new media? (6) What actual technological tools should be used: computers, apps, tablets, and so on?

These questions remove the walls from the classroom, since they put the onus on teachers to literally “reach out” into cyberspace to put together a self-styled curriculum and teaching materials and supports. For example, manipulatives have been shown to enhance learning, ever since the Montessori method made it obvious that the use of the senses, including touch, stimulates the brain’s inbuilt resources to acquire knowledge especially in childhood. A manipulative is any object designed to get a learner to “grasp” (through touch) some mathematical concept by manipulating relevant objects. For example, putting more marbles in one of the student’s hands and then asking him or her to indicate which hand holds the greater number will lead the student to understand the concept of “greater” through manipulation. “Virtual manipulatives” now exist that allow for more versatility and openness in using this technique. As Moyer et al. (2002: 372) define it, a virtual manipulative is a “visual representation of a dynamic object that presents opportunities for construction of mathematical knowledge.” Virtual manipulative sites, such as the National Library of Virtual Manipulatives, provide prompts and feedback to problems.

The above example brings us to McLuhan’s idea of the sense ratios. These are the inbuilt sensory modes for knowing:

1. *Auditory-vocality*: learning that occurs through speaking and listening.
2. *Visuality*: learning that occurs through the sense of sight, including the use of diagrams, illustrations, pictures, alphabets, and the like.

3. *Tactility*: learning through the sense of touch, of which manipulatives are an example.
4. *Olfaction*: learning through the sense of smell, which is now a specialized sense ratio, useful primarily for subjects such as cuisine or chemistry.
5. *Gustation*: learning through the sense of taste, which is now also a specialized sense ratio, useful, again in courses that involve cuisine.

McLuhan claimed that the dominant sense ratio employed for learning and communication varied according to culture or period of time. Human beings are endowed by nature to decipher information with all the senses. The sense ratios are equally calibrated at birth to receive information in a balanced, complementary fashion. However, in social settings, one or the other is given preeminence—that is, one sense ratio or the other increases according to the modality emphasized in a culture. In an oral culture, the auditory-vocal sense ratio is the one that largely shapes information processing and knowledge understanding; in a print culture, on the other hand, the visual sense ratio is the primary one. This raising or lowering of sense ratios is not, however, preclusive. Indeed, we can have various sense ratios activated in tandem. For example, if one were to hear the word “dog” uttered, the auditory sense ratio would process the meaning of the word. If, however, one were to see the word written on a sheet of paper, then the visual sense ratio would be activated instead. A visual depiction of the dog accompanied by an utterance of the word would activate the auditory and visual sense ratios in tandem.

In the Digital Galaxy, several sense ratios can be activated in tandem through multimodality. In fact, online reading has become a new *sensorium*, where several sense ratios interact in the processing of information. Print literacy favors, as mentioned, the sense of sight, which in turn causes us to follow a text in a linear way. This has the cumulative effect of shaping how we perceive and understand things—an effect known as the “alphabet effect” (Logan 1986). Online literacy favors different sense ratios in tandem, but in so doing, diminishes somewhat the ability to concentrate on the content of a text. This does not imply in any way that print literacy is superior to media or multimodal literacy; it simply is intended to convey that multimodal literacy has its own means of imparting understanding, and reflection or concentration is gained in other ways.

The last point brings us to consider what can be called “blended learning and teaching,” which will be discussed in more detail in the final chapter. Essentially, this means that the old must be integrated with the new in order to preserve what is of value and to expand on it in meaningful ways. As the ancients knew, the dialogue is a primary medium for fostering true learning and self-knowledge. The online social media do generate meaningful dialogical interaction initially, but the novelty seems to wear off quickly, since it is an ever-present possibility. Stability can thus be introduced into any learning environment—offline, online, or integrated (online-with-offline)—by a retrieval of the dialogue. Another feature that should be retrieved is the authority of the teacher. This is not meant in any rigid or imperious sense, but in the original sense of “expert *magister*.” Some students are motivated to seek information online by themselves and are able to use it effectively; others are

not, using the Internet mainly for other kinds of reasons. This is where the teacher as *magister* comes into the picture, mediating between sources of knowledge and students and guiding them through a new form of dialogue.

As we saw from a survey in the previous chapter, there are no overarching solutions to math pedagogy (or any pedagogy for that matter). The best results are obtained by a combination and integration of the online and offline options. Burns and Hamm (2011) found that the best learning results in the use of manipulatives, for example, is through a combination of virtual and real (concrete) ones. This finding can likely be extended to other areas. Returning to the typology of uses of technology above, the most effective use of technology is, therefore, as an ancillary and integrative tool, rather than as a substitutive one.

