

Chapter 2

Commentary

2.1 Introduction

[152] *At the very beginning of his monograph, Tarski puts forward his goal. It is to construct “a materially adequate and formally correct definition of the term ‘true sentence’”. The conditions for defining certain concepts are also given here. The bibliographical note is worth of notice, especially since in this form it only appears in the English version. Only the first paragraph of the note can be read in the German translation.*

It is worth noticing that the Polish title *Pojęcie prawdy w językach nauk dedukcyjnych* has not been literally translated, as *The concept of truth in the languages of deductive sciences*. On March 26th 1935, as the German translation was almost finished, Tarski wrote a post card to Kazimierz Twardowski with 2 requests:

1° I would like (if it is still possible) for the title of my paper to be “Der Wahrheitsbegriff in den formalisierten Sprachen”. 2° My paper was written three years ago; since then my positions regarding a few points have changed. I would like for that circumstance to be reflected in my paper. For this purpose, I could change the ending. However, I would much rather prefer to settle this matter otherwise – after the “Summary” I would like to add “Nachwort”, which would approximately take 2 pages. Is it possible, and could I sent the text in Polish? [Translation M.G.]¹

The original title broadens the spectrum of the languages for which truth can be defined. Colloquial language is not explicitly excluded from languages of deductive sciences, whereas it is certainly excluded from the set of formalized languages.

¹“1° Pragnąłbym (o ile to jeszcze możliwe), by tytuł mojej pracy brzmiał “Der Wahrheitsbegriff in den formalisierten Sprachen”. 2° Praca moja była napisana przed trzema laty; od tego czasu poglądy moje w kilku punktach uległy zmianie. Pragnąłbym, by ten stan rzeczy znalazł swój wyraz w pracy, Mógłbym w tym celu zmienić zakończenie. Bardziejby mi jednak odpowiadało inne załatwienie sprawy – chciałbym po “Zakończeniu” umieścić jeszcze dodatek (“Nachwort”), który by zajął w przybliżeniu 2 strony druku. Czy to możliwe i czy mógłbym nadesłać tekst po polsku?” Letter L. 105/35 archived in Polskie Towarzystwo Filozoficzne, Poznań.

Already on the next page, however, i.e. p. 153 of the English text, Tarski writes explicitly that his investigations will exclusively consider only the formalized languages of the deductive sciences. Furthermore, certain apparently overlooked passages suggest Tarski's understanding of the languages for which he intends to define truth.

In the Polish original Tarski writes "zdanie prawdziwe" [p. 14] which means the same as a "true sentence" and has been translated as such into English. In the German translation we read "wahre Aussage" [p. 265], see also [1.2.3].

Tarski had legitimate reasons for choosing sentences as truth bearers. The predicate 'true' can be used to refer to psychological phenomena like judgements or beliefs, which are, however, more difficult to determine precisely, and therefore have been rejected by Tarski as primary truth bearers. Furthermore, truth has sometimes been ascribed to certain ideal entities called 'propositions', but the meaning of this term is even more difficult to grasp and it remains unclear and ambiguous. Therefore, Tarski decided to use 'sentences', understood as 'declarative sentences', as the bearers of truth.

Translational Remarks

The second sentence on this page begins – "**Its** task is to construct ...", where the possessive pronoun '**Its**' in the English translation refers to the article mentioned in the previous sentence. In Polish "**jego** istota" [p. 14], and in German "**sein** Wesen" [p. 264] are used, however, which can both be translated as "**its** essence", relating to the problem of the article, not to the article itself. The following sentence, beginning with the phrase "this problem", highlights the mistake. This, however, is a minor discrepancy, which does not influence the content that much.

The phrase "which started with apparently **evident** premisses" is a translation of German "welche von scheinbar **evidenten** Prämissen" [p. 265] which comes from Polish "oparte na **intuicyjnych** na pozór przesłankach" [p. 14]. In comparison with the original, both versions neglect to literally translate the term 'intuicyjnych' meaning 'intuitive', and they replace it with 'evident', see also [1.2.1].

[153] *Tarski emphasizes that he will be concerned exclusively with the classical, as opposed to the utilitarian, conception of truth. Several languages will be considered, each of them separately, with regard to the problem of constructing the definition of a true sentence. In regard to colloquial languages the conclusion will be negative. The languages are divided into 'poorer' and 'richer', with the conclusion being positive in the first case.*

For the readers to better understand the objective of this monograph, Tarski emphasizes on this page that his goal is not "a thorough analysis of the current meaning in everyday life of the term 'true' ". This task belongs to philosophers. Tarski considered himself a "mathematician, (as well as a logician, and perhaps a philosopher of a sort)" (Tarski 1944, p. 369). Tarski's goal was to lay down a rigorous logical apparatus which would serve as a tool for expressing 'intuitively' clear concepts for dealing with philosophical problems. According to Steven Givant² "three things were

²Givant (1991, 18).

especially important to Tarski: correctness, preciseness, and conciseness”. They all are reflected in this monograph.

The following phrasing has been the source of major misinterpretations of Tarski’s words and the reason why many philosophers considered him to be a propounder of the correspondence theory of truth: Polish “prawdziwie – to tyle co zgodnie z rzeczywistością” [p. 15] became in German “wahr – mit der Wirklichkeit übereinstimmend” [p. 265] which was translated into English as “true – corresponding with reality”. Although, both translations grasp the meaning of Tarski’s words, the English one, especially, has been the source of some confusion. Polish ‘prawdziwie’ is an adverb which means the same as ‘truly’, “prawdziwie – to tyle co zgodnie z rzeczywistością” means no more than “truly – is just agreeing with reality”. The correspondence theory is based on the same principle, i.e., the truth of a sentence consists in its correspondence to (agreement with) reality. Hence, the question whether Tarski was a correspondence theorist is legitimate. In order to answer it, we have to specify our understanding of the correspondence theory of truth. We will follow Patterson’s brief and pointed interpretation:

...there is a fairly well established tradition of distinguishing “weak” from “strong” correspondence theories, where “strong” theory posits some sort of structural relationship between sentences and the other relata of the correspondence relation, and a weak one merely insists that whether or not a sentence is true depends on whether what it says is the case...(Patterson 2012, p. 157)³

Accepting this reading, it is clear that Tarski was a correspondence theorist in a weak sense, as also the above translated sentence illustrates. In the strong sense, the relation of correspondence between sentences and reality is the essence of this theory. Patterson notices accurately that T-sentences of Tarski’s **Convention T** are not even of the right form to ascribe a structural relation.

Tarski was convinced that a *semantical definition* captures our intuitions in the clearest and at the same time most precise way, as he explicates in Sect. 1 of his monograph. Further, Tarski applies precise and unambiguous terminology as well as a coherent axiomatic system, and develops it into a comprehensive formal theory of truth which, in this respect, cannot be compared with other correspondence theories.

Translational Remarks

The phrase “**intuitive** knowledge” is, exceptional in this case, the correct translation of Polish “**intuicyjną** znajomość” [p. 14], and of German “**intuitive** Kenntnis” [p. 265], see also [1.2.1].

“**The extension of the concept**” is a translation of the German phrase “**der Umfang des Begriffes**” [p. 265], both of which are not very accurate translations of Tarski’s “**zakres terminu**” [15], which means the same as the “scope of the term”. The terms ‘concept’ and ‘Begriff’ are ambiguous. The word ‘term’ exists in both languages, English and German, and that is the accurate translation of the Polish

³Patterson’s book version used here, is the version available online in 2011, therefore it may happen that the page numbers do not have a one to one correspondence with the edition printed in 2012.

‘termin’. Thus, the German translation should be “der Anwendungsbereich des Terminus”, and the English “the scope of the term”.

Referring to colloquial language as the object of our investigations, Tarski closes in Polish with the statement that “ostateczna konkluzja **tych rozważań** jest wybitnie negatywna” [p. 15], which has been accurately translated into German as “das Schlussergebnis **dieser Erwägungen** ist gänzlich negativ” [p. 266]. A thorough English translation should be “the final conclusion **of these considerations** is totally negative”, however, the phrase written in boldface is missing in the English version.

Tarski divides the formalized languages of the deductive sciences “**na dwie wielkie grupy**” [16], which is accurately translated into German as “**in zwei große Gruppen**” [p. 266], whereas in English we simply have the division “**into two groups**”, omitting the adjective ‘large’. It seems important to add the adjective ‘large’, since it implies the size of both groups, and does not allow us to regard one of them as considerably larger than the other. It is also vital that in connection with one of the groups the problem of the definition of truth has a positive solution, and within the other group the solution is negative.

