Chapter 2
Detection and Isolation of Actuator Faults

This chapter presents an SMO-based actuator FDI approach for uncertain Lipschitz nonlinear systems. It is assumed in this chapter that only actuator faults occur in the system.

2.1 Introduction

The research on FDI has received considerable attention during the past two decades due to increasing demand for safety and reliability of an automatic control system. Fruitful results can be found in [1–6] and the references therein.

Due to the robustness to uncertainties such as system deviation, disturbances, and unknown nonlinearities, sliding-mode control (SMC) has been recognized as a promising robust control approach to confront uncertain systems. Since early work of applying SMOs for FDI in [7], the SMOs-based FDI methods have been developed extensively [8–13]. However, almost all these approaches mainly focus on relatively large-sized faults. The research on the detection and isolation of incipient faults has been less studied and still remains a challenge to model-based FDI techniques, because they are almost unnoticeable during their initial stage and their effects to residuals are most likely to be concealed by system uncertainties. Moreover, the robustness of the SMO to uncertainties makes it also robust to these type of faults.

Inspired by the work presented in [14], we develop a novel method to detect and isolate incipient actuator faults for uncertain Lipschitz nonlinear systems. The proposed method essentially transforms the original system into two subsystems (subsystem-1 and 2), where subsystem-1 includes the effects of system uncertainties and actuator faults while subsystem-2 only has actuator faults. For the purpose of fault detection, we design an SMO for subsystem-1 and a traditional Luenberger observer for subsystem-2. Taking the output estimation error of subsystem-2 as the residual and comparing it with a predefined threshold, the occurrence of an actuator fault can then be detected if the residual goes over the threshold.

J. Zhang et al., *Robust Observer-Based Fault Diagnosis for Nonlinear Systems Using MATLAB®*, Advances in Industrial Control,
DOI 10.1007/978-3-319-32324-4_2

After a fault is detected, the next step is to determine the location of the fault, which is the purpose of FI. In principle, the use of one single observer may permit the isolation of faults if their effect has independent projections onto the residual space. However, if the system has significant nonlinearities, it is difficult to assure this independence. Therefore, a bank of observers are needed to isolate faults if they occur on different actuators at the same time. There are two schemes for FI. The first one is called dedicated observer scheme [15]. In this scheme, N observers are designed to generate N residuals and the *i*th residual is expected to be only sensitive to the *i*th fault, but is insensitive to others. The other scheme is called generalized observer scheme [16], where N observers are also designed to produce N residuals. However, the difference is that the *i*th residual is sensitive to all possible faults except the *i*th one. In this chapter, the actuator FI is carried out using the modified dedicated observer scheme to subsystem-2. Multiple SMOs, one for each possible actuator fault, are used to generate the estimated output vector. The estimated output vector is then compared with the actual output vector in order to determine which actuator is affected by the fault.

The remaining sections of this chapter are organized as follows: Sect. 2.2 briefly describes the mathematical preliminaries required for designing observers. Section 2.3 designs observers for FD and derives the sufficient condition for the stability of the proposed observers based on Lyapunov approach. The actuator FI scheme is presented in Sect. 2.4. Simulation results are shown in Sect. 2.5 with conclusions in Sect. 2.6.

2.2 Problem Formulation

Consider a nonlinear system described by

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x,t) + Bu(t) + Df_a(t) + E\Delta\psi(t), \\ y(t) = Cx(t), \end{cases} \tag{2.1}$$

where $x \in \mathscr{R}^n$, $u \in \mathscr{R}^m$, and $y \in \mathscr{R}^p$ denote respectively the vector of state variables, inputs and outputs. $f_a \in \mathscr{R}^h$ is the vector of unknown actuator faults. $\Delta\psi \in \mathscr{R}^r$ models the lumped uncertainties and disturbances experienced by the system and $f(x,t)$ represents the known nonlinear continuous term. $A \in \mathscr{R}^{n\times n}$, $B \in \mathscr{R}^{n\times m}$, $C \in \mathscr{R}^{p\times n}$, $D \in \mathscr{R}^{n\times h}$, and $E \in \mathscr{R}^{n\times r}$ are known constant matrices with C and E both being of full rank. Note that a nonlinear system of the form $\dot{x}(t) = \Omega(x,u,t)$ can be expressed as $\dot{x}(t) = Ax(t) + f(x,t)$ if $\Omega(x,u,t)$ is continuously differentiable with respect to x.

Remark 2.1 It is assumed in the book that the fault distribution matrix D is known. This is not a restrictive assumption since much work has been done on estimating the fault distribution matrix when it is not fully known. Some examples can be seen in [17, 18]. In this chapter, we assume that the actuator faults could occur in each input channel, and therefore we have $D = B$ and $f_a \in \mathscr{R}^m$.

Before starting the main results of this chapter, we make the following assumptions on System (2.1).

Assumption 2.1 $rank(CE) = rank(E)$.

Assumption 2.2 For every complex number s with nonnegative real part

$$rank \begin{bmatrix} sI - A & E \\ C & 0 \end{bmatrix} = n + rank(E). \tag{2.2}$$

Assumption 2.3 The nonlinear continuous term $f(x, t)$ is assumed to be known and Lipschitz about the state x uniformly, i.e.,

$$\| f(x, t) - f(\hat{x}, t)\| \leq \mathscr{L}_f \|x - \hat{x}\|, \quad \forall x, \hat{x} \in \mathscr{R}^n \tag{2.3}$$

where $\mathscr{L}_f$ is the known Lipschitz constant.

Assumption 2.4 The actuator fault vector f_a and uncertainty vector $\Delta\psi$ satisfies the following constraint:

$$\| f_a\| \leq \rho_a \text{ and } \|\Delta\psi\| \leq \xi, \tag{2.4}$$

where ρ_a and ξ are two known positive constants.

Lemma 2.1 *Under Assumption 2.1, there exist state and output transformations*

$$z = Tx = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad w = Sy = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{2.5}$$

such that in the new coordinate, the system matrices become,

$$TAT^{-1} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \; TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \; TE = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}, \; SCT^{-1} = \begin{bmatrix} C_1 & 0 \\ 0 & C_4 \end{bmatrix} \tag{2.6}$$

where $T \in \mathscr{R}^{n\times n}$, $S \in \mathscr{R}^{p\times p}$, $z_1 \in \mathscr{R}^r$, $z_2 \in \mathscr{R}^{n-r}$, $w_1 \in \mathscr{R}^r$, $w_2 \in \mathscr{R}^{p-r}$, $A_1 \in \mathscr{R}^{r\times r}$, $A_2 \in \mathscr{R}^{r\times(n-r)}$, $A_3 \in \mathscr{R}^{(n-r)\times r}$, $A_4 \in \mathscr{R}^{(n-r)\times(n-r)}$, $B_1 \in \mathscr{R}^{r\times m}$, $E_1 \in \mathscr{R}^{r\times r}$, $C_1 \in \mathscr{R}^{r\times r}$, $C_4 \in \mathscr{R}^{(p-r)\times(n-r)}$ *and* $D_2 \in \mathscr{R}^{(p-r)\times q}$. E_1 *and* C_1 *are invertible.*

Partition T and S as,

$$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}, \quad S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \tag{2.7}$$

where $T_1 \in \mathscr{R}^{r\times n}$, $T_2 \in \mathscr{R}^{(n-r)\times n}$, $S_1 \in \mathscr{R}^{r\times p}$, *and* $S_2 \in \mathscr{R}^{(p-r)\times p}$. *After introducing the state and output transformations (2.5), System (2.1) is expressed as,*

$$\begin{aligned} \dot{z} &= TAT^{-1}z + Tf(T^{-1}z, t) + TB(u + f_a) + TE\Delta\Psi \\ y &= CT^{-1}z. \end{aligned} \tag{2.8}$$

Using the relations in (2.6), System (2.8) is converted into two subsystems as

$$\begin{cases} \dot{z}_1 = A_1 z_1 + A_2 z_2 + f_1(T^{-1}z, t) + B_1 (u + f_a) + E_1 \Delta\psi \\ w_1 = C_1 z_1 \end{cases} \tag{2.9}$$

$$\begin{cases} \dot{z}_2 = A_3 z_1 + A_4 z_2 + f_2(T^{-1}z, t) + B_2 (u + f_a) \\ w_2 = C_4 z_2, \end{cases} \tag{2.10}$$

where $f_1(T^{-1}z, t) = T_1 f(T^{-1}z, t)$ *and* $f_2(T^{-1}z, t) = T_2 f(T^{-1}z, t)$.