To summarize the foregoing discussion, describing the potential and varied uses of math online, the following diagram can be used. In it, there are three options: (1) the use of digital technology as substitutive of the classroom, as in the MOOC case; (2) the use of technology for ancillary tutoring (websites that provide follow-up and complementary instruction); (3) the use of websites for integrative practice (websites with problems explained and answered, with mathematical objects such as manipulatives, for recreational practice aspects such as puzzles, and for access to various materials, such as the pop math materials discussed above (Fig. 2.2).

Online math pedagogy does not solve every problem. A sense of balance with the past is required. Martinovic (2015: 106) has provided a comprehensive assessment of the pedagogy-technology paradigm:

The mere availability of technology does not mean that it will be used for learning, unless the students (a) are encouraged by their teachers to do so, (b) find technology compatible with their learning style and ability, and (c) new pedagogies are developed and implemented. In other words, there exists a tension between the students' informal social use of digital technologies and the more formal, teacher-led forms of learning in schools, which goes against the smooth transition between these two uses.

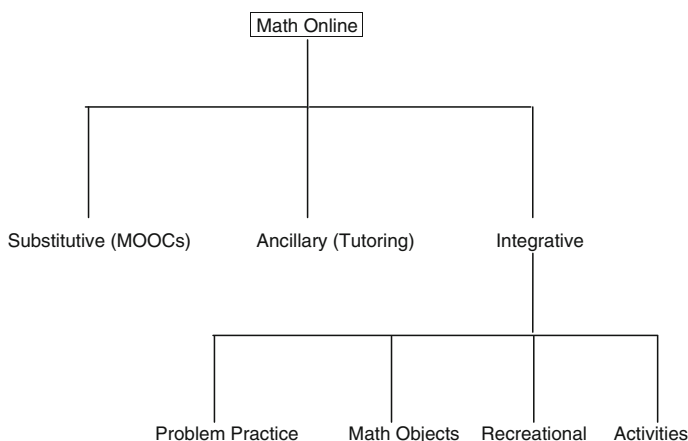


Fig. 2.2 Math online

The best model of technology use in the classroom, she asserts, is virtually identical to the one being proposed here (integrative), noting that when users are able to critically examine information and interactive events produced by technology then the advantages of technology emerge. Overall, this means that technology can be used (Martinovic 2015: 108):

1. as a cognitive tool
2. as an extensive tool (virtually a corollary of the first point)
3. as a catalyst for exploring ideas
4. as an “agent provocateur” challenging students to discover things on their own.

Martinovic follows up cogently as follows: “Technology does not have to have all the answers; its limitations may provide an opportunity to have ‘a teaching moment,’ to engage in critical and higher-order thinking” (2015: 113).

Math and Computer Science

In an interesting article, Mumford and Garfunkel (2013: 174) make the following observation that resonates with many teachers today:

Everyone says computer technology should be used in schools, but why let the computer be another incomprehensible technological mystery? Teach everyone the rudiments of programming and what goes on inside that box. “But is this math?” we hear you saying. Yes; writing computer code teaches you how to be precise and formal and makes concrete mathematical recipes like that for long division. They are what we call algorithms, and this sort of training is a paradigm for rational thinking.

The key to the use of computers in the math classroom is indeed the algorithm, which many consider the core of mathematical method. Computer science offers a potentially powerful learning-enhancing way to develop mathematical competence. Even if writing an algorithm does not produce the required output, the process of writing it in itself leads to understanding just the same. By exploring the “fault” in the algorithm we can better understand the mathematical principles needed to make it run better. When nothing works, then the student will have to go back to the principles and re-examine them critically.

Computer programming is thus a very useful tool for math pedagogy. And it is now consistent with how some mathematical proofs are being undertaken. The most famous one is the proof of the Four Color Problem, known as *proof by exhaustion*. This was conducted by breaking down the problem into a finite number of cases and then devising an algorithm for proving each one separately. If no exception emerges after an exhaustive search of potential cases, then the theorem is established as valid. The number of cases sometimes can become very large. The first proof of the Four Color theorem was based on 1,936 cases, all of which were checked by the algorithm. It was published in 1977 by Haken and Appel and it astonished the world of mathematics, since it went against the traditional methods of proof (Haken and Appel 1977; Appel and Haken 1986).