[154] *Tarski mentions that in Sect. 5 it will be shown that it is impossible to define the concept of truth in connection with the ‘richer’ languages. There is a note regarding Sect. 5 sending us to the Postscript, which has only been added to the German version and then translated into English. Sect. 1 begins here and its main task is to emphasize the difficulties of constructing the definition of truth for colloquial languages.*

Tarski draws attention to the fact that regarding the ‘poorer’ languages “there is a uniform method for the construction of the required definition in the case of each of these languages” separately. The word ‘separately’ has also been left out of the English translation, while it clearly has a place in the Polish original “istnieje jednolita metoda umożliwiająca konstrukcję żądanej definicji **dla każdego z tych języków z osobna**” [p. 16] and in the German translation “es gibt eine einheitliche Methode, welche die Konstruktion der geforderten Definition **für jede dieser Sprachen gesondert** ermöglicht” [p. 266]. It emphasizes the fact that, for each of the languages of the ‘poorer’ group, the required definition can be constructed, but it has to be done separately for each of them.

“The problem of defining truth **in regard to colloquial language**” is an accurate translation of Polish and German versions, in which we read “problem definicji prawdy **w zastosowaniu do języka potocznego**” [p. 17], and “(Betrachtung) des Problems der Wahrheitsdefinition **in Bezug auf die Umgangssprache**” [p. 267], respectively. In the English edition we read about the problem of defining truth **in** colloquial language, however, instead of **in regard to** colloquial language. “To define truth in colloquial language” has a totally different meaning from “defining truth in regard to colloquial language”.

The footnote which begins on this page is of crucial importance. Here, Tarski explicitly credits Leśniewski for the negative results which will be presented in Sect. 1 of this epoch-breaking monograph. The idea that semantical paradoxes demonstrate how inadequate natural languages are for scientific investigations came actually from Tarski’s Doktorvater, not from himself.

The considerations which I shall put forward in this connexion are, for the most part, not the results of my own studies. Views are expressed in them which have been developed by S. Leśniewski in his lectures at the University of Warsaw (from the year 1919/1920 onwards), in scientific discussions and in private conversations; this applies, in particular, to almost everything which I shall say about quotation-mark expressions (M.G. correction) and the semantical antinomies. It remains perhaps to add that this fact does not in the least involve Leśniewski in the responsibility for the sketchy and perhaps not quite precise form in which the following remarks are presented (Tarski 2006g, pp. 154-5).

Nevertheless, the negative results presented in Sect. 1 have been from the beginning, and sometimes still are, attributed to Tarski. This is partially due to the lack of knowledge of Polish analytic philosophy, in this case of Leśniewski's work outside of Poland, at least during the first half of the twentieth century.⁴

Translational Remarks

In 1933 Tarski did not consider it possible to construct a correct definition of the notion of truth with regard to the languages of the 'richer' group. Even then, however, he was positive that a "**consistent and accurate**" use of this notion was possible. This is an accurate translation of the Polish "**konsekwentnego i trafnego** operowania" [p. 16], which has been translated into German as "**konsequenten und richtigen** Gebrauch" [p. 266], which can be misleading, see also [1.2.5].

The correct translation of Polish "**znajomości podstaw**" [p. 16], and of the German "**die Kenntnis der Grundzüge**" [p. 266] should be "a knowledge of the **fundamentals**" and not, as the English translation has it, "a knowledge of the **principles** of modern formal logic".

In the first footnote, instead of the term '**considerations**', '**remarks**' would be a much better and an accurate translation of Polish '**uwagi**' [p. 17], and of German '**die Bemerkungen**' [p. 267].

2.2 Section 1. The Concept of True Sentence in Everyday or Colloquial Language

[155] *A semantical definition of truth for sentences in everyday language seems to be the most natural one. A possible formulation of such a definition is given, followed by a general scheme of this kind of sentence – the first formulation of what later becomes known as equivalence of the form T.*

At the beginning of the twentieth century semantics was considered to be a branch of linguistics and a part of the theory of language. Its importance in formal logic was not recognized until later, and only areas such as programming languages acknowledged its significance from the beginning. As Woleński points out, semantics became popular only in the thirties, until then there was no uniform definition of this term. As examples of this he mentions C.K. Ogden and I.A. Richards, who spoke of a science

⁴Cf. Betti (2004).

of semantics as dealing with the relation between words and facts, or Quine who used the term ‘semantic’ as a noun, not as an adjective. Moreover, in Rudolf Eisler’s *Wörterbuch der Philosophische Begriffe* there is no entry on semantics, even in its 4th edition (completely revised by Karl Roretz), published in 1930.⁵

The lack of the word ‘semantics’ indicates that this term was not on the tongue of philosophers. Poland was an exception in this respect. In the twenties, Polish philosophers began to use the word ‘semantyka’ (the Polish counterpart of ‘semantics’) for considerations on the meaning-aspect of language. In particular, a very influential book by Tadeusz Kotarbiński, *Elements of Theory of Knowledge, Logic and Methodology of Science* (1929) spoke about semantics understood in this way.⁵ At the same time, Stanisław Leśniewski introduced the term ‘semantic categories’ for what Edmund Husserl understood by *Bedeutungskategorien*. Kazimierz Ajdukiewicz employed the term ‘semantics’ in his review of the above mentioned book by Kotarbiński.⁶ The content of the relevant sections shows that Ajdukiewicz considered semantics to be occupied with various functions of language (meaning, denotation, etc.) (Woleński 1999, pp. 1–2).

Tarski gives his understanding of this concept by the end of Sect. 5 of this monograph. Belonging to the domain, Tarski calls, *semantics of language* are such concepts as satisfaction, denoting, truth and definability.

A characteristic feature of the semantical concepts is that they give expression to certain relations between the expressions of language and the objects about which these expressions speak, or that by means of such relations they characterize certain classes of expressions or other objects. We could also say (making use of the *suppositio materialis*) that these concepts serve to set up the correlation between the names of expressions and the expressions themselves (Tarski 2006g, p. 252).

Tarski sustained his expositions regarding semantics in *The Establishment of Scientific Semantics*, which is a summary of an address Tarski gave at the International Congress of Scientific Philosophy in Paris in 1935, where he also presented his monograph on the concept of truth.

The word ‘semantic’ is used here in a narrower sense than usual. We shall understand by semantics the totality of considerations concerning those concepts which, roughly speaking, express certain connexions between the expressions of a language and the objects and state of affairs referred to by these expressions. As typical examples of semantical concepts we may mention the concepts of *denotation*, *satisfaction*, and *definition*, which appear, for example, in the following statements:

The expression, ‘the victor of Jena’ denotes Napoleon; snow satisfies the condition ‘x is white’; the equation ‘ $x^3 = 2$ ’ defines (determines uniquely) the cube root of the number 2.

The concept of *truth* also—and this is not commonly recognized—is to be included here, at least in its classical interpretation, according to which ‘true’ signifies the same as ‘corresponding with reality’ (Tarski 2006h, p. 401).

As Tarski points out, formulations of the definition of truth similar to (1) are found in Kotarbiński’s book (1926), but also as far back as Aristotle. In his later publication (Tarski 1944), Tarski recalls formulations of a few conceptions of truth, among them the correspondence theory, none of which he considers to be sufficiently precise

⁵Cf. Woleński (1999), p. 1.

and clear to be a satisfactory definition of truth, though, as he notes, “this applies much less to the original Aristotelian formulation than to either of the others” (Tarski 1944, p. 343). Tarski quotes the famous words of Aristotle’s *Metaphysics* because he believes that they capture the intuitions which are also the basis of his own definition:

To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true. (Aristotle 1908)

The formulation (1) of the definition is no more than a draft, which Tarski makes explicit by adding “**w pierwszym rzucie**” [p. 17], and in German “**zunächst**” [p. 268], which could be translated as “**in the first instance**”. In English any such emphasis is missing. It is also important to notice the phrasing within this formulation. Namely, Tarski was very cautious with his choice of words. He deliberately used a colloquial formulation, which is clear in Polish:

(1) “zdanie prawdziwe jest to zdanie, które wyraża, że tak a tak **rzeczy się mają i rzeczy mają się** tak właśnie” [p. 18], and also in German:

(1) “eine wahre Aussage ist eine Aussage, welche besagt, dass **die Sachen sich** so und so **verhalten, und die Sachen verhalten sich** eben so und so” [p. 268].

This sentence should have been translated as

(1) “a true sentence is one which says that **the things are** so and so, and **the things are** indeed so and so.”