Lemma 2.2 *The pair* (A_4, C_4) *is detectable if and only if Assumption 2.2 holds.*

Proof See [19, 20]. □

It follows from Lemma 2.2 that there exists a matrix $L \in \mathscr{R}^{(n-r)\times(p-r)}$ such that $A_4 - LC_4$ is stable, and thus for any $Q_2 > 0$, the Lyapunov equation,

$$(A_4 - LC_4)^T P_2 + P_2(A_4 - LC_4) = -Q_2, \tag{2.11}$$

has a unique solution $P_2 > 0$ [21].

Remark 2.2 It is seen from Lemma 2.1 that the satisfaction of Assumption 2.1 ensures the existence of coordinate transformations T and S, such that in the new coordinate, the subsystem-1, formulated in (2.9), is prone to both actuator faults and system uncertainties, while the subsystem-2, formulated in (2.10), is only prone to actuator faults but free from system uncertainties. It follows from Assumption 2.2 that the pair (A_4, C_4) is detectable, which provides the necessary condition for the existence of an observer for system (2.10). Assumption 2.3 states that the nonlinear systems considered is Lipschitz. Many practical systems satisfy the Lipschtiz condition, at least locally. For example, trigonometric nonlinearities occurring in robotic applications and the nonlinearities which are square or cubic in nature, can be assumed to be Lipschitz.

2.3 Actuator FD Scheme

FD is the first step of fault diagnosis to determine whether a fault has occurred or not. The decision on the occurrence of a fault can be made if a significant residual change is observed. If we design SMOs directly for the original system, the effect of actuator faults, especially the ones with small magnitudes, on state estimation errors could be attenuated or even eliminated by the variable structure term [14]. The detection of faults therefore becomes difficult. Observing subsystem-2 in (2.10), one can find out that the state z_2 is neither subject to system uncertainties nor faults before the occurrence of any actuator fault. If we can design an observer for this particular subsystem and take the output estimation error $w_2 - \hat{w}_2$ ($\hat{w}_2$ is the estimation of w_2) as the residual, then the problem caused by designing conventional SMOs for the original system can be solved. This intuition inspires the proposed FD scheme in this section.

For Subsystem (2.9), we construct the following SMO:

$$\begin{cases} \dot{\hat{z}}_1 = A_1\hat{z}_1 + A_2\hat{z}_2 + f_1(T^{-1}\hat{z}, t) + B_1u + (A_1 - A_1^s)C_1^{-1}(w_1 - \hat{w}_1) + \nu_1 \\ \hat{w}_1 = C_1\hat{z}_1, \end{cases} \tag{2.12}$$

where $A_1^s \in \mathscr{R}^{r\times r}$ is a stable matrix which needs to be determined and $\hat{z}$ is defined as $\hat{z} := col(C_1^{-1}w_1, \hat{z}_2)$. It is worth noting that $\hat{z}$ does not represent the state estimate vector $col(\hat{z}_1, \hat{z}_2)$. The discontinuous output error injection term ν_1, that is used to eliminate the effects of uncertainties, is defined by

$$\nu_1 = \begin{cases} k_1 \frac{P_1(C_1^{-1}w_1 - \hat{z}_1)}{\|P_1(C_1^{-1}w_1 - \hat{z}_1)\|} & \text{if } C_1^{-1}w_1 - \hat{z}_1 \neq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{2.13}$$

where $k_1 = \|E_1\|\xi + \eta_1$. The parameter η_1 is a positive scalar which is to be determined for System (2.15) to be driven to the predefined sliding surface (2.32). $P_1 \in \mathscr{R}^{r\times r} > 0$ is the Lyapunov matrix of A_1^s. It is worth noting that state z_1 can be obtained by the measured output y as $z_1 = C_1^{-1}w_1 = C_1^{-1}S_1y$.

For Subsystem (2.10), a Luenberger observer with the following form is designed:

$$\begin{cases} \dot{\hat{z}}_2 = A_4\hat{z}_2 + A_3C_1^{-1}w_1 + f_2(T^{-1}\hat{z}, t) + B_2u + L(w_2 - \hat{w}_2) \\ \hat{w}_2 = C_4\hat{z}_2, \end{cases} \tag{2.14}$$

where $L \in \mathscr{R}^{(n-r)\times(p-r)}$ is the gain of a traditional Luenberger observer.

If the state estimation errors are defined as $e_1 = z_1 - \hat{z}_1$ and $e_2 = z_2 - \hat{z}_2$, then the state estimation error dynamics, before the occurrence of actuator faults, can be obtained as

$$\begin{aligned} \dot{e}_1 &= \dot{z}_1 - \dot{\hat{z}}_1 \\ &= A_1z_1 + A_2z_2 + f_1(T^{-1}z, t) + B_1u + E_1\Delta\psi \\ &\quad -A_1\hat{z}_1 - A_2\hat{z}_2 - f_1(T^{-1}\hat{z}, t) - B_1u - (A_1 - A_1^s)C_1^{-1}(w_1 - \hat{w}_1) - \nu_1 \\ &= A_1^s e_1 + A_2e_2 + \left[f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t)\right] + E_1\Delta\psi - \nu_1 \\ &= A_1^s e_1 + A_2e_2 + \Delta f_1 + E_1\Delta\psi - \nu_1 \end{aligned} \tag{2.15}$$

$$\begin{aligned} \dot{e}_2 &= \dot{z}_2 - \dot{\hat{z}}_2 \\ &= A_3z_1 + A_4z_2 + f_2(T^{-1}z, t) - A_4\hat{z}_2 - A_3C_1^{-1}w_1 - f_2(T^{-1}\hat{z}, t) - L(w_2 - \hat{w}_2) \\ &= (A - LC_4)e_2 + \left[f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t)\right] \\ &= (A - LC_4)e_2 + \Delta f_2, \end{aligned} \tag{2.16}$$

where $\Delta f_1 = f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t)$ and $\Delta f_2 = f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t)$.

We now present Theorem 2.1 which establishes the sufficient condition for the stability of the above error dynamics (2.15) and (2.16).

Theorem 2.1 *Given System (2.1) with Assumptions 2.1–2.4. When the system is free of actuator faults, the error dynamics (2.15) and (2.16) are asymptotically stable, if there exist matrices* $A_1^s < 0$, L, $P_1 = P_1^T > 0$ *and* $P_2 = P_2^T > 0$, *and positive scalars* α_1 *and* α_2 *such that*

$$\Lambda := \begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1} P_1 P_1 & P_1 A_2 \\ A_2^T P_1 & \Pi_2 + \frac{1}{\alpha_2} P_2 P_2 + a I_{n-r} \end{bmatrix} < 0 \tag{2.17}$$

where $\Pi_1 = A_1^{sT} P_1 + P_1 A_1^s$, $\Pi_2 = (A_4 - LC_4)^T P_2 + P_2(A_4 - LC_4)$ *and* $a = \alpha_1 \mathscr{L}_{f_1}^2 \|T^{-1}\|^2 + \alpha_2 \mathscr{L}_{f_2}^2 \|T^{-1}\|^2$.

Proof Assume $V_1(e_1) = e_1^T P_1 e_1$ and $V_2(e_2) = e_2^T P_2 e_2$, and consider $V = V_1 + V_2$ as the Lyapunov candidate.