The central idea in traditional proof is to show that something is always true by the use of logic, rather than enumerate all potential cases and then test them—as does proof by exhaustion, where there is no upper limit to the number of cases allowed. Some mathematicians prefer to avoid computer proofs, since they tend to leave the impression that a theorem is only true by coincidence, or more accurately exhaustion, and not because of some underlying principle or pattern. However, there are many conjectures and theorems that cannot be proved (if proof is the correct notion) in any other way. These include: the proof that there is no finite projective plane of order 10, the classification of finite simple groups, and the so-called Kepler conjecture. The computer is thus a device for doing mathematics, not just for crunching numbers. An algorithm is a product of the human mind and is based on making various analogies as it is being constructed (Hofstadter 1979; Hofstadter and Sander 2013).

Computer science has thus revolutionized mathematics. A relevant case-in-point is the $P = NP$ problem. The problem can be understood anecdotally as follows (using an example by Fortnow 2013). If one is asked to solve a 9-by-9 Sudoku puzzle, the task is considered to be a fairly simple one. The complexity arises when asked to solve, say, a 25-by-25 Sudoku puzzle. And by augmenting the grid to 1000-by-1000 the solution to the puzzle becomes a gargantuan task. Computer algorithms can easily solve complex Sudoku puzzles, but start having difficulty as the degrees of complexity increase. The idea is, therefore, to devise algorithms to find the shortest route to solving complex problems. So, the issue of complexity raises the related issue of decidability, since there would be no point in tackling a complex problem that may turn out *not* to have a solution. If we let P stand for any problem with an easy solution, and NP for any problem with a difficult complex solution, then the whole question of decidability can be represented in a simple way. If P were equal to NP , $P = NP$, then problems that are complex (involving large amounts of data) could be tackled easily as the algorithms become more efficient (which is what happened in the Four-Color solution). The $P = NP$ problem is the most important open problem in computer science and formal mathematics today. It seeks to determine whether every problem whose solution can be quickly checked by computer can also be quickly solved by computer; and this has implications for solvability and thus decidability. Work on this problem has made it evident that a computer would take hundreds of years to solve some NP questions and sometimes go into a loop (the halting problem). Indeed, to prove $P = NP$ one would have to use, ironically, one or more of the classic methods of proof. We seem to be caught in a circle where algorithms are taking over from proofs and vice versa, proofs are informing algorithms.

The $P = NP$ problem is a profound one for mathematics. It is relevant to note that a famous computing challenge was issued by the security company, RSA Laboratories, in 1991, which published a list of fifty-four numbers, between 100 and 617 digits long, offering prizes of up to two hundred thousand dollars to whoever could factor them. The numbers were semiprimes, or almost-prime numbers, defined as the product of two (not necessarily different) prime numbers. Examples under 10 are 4, 6, 9 (and 10 itself). By definition, semiprimes have no

composite factors other than prime factors and themselves. For example, 26 is semiprime and its only factors are 1, 2, 13, and 26. In 2007 the company retracted the challenge and declared the prizes inactive, since the problem turned out to be impracticable. But the challenge has not receded from the mathematician's radar screen, as many try to factor the numbers using computers. The largest factorization of an RSA semiprime, known as RSA-200, because it consists of 200 digits, was carried out in 2005. Its factors are two 100-digit primes, and it took nearly 55 years of computer time, employing the number field sieve algorithm, to carry out.

The enormity of the RSA challenge brings us directly into the core of the $P = NP$ problem. Can a problem, such as the RSA one, be checked and solved quickly? The problem is still an outstanding one, and it too carries a price tag of one million dollars, offered by the Clay Institute. To reiterate here, the $P = NP$ problem entails asking whether every problem whose solution can be determined to be possible by computer can also be solved quickly by the computer. Not surprisingly, the problem was mentioned by Gödel in a letter he sent to John von Neumann in 1956, asking him whether an NP-complete problem could be solved in quadratic or linear time. The formal articulation of the problem came in a 1971 paper by Stephen Cook. Of course, it could well turn out that a specific problem itself will fall outside all our mathematical assumptions and techniques.