The usage of the term “state of affairs” in the English translation is misleading. There is a fair amount of literature regarding the possible truth-bearers and truth-makers i.e. sentences, propositions, pro-sentences, facts, states of affairs etc. We will not get into detailed discussions on this topic here, instead we will just quote a passage from Betti’s article which emphasizes the complexity of this discussion.

On this much there is fair agreement among philosophers: a state of affairs is a complex object (minimally, in the sense that it is *not simple*), it has a fixed number of *constituents* arranged in a special way, of which at least one is an individual, and it belongs to metaphysics, not to semantics.

How many constituents does (*basic or atomic*) state of affairs have, involve or, as I will say, *reticulate*? At least two, an individual and a property, and at most three, an individual, a property and some connection between them. (Betti 2006, p. 2)

For our present investigations, it is important to notice that the above mentioned English translation may have caused unnecessary ambiguity and misunderstandings. State of affairs is a complex entity, on the meaning and the role of which there is not much consensus in today’s philosophical discussions. Therefore, it is the opposite of Tarski’s intentions in the above, intuitively clear and simple formulation.

Translational Remarks

The sentence Tarski thinks “could serve as partial definitions of the truth of a sentence” are called sentences of a ‘more special kind’. Before the adjective ‘special’, there is a comparative form ‘**more**’ missing. In Polish we have “zдания o **bardziej** specjalnym charakterze” [p. 18], and in German “Sätze **spezielleren** Charakters”

[p. 268]. Additionally, the German translator decided to use the term ‘Satz’, which he uses to denote provable sentences in the most of the translation, see [1.2.3].

Having stated the sentence (2), Tarski instructs the readers how to obtain concrete ‘**explanations**’, for that is the accurate translation of the Polish term ‘**wyjaśnień**’ [p. 18], and of the German ‘**Erklärungen**’ [p. 268]. The English translation: ‘definitions’ is most confusing and simply incorrect, since these explanations can be regarded only as partial definitions. On the following pages, the English translator uses the correct term ‘explanations’ in this context.

The second part of the footnote has not been translated from the original but added directly to the English translation.

[156] “*Quotation-mark names*” and “*structural-descriptive names*” for expressions in general, and for sentences in particular, are introduced into the investigation. With their help further examples of the general scheme of (2) are formulated. In the footnote, the understanding of the term ‘sentence’ is specified as a certain kind of a linguistic entity.

If a person speaking Polish, German and English was asked to translate Polish “**śnieg pada**” [p. 19] into the two remaining languages, they would surely say “**es schneit**” [p. 269] and “**it is snowing**”, for that is the most natural translation grasping the colloquial meaning of the sentence. If we were to translate it literally, word for word, however, we would arrive at a correct, but rather unusual “**der Schnee fällt**” and “**the snow is falling**”. The important feature of these awkward sounding sentences is that they have the correct grammatical form, i.e., they have a subject and a predicate. That is probably the reason why Tarski chose such a grammatically simple sentence as an example. Using a mass term as a subject was not an obstacle, as it might have been in a formalized language where it could have created a rather complex logical structure. Tarski did not intend to formalize the colloquial language, his goal was precisely the opposite. He showed it was impossible to construct a correct definition of the term ‘true sentence’ in regard to colloquial languages and using such an example only made his point stronger. What is more important, he never uses the sentence “**śnieg pada**” in further paragraphs in connection with the formalized languages.

As previously mentioned, there is a fair amount of literature regarding the different truth-bearers, and in today’s philosophical debate on truth pretty much every theory of truth introduces its own truth-bearers giving the reader convincing reasons for agreeing with the author’s choice and for rejecting those of different truth theories. So why did Tarski decide to use sentences? He does not make explicit claims on this topic in this original monograph on the *Concept of Truth*. Perhaps, he thought the matter to be so obvious that any additional justification for using sentences as truth-bearers seemed redundant. In the short version of the CTFL, however, (1944) he makes it explicit.

The predicate “true” is sometimes used to refer to psychological phenomena such as judgments or beliefs, sometimes to certain physical objects, namely, linguistic expressions and specifically sentences, and sometimes to certain ideal entities called “propositions.” By “sentence” we understand here what is usually meant in grammar by “declarative sentence”; as regards the term “proposition,” its meaning is notoriously a subject of lengthy disputations

by various philosophers and logicians, and it seems never to have been made quite clear and unambiguous. For several reasons it appears most convenient to apply the term “true” to sentences, and we shall follow this course.⁵ [For our present purposes it is somewhat more convenient to understand by “expressions,” “sentences,” etc., not individual inscriptions, but classes of inscriptions of similar form (thus, not individual physical things, but classes of such things).] (Tarski 1944, p. 342)

This quote makes the matters clear. It is followed by a statement which emphasizes the necessity of always relating both notions: of truth and of a sentence to a given language which is being considered. The fact that Tarski considers ‘sentences’ to be physical objects calls for a comment, and in fact, it has been commented on. It has been widely discussed in many philosophical essays. Tarski’s choice of sentences as truth-bearers and regarding them as physical objects has been labeled as physicalism, and it has been ascribed to the influence of the Vienna Circle in the 30’s.⁶ It has also been argued, quite convincingly, that it has its roots in the tradition of Lvov-Warsaw School and its connection to Brentanism.⁷ Rojszczak presents a very detailed and historically sound analysis of Tarski’s philosophical background and the ideas of scholars surrounding him. He compares the original Polish version of *The Establishment of Scientific Semantics* with its English translation, and points to an inaccurate translation as the cause of the misinterpretation of Tarski’s ideas. He finishes with a statement repeatedly referring to the CTFL where the reader can follow up on Tarski’s thoughts herself.

For Tarski himself, assuming that I am right about his philosophical background knowledge, the good candidate for the truth-bearer would be an inscription or an utterance in the sense of a psycho-physical product. On the one hand, the structure of such bearers can be given by the syntax: the sentence is a sentence-function without variables. [37] On the other hand, for metalogical reasons, truth-bearers should be types or names. [38] And because metamathematics needs infinitely many inscriptions (or utterances) they should be taken as physical bodies: there is no possibility for people to produce infinitely many psycho-physical products. But it is reasonable for Tarski to say there are infinitely many physical bodies. [39] And this is the decisive argument of his in favor of the physicality of the sentences. (Rojszczak 1999, p. 122)⁸

Another statement in this matter is taken by Woleński (2014). There, he refers to the correspondence between Tarski and Popper in which Tarski once again explicitly insists on translating the Polish term ‘zdanie’ as ‘sentence’ and not as ‘statement’. According to Popper, there was not much difference between the two English translations. However, this cannot be true, as Woleński notices, since the term ‘statement’ has a clear pragmatic connotation. In the German translation of Tarski’s paper, we have ‘Aussage’ and in the brackets ‘Satz’. According to Woleński, this means that both terms were seen by Tarski as synonymous. However, as already mentioned in

⁶E.g. Field (1972), McDowell (1978).

⁷Rojszczak (1999).

⁸All the references refer to the CTFL: [37] (Tarski 1933, pp. 12–13, 27–29, 63–65), (Tarski 2006g, pp. 162–164, 176–178, 212–214); [38] (Tarski 1933, p. 5), (Tarski 2006g, p. 156); [39] (Tarski 1933, p. 25), (Tarski 2006g, p. 174).

[1.2.3], the term ‘Satz’ is ambiguous because apart from meaning sentence in a grammatical sense, it has often been used to mean a provable sentence, i.e., a theorem. This may be the reason for Popper’s statement. Woleński sums up that no matter which term is better as a translation of the Polish ‘zdanie’, it remains a decisive fact, that sentences are linguistic expressions having meanings. Ignoring this fact caused much resistance towards Tarski’s semantic approach to truth and its misinterpretation.

It should be pointed out, that there is an ongoing discussion regarding Tarski’s choice of *sentences* as truth-bearers. Perhaps, it remains to add that Tarski would no doubt be disappointed, but not necessarily surprised, to find out that even eighty years after his master piece on truth, there still is no precise and unambiguous definition of the term ‘proposition’.

Translational Remarks

The quotation-mark names are the “most important and common **category** of names” is the complete translation of the Polish “Najważniejszą i najczęściej spotykaną **kategorią** nazw” [p. 19], and of German “Die wichtigste und die häufigste **Kategorie** von Namen” [p. 268].

Directly after the sentence (3), Tarski speaks of “another category of **individual** names of sentences”. The term ‘individual’ is missing from the English translation. In Polish we read “Inną kategorię nazw **jednostkowych** zdań” [p. 20], and also in German “Eine andere Kategorie der **Einzelnamen**” [p. 269].