The time derivative of V_1 along the trajectory of system (2.15) can be shown to be

$$\begin{aligned} \dot{V}_1 &= e_1^T P_1 \dot{e}_1 + \dot{e}_1^T P_1 e_1 \\ &= e_1^T P_1 (A_1^s e_1 + A_2 e_2 + \Delta f_1 + E_1 \Delta\psi - \nu_1) \\ &\quad + (e_1^T A_1^{sT} + e_2^T A_2^T + \Delta f_1^T + \Delta\psi^T E_1^T - \nu_1^T) P_1 e_1 \\ &= e_1^T (A_1^{sT} P_1 + P_1 A_1^s) e_1 + 2e_1^T P_1 A_2 e_2 + 2e_1^T P_1 E_1 \Delta\psi + 2e_1^T P_1 \Delta f_1 - 2e_1^T P_1 \nu_1. \end{aligned}$$

Consider the term $2e_1^T P_1 \Delta f_1$ and apply the inequality $2X^T Y \le \frac{1}{\alpha_1} X^T X + \alpha_1 Y^T Y$, which holds true for any scalar $\alpha_1 > 0$ [22]. Taking $X = P_1^T e_1$ and $Y = \Delta f_1$, we can obtain

$$2e_1^T P_1 \Delta f_1 \le \frac{1}{\alpha_1} e_1^T P_1 P_1^T e_1 + \alpha_1 \Delta f_1^T \Delta f_1. \tag{2.18}$$

Note that $\hat{z} := [(C_1^{-1} w_1)^T, (\hat{z}_2)^T]^T$. Then, before the occurrence of actuator faults we have,

$$z - \hat{z} = \begin{bmatrix} 0 \\ e_2 \end{bmatrix} \tag{2.19}$$

Therefore $\|T^{-1} z - T^{-1} \hat{z}\| = \|T^{-1} e_2\|$ and

$$\begin{aligned} \|\Delta f_1\| &\le \mathscr{L}_{f_1} \|T^{-1}\| \|e_2\| \\ \|\Delta f_2\| &\le \mathscr{L}_{f_2} \|T^{-1}\| \|e_2\|, \end{aligned} \tag{2.20}$$

where $\mathscr{L}_{f_1} = \|T_1\| \mathscr{L}_f$ and $\mathscr{L}_{f_2} = \|T_2\| \mathscr{L}_f$.

Then we have

$$\begin{aligned}\dot{V}_1 &\le e_1^T(A_1^{sT}P_1 + P_1A_1^s)e_1 + 2e_1^TP_1A_2e_2 + \frac{1}{\alpha_1}e_1^TP_1P_1^Te_1 + \alpha_1\Delta f_1^T\Delta f_1 \\ &\quad + 2e_1^TP_1E_1\Delta\psi - 2e_1^TP_1\nu_1 \\ &\le e_1^T(A_1^{sT}P_1 + P_1A_1^s)e_1 + 2e_1^TP_1A_2e_2 + \frac{1}{\alpha_1}e_1^TP_1P_1^Te_1 + \alpha_1\mathscr{L}_{f_1}^2\|T^{-1}\|^2\|e_2\|^2 \\ &\quad + 2e_1^TP_1E_1\Delta\psi - 2e_1^TP_1\nu_1. \end{aligned} \tag{2.21}$$

From the definition of ν_1 in (2.13), it can be shown that

$$e_1^TP_1\nu_1 = k_1e_1^TP_1\frac{P_1e_1}{\|P_1e_1\|} = \frac{k_1\|P_1e_1\|^2}{\|P_1e_1\|} = k_1\|P_1e_1\|.$$

Now

$$2e_1^TP_1E_1\Delta\psi \le 2\|E_1\|\|\Delta\psi\|\|P_1e_1\| \le 2\|E_1\|\xi\|P_1e_1\|. \tag{2.22}$$

Since $k_1 = \|E_1\|\xi + \eta_1$, then

$$\begin{aligned}2e_1^TP_1E_1\Delta\psi - 2k_1\|P_1e_1\| &\le 2\|E_1\|\xi\|P_1e_1\| - 2(\|E_1\|\xi + \eta_1)\|P_1e_1\| \\ &= -2\eta_1\|P_1e_1\|.\end{aligned} \tag{2.23}$$

Therefore (2.21) can further be simplified as

$$\begin{aligned}\dot{V}_1 &\le e_1^T\Pi_1e_1 + 2e_1^TP_1A_2e_2 + \frac{1}{\alpha_1}e_1^TP_1P_1e_1 + \alpha_1\mathscr{L}_{f_1}^2\|T^{-1}\|^2\|e_2\|^2 - 2\eta_1\|P_1e_1\| \\ &\le e_1^T\left(\Pi_1 + \frac{1}{\alpha_1}P_1P_1\right)e_1 + 2e_1^TP_1A_2e_2 + \alpha_1\mathscr{L}_{f_1}^2\|T^{-1}\|^2\|e_2\|^2.\end{aligned} \tag{2.24}$$

Similarly, the time derivative of $V_2 = e_2^TP_2e_2$ along the trajectory of System (2.15) can be computed as

$$\begin{aligned}\dot{V}_2 &= e_2^TP_2\dot{e}_2 + \dot{e}_2^TP_2e_2 \\ &= e_2^T\left[P_2(A_4 - LC_4) + (A_4 - LC_4)^TP_2\right]e_2 + 2e_2^TP_2\Delta f_2 \\ &\le e_2^T\left(\Pi_2 + \frac{1}{\alpha_2}P_2P_2\right)e_2 + \alpha_2\mathscr{L}_{f_2}^2\|T^{-1}\|^2\|e_2\|^2.\end{aligned} \tag{2.25}$$

Combining (2.24) and (2.25) yields

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_2 \\ &\leq e_1^T\left(\Pi_1 + \frac{1}{\alpha_1}P_1P_1\right)e_1 + e_2^T\left(\Pi_2 + \frac{1}{\alpha_2}P_2P_2 + aI_{n-r}\right)e_2 + 2e_1^TP_1A_2e_2 \\ &= \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}^T \Lambda \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \end{aligned} \tag{2.26}$$

If there exist matrices $A_1^s < 0$, L, $P_1 = P_1^T > 0$ and $P_2 = P_2^T > 0$, and positive scalars α_1 and α_2 such that Inequality (2.17) is satisfied, then $\dot{V} < 0$ for any $e \neq 0$, where $e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$. This implies that the error dynamics (2.15) and (2.16) are asymptotically stable.

This completes the proof. □

Remark 2.3 Using the Schur complement result, the Inequality (2.17) can be transformed into the following LMI feasibility problem: there exist matrices $X, Y, P_1 > 0$, $P_2 > 0$ and positive scalars α_1, α_2 such that

$$\begin{bmatrix} X + X^T & P_1 & P_1A_2 & 0 \\ P_1 & -\alpha_1 I & 0 & 0 \\ A_2^T P_1 & 0 & A_4^T P_2 + P_2A_4 - C_4^T Y^T - YC_4 + aI & P_2 \\ 0 & 0 & P_2 & -\alpha_2 I \end{bmatrix} < 0 \tag{2.27}$$

where $X = P_1A_1^s$ and $Y = P_2L$.

Lemma 2.3 *Let a_0 and c_0 be positive constants such that $\|e^{(A_4-LC_4)t}\| \leq c_0e^{-a_0t}$. If $a_0 \geq c_0\mathscr{L}_{f_2}\|T^{-1}\|$, then before the occurrence of actuator faults, the state estimation error $e_2(t)$ is bounded by*

$$\|e_2(t)\| \leq \varepsilon = c_0\|e_2(0)\|e^{(c_0\mathscr{L}_{f_2}\|T^{-1}\|-a_0)t} \tag{2.28}$$

Proof From (2.16), the solution of $e_2(t)$ can be obtained as

$$e_2(t) = e^{(A_4-LC_4)t}e_2(0) + \int_0^t e^{(A_4-LC_4)(t-\tau)} \cdot \left(f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t)\right) d\tau. \tag{2.29}$$

Applying the triangle inequality to (2.29), gives

$$\|e_2(t)\| \leq c_0e^{-a_0t}\|e_2(0)\| + c_0\mathscr{L}_{f_2}\|T^{-1}\| \int_0^t e^{-a_0(t-\tau)}\|e_2(\tau)\|d\tau. \tag{2.30}$$

where the positive constants a_0 and c_0 are chosen such that $\|e^{(A_4-LC_4)t}\| \leq c_0e^{-a_0t}$. Note that a_0 and c_0 can always be found, since $A_4 - LC_4$ is Hurwitz [23]. Applying Gronwall–Bellman inequality [23] to (2.30) yields

$$\|e_2(t)\| \le c_0\|e_2(0)\|e^{(c_0\mathscr{L}_{f_2}\|T^{-1}\|-a_0)t}. \tag{2.31}$$

This completes the proof. □

Theorem 2.1 implies that the error dynamics (2.15) and (2.16) are asymptotically stable. Now, we are in the position to investigate the reachability of a sliding surface in the estimation error space. For the error dynamics (2.15) and (2.16), define the sliding-mode surface as

$$\mathscr{S} = \{(e_1, e_2)|e_1 = 0\}. \tag{2.32}$$

The following theorem shows that with the proper choice of $k_1(.)$ in (2.13), the error system can be driven to the sliding surface (2.32) and a sliding motion can be maintained on it thereafter.