At the core of problems such as these is the algorithm. As is well known, the concept (although not named in this way) goes back to Euclid. His algorithm is worth revisiting here because it brings out the essence of what algorithms are all about and how they can be used in the math classroom. It is called the *Fundamental Theorem of Arithmetic*. Given any composite number, such as 14 or 50, the theorem states that it is decomposable into a unique set of prime factors:

$$14 = 2 \times 7$$

$$50 = 2 \times 5 \times 5.$$

Let's look more closely at how the unique set of prime factors of a composite number, such as 24, can be identified using a version of Euclid's algorithm, consisting of 5 rules:

1. $24 = 12 \times 2$
2. But $12 = 6 \times 2$
3. Plug this in (1) above: $24 = (6 \times 2) \times 2 = 6 \times 2 \times 2$
4. But $6 = 3 \times 2$
5. Plug this in (3) above: $24 = 6 \times 2 \times 2 = (3 \times 2) \times 2 \times 2 = 3 \times 2 \times 2 \times 2.$

We have now uncovered the prime factors of 24. They are 2 and 3. Of course, the 2 occurs three times: $24 = 3 \times 2^3$. We also note that each of the prime factors that produces a composite number also divides evenly into it: 3 divides into 24 as does 2. This is then the basis for a generalization of the algorithm:

1. Start by checking if the smallest prime number, 2, divides into the number evenly.
2. Continue dividing by 2 until it is no longer possible to do so evenly.

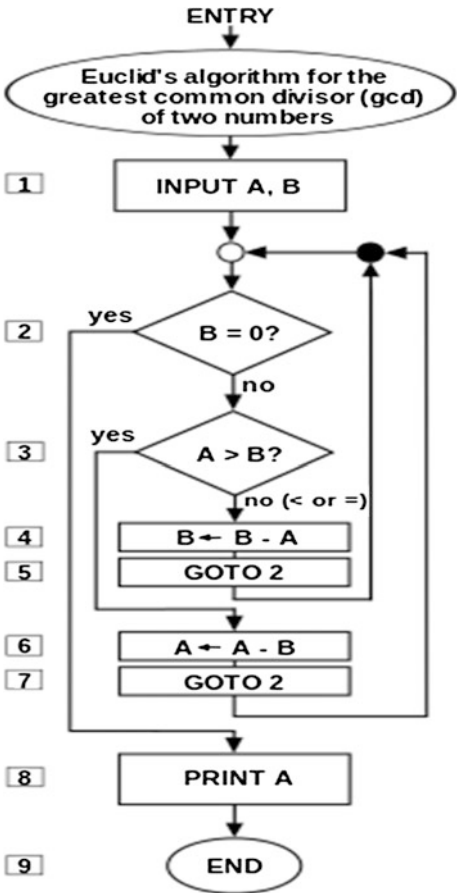
- 3. Go to the next smallest prime, 3.
- 4. Continue in this way.

This method will work every time, although it may take time and effort to determine the prime factors of large numbers (the $N = NP$ problem). Of course, a computer algorithm will take much less time. Euclid’s algorithm shows saliently that an algorithm is a logical step-by-step sequence of procedures. Moreover, it becomes simultaneously a *model* of factorization itself, since it breaks the operation down into its essential steps. By modeling mathematical problems in the form of algorithms, we are in effect gaining insight into the problems themselves.

Euclid’s algorithm above can be easily transformed into a computer program via a flowchart, such as the one constructed by Scott (2009: 13) (Fig. 2.3).

This breaks down the steps used in calculating the greatest common divisor of numbers a and b in locations named A and B . The algorithm proceeds by subtractions in two loops: If the number b in location B is greater than or equal to the

Fig. 2.3 A flowchart of Euclid’s algorithm



number a in location A , then, the algorithm specifies $B \leftarrow B - A$ (meaning the number $b - a$ replaces the old b). Similarly, if a is greater than b , then $A \leftarrow A - B$. The process terminates when (the contents of) B is 0, yielding the greatest common divisor in A .

So, if an algorithm can be written for some problem, then the problem is computable (that is, it can be carried out). When the algorithm above is run on a computer, and a specific number is the input, the algorithm will then: (1) decide if it can be factored and (2) if so, carry out the factorization.

By trying to figure out how to design a computer program to solve problems of various sorts, the learner can thus discover certain unexpected patterns. Kosslyn (1983: 116) describes the utility of computers aptly as follows:

The computer model serves the function of a note pad when one is doing arithmetic: It helps keep track of everything so that you don't get a headache trying to mentally juggle everything at once. Sometimes the predictions obtained in this way are surprising, which often points out an error in your thinking or an unexpected prediction.

It is useful here to discuss what is involved in programming a computer to model or simulate some activity, even though this brief digression will necessarily be reductive and generic. The goal is to show how knowledge of something can be deconstructed into algorithms and thus grasped more concretely. The deconstruction is represented in flowcharts (like the one above). This is a complete description of the operation that the computer is intended to carry out. This set of instructions explains what information must be inputted, what system of instructions and types of computing processes must be carried out, and what form the required output should take. The flowchart is essentially a model of some knowledge task, showing all the steps involved in putting the instructions together in a coherent way. The flowchart is converted into a program that is then typed into a text editor, a program used to create and edit text files.