In the Polish footnote we read that ‘Zdania’ [p. 19], which means the same as ‘Sentences’, “are always treated here as a particular kind of expressions”, see also [1.2.3]. It is also important to notice in this footnote, that Tarski considers linguistic expressions, and in particular sentences, not as tokens, but as types, and that he identifies types with classes of tokens of the same shape. Quotation-mark names are therefore individual names of types, i.e. of classes of tokens of the same shape.

[157] *Now a structural-descriptive name is used in order to construct an instance of the general scheme. Caution must be taken in order to avoid the antinomy of the liar, presented on the following page.*

By naming the letters of the alphabet in the Polish original, Tarski distinguishes between the consonants and the vowels, which is perfectly understandable. Every Polish consonant, when pronounced individually, receives an additional prefix or a suffix, which is also the case in English and German. For example, the sounds we make when pronouncing the consonants ‘m’ or ‘n’, are ‘em’ or ‘en’ respectively, in all three languages alike. Tarski proposes to use these sounds as the names of particular consonants. What distinguishes Polish and German from English, however, are the vowels, i.e., the sounds made when pronouncing ‘a’ or ‘e’ are not different from the written form, except maybe for the length of the sound in certain situations, in the first two languages. Therefore, in order to avoid ambiguity, Tarski suggests to use as the names of the vowels ‘a’, ‘e’, ‘i’...— ‘aj’, ‘ej’, ‘ij’...: “jako nazwy samogłosek ‘a’, ‘e’, ‘i’...można by obrać ‘aj’, ‘ej’, ‘ij’...(nie zaś ‘a’, ‘e’, ‘i’...— dla uniknięcia wieloznaczności)”. [p. 20]. Interestingly, German does not differ in this matter from Polish, i.e., the sounds made when pronouncing vowels remain the same, except for

the length of the sound sometimes, as their written form. The author of the German translation, however, decided to use a different notation, which has been carried onto the English version as well. Perhaps the translator's version is clearer in this respect, since the names of all the letters begin with a capital letter, they are the letters of the so-called spelling alphabet.

Translational Remarks

It should be noticed that considering the footnote from the previous page the bold-faced article must be definite. 'For example, corresponding to the name "snow" we have the name 'a word which consists of the four letters: Es, En, O, Double-U (in that order)' ". Furthermore, the article at the beginning of the example (4) should also be definite "the expression consisting of three words...". In the German translation the articles are indefinite as well, and on the basis of Polish it is undecidable, see also [1.2.6].

On the same page, in both translations the Polish word '**twierdzenia**' [p. 20], which can mean 'statements', has acquired two different translations. First, it became 'sentences' and in German 'Sätze' [p. 270]. Later on, the German translator uses the term 'Satz' in most cases when Tarski means in Polish 'a provable sentence' which is clearly not the case here. This influenced the English translation where, with a few exceptions, we read 'statement' where a 'theorem' is meant. Unfortunately, there is little consistency within both translations, regarding the use of these terms. Later, '**twierdzenia**' [p. 21] has been translated as 'Behauptungen' [p. 270] and 'assertions', which adds to the confusion and, moreover, it is disputable whether both translations are equivalent with each other. It seems that here the term 'assertion' would be most appropriate, see also [1.2.3].

When describing the sentences which are analogous to (3) and (4), Tarski writes in Polish [pp. 20–21] that they "wydają się **intuicyjnie oczywiste i najzupełniej zgodne z tą intuicją prawdziwości, która tkwi w wypowiedzeniu (1)**", which should be correctly translated as "seem **intuitively evident and completely in accordance with the intuition of truth** which is expressed in the formulation (1)". The German translation is inaccurate and probably directly responsible for the English translation "scheinen **evident** zu sein und **vollkommen mit der Bedeutung des Wortes "wahr"** übereinstimmen, welche in der Formulierung (1) ihren Ausdruck gefunden hat." [p. 270], see also [1.2.1].

[158] *A simple formulation of this antinomy, owed to Jan Łukasiewicz, is presented. Following is the explanation of the contradiction. Later, an attempt to define a true sentence is presented, this time by generalizing the explanation (3), containing a quotation-mark name.*

An interesting fact, which has occurred in both translations is the exchange of the two premisses (α) and (β). What Tarski wrote in Polish as (α) was translated as (β) and vice-versa. The reason for this switch remains unclear, but at least it remains consistent throughout the following pages.

Following is a formal exposition of the Liar Antinomy employing the quotation-mark names.

- (α) ' $c \notin Tr$ ' = c
 (β) Tr ' $c \notin Tr$ ' $\leftrightarrow c \notin Tr$
 (γ) $c \in Tr \leftrightarrow c \notin Tr$

Keeping in mind the meaning of the symbol ' c ', we establish empirically (α). For the quotation-mark name of the sentence c we can easily set up an explanation of type (2), given on page 155, arriving at (β), which will later become known as the equivalence of the form (T). By the rule of substitution of identicals we arrive at a contradiction.

[159] *First, sentence (5) is presented, which encompasses all assertions of type (3) as special cases. Then, the generalization of (5) follows as sentence (6).*

Following are sentences (5), (5') and (6), presented in a formal notation:

- (5) $\forall p(Tr('p') \leftrightarrow p)$ generalization of (3)
 (5') $\forall x(Tr(x) \rightarrow \exists p(x = 'p'))$
 $\therefore$ (6) $\forall x(Tr(x) \leftrightarrow \exists p(x = 'p' \wedge p))$ generalization of (5)

Translational Remarks

At the very top of this page Tarski writes in Polish [p. 22] “dochodzi się z miejsca do **zdania**, obejmującego wszystkie **twierdzenia** typu (3) jako szczególne przypadki”, an accurate translation of which is the following “we reach at once a **sentence** which comprehends all **assertions** of type (3) as special cases”. Here, the English translation is not accurate and we have the term ‘sentence’ twice, while in Polish Tarski distinguishes between the two terms, see also [1.2.3].

The expression “**well-known fact** that to every true sentence [...] there corresponds a quotation-mark name” is a translation of the German “**die bekannte Tatsache**” [p. 272], which, however, in Polish [p. 22] is “**znanego intuicyjnie faktu**” – “intuitively known fact”, see also [1.2.1]. While one can argue how intuitive this fact actually is, the point is that Tarski considered it to be intuitive and it should have been translated as such.

The English terms ‘**significance**’ and ‘**meaning**’ can, in Polish as well as in German, be translated with one word. In Polish it is ‘**znaczenie**’ [p. 23], and in German ‘**Bedeutung**’ [p. 272], which can have these two meanings. That is no doubt the reason for the confusion which occurred in the English version. At first, ‘Bedeutung’ was translated as “**significance** of the quotation-mark names which occur in (5) and (6)”, and is not what Tarski had in mind. The second time, it was correctly translated as ‘meaning’. Since in both cases we are dealing with the meaning of the quotation-mark names, as confusing as it may be, it is enough to read the two paragraphs to realize the mistake.

[160] *The sentences (5) and (6) cannot be accepted as the generalizations of the partial definitions of the type (3). (5) leads to a contradiction and from both (5) and (6) senseless conclusions are derivable. Hence, the quotation-mark names are later interpreted as syntactically composite expressions.*

Tarski first proposes to regard the quotation-mark names to be syntactically simple expressions. As a consequence, however, we have to accept that each constituent of those expressions, quotation marks and the expressions within them, fulfil the same function as the letters of the alphabet, and hence have no independent meaning.

Translational Remarks

Where in Polish we read “Przy tej interpretacji – która nb. wydaje się **najbardziej naturalna i najzupełniej zgodna z intuicją potoczną**” [p. 24], it should be translated as “With this interpretation, which nota bene seems to be **the most natural one and completely in accordance with common intuition**”. Also here, the German version was used as the basis for the English one, and that is where the discrepancy originated: “**der gewöhnlichen Gebrauchsweise der Anführungszeichen** vollkommen zu entsprechen scheint” [p. 273] became “the customary way of using quotation marks”, which is not what Tarski meant here, see also [1.2.1]. Here, Tarski refers to the interpretation of quotation-mark names as syntactically simple expressions, meaning that quotation-marks and the expressions standing between them can be interpreted as the letters, or complexes of successive letters, in single words. He does not speak of the usual way of using quotation marks, such a translation is misleading.