Theorem 2.2 *Given System (2.1) with Assumptions 2.1–2.4 and the proposed observers (2.12)–(2.14). Then, the error dynamics (2.15) can be driven to the sliding surface given by (2.32) in finite time if the LMI formulated in (2.27) is solvable and the gain η_1 from (2.13) is chosen to satisfy*

$$\eta_1 \ge \left(\|A_2\| + \mathscr{L}_{f_1}\|T^{-1}\|\right)\varepsilon + \eta_2, \tag{2.33}$$

where η_2 is a positive scalar.

Proof Consider a Lyapunov candidate function $V_1 = e_1^T P_1 e_1$. Then its time derivative is given as

$$\dot{V}_1 = e_1^T({A_1^s}^T P_1 + P_1 A_1^s)e_1 + 2e_1^T P_1 A_2 e_2 + 2e_1^T P_1 E_1 \Delta\psi + 2e_1^T P_1 \Delta f_1 - 2e_1^T P_1 \nu_1. \tag{2.34}$$

It is obvious that the term ${A_1^s}^T P_1 + P_1 A_1^s < 0$, since A_1^s is a stable matrix. From Assumption 2.4 and the definition of ν_1, it follows that

$$\begin{aligned}
\dot{V}_1 &\le 2\|P_1e_1\|\|A_2\|\|e_2\| + 2\|P_1e_1\|\|E_1\|\xi + 2\|P_1e_1\|\mathscr{L}_{f_1}\|T^{-1}\|\|e_2\| - 2k_1\|P_1e_1\| \\
&\le 2\|P_1e_1\|\|A_2\|\|e_2\| + 2\|P_1e_1\|\|E_1\|\xi + 2\|P_1e_1\|\mathscr{L}_{f_1}\|T^{-1}\|\|e_2\| \\
&\quad - 2\|P_1e_1\|(\|E_1\|\xi + \eta_1) \\
&\le 2\|P_1e_1\|\left[(\|A_2\| + \mathscr{L}_{f_1}\|T^{-1}\|)\|e_2\| - \eta_1\right].
\end{aligned} \tag{2.35}$$

It follows from Lemma 2.3 that $\|e_2\| \le \varepsilon$. Thus, if (2.33) is satisfied, we have

$$\dot{V}_1 \le -2\eta_2\|P_1e_1\| \le -2\eta_2\sqrt{\lambda_{\min}(P_1)}\sqrt{V_1}. \tag{2.36}$$

This shows that the reachability condition [24] is satisfied and a sliding motion is achieved and maintained after some finite time $t_s > 0$.

This completes the proof. □

Let the actuator fault occurs at time instant t_f. Then the error dynamics (2.15) and (2.16) become

$$\dot{e}_1 = A_1^s e_1 + A_2 e_2 + \left(f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t)\right) + E_1 \Delta\psi + B_1 f_a - \nu_1 \quad (2.37)$$

$$\dot{e}_2 = (A_4 - LC_4)e_2 + \left(f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t)\right) + B_2 f_a. \quad (2.38)$$

Observing (2.38), one can find out that e_2 is only affected by actuator faults f_a, but not subject to system uncertainties $\Delta\psi$ as well as the error injection term ν_1. From (2.31), it can be seen that the bound of the norm of $e_2(t)$ depends on the bound of the unknown initial condition $e_2(0)$. Since $\|e_2(0)\|$ is multiplied by $e^{(c_0 \mathscr{L}_{f_2} \|T^{-1}\| - a_0)t}$, its effect will decrease exponentially and e_2 will approach to zero if there is no actuator fault. Otherwise it will deviate from zero. Therefore $\|e_{w2}\| = \|C_4 e_2\|$ provides a good choice, as the residual, to detect the occurrence of actuator faults. The actuator FD scheme can be devised as follows:

Actuator FD scheme: *Actuator faults can be detected if the residual $\|e_{w_2}\|$ exceeds a predefined threshold ς. Otherwise the system is healthy within the considered time. The detection time $t_d (t_d \geq t_f)$ is defined as the first time instant such that $\|e_{w_2}\|$ is observed greater than ς.*

Remark 2.4 It follows from Lemma 2.3 that e_2 will approach to zero when System (2.1) is healthy. This implies that a small threshold ς can be selected. The value of ς does not significantly affect the performance of the proposed FD scheme.

2.4 Actuator FI Scheme

After detecting the occurrence of actuator faults, the next objective is to determine their locations, if the system suffers from multiple faults simultaneously. Denote f_a as $f_a = [f_a^{1^T}, f_a^{2^T}, \ldots, f_a^{m^T}]^T$. If we can decide whether or not $f_a^i = 0, \; i = 1, 2, \ldots, m$, then the actuator fault isolation can be achieved according to the known fault distribution matrix B. In order to do this, we adopt the modified dedicated observer scheme. More specifically, for each possible $f_a^i \neq 0, \; i = 1, 2, \ldots, m$, we design two SMOs (one is designed for subsystem-1 and the other is designed for subsystem-2) and a total number of $2m$ SMOs are designed. The observer that is designed for f_a^i is required to satisfy the following constraint: the obtained residual is only sensitive to f_a^i, but is insensitive to all other faults.

For the ith actuator fault $f_a^i, \; i = 1, 2, \ldots, m$, we design the following SMO for Subsystem (2.9):

$$\begin{cases} \dot{\hat{z}}_1^i &= A_1 \hat{z}_1^i + A_2 \hat{z}_1^i + f_1(T^{-1}\hat{z}^i, t) + B_1 u + (A_1 - A_1^s) C_1^{-1} (w_1^i - \hat{w}_1^i) + \nu_1^i \\ \hat{w}_1^i &= C_1 \hat{z}_1^i, \end{cases} \quad (2.39)$$

where $\hat{z}^i$ and $\hat{w}^i$ denote respectively the estimated state and output obtained by this isolation estimator. $\hat{z}^i$ is defined as $\hat{z}^i := col(C_1^{-1}w_1, \hat{z}_2^i)$. The output error injection term ν_1^i is defined as

$$\nu_1^i = \begin{cases} (\|E_1\|\xi + \|B_1\|\rho_a + \eta_1)\frac{P_1(C_1^{-1}w_1 - \hat{z}_1^i)}{\|P_1(C_1^{-1}w_1 - \hat{z}_1^i)\|} & \text{if } C_1^{-1}w_1 - \hat{z}_1^i \neq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{2.40}$$

where $P_1 \in \mathscr{R}^{r\times r}$ is a symmetric positive definite matrix which is to be determined and η_1 is a positive scalar defined by (2.33).