Flowcharts use simple symbols and arrows to specify relationships. The beginning or end of a program is represented by an oval; a process is represented by a rectangle; a decision is represented by a diamond; and an I/O process is represented by a parallelogram or wavy parallelogram. The flowchart below shows how to build a computer program to find the largest of three numbers, A , B , and C (Fig. 2.4).

This breaks down the steps in the comparison of the magnitudes of numbers in a precise way. Basically it mimics what we actually do, comparing two numbers at a time and deciding when to determine the largest magnitude along the way.

Programs are written with high-level languages, which include symbols, linguistic expressions, and/or mathematical formulas. Some programming languages support the use of objects, such as a block of data and the functions that act upon the given data. These relieve programmers of the need to rewrite sections of instructions in long programs. Before a program can be run, special translator programs must translate the programming language text into a machine language, or low-level language, composed of numbers. Sophisticated systems today combine a whole series of states and representational devices to produce highly expert systems for processing input.

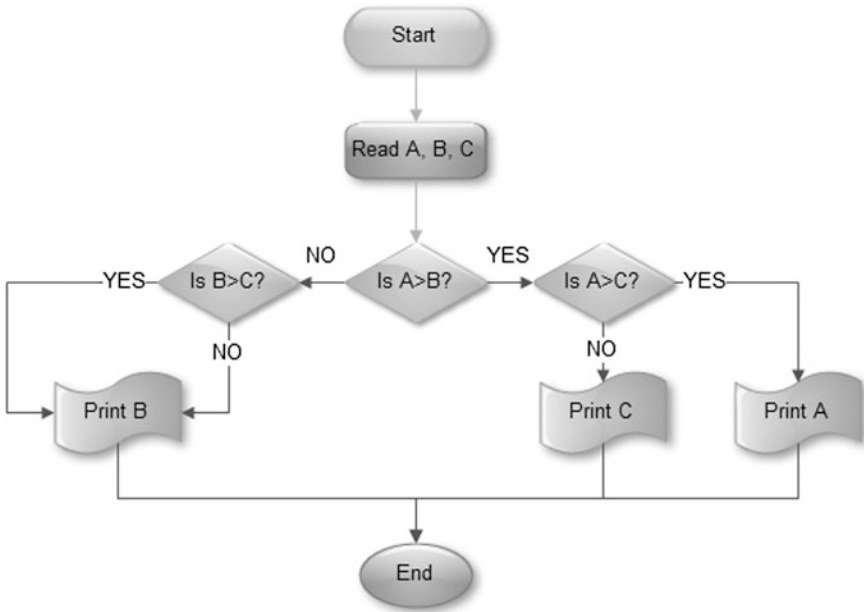


Fig. 2.4 Flowchart for determining the largest number

The computer-science-math partnership can also be used centrifugally (outside of the classroom) to show its utility in the Digital Galaxy. One of the most intriguing domains of application lies in the use of the natural logarithm, $\ln$, which is the logarithm with base $e = 2.718281828$, defined as follows:

$$\ln x \equiv \int_1^x \frac{dt}{t}$$

for $x > 0$.

This means that e is the unique number with the property that the area of the region bounded by the hyperbola $y = 1/x$ and the x -axis, and the vertical lines $x = 1$ and $x = e$ is 1:

$$\int_1^e \frac{dx}{x} = \ln e = 1.$$

The natural logarithm shows up in various branches of mathematics and in domains such as accounting, but the most relevant educational one is perhaps that it is used by Google to give every page on the WWW a score (PageRank), which is a rough measure of importance. The score is a logarithmic scale. So, a site with PageRank 2 (2 digits) is ten times more popular than a site with PageRank 1. This

example falls under the rubric of anecdotal math (previous chapter) since it relates math to some event in the world and thus allows the student to make connections that are often critical to understanding a topic.

Another anecdotal math application is in the area of Markov chains. Take, for example, the probability laws describing stochastic processes—those that have a random probability distribution or pattern that may be analyzed statistically but may not be predicted precisely. These develop over time as probabilistic rules. One of these is the *random walk*, introduced by mathematics educator George Polya in 1921:

Choose a point on a graph at the beginning. What is the probability that a random walker will reach it eventually? Or: What is the probability that the walker will return to his starting point?