In the same sentence we read in Polish that “częstkowe definicje tego typu co (3) nie są podatne do jakichkolwiek **rozsądnych uogólnień**” [p. 24], just like we read in the German version “sind Teildefinitionen von Typus (3) für irgend welche **vernünftige Verallgemeinerungen** nicht verwendbar” [p. 273]. This means that we cannot use such partial definitions for “any **reasonable generalizations**” and not for ‘any significant generalizations’. Although, it does not influence the context that much, it should be noted that the two words, ‘significant’ and ‘reasonable’ are not synonymous.

Again, Tarski’s ‘intuition’ has been disregarded. In Polish [p. 24] regarding the formulation of the sentences (5) and (6) he writes “jawne niedorzeczności **z intuicyjnego punktu widzenia**”, which means that they are “obviously senseless **from the intuitive point of view**”. In German we read that the formulations are simply “offenbar unsinngig” [p. 273], see also [1.2.1].

Tarski points out that with our understanding of the quotation-mark names “they can be eliminated **from the language...**”, the part in bold-face type is missing in the English translation, in Polish we read “można je w ogóle wyrugować **z języka**” [p. 24], and in German “überhaupt **aus der Sprache** eliminieren” [p. 273].

Here, in the English version the symbol “ ‘ p ’ ”, has been omitted, which creates unnecessary obscurity and forces the reader to fill in the blanks herself. In an accurate translation of the Polish original, and of the German version we would read that the quotation-mark name ‘p’ is to be replaced by the structural-descriptive name ‘Pe’.

[161] *Since the attempt to regard the quotation-mark names as syntactically simple expressions ended up as a complete fiasco, Tarski tries to interpret them as syntactically composite expressions. Here, the quotation marks and the expressions within them are the constituents of a whole quotation-mark expression. Quotation marks themselves are thus independent words.*

The first problem with this interpretation is that not all quotation-mark expressions are constant names, i.e., the expression “*p*” occurring in (5) or (6) has to be regarded as a function, in this case as a quotation-function. The intuitive meaning of such functions is not clear enough. They can neither be considered extensional nor intensional, since both these terms are normally used in connection with sentence-building functors, and not, like here, name-building functors. Further, using quotation-functions may also lead to semantical antinomies. These arguments alone are enough to consider the attempt of constructing a correct semantical definition of a ‘true sentence’ hardly possible on the grounds of the colloquial language.

In the last footnote on this page Tarski refers to Carnap’s work. There, Carnap presents his ideas considering extensionality.

Eine Aussagefunktion, deren Argumente Aussagen oder Aussagefunktionen sind, und deren Aussagewert derselbe bleibt, wenn für eine Argument-Aussagefunktion eine generell äquivalente tritt, heißt eine ‘*extensionale*’ Funktion. Die Wahrheitsfunktionen (3a) sind somit extensional. Als ‘*ExtensionalitätsThese*’ bezeichnen wir die (noch umstrittene) Behauptung, daß alle Aussagefunktionen, deren Argumente Aussagen oder Aussagefunktionen sind, extensional sind. Hat die These recht, so sind generell äquivalente Funktionen identisch; denn sie verleihen dann allen Funktionen, in denen sie als Argument auftreten, übereinstimmenden Aussagewert (Carnap 1929, p. 22).

Translational Remarks

On this page the translator of the English version decided to omit the word ‘intuitive’, just as it has been omitted in the German translation, in the following instances:

“The sense of the quotation-functions and of quotation marks” is for Tarski intuitive, in Polish [p. 25] “**Sens intuicyjny** funkcji cudzysłowowej”.

Also in the sentence directly following the above one, the meaning of the discussed functors is “in palpable contradiction to the **common intuition**”, in Polish [pp. 25–26] “pozostaje niewątpliwie w jaskrawej sprzeczności z **potoczną intuicją**”.

And finally, we will be making “use only of those properties of quotation-functions which seem **intuitively almost evident**”, in Polish [p. 26] “**intuicyjnie niemal oczywiste**”.

It is also worth noticing that, in the first footnote of the English version we read that the quotation marks are an example of a name-forming functor with *one expression argument*. In the Polish original and in the German translation we read that it is a name-forming functor with *one sentence argument*; in Polish “cudzysłowy byłyby przykładem funktora nazwtwórczego o *jednym argumentcie zdaniowym*” [p. 25], and in German “die Anführungszeichen sind ein Beispiel für einen namenbildenden Funktor mit *einem Aussageargument*” [p. 274]. This English translation is an improvement of the text compared to the Polish original and to the German translation.

[162] *Yet another formulation of a definition of truth is given, involving quotation-functions with variable arguments. Certain expressions, however, prove ambiguous for they sometimes have to be considered functions with a variable argument and sometimes a constant name denoting one of the letters of the alphabet. Furthermore, certain linguistic expressions would have to be allowed, which, however, do not comply with the fundamental laws of syntax. This allows for the conclusion that the construction of a correct semantical definition of a true sentence in connection with colloquial languages is very difficult.*

Following is a formal exposition of the Liar Antinomy, without using the word ‘true’, but employing the quotation-functions with variable arguments.

(α) $\forall p(c = 'p' \rightarrow \neg p) = c$

(β) $\forall p \forall q ('p = 'q' \rightarrow (p \iff q))$

(γ) $\forall p(c = 'p' \rightarrow \neg p) \iff \neg \forall p(c = 'p' \rightarrow \neg p)$

Starting with an empirical statement (α) and adding (β) – a supplementary assumption concerning the quotation-function, we derive from premisses (α) and (β) a contradiction without any trouble.

[163] *The construction of a structural definition, i.e., a definition in which certain laws of formal logic allow one to infer the truth of a sentence from its structural properties, is considered as the last option. With the help of these laws, every fragmentary definition of truth, an extension of which embraces an arbitrary category of sentences, can be extended to all complex sentences of the category in question, built up from them by logical particles such as sentential connectives.*

Tarski's attempts to construct the desired definition of a true sentence by generalizing explanations of type (3) involved different interpretations of quotation-mark names, which were illustrated on the last few pages of CTFL. As Tarski showed, all these attempts have failed. At the very beginning of Sect. 1 he explicitly credited Leśniewski for his development of the views which were presented in this paragraph. As has already been thoroughly discussed,⁹ Tarski's work, at least in the original Polish version of this manuscript, was determined by Leśniewski's influence. Not only was Leśniewski responsible, in a positive manner, of course, for the negative results regarding colloquial languages, but he also passed onto Tarski an attitude he called *Intuitionistic Formalism*. In order to understand the complex relationship between these two brilliant minds, and especially to understand the influence the master – Leśniewski, had on the work of his prodigy student – Tarski, some knowledge of their professional backgrounds would be advantageous. To present even a sketch of their relationship and the academic and political situation surrounding them, however, would go beyond the scope of this work. It will have to suffice for the present purpose just to mention that Leśniewski was an educated philosopher mainly interested in finding answers to philosophical problems. Tarski was a mathematician in the first place, which makes it even more astounding that he got his doctorate under Leśniewski, being his only doctoral student. Perhaps, this passage reflects

⁹Cf. Woleński (1989), Sundholm (2003), Betti (2008), Patterson (2012).

Leśniewski's attitude towards mathematics best, at the same time clarifying Tarski's decision to write his doctor ate under Leśniewski's supervision.

Da ich keine Vorliebe für verschiedene "Mathematikspiele" habe, welche darin bestehen, dass man nach diesen oder jenen konventionellen Regeln verschiedene mehr oder minder malerische Formeln aufschreibt, die nicht notwendig sinnvoll zu sein brauchen oder auch sogar, wie es einige der "Mathematikspieler" lieber haben möchten, notwendig sinnlos sein sollen, – hätte ich mir nicht die Mühe der Systematisierung und vielmaligen skrupulösen Kontrollierung der Direktiven meines Systems gegeben, wenn ich nicht in die Thesen dieses Systems einen gewissen ganz bestimmten, eben diesen und nicht einen anderen, Sinn legen würde, bei dem für mich die Axiome des Systems und die in den Direktiven zu diesem System kodifizierten Schluss- und Definitionsmethoden eine unwiderstehliche intuitive Geltung haben. Ich sähe keinen Widerspruch darin, wenn ich behaupten wollte, dass ich eben deshalb beim Aufbau meines Systems einen ziemlich radikalen "Formalismus" treibe, weich ich ein verstockter "Intuitionist" bin: indem ich mich beim Darstellen von verschiedenen deduktiven Theorien bemühe, in einer Reihe sinnvoller Sätze eine Reihe von Gedanken auszudrücken, welche ich über diese oder jenes Thema hege, und die einen Sätze aus den anderen Sätzen auf eine Weise abzuleiten, die mit den Schlussweisen harmonisieren würde, welche ich "intuitiv" als für mich bindend betrachte, kenne ich keine wirksamere Methode, das Leser mit meinen "logischen Intuitionen" bekannt zu machen, als die Methode der "Formalisierung" der darzulegenden deduktiven Theorien, die jedoch keineswegs unter dem Einfluss solch einer "Formalisierung" aufhören, aus lauter sinnvollen Sätzen zu bestehen, welche für mich intuitive Geltung haben. Die Methode der Durchrührung von mathematischen Deduktionen auf der "intuitionistischen" Basis verschiedener logischer Geheimnisse sehe ich auf jeden Fall als eine bedeutend weniger zweckmässige Methode an. (Leśniewski 1929, p. 29)¹⁰

In spite of this exaggerated picture of formalism Leśniewski's concludes that he knows no other method which is more effective for acquainting the reader with his "logical intuitions" than the method of formalizing the needed deductive theory. This was precisely Tarski's aim in this monograph – to express the intuitive meaning of the concept of truth by the means of infallible logical apparatus – deductive theory constructed exactly for this puprose. Tarski stated his own view regarding *intuitionistic formalism* in "Fundamental Concepts of the Methodology of the Deductive Sciences".