For Subsystem (2.10), an SMO is designed instead of a normal Luenberger observer that has been used in Sect. 2.3 for FD. The proposed SMO has the following form:

$$\begin{cases} \dot{\hat{z}}_2^i &= A_4\hat{z}_2^i + A_3C_1^{-1}w_1^i + f_2(T^{-1}\hat{z}^i, t) + B_2u + L(w_2^i - \hat{w}_2^i) + \bar{B}_2^i\nu_2^i \\ \hat{w}_2^i &= C_4\hat{z}_2^i, \end{cases} \tag{2.41}$$

where L is the observer gain to be determined. Partition $\bar{B}_2$ into $\bar{B}_2 = [B_2^1, \ldots, B_2^m]$. Then B_2^i represents the ith column of B_2 and $\bar{B}_2^i$ denotes the rest of the columns. The output error injection term ν_2^i is defined by

$$\nu_2^i = \begin{cases} (\rho_a + \eta_3)\frac{\bar{F}^i(w_2^i - \hat{w}_2^i)}{\|\bar{F}^i(w_2^i - \hat{w}_2^i)\|} & \text{if } w_2^i - \hat{w}_2^i \neq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{2.42}$$

where η_3 is a positive scalar and $F \in \mathscr{R}^{m\times(p-r)}$ is a matrix to be determined. F^i represents the ith row of F and $\bar{F}^i$ consists of all other rows.

If the state estimation errors obtained from the SMOs, which are designed for the f_a^i, are defined as $e_1^i = z_1^i - \hat{z}_1^i$ and $e_2^i = z_2^i - \hat{z}_2^i$, then the error dynamics after the occurrence of actuator faults can be obtained as

$$\dot{e}_1^i = A_1^s e_1^i + A_2 e_2^i + \left[f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}^i, t)\right] + B_1 f_a + E_1\Delta\psi - \nu_1^i \tag{2.43}$$

$$\begin{aligned} \dot{e}_2^i &= (A_4 - LC_4)e_2^i + \left[f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}^i, t)\right] + B_2 f_a^i - \bar{B}_2^i\nu_2^i \\ &= (A_4 - LC_4)e_2^i + \left[f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}^i, t)\right] + B_2^i f_a^i + \bar{B}_2^i(\bar{f_a}^i - \nu_2^i), \end{aligned} \tag{2.44}$$

where $\bar{f_a}^i$ represents the vector of actuator faults excluding f_a^i.

The sufficient conditions for the stability of the above error dynamics are presented in the following result:

Theorem 2.3 *Given System (2.1) with Assumptions 2.1–2.4. If there exist matrices $A_1^s < 0$, L, $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, and F, and positive scalars α_1 and α_2 such that*

$$B_2^T P_2 = F C_4 \tag{2.45}$$

$$\begin{bmatrix} \varPi_1 + \frac{1}{\alpha_1} P_1 P_1 & P_1 A_2 \\ A_2^T P_1 & \varPi_2 + \frac{1}{\alpha_2} P_2 P_2 + a I_{n-r} \end{bmatrix} < 0 \tag{2.46}$$

where $\varPi_1 = A_1^{s\,T} P_1 + P_1 A_1^s$, $\varPi_2 = (A_4 - LC_4)^T P_2 + P_2(A_4 - LC_4)$ *and* $a = \alpha_1 \mathscr{L}_{f_1}^2 \|T^{-1}\|^2 + \alpha_2 \mathscr{L}_{f_2}^2 \|T^{-1}\|^2$, *then the state estimation error* e_2^i *will exponentially tend to zero if* $f_a^i = 0$*; otherwise* e_2^i *satisfies* $(A_4 - LC_4)e_2^i + \left[f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}^i, t)\right] + B_2^i f_a^i + \bar{B}_2^i(\bar{f}_a{}^i - \nu_2^i)$ *if* $f_a^i \neq 0$.

Proof Assume $V_1^i = e_1^{i\,T} P_1 e_1^i$ and $V_2^i = e_2^{i\,T} P_2 e_2^i$. Consider $V^i = V_1^i + V_2^i$ as the Lyapunov candidate.

The time derivative of V_1^i along the trajectory of System (2.43) can be shown to be

$$\dot{V}_1^i \leq e_1^{i\,T} \varPi_1 e_1^i + 2 e_1^{i\,T} P_1 A_2 e_2^i + \frac{1}{\alpha_1} e_1^{i\,T} P_1 P_1 e_1^i + \alpha_1 \mathscr{L}_{f_1}^2 \|T^{-1}\|^2 \|e_2^i\|^2. \tag{2.47}$$

If $f_a^i = 0$, the error dynamics of e_2^i of (2.44) can be rewritten as

$$\dot{e}_2^i = (A_4 - LC_4)e_2^i + \left[f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}^i, t)\right] + \bar{B}_2^i(\bar{f}_a{}^i - \nu_2^i). \tag{2.48}$$

Then, it follows from (2.45) and (2.48) that the derivative of V_2^i can be obtained as

$$\begin{aligned} \dot{V}_2^i &= e_2^{i\,T} \varPi_2 e_2^i + 2 e_2^{i\,T} P_2 \left[f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}^i, t)\right] + 2 e_2^{i\,T} P_2 \bar{B}_2^i (\bar{f}_a{}^i - \nu_2^i) \\ &\leq e_2^{i\,T} \varPi_2 e_2^i + 2 e_2^{i\,T} P_2 \left[f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}^i, t)\right] + 2\|\bar{F}^i e_{w2}^i\|(\|\bar{f}_a^i\| - \rho_a - \eta_3) \\ &\leq e_2^{i\,T} \varPi_2 e_2^i + \frac{1}{\alpha_2} e_2^{i\,T} P_2 P_2 e_2^i + \alpha_2 \mathscr{L}_{f_2}^2 \|T^{-1}\|^2 \|e_2^i\|^2. \end{aligned} \tag{2.49}$$

Combining (2.47) and (2.49) yields

$$\begin{aligned} \dot{V}^i &= \dot{V}_1^i + \dot{V}_2^i \\ &= \begin{bmatrix} e_1^i \\ e_2^i \end{bmatrix}^T \begin{bmatrix} \varPi_1 + \frac{1}{\alpha_1} P_1 P_1 & P_1 A_2 \\ A_2^T P_1 & \varPi_2 + \frac{1}{\alpha_2} P_2 P_2 + a I_{n-r} \end{bmatrix} \begin{bmatrix} e_1^i \\ e_2^i \end{bmatrix} \end{aligned} \tag{2.50}$$

If there exist solutions such that Inequality (2.46) is satisfied, then $\dot{V}^i < 0$, which implies that e^i will tend to zero exponentially when $f_a^i = 0$, even after the occurrence of actuator faults $f_a^j \neq 0$, $j \in \{1, 2, \ldots, m\} \backslash \{i\}$. On the other hand, if $f_a^i \neq 0$, the term $B_2^i f_a^i$ in (2.44) can not be attenuated by $\bar{B}_2^i(\bar{f}_a{}^i - \nu_2^i)$, because B_2 is of full column rank. Therefore, we can conclude that $\lim_{t \to \infty} e_0^i \neq 0$ if $f_a^i \neq 0$.

This completes the proof. □

Remark 2.5 It is noted that the condition proposed in Theorem 2.3 includes a linear matrix equality (2.45), which is difficult to solve directly by MATLAB toolbox. This equality together with the Lyapunov equation (2.11) is a passivity condition for system $((A_4 - LC_4), B_2, C_4)$ [21]. The sufficient and necessary condition for the existence of F satisfying (2.45) is $rank(C_4 B_2) = rank(B_2)$. Here, we employ the algorithm introduced in [25] to determine the value of F.

Equation (2.45) can be rewritten as

$$\text{Trace}\left[(B_2^T P_2 - FC_4)^T (B_2^T P_2 - FC_4)\right] = 0.$$

Introduce the matrix inequality:

$$(B_2^T P_2 - FC_4)^T (B_2^T P_2 - FC_4) < \gamma^2 I_{n-r}, \tag{2.51}$$

where γ is a positive scalar.