Polya proved that the answer is 1, making it a virtual certainty. He called it a 1-dimensional outcome. But in higher dimensions this is not the case. A random walker on a 3-dimensional lattice, for instance, has a much lower chance of returning to the starting point ($P = 0.34$). This brings us to the notion of Markov chain as a relevant model. Suppose that at any stage of a random walk we flip a coin to decide in which direction to go next. In this case the type of analysis involved is of the *PI* variety. The defining characteristic of a Markov chain is that the probability distribution at each stage depends only on the present, not the past. Markov chains are thus perfect models for random walks and random events. The following figure (from Wikipedia) shows a walk whereby a marker is placed at zero on the number line and a coin is flipped—if it lands on heads (H) the marker is moved one unit to the right (1); if it lands on tails (T), it is moved one unit to the left (−1).

There are 10 ways of landing on 1 (by 3H and 2T), 10 ways of landing on −1 (2H and 3T), 5 ways of landing on 3 (4H and 1T), 5 ways of landing on −3 (1H and 4T), 1 way of landing on 5 (5H), and 1 way of landing on −5 (5T) (Fig. 2.5).

As these example show, computer flowcharts, algorithms, and computer-linked anecdotal math example are all useful devices for bringing math into the domain of

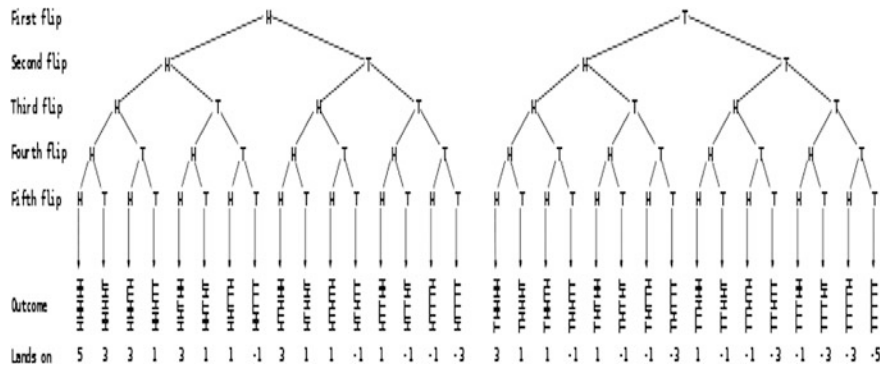


Fig. 2.5 Markov chain analysis of the random walk problem (from Wikipedia)

technology. It has all the four characteristics mentioned above. The computer is an ancillary device (helping students grasp certain concepts already taught); it is an integrative tool, allowing students to explore ideas through simulation and modeling; it is collaborative because it requires collaboration among teacher and students in discussing algorithms; and of course the use of computers for various purposes is part of students' embedded knowledge.

A Pedagogical Epilogue

The idea that computers can enhance learning emerged after computers became powerful at around the middle part of the twentieth century. This led to the notion of *cybernetics*, conceived by mathematician Norbert Wiener, who coined the term in 1948 in his book *Cybernetics, or Control and Communication in the Animal and Machine*. Actually, the same word was used in 1834 by the physicist André-Marie Ampère to denote the study of government in his classification system of human knowledge. Ampère, in turn, had probably taken it from Plato, who used it to signify the governance of people. Wiener popularized the social and educational implications of the merger between humans and machines in his best-selling 1950 book *The Human Use of Human Beings: Cybernetics and Society*.

In 1951, McLuhan used the term “mosaic approach” to encapsulate what this cybernetic blending of different systems might imply. But at the same time he warned that our modern cybernetic world is a two-edged sword. Merging with machines is the opposite of using machines as extensions. This can lead to totalitarianism in many surreptitious forms, whereby machines are used to control human behavior—a danger foreseen as well by the cyberneticians. So, we must always keep in mind that our machines are our extensions, not our counterparts or our amalgams.

In print culture, as mentioned, the kind of consciousness that develops is shaped by the written page, with its edges, margins, and sharply defined characters organized in neatly-layered rows or columns, inducing a linear-rational way of thinking in people. In such cultures, the knowledge encoded in writing is perceived as separable from the encoder of that knowledge primarily because the maker of the written text is not present during the reading and understanding of the text. Because electronic channels of communication increase the rapidity at which people can interact and because they make it possible to reach many people, the world has once again changed. As Davis (2015: 11) points out, late Renaissance math educators, like Robert Recorde, “were finding ways to take advantage of the new technology of the printing press—which might be argued in its own right to be a major contributor to the emergence of the very possibility of a standardized curriculum.” That paradigm shift was caused, as Davis correctly asserts, by print technology. We must in fact be always aware of how education, technology, and social ideologies are intertwined. Davis (2015: 12) puts it as follows:

In fact, I am aware of no research into learning that demonstrates the “fitness” of the curriculum structures and contents we have inherited. There is, however, a growing body of commentary and research that renders the above assumptions problematic. Humans, for example, are not logical creatures, but association-making beings whose capacity for logic rides atop irrepressible tendencies to see connections and make intuitive leaps (cf., Lakoff and Johnson 1999; Lakoff and Núñez 2000). Further, the contents of curriculum that were selected in the 1600s (and that have changed surprisingly little in the intervening centuries) reflect an impoverished view of the field of mathematics.