In conclusion it should be noted that no particular philosophical standpoint regarding the foundations of mathematics is presupposed in the present work. Only incidentally, therefore,

¹⁰"I have no predilection for various 'mathematical games' that consist in writing out, according to one or another conventional rule, various more or less picturesque formulae which need not be meaningful or even – as some of the 'mathematical gamers' might prefer – which should necessarily be meaningless, I would not have taken the trouble to systematize and to often check quite scrupulously the directives of my system, had I not imputed to its theses a certain specific and completely determined sense, in virtue of which its axioms, definitions, and final directives (as encoded for SS5), have for me an irresistible intuitive validity. I see no contradiction, therefore, in saying that I advocate a rather radical 'formalism' in the construction of my system even though I am an obdurate 'intuitionist'. Having endeavoured to express some of my thoughts on various particular topics by representing them as a series of propositions meaningful in various deductive theories, and to derive one proposition from others in a way that would harmonize with the way I consider finally intuitively binding, I know no method more effective for acquainting the reader with my logical intuitions than the method of formalizing any deductive theory to be set forth. By no means do theories under the influence of such a formalization cease to consist of genuinely meaningful propositions which for me are intuitively valid. (Leśniewski 1992, p. 487)

I may mention that my personal attitude towards this question agrees in principle with that which has found emphatic expression in the writings of S. Leśniewski² and which I would call *intuitionistic formalism*. [†] [This last sentence expresses the views of the author at the time when this article was originally published and does not adequately reflect his present attitude.] (Tarski 2006a, p. 62)

Tarski's footnote [2] refers to Leśniewski's passage cited above. It has been argued¹¹ that *Intuitionistic Formalism* was a 'theme' which determined Tarski's work at least until 1935. Keeping this fact in mind, the reader should be able to understand certain essential aspects of Tarski's work more clearly.¹² Considering the last footnote [†], we can assume that Tarski shared Leśniewski's views in 1930, when the above quoted article was published, and all the way through his work on the original Polish version of his monograph on the concept of truth, that is until 1933. In 1935 the German translation appeared, together with the newly written *Nachwort*, however, which evidences that Tarski no longer sympathised with his mentor's views.

Translational Remarks

In the Polish original Tarski concludes that “Nie znamy nawet ogólnej metody, która by pozwalała ustalić znaczenie dowolnego konkretnego zwrotu typu ‘*x* jest zdaniem prawdziwym’, gdy zamiast ‘*x*’ występuje **jakakolwiek nazwa jednostkowa zdania**” [p. 27] which has been correctly translated into English with the exception of the last part of this sentence. As we read in the correctly translated German text “wo an Stelle von ‘*x*’ **irgend ein Einzelname einer Aussage** steht” [p. 276]. This part concerns “an arbitrary **individual name** of a sentence”. The boldfaced part is missing from the English translation.

[164] *In order to construct a correct structural definition of the expression ‘*x* is a true sentence’, it would be necessary to set up an infinite number of general logical laws for every sentence. It is clear that also this attempt to construct a correct definition of ‘true sentence’, considering colloquial languages, is hopeless.*

Translational Remarks

As Tarski points out, a characteristic feature of colloquial language is its universality. This means that all “**words or expressions**”, in Polish “**wyrazy lub zwroty**” [p. 30], occurring in one such language can easily be translated into another. The German translation provides us with both terms, i.e., “**Worte oder Ausdrücke**” [p. 278], however the English version speaks only of the possibility of translating each ‘word’. Even though expressions are clusters of words, it is evident that certain expressions lose their meaning if, from one language, they are translated literally into another. Fortunately, when the literal translation fails, it is still possible for a translator to grasp the meanings of expressions. That is, perhaps, the reason why Tarski thought it important to include the expressions in his considerations.

¹¹Cf. Patterson (2012).

¹²A thorough discussion regarding *Intuitionistic Formalism* goes beyond the scope of this project as well as beyond my competence. For an exhausting presentation on this matter the reader is referred to Patterson (2012).

[165] *Sect. 1 ends with the negative conclusion regarding the possibility of constructing a correct definition of a true sentence in regard to colloquial languages. Sect. 2 begins with the considered issue presented in connection with formalized languages, with special attention to the language of the calculus of classes. As it will be shown, the results obtained here can also be of some validity for colloquial languages.*

At the end of the first chapter, Tarski concludes that the possibility of constructing a correct definition of the expression ‘true sentence’ in regard to colloquial language is very questionable. This conclusion is based on Tarski’s conviction that no consistent language for which the usual laws of logic hold can satisfy the three conditions (I), (II), and (III). In his paper “The Semantic Conception of Truth and the Foundations of Semantics” (1944), Tarski also states three conditions which, however, do not conform to the original ones.

(I) We have implicitly assumed that the language in which the antinomy is constructed contains, in addition to its expressions, also the names of these expressions, as well as semantic terms such as the term “true” referring to sentences of this language; we have also assumed that all sentences which determine the adequate usage of this term can be asserted in the language. A language with these properties will be called “*semantically closed*.”

(II) We have assumed that in this language the ordinary laws of logic hold.

(III) We have assumed that we can formulate and assert in our language an empirical premise such as the statement (2) which has occurred in our argument. (Tarski 1944, p. 348)

For our further discussion, we will add an asterisk to the conditions in the 1944 paper to avoid confusion. As can be observed, the original conditions (I) and (II) are represented together by (I*). Conditions (III) and (III*) are parallel. Condition (II*), on the other hand, figures in the original paper as a requirement but is not numbered among the three conditions. Tarski remarks that condition (III*) is not essential since the antinomy of the liar can be constructed without it. The two remaining conditions (I*) and (II*), however, are crucial. A language satisfying these two conditions – such as everyday language – is doomed to inconsistency. Therefore, at least one of them must be rejected if consistency is required, but both conditions are indispensable if we are searching for a formally correct and materially adequate definition of the concept of truth. Tarski therefore decides to dispense of any semantically closed language in the further course of his investigations. Since everyday language possesses no exactly specified structure, the question of its consistency does not even have an exact meaning; thus we must be content with the assumption that a language resembling our everyday language, but possessing an exactly specified structure, would be inconsistent. (cf. Tarski 1944, p. 349). Therefore, for the rest of his paper Tarski concentrates his work entirely on formalized languages.

Translational Remarks

In (I) of [165] we read that “for any sentence which occurs in the language, **a definite name** of this sentence also belongs to the language”. The expression in bold-face type reads in Polish “**pewna nazwa jednostkowa** tego zdania” [p. 30] and in German “**ein gewisser Einzelname** dieser Aussage” [p. 279], which means the same as a “**certain individual name** of this sentence”. First of all, a more accurate translation of the

Polish term ‘**pewna**’ and of the German ‘**gewisser**’ would be ‘**certain**’, definitely not ‘definite’. Furthermore, the translator of the English version omitted the important adjective ‘**individual**’ standing before the term ‘name’.

Also in the second condition we read that the symbol ‘ x ’ is replaced by “a name of this sentence” instead of, correctly, by “an **individual name** of this sentence”, as we clearly see in Polish “zaś symbolu ‘ x ’ **nazwą jednostkową** tego zdania” [p. 30] and German “‘ x ’ – durch einen **Einzelnamen** dieser Aussage” [p. 279]. Once more, the term ‘individual’ is omitted in the English version.