Using the Schur complement result, (2.51) is equivalent to the following LMI:

$$\begin{bmatrix} -\gamma I_{n-r} & (B_2^T P_2 - FC_4)^T \\ B_2^T P_2 - FC_4 & -\gamma I_m \end{bmatrix} < 0 \tag{2.52}$$

Therefore, the solvability of (2.45) can be converted into finding the minimum of γ satisfying the above inequality. Subsequently, the design of the proposed SMOs is now converted into a problem of finding a global solution of the following minimization problem:

$$\text{Minimize } \gamma \text{ subject to } P_1 > 0,\ P_2 > 0,\ (2.27) \text{ and } (2.52)$$

Theorem 2.3 presents the sufficient conditions for the stochastic stability of the dynamics (2.43) and (2.44) when $f_a^i = 0$ and also forms the intuitive principle of the FI scheme as follows: the decision on which actuator is faulty is equivalent to the problem of determining which f_a^i is not equal to zero. After the system is detected to be faulty at some time instant t_d, a bank of $2m$ SMOs are designed according to all possible faulty models. More specifically, for each $f_a^i, i = 1, 2, \ldots, m$, two observers given by (2.39) and (2.41) are designed to estimate states and outputs. It can be seen from the proof of Theorem 2.3 that the output error injection term ν_2^i can attenuate the effect of f_a^j, $j \in \{1, 2, \ldots, m\}\backslash\{i\}$ to the residual, but can not eliminate the effect of f_a^i. Therefore, if $f_a^i = 0$, the state estimation error e_2^i obtained by the observers which are designated for f_a^i will converge to zero. Otherwise e_2^i will go beyond a predefined threshold for some finite time $t_i > t_d$ if $f_a^i \neq 0$. Based on this analysis, we can choose $\|e_{w_2}^i\| = \|C_4 e_2^i\|$ as the residual and compare it with the corresponding threshold ς_i, then the location of actuator faults can be concluded. The selection of the isolation threshold ς_i is similar to the selection of the detection threshold ς. Since the residual $\|e_{w_2}^i\|$ obtained from the observer which is designed

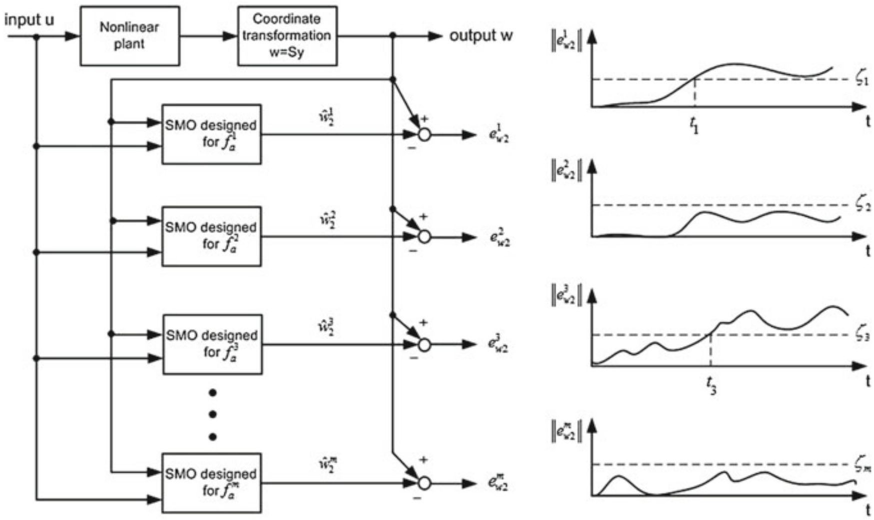

Fig. 2.1 An example of actuator FI using a bank of SMOs

to isolate f_a^i, is close to zero if $f_a^i = 0$, a small value of ς_i can be chosen. The FI scheme can be summarized as follows:

Actuator FI scheme: *Multiple actuator faults can be isolated by comparing the residual* $\|e_{w_2}^i\|$, $(i = 1, 2, \ldots, m)$ *with a predefined threshold* ς_i. *It can be concluded that* $f_a^i \neq 0$ *if* $\|e_{w_2}^i\|$ *goes over the threshold for some finite time* $t_i > t_d$. *Otherwise* $f_a^i = 0$ *if* $\|e_{w_2}^i\|$ *is always below the threshold* ς_i *during the time studied. Considering the structure of* B, *the decision on which actuator is faulty can then be made.*

To further illustrate this scheme, we consider an example of a system with two actuator faults. As shown in Fig. 2.1, f_a^1 and f_a^3 can be isolated at time t_1 and t_3 respectively.

2.5 Simulation Results

In this section, the effectiveness of the proposed schemes in detecting and isolating actuator faults has been demonstrated by an example of a modified seventh-order aircraft model used in [26], in which the states are defined as

$$
\begin{aligned}
x_1 &= \phi - bank\ angle(rad)\\
x_2 &= r - yaw\ rate(rad/s)\\
x_3 &= p - roll\ rate(rad/s)\\
x_4 &= \delta - sideslip\ angle(rad)\\
x_5 &= x_7 - washout\ filter\ state\\
x_6 &= \delta_r - rudder\ deflection(rad)\\
x_7 &= \delta_a - aileon\ deflection(rad)
\end{aligned}
$$

The inputs are

$$
\begin{aligned}
u_1 &= \delta_{rc} - rudder\ command(rad)\\
u_2 &= \delta_{ac} - aileon\ command(rad)
\end{aligned}
$$

and outputs are

$$
\begin{aligned}
y_1 &= r_a - roll\ acceleration(rad/s)\\
y_2 &= p_a - yaw\ acceleration(rad/s)\\
y_3 &= \phi - bank\ angle(rad)\\
y_4 &= x_7 - washout\ filter\ state
\end{aligned}
$$

The system is in the form of (2.1) with

$$
A = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & -0.154 & -0.04 & 1.54 & 0 & -0.744 & -0.032\\
0 & 0.249 & -1 & -5.2 & 0 & 0.337 & -1.12\\
0.0386 & -0.996 & 0 & -2.117 & 0 & 0.02 & 0\\
0 & 0.5 & 0 & 0 & -4 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -20.000 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -25
\end{bmatrix}, \ B = \begin{bmatrix}
0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ 20 & 0\\ 0 & 25
\end{bmatrix}
$$

$$
C = \begin{bmatrix}
0 & -0.154 & -0.04 & 1.54 & 0 & -0.744 & -0.032\\
0 & 0.249 & -1 & -5.2 & 0 & 0.337 & -1.12\\
1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
$$

$$
E = \begin{bmatrix} 1\ 1\ 0\ 0\ 1\ 0\ 0 \end{bmatrix}^T, \ f(x,t) = \begin{bmatrix} \sin x_3\ \sin x_3\ 0\ 0\ \sin x_3\ 0\ 0 \end{bmatrix}^T, \ \Delta\psi = 2\sin t.
$$

Notice that in [26], the original model is linear and there is no system uncertainty. In our simulation, the terms associated with the nonlinearity and system uncertainty are added to show the effectiveness of the proposed actuator FDI method for uncertain Lipschitz nonlinear systems.

The actuator fault $f_a = col(f_{a_1}, f_{a_2})$ is applied to the system and defined as

$$f_{a_1} = \begin{cases} 0 & , \quad t \leq 15\,\mathrm{s} \\ 0.05\exp(0.01t) & , \quad t \geq 15\,\mathrm{s} \end{cases}$$

$$f_{a_2} = \begin{cases} 0 & , \quad t \leq 20\,\mathrm{s} \\ 0.07\exp(0.03t) & , \quad t > 20\,\mathrm{s} \end{cases}$$

The nonsingular transformation matrices T and S are selected as

$$T = \begin{bmatrix} 0.8440 & 0.1560 & 0.0405 & -1.5598 & 0 & 0.7535 & 0.0324 \\ -1.0000 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ -1.0000 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$S = \begin{bmatrix} 1.0000 & 0 & -0.8333 & 0 \\ -1.4359 & 1.0000 & -0.4701 & 0 \\ 1.0128 & 0 & 0.1560 & 0 \\ 1.0128 & 0 & -0.8440 & 1.0000 \end{bmatrix}$$