As Davis goes on to show, life in the Digital Galaxy has led to several principles that are fast becoming standard ones in the educational mindset:

1. *Spatial reasoning*: is now seen as a necessary skill for mathematics learning. This is not surprising since in the Digital Galaxy the visual modality of understanding dominates as a sense ratio. And unlike the visibility of print cognition, processing web pages is more holistic and economical, making it possible to take in great amounts of information at once (unlike the linear printed page).
2. *Complexity reasoning*: The Digital Age has brought about a shift in mathematical research and conceptualization that can be called conceptual reasoning or modeling, which as Arnheim (1969) pointed out years ago, is a derivative of visual reasoning. So, topics like fractals and chaos theory are now inserted in curricula alongside the traditional ones from the past. Also emphasized are exponentiation, logarithms, and power law distributions. These topics fit in nicely with the new world of technology.
3. *Coding*: “Already mandatory from the early grades in some nations, including the United Kingdom, the inclusion of coding is seen as vital on both individual and collective levels. For the individual, digital literacy skills afford a powerful conceptual and pragmatic currency in today’s world. For society, it addresses an accelerating need in a digital economy” (Davis 2015: 15).
4. *Curriculum models based on networks*: In the past curriculum models were both linear and hierarchical; that is, they both reflected a logical sequence of learning and an organization of concepts in a hierarchical way. The idea of networks has changed this picture; in network theory goals are centralized and the topics connected to them are connected to it like spikes in a wheel. “A curriculum that is organized around hub ideas in grander networks of ideas might relieve the compulsion to organize the curriculum as a sequence to follow and present possibilities of, e.g., a territory to explore” (Davis 2015: 15).

Network models of education are often integrated with so-called parallel processing ones. These are traced to *Parallel Distributed Processing* (PDP) theory which is based on writing computer programs designed to show how, potentially, brain networks are interconnected with each other in the processing of information. The PDP model appears to perform the same kinds of tasks and operations that language and problem-solving does (MacWhinney 2000). As Obler and Gjerlow (1999: 11) put it, in PDP theory, “there are no language centers per se but rather ‘network nodes’ that are stimulated; eventually one of these is stimulated enough that it passes a certain threshold and that node is ‘realized,’ perhaps as a spoken word.”

Studies of various kinds, some of which have been mentioned above, have shown that all these movements have started to influence teacher preferences. Many teachers use technology in an ancillary-integrative way, including it in a classroom system that branches out like a network. Factors such as access to adequate technology and teacher expertise in using technology stand out as interfering ones, needless to say. Philosophical opposition, on the other hand, rarely shows up today in the studies. Most teachers are not averse to technology, seeing it as part of the Digital Galaxy, in the same way that the book was the main technology in the Gutenberg Galaxy. I myself conducted an informal survey of 20 high school math teachers (most being ex-students of mine who went on to become math teachers). I asked the following five questions through an email questionnaire:

1. Do you use technology in your classroom?
2. How do you use it (for tutoring, extra activities, as a source of interest for students)?
3. Do you use a print textbook?
4. If so, how do you integrate it with technology? If not, what do you use?
5. Are you for or against expanding the use of technology in math education?

The study was, of course, informal and uncontrolled. I simply wanted to get a sense of what was in the minds of math teachers today, all of them relatively young. Although anecdotal, there is no reason to believe that the sample, if increased and subjected to statistical controls, would yield different results. Here is what the survey revealed:

1. Eighteen of the 20 used some form of technology; only 2 said that they didn't—one for philosophical reasons (believing that only face-to-face dialogue and textbook learning were effective) and the other because the classroom did not have the appropriate equipment.
2. All those who used it (18 out of 20) favored the use of websites for follow-up activities; 3 used the Internet as well for tutoring functions.
3. Everyone used a print textbook; 10 of the 20 said that it was the main component of the course(s) they taught, although online resources were often used to supplement the book; the others used it mainly as a resource, utilizing self-made materials and online resources in conjunction with the textbook.
4. Answered in (3).
5. Nineteen of the 20 said they supported the expansion of technology in math education, given that students relate well to it and come to class with a high level of media literacy.