2.3 Section 2. Formalized Languages, Especially the Language of the Calculus of Classes

[166] *The essential properties of all formalized languages are listed. Tarski emphasizes that such languages are constructed in order to study deductive sciences formalized on their grounds. Further crucial notions are introduced, i.e., the concepts of an axiom, a provable or asserted sentence, and of rules of inference.*

Tarski draws our attention to the essential properties of the formalized languages. Here, he emphasizes that since the formalized languages are constructed in order to study a particular deductive science, it is a fact that “the language and the science grow together to a single whole, so that we speak of the language of a particular formalized deductive science.” (Tarski 2006g, p. 166). In his paper from 1930 on the “Fundamental Concepts of the Methodology of the Deductive Sciences”, he explicates his understanding on the basic concepts which constitute the *deductive science*.

Deductive systems are, so to speak, organic units which form the subject matter of meta-mathematical investigations. Various important notions, like consistency, completeness, and axiomatizability, which we shall encounter in the sequel, are theoretically applicable to any sets of sentences, but in practice are applied chiefly to systems. (Tarski 2006a, p. 70)

Furthermore, in the same paper, Tarski refers to Hilbert as the father of the modern metamathematics. Jan Woleński describes the work on metamathematics in the Lvov-Warsaw School and how it determined Tarski’s investigations on his theory of truth (Woleński 1989). He also illustrates the fundamental differences between Tarski’s methods and those of Hilbert.

Finally we must point out that Tarski in his proposal of metamathematics referred to the ideas of Hilbert, who postulated the formulation of a theory of deductive systems and suggested the term ‘metamathematics’. The important point is that metamathematics is to be an investigation of deductive systems by a definite method. Hence the various reflections on deductive disciplines made in philosophical language do not belong to metamathematics. This is the point that Tarski wanted to emphasize when stating that metamathematics is a scientific investigation of deductive systems by rigorous mathematical methods. But there is a fairly essential difference between the conceptions of Tarski and those of Hilbert. Hilbert developed metamathematics in connection with proofs of consistency, whereas in the Warsaw School metamathematical research was not defined by an definite aim. The point was

to analyse the various aspects of deductive systems. Moreover and this is of special significance, Hilbert admitted in metamathematical research only a fixed repertory of methods, namely the so-called finitistic methods.¹ As a result Hilbert's metamathematics had strong undertones of the formalist philosophy of mathematics. Nothing of the sort took place in the Warsaw School. Metamathematics was treated there as a certain science, independent of this or that philosophy of mathematics. In particular the methods used included those banned by formalism, i.e., infinitistic methods, if their use was fertile in the study of definite problems. (Woleński 1989, p. 163)

In the last short paragraph on this page Tarski emphasizes the fact that he excludes from his further considerations formal languages and sciences which are 'formal' "in one special sense of the word 'formal', namely sciences to the signs and expressions of which no [Pl.: **intuitive**] meaning is attached."

Zbyteczne jest może dodawać, że nie interesują tu nas wcale języki i nauki "formalne" w pewnym specyficznym znaczeniu tego wyrazu, a mianowicie tego rodzaju nauki, iż występującym w nich znakom i wyrażeniom nie przypisuje się żadnego intuicyjnego sensu; w odniesieniu do takich nauk postawione tu zagadnienie traci wszelką rację bytu i przesztaje być po prostu zrozumiałe. Znakom, występującym w tych językach, których dotyczą niniejsze rozważania, przypisujemy zawsze całkiem konkretne i zrozumiałe dla nas znaczenie; (Tarski 1995c, p. 33)¹³

This is a very relevant passage and it definitely is not unnecessary. What Tarski means is that the languages he is interested in must be interpreted and not strictly formal, in the sense of uninterpreted. Additionally, the German and English titles of this monograph can be understood as implying that Tarski was considering formal, not only formalized, but interpreted languages.¹⁴ It should also be noted that Tarski's negative results in regard to colloquial languages have often been understood as his negative attitude towards everyday language. Tarski's statements from pp. 164–165 regarding the universal character of the colloquial language which leads to semantical antinomies have been seen as supporting such claims. Nonetheless, even though it proved impossible to construct a correct definition of a "true sentence" in natural languages, they remain essential within the theory of meaning, serving as a semiotic link to every other language system.¹⁵

Translational Remarks

As we read in condition (γ) in a deductive science, there is a list, or a structural description of a certain category of sentences called *axioms* or *primitive sentences*, as we read in the Polish original '**zdaniami pierwotnymi**' [p. 32]. In the English translation we read '**primitive statements**', which comes probably from the German

¹³"Perhaps, it is unnecessary to add, that we are not interested here in languages and sciences which are 'formal' in certain specific sense of this term, namely such sciences to the signs and expressions occurring in them no intuitive meaning is attached; in regard to such sciences the issue raised here ceases to apply and it becomes no longer intelligible. To the signs occurring in the languages considered here we shall always ascribe quite concrete and, for us, intelligible meaning;" (translation M.G.)

¹⁴See the commentary to [153].

¹⁵For further discussion on this topic see Woleński (2003).

‘**Grundsätze**’ [p. 280]. As already mentioned, the German translator decided to use the term ‘**Satz**’ not simply for a sentence, but for a provable or asserted sentence. Consequently, in the English version we read ‘statement’ in most contexts where in the German version the term ‘**Satz**’ appears. Unfortunately, the translators have not always been as consistent as may be wished, see also [1.2.3].

Later, Tarski describes special rules, called *rules of inference*, by the means of which certain operations of a structural kind are performed, which permit the transformation of sentences into other sentences. The sentences which we obtain in this way, by one or more applications of these operations, are called *consequences* of the given sentences. “In particular the consequences of the axioms are called *provable* or *asserted sentences*”. In Polish “noszą nazwę *tez* lub *zdań uznanych*” [p. 32], and in German “*beweisbare* oder *anerkannte Sätze*” [p. 280]. Once again the translation is neither accurate nor precise enough, which makes the understanding of the text more difficult, see also [1.2.3].

In Polish, [p. 33] “tego rodzaju nauki, iż występującym w nich znakom i wyrażeniom nie przypisuje się **żadnego intuicyjnego sensu**”, which means the same as “sciences to the signs and expressions of which “**no intuitive meaning**” is attached”. The German translator decided to omit the word ‘intuitive’ and wrote “solche Wissenschaften, deren Zeichen und Ausdrücken **kein inhaltlicher Sinn** zukommt” [p. 281], see also [1.2.1].

[167] *This paragraph begins with a description of the languages which are to be considered. The signs occurring in them always have a concrete and intelligible meaning. The sentences of the formal language remain sentences after they have been translated into colloquial language. The importance of distinguishing between the language about which we speak – the object language, and the language in which we speak – the metalanguage is emphasized. The metalanguage contains the names of all the expressions of the object language and of the relations between them. Following is the description of their features.*

It is worthy of notice that Tarski credits Leśniewski for recognizing the importance of distinguishing between the object language and the metalanguage. Following up on the topic of the semantical antinomies, Tarski writes in “Establishment of Scientific Semantics”:

The main source of the difficulties met with seems to lie in the following: it has not always been kept in mind that the semantical concepts have a relative character, that they must always be related to a particular language. People have not been aware that the language *about which* we speak need by no means coincide with the language *in which* we speak. They have carried out the semantics of a language in that language itself and, generally speaking, they have proceeded as though there was only one language in the world. The analysis of the antinomies mentioned shows, on the contrary, that the semantical concepts simply have no place in the language to which they relate, that the language which contains its own semantics, and within which the usual logical laws hold, must inevitably be inconsistent. Only in recent years has attention been given to all these facts (as far as I know Leśniewski was the first to become fully aware of them). (Tarski 2006h, p. 402)

The credit Tarski gives Leśniewski here is given definitely in the aftermath. The above quoted article was published in 1936 in Polish, and later that year in German. In the

CTFL, Tarski does not mention Leśniewski in connection with this crucial distinction. Perhaps, this is due to the fact that Leśniewski himself makes no statement on this topic in any of his writings.¹⁶

Translational Remarks

The sentences which are distinguished as axioms seem to be ‘**intuitively true**’. The Polish ‘**intuicyjnie prawdziwe**’ [p. 33] has been translated into German as ‘**inhaltlich wahr**’ [p. 281], which later became ‘materially true’ in the English version, see also [1.2.1].