The system matrices under the new coordinate become

$$TAT^{-1} = \left[\begin{array}{c|cccccc} 1.4794 & 1.3088 & 0.7373 & 5.6393 & 0 & -16.3183 & -0.9083 \\ \hline -0.1540 & -0.1300 & -1.0338 & 1.2998 & 0 & -0.6280 & -0.0270 \\ 0.2490 & 0.2102 & -1.0101 & -4.8116 & 0 & 0.1494 & -1.1281 \\ -0.9574 & -0.8466 & 0.0388 & -3.6104 & 0 & 0.7414 & 0.0310 \\ -3.5000 & 1.0460 & -0.8583 & -5.4593 & -4.0000 & 2.6372 & 0.1134 \\ 0 & 0 & 0 & 0 & 0 & -20.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -25.0000 \end{array}\right]$$

$$SCT^{-1} = \left[\begin{array}{c|cccccc} -0.9873 & 0.0000 & -0.0000 & 0.0000 & 0 & -0.0001 & -0.0000 \\ \hline 0.0000 & 0.4701 & -0.9426 & -7.4112 & 0 & 1.4053 & -1.0741 \\ 0.0000 & -0.1560 & -0.0405 & 1.5598 & 0 & -0.7535 & -0.0324 \\ 0.0000 & -0.1560 & -0.0405 & 1.5598 & 1.0000 & -0.7535 & -0.0324 \end{array}\right]$$

$$TB = \left[\begin{array}{cc} 15.0700 & 0.8100 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20.0000 & 0 \\ 0 & 25.0000 \end{array}\right], \quad TE = \left[\begin{array}{c} 1 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right]$$

Imposing the stability constraint to the transformed system and formulating the problem in an LMI framework gives the values of the parameters of the proposed observers.

The simulation results are obtained by running successively the files **chapter2_lmi.m**, **chapter2.mdl**, and **chapter2_plot.m**. The solutions of various LMIs are obtained by running the M-file **chapter2_lmi.m** which are given below.

File Chapter2_lmi.m

```
clear all
clc

% Oringinal system matrices
A=[0          0       1      0    0    0       0
   0       -0.154  -0.04  1.54   0  -0.744  -0.032
   0        0.249    -1   -5.2   0   0.337  -1.12
   0.0386  -0.996     0  -2.117  0   0.02     0
   0        0.5       0      0  -4    0       0
   0         0        0      0   0  -20       0
   0         0        0      0   0    0      -25];
B=[0 0 0 0 0 20 0;0 0 0 0 0 0 25]';
C=[0  -0.1540   -0.04    1.54  0 -0.7440  -0.0320
   0   0.249     -1    -5.2000 0  0.3370  -1.1200
   1     0         0         0  0    0         0
   0     0         0         0  1    0         0];
E=[1 1 0 0 1 0 0]';

% Dimensions of system matrices
n=size(A,1);
m=size(B,2);
p=size(C,1);
r=size(E,2);

% Transformation matrices T and S
T =[0.8440 0.1560 0.0405 -1.5598 0 0.7535 0.0324
      -1      1      0        0    0    0       0
       0      0      1        0    0    0       0
       0      0      0        1    0    0       0
      -1      0      0        0    1    0       0
       0      0      0        0    0    1       0
       0      0      0        0    0    0       1];
```

```
S =[ 1.0000   0   -0.8333    0
   -1.4359    1   -0.4701    0
    1.0128    0    0.1560    0
    1.0128    0   -0.8440    1];

% Transformed system matrices (2.6)
A=T*A*inv(T);
B=T*B;
E=T*E;
C=S*C*inv(T);

A1=A(1:r,1:r);
A2=A(1:r,r+1:n);
A3=A(r+1:n,1:r);
A4=A(r+1:n,r+1:n);

B1=B(1:r,:);
B2=B(r+1:n,:);

E1=E(1:r,:);

C1=C(1:r,1:r);
C4=C(r+1:p,r+1:n);

lf1=1;  % Lipschitz constant for the nonlinear term f1.
        % Note that after the transformation the nonlinear
        % term f2=0, therefore lf2=0 and alpha2=0

% Define unknown variables
P1=sdpvar(r,r);
P2=sdpvar(n-r,n-r);
X=sdpvar(r,r);
Y=sdpvar(n-r,p-r);
F=sdpvar(m,p-r);
alpha1=sdpvar(1,1);
gamma=sdpvar(1,1);

a=alpha1*lf1^2*(norm(inv(T)))^2;  % The positive scalar in (2.17)

% LMI (2.27)
% Note that since lf2=0, the last column and last row of (2.27)
% are no longer present. Therefore (2.27) becomes a 3-by-3 matrix
```

```
% instead of 4-by-4.
M1=[X+X' P1 P1*A2
      P1 -alpha1 zeros(r,n-r)
      A2'*P1 zeros(n-r,r) A4'*P2+P2*A4-C4'*Y'-Y*C4+a*eye(n-r)];

% LMI (2.52)
M2= [-gamma*eye(n-r) (B2'*P2-F*C4)'
       B2'*P2-F*C4 -gamma*eye(m)];

% minimize gamma subject to P1>0, P2>0,
% alpha1>0, (2.27) and (2.52)
const = [M1<0,M2<0,P1>0,P2>0,alpha1>0];
solvesdp(const,gamma);

P1=double(P1)
P2=double(P2)
F=double(F)
X=double(X);
Y=double(Y);
A1s=inv(P1)*X
L=inv(P2)*Y
alpha1=double(alpha1)
gamma=double(gamma)
```

Parameters are obtained as

$$P_1 = 0.0048,\ A_1^s = -25.8618,\ \alpha_1 = 0.0021,\ \gamma = 2.0190 \times 10^{-11}$$

$$P_0 = \begin{bmatrix} 0.3072 & 0.0109 & -0.0689 & -0.2578 & 0.0144 & 0.0058 \\ 0.0109 & 0.2432 & 0.0527 & 0.0969 & 0.0506 & 0.0342 \\ -0.0689 & 0.0527 & 0.4599 & 0.1648 & -0.0253 & 0.0254 \\ -0.2578 & 0.0969 & 0.1648 & 0.4662 & -0.0008 & 0.0002 \\ 0.0144 & 0.0506 & -0.0253 & -0.0008 & 0.1079 & 0.0545 \\ 0.0058 & 0.0342 & 0.0254 & 0.0002 & 0.0545 & 0.0372 \end{bmatrix}$$

$$L = \begin{bmatrix} 2.3497 & 6.3662 & 1.6347 \\ -3.1985 & -13.8943 & -1.7068 \\ -3.8256 & -11.5108 & -1.5464 \\ 3.6515 & 10.8768 & -1.9943 \\ 58.2398 & 282.3094 & -3.8605 \\ -57.1666 & -394.4680 & 8.0411 \end{bmatrix}$$

$$F = \begin{bmatrix} -0.8797 & -4.4882 & -0.0156 \\ -0.7677 & -3.2451 & 0.0039 \end{bmatrix}$$

It is worth noting that the parameters obtained from LMI may differ from that shown here. This is expected because these are obtained by solving LMIs which does not give unique solutions. Note that although these parameters are computed for fault isolation, they can also be applied for fault detectors (2.12) and (2.14). The above obtained parameters are used to simulate the system described in Simulink model **chapter2.mdl** and the figures are plotted by running the file **chapter2_plot.m**. In the simulation, we have selected the initial state as $x(0) = [0, 0, 0, 0, 0, 0, 0]^T$ and $\hat{x}(0) = [-0.1, 0, 0.2, -0.1, -0.1, -0.1, -0.1]^T$. In the Simulink model, a parameter "δ" has been added to the denominator of (2.40) and (2.42) to reduce the chattering effect. We have selected $\delta = 0.01$ in the simulation.

The detectability of the proposed scheme is shown in Fig. 2.2. It shows that the proposed method could successfully detect the occurrence of a fault at around $t = 10.05$ s (the fault occurs at 10 s) with the corresponding threshold being chosen as 0.05.

After the detection of faults, the next stage is to determine which actuator, amongst the various actuators, is faulty. This is carried out using two isolation observers. Results of the simulation are shown in Figs. 2.3 and 2.4. The residual generated by the first isolation observer, which is designed for f_a^1, is compared with the threshold that is set to 0.02 in Fig. 2.3. From the figure it is observed that the residual exceeds the threshold at about 10.1 s, which implies that f_a^1 can be found to be nonzero at approximately 10.1 s. Figure 2.4 shows the simulation result when the isolation observer designed for f_a^2 is used. It is seen from the figure that the residual obtained by the this observer exceeds the threshold (0.06) at approximately 20.1 s, which denotes that f_a^2 can be found to be nonzero at about 20.1 s.