A more comprehensive study would ask teachers to report on how frequently they used different types of technologies using a graded scale to: (1) rate the level of different types of access to technology in their schools, (2) rate their own level of expertise, and (3) provide a rating for their peer teachers at their school. Several national studies have addressed these issues (some of these are mentioned in Martinovic 2015) and the main hurdles include access to adequate technology in the classroom, the availability of appropriate content and, more importantly, its

organization for practical pedagogy—that is, the resources on the web need to be categorized into a kind of cyber-curriculum that allows teachers to go to the relevant sites. Organizations such as the NCTM provide relevant suggestions and websites.

Several other issues that can be mentioned here include the time required to integrate technology with print materials and classroom instruction. Moreover, since the selection of the online resources is essentially a subjective one, there seems to be a need for “scientific validation” on the part of many teachers that what they are doing is consistent with the psychology of learning. It would seem that in courses that are based on an inductive methodology, technology tends to fit in more easily with the course objectives because it is seen as another aspect of the process of generalizing knowledge through doing; deductive-based courses see technology more “loosely” as an adjunct rather than a complement.

The following selection of websites shows that in the online universe there are indeed resources that extend not only the textbook and curricula, but also learning itself beyond the classroom. Whereas in the Gutenberg Galaxy, individualism was stressed (given that printed books were read by one person at a time), in the Digital Galaxy collectivity is stressed instead. The psychology of individualism-versus-collectivity will be taken up subsequently. It is relevant to note here that a student in Karaali’s (2015: 129) course, which integrates math with the humanities, encapsulated the new mindset as follows: “Why would you want to even do math if you could not talk about it? The misery of mathematics is that you know something but you cannot tell others. The joy of mathematics is when you can share and extend.”

The following selection of websites gives a generic indication of how the classroom today can reach out into cyberspace for various reasons. There are many more sites of a similar nature, needless to say:

1. *NRICH* (www.nrich.maths.org.uk) provides very useful mathematical problems and games as well as a forum where teachers can ask specific pedagogical questions and have access to various articles and studies.
2. *MATH CENTRAL* (mathcentral.uregina.ca) provides tutoring for elementary students and a section where pedagogical issues are discussed.
3. *MATH FORUM* (www.mathforum.com) provides many problems, student responses to problems, and general commentaries on the responses.
4. *PBS* (www.pbs.org/teachersource/math) is a teacher resource site providing materials for assessment, complementary learning tasks, questions, and so on; it also identifies new math research and its implications along with changing issues in math education.
5. *DR. MATH* (www.mathforum.com/dr.math) is a site where answers to pedagogical and actual math questions are discussed.
6. *MEGAMATH* (www.c3.lanl.gov/mega-math/) presents classic problems in math at an elementary level, making math very interesting and connected to various aspects of life.
7. *MATHMAGIC* (<http://www.mathforum.com//mathmagic>) allows students to work together in solving problems.

The Global Village, McLuhan suggested, is where the central nervous system is distributed in a global network and thus where awareness of others is amplified and where humans actually retrieve tribal conditions, wanting to be part of communities rather than be distinctively individualistic (secondary orality). To paraphrase McLuhan: “You can take the human being out of the tribe, but you can’t take the tribe out of the human being.”

As McLuhan stated in one of his last television interviews on the CBC, this can be a serious problem: “tribal people, one of their main kinds of sport is butchering each other” (cited in Gibson and Murray 2013). War, torture, terrorism, and other violent acts are “quests for identity” in the Global Village. So, balancing the positive potential and the negative effects of the Global Village has become one of the pressing challenges for everyone today.

McLuhan’s concept of a Global Village was not completely original. Lewis Mumford had expressed a similar idea in *Technics and Civilization* (1934). And in 1948, Wyndham Lewis had observed that the “earth has become one big village, with telephones laid from one end to the other” (Lewis 1948: 21). His colleague at the University of Toronto, Harold Innis, was also discussing the effects of mass communication systems on society in his *Bias of Communication*, published in 1951, the same year in which McLuhan published *Mechanical Bride*. In both, there are warnings that we should not ignore. These will be discussed again in the final chapter.

A fitting epilogue to this chapter comes from McLuhan himself as a guest on a CBC report on the effects of the mass media on young people, broadcast in May of 1960. He quipped that the “book is no longer king,” and that television has transformed the world into a Global Village. His examples resonate even more today in the Digital Age. If there’s an earthquake, no matter where we live, we all get the message. And the young person today, who feels especially at home with the new gadgets, will bring our tribe even closer together.

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