In the 2nd footnote, Tarski describes the purpose which definitions serve. Namely, they are constructed in such a way that “by wyjaśniały i ustalały znaczenie znaków” [p. 33, footnote 13], i.e., they “elucidate **and** determine the meaning of the signs which are introduced into the language by the means of primitive signs or signs previously defined”. This discrepancy originated in the German version where we read that the definitions “erläutern **oder** bestimmen” [p. 281] the meaning of these signs, instead of assigning them both functions.

[168] *Employing a certain method, it is possible to construct a correct definition of truth for many formalized languages. The language of the calculus of classes is chosen as the simplest language which can serve as the object of the following investigations. Later on, the signs (constants and variables) of the language are listed and the expressions of the language are described. Łukasiewicz’s notation is employed.*

The notion of *definability* is not easily and unequivocally understood. Tarski was aware of the ambiguity surrounding it and he gave a talk to the Polish Mathematical Society in 1930, which became the basis of an article “On definable sets of real numbers”. There he wrote:

Mathematicians, in general, do not like to deal with the notion of definability; their attitude toward this notion is one of distrust and reserve. The reasons for this aversion are quite clear and understandable. To begin with, the meaning of the term ‘definable’ is not unambiguous: whether a given notion is definable depends on the deductive system in which it is studied, in particular, on the rules of definition which are adopted and on the terms that are taken as primitive. It is thus possible to use the notion of definability only in a relative sense.[...] The problems of making its meaning more precise, of removing the confusions and misunderstandings connected with it, and of establishing its fundamental properties belong to another branch of science – metamathematics. (Tarski 2006d, p. 110)

It was clear to Tarski that the notion of definability belongs to metamathematics. He believed to have found “a general method which allows us to construct a rigorous metamathematical definition of this notion”(Tarski 2006d, p. 111). Furthermore, Tarski is convinced that “an analogous method can be successfully applied to define other concepts in the field of metamathematics, e.g., that of *true sentence* or of a *universally valid sentential function*”(Tarski 2006d, p. 111).¹⁷

¹⁶Cf. Betti (2004), p. 280.

¹⁷For further discussions on this topic see for example Tarski (2006f), Tarski (2006d), Coffa (1987, p. 556 ff.), Patterson (2012).

The second footnote of this page is worth noticing. Tarski uses a parenthesis-free logical notation, which was conceived by Łukasiewicz. The basic idea is to write the logical constants before their arguments, which allows one to avoid technical signs such as brackets, dots etc. Every well-formed formula begins with a capital letter, which is the main functor of the entire formula. The structure of every formula in this notation is uniquely determined by the position of the letters. The parenthesis-free notation is unambiguous since every formula written in it has only one translation into the standard symbolism. Łukasiewicz's notation has many advantages, the most important of them, perhaps, is its economy.¹⁸

Unlike Łukasiewicz, Tarski introduces only four constant signs: the negation, the logical sum (disjunction), the universal quantifier and the inclusion. Although, in the following footnote, he notes that also the inclusion sign could be eliminated. However, the more important comment concerns Tarski's formalized language. As the author notes himself, this language is only fragmentary, in the sense that it does not include other constants normally occurring in the calculus of classes. Nevertheless, the missing constants could easily be introduced into the languages by means of definition, i.e., as defined terms. Tarski concludes in the footnote on this page that "Owing to this fact our fragmentary language already suffices for the expression of every idea which can be formulated in the complete language of this science".¹⁹

Translational Remarks

In the English translation, the sentence beginning on the previous page informs us that, for an extensive group of languages, there is a possibility of giving a method by which a correct definition of **truth** can be constructed, for each of these language. An accurate translation should state that it is possible to give a method by which a correct definition of **true sentence** can be constructed for each of these languages **separately**. In Polish "Dla pewnej dość obszernej kategorii sformalizowanych języków można wskazać metodę, umożliwiającą skonstruowanie poprawnej definicji **zdania prawdziwego** w odniesieniu do każdego z tych języków **z osobna**." [p. 34]. The essential part, that it has to be constructed for each of these languages **separately**, has been, once again, left out (see the commentary to [154]). In the German version we read "Für eine recht umfangreiche Gruppe von formalisierten Sprachen kann man eine Methode angeben, welche die Konstruktion korrekter Definitionen der **wahren Aussage für jede einzelne dieser Sprachen** ermöglicht" [p. 282]. It is perhaps worth mentioning here that the expressions 'definition of truth' and 'definition of a true sentence' have been considered synonymous and hence, used interchangeably within the English translation. Here, Tarski wrote in Polish 'definicja zdania prawdziwego', which means the same as "a definition of a true sentence". Tarski uses the term "definicja prawdy" – "definition of truth" [cf. pp.152,154] a few times, but he always does it in an informal context. Whenever speaking in a formal manner, he uses the term "definicja zdania prawdziwego" – "definition of a true sentence".

¹⁸Cf. Woleński (2013).

¹⁹For a detailed discussion regarding Polish notation see Woleński (2013).

[169] *After the list of constants the description of the variables and the expressions of the language follows.*

Tarski emphasizes the importance of specifying the form of variables, so that they could easily be ordered in a sequence, and hence numbered. He does this by adding a certain number of small strokes below the signs.

Translational Remarks

Tarski emphasizes the importance of specifying the form of the variables used “so that these signs could easily be ordered in a sequence (**numbered**)”. In Polish [p. 36] “by znaki te łatwo było ustawić w ciąg (**ponumerować**)” and in German [p. 283] “dass man diese Zeichen leicht in eine Folge anordnen (**abzählen**) kann” we read the additional term in brackets.

After giving the examples of expressions, Tarski mentions ‘sentences’ and ‘sentential functions’, adding in brackets that “the meaning of **these terms** will be explained below”, which is an accurate translation of Polish [p. 36] “znaczenie **tych terminów** będzie wyjaśnione poniżej”. In German [p. 284] “die Bedeutung **dieser Termini** wird unten erklärt werden”. It is clear that in the Polish original, as well as in the German translation, the text refers to the meaning of both terms ‘sentence’ and ‘sentential function’. Tarski intends to clarify his use of the terms in question within the metalanguage later on.

[170] *A short account is given of the structure of the metalanguage and of the meta-science constructed upon it. The focus is restricted to two points, i.e., (1) listing of all the signs and expressions used in the metalanguage, and (2) setting up of an axiomatic system. The distinction between two kinds of expressions of the metalanguage follows. The description of the first kind, namely expressions of a general logical character which can be found in any sufficiently developed system of mathematical logic is given later on. Tarski refers to the works of Whitehead and Russell (1925) and of Carnap (1929) as providing examples of such systems.*

Tarski refers here to the work of Whitehead and Russell as presenting a “sufficiently developed system of mathematical logic”, which could perhaps also be used for his theory. A major difference between the systems of Whitehead and Russell and that of Tarski, however, should be noticed. Namely, Tarski’s theory applies to *expressions*, whereas Whitehead and Russell’s theory of types applies to *objects*.²⁰

[171] *Further terms belonging to the expressions of a general logical character are listed. These include the expressions from the domain of the sentential calculus, of the first order functional calculus, and of the calculus of classes. Also some expressions from the theory of the equinumerosity of classes and of the logic of relations are found here.*

²⁰ As will be discussed later (within the commentary to [215] and to *Postscript*), there is an open debate regarding Tarski’s choice of logical framework and his alleged change of it from the one used in the main text to the one used in the postscript. Although his theory referred to expressions, the notion of *order* introduced on [218] is not without ambiguity and can be interpreted as applying to objects. On the other hand, Whitehead and Russell’s theory is also difficult to assess unequivocally.

Among the terms belonging to the logic of relations, the concept of *sequence* deserves special attention. This notion plays a crucial role in Tarski's further investigations. He defines

a '*finite sequence of n terms*' as: a one-many relation whose codomain is the class of all natural numbers k such that $1 \leq k \leq n$.

R_k (i.e. the k th term of the sequence R) is the unique x which satisfies the formula ' $x R k$ '.

Translational Remarks

Among the expressions of the metalanguage of a general logical kind, Tarski counts also "some expressions from the domain of the theory of the **equinumerosity of classes**, and of the arithmetic of cardinal numbers". As we read in the Polish original "spotykamy tu pewne wyrażenia z zakresu **teorii równości mocy** i arytmetyki liczb kardynalnych" [p. 38], which has been correctly translated into German as "finden wir hier manche Ausdrücke aus dem Bereich der **Gleichmächtigkeitstheorie** und der Arithmetik der Kardinalzahlen" [p. 286]. The term "equivalence of classes", was used at that time synonymously with the term "equinumerosity of classes", which is commonly used today; for more detail see also (Tarski 