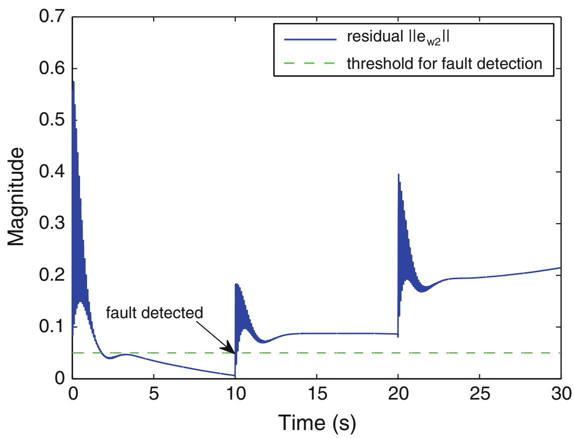

Fig. 2.2 Detection of the occurrence of actuator faults

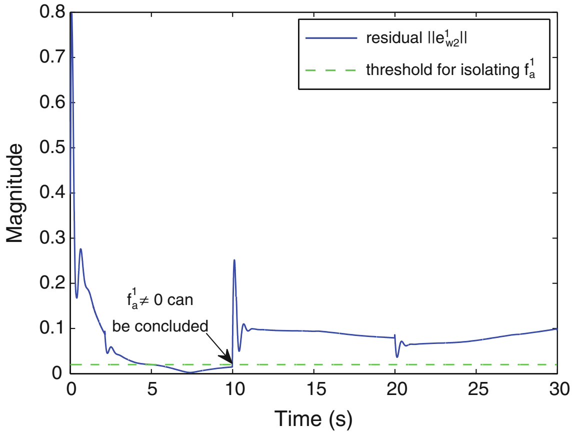

Fig. 2.3 Isolation of f_a^1

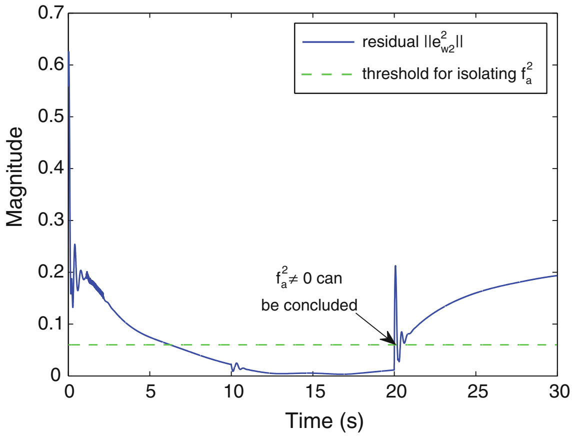

Fig. 2.4 Isolation of f_a^2

2.6 Conclusions

In this chapter, we propose a new scheme to robustly detect and isolate incipient actuator faults for uncertain Lipschitz nonlinear systems. The proposed FDI scheme essentially transforms the original system into two subsystems where subsystem-1 includes both actuator faults and system uncertainties while subsystem-2 has actuator faults but without uncertainties. Actuator faults can be detected by applying a Luenberger observer for subsystem-2, and isolated using a bank of SMOs for both subsystems based on the modified dedicated observer scheme. The most distinct feature of the proposed FDI scheme is that, by imposing a coordinate transformation to the original system, the effects of system uncertainties to the residual of subsystem-

2 are completely decoupled, which makes the scheme sensitive to incipient faults while still robust to modelling uncertainty. Thus, early detection can be achieved and a false alarm caused by modeling uncertainties can be totally avoided. The sufficient conditions of stability of the proposed observers have been studied and represented in the form of LMI. Its effectiveness has been demonstrated considering the example of a modified aircraft model.

References

1. Yeu TK, Kim HS, Kawaji S (2005) Fault detection, isolation and reconstruction for descriptor systems. Asian J Control 7:356–367
2. Ding SX, Zhang P, Naik A, Ding EL, Huang B (2009) Subspace method aided data-driven design of fault detection and isolation systems. J Process Control 19:1496–1510
3. Zhang P, Zou J (2012) Observer-based fault diagnosis and self-restore control for systems with measurement delays. Asian J Control 14:1717–1723
4. Wang Z, Shen Y, Zhang X (2014) Actuator fault estimation for a class of nonlinear descriptor systems. Int J Syst Sci 45:487–496
5. Zhang J, Swain AK, Nguang SK (2012) Detection and isolation of incipient sensor faults for a class of uncertain nonlinear systems. IET Control Theory Appl 6:1870–1880
6. Zhang J, Swain AK, Nguang SK (2013) Robust sensor fault estimation scheme for satellite attitude control systems. J Frankl Inst 350:2581–2604
7. Sreedhar R, Fernandez B, Masada G (1993) Robust fault detection in nonlinear systems using sliding mode observers. In: Proceedings of IEEE conference on control application
8. Edwards C, Spurgeon SK, Patton RJ (2000) Sliding mode observers for fault detection and isolation. Automatica 36:541–553
9. Tan CP, Edwards C (2002) Sliding mode observers for detection and reconstruction of sensor faults. Automatica 38:1815–1821
10. Tan CP, Edwards C (2003) Sliding mode observers for robust detection and reconstruction of actuator and sensor faults. Int J Robust Nonlinear Control 13:443–463
11. Tan CP, Edwards C (2007) Sensor and/or actuator fault reconstruction plays a key role in the ftc design. Asian J Control 9(3):340–344
12. Yan XG, Edwards C (2007) Sensor fault detection and isolation for nonlinear systems based on a sliding mode observer. Int J Adapt Control Signal Process 21:657–673
13. Raoufi R, Marquez HJ (2010) Simultaneous sensor and actuator fault reconstruction and diagnosis using generalized sliding mode observers. In: Proceedings of American control conference, pp 7016–7021
14. Chen W, Chowdhury FN (2010) A synthesized design of sliding-mode and Luenberger observers for early detection of incipient faults. Int J Adapt Control Signal Process 24:1021–1035
15. Clark R (1978) Instrument fault detection. IEEE Trans Aerosp Electron Syst 14:456–465
16. Frank PM (1990) Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy-a survey and some new results. Automatica 26:459–474
17. Chen J, Patton RJ (1991) Optimal selection of unkown input distribution matrix in the design of robust observers for fault diagnosis. Automatica 29:837–841
18. Gertler J (1995) Optimal residual decoupling for robust fault diagnosis. Int J Control 61:395–421
19. Corless M, Tu J (1998) State and input estimation for a class of uncertain systems. Automatica 34:757–764
20. Hui S, Zak SH (2005) Observer design for system with unknown inputs. Int J Appl Math Comput Sci 15(4):431–446

21. Yan XG, Edwards C (2008) Robust sliding mode observer-based actuator fault detection and isolation for a class of nonlinear systems. Int J Syst Sci 39(4):349–359
22. Yan XG, Edwards C (2007) Nonlinear robust fault reconstruction and estimation using a sliding mode observer. Automatica 43:1605–1614
23. Ioannou PA, Sun J (1996) Robust adaptive control. Prentice Hall, Englewood Cliffs
24. Utkin VI (1992) Sliding modes in control optimization. Springer, Berlin
25. Zhang K, Jiang B, Shi P (2009) Fast fault estimation and accommodation for dynamical systems. IET Control Theory Appl 3:189–199
26. Tan CP, Edwards C (2000) An LMI approach for designing sliding mode observers. In: Proceedings of 39th IEEE conference on decision and control, Sydney, Australia

Springer

http://www.springer.com/978-3-319-32323-7

Robust Observer-Based Fault Diagnosis for Nonlinear Systems Using MATLAB®
Zhang, J.; Swain, A.; Nguang, S.K.
2016, XIII, 224 p. 44 illus., 42 illus. in color. With online files/update., Hardcover
ISBN: 978-3-319-32323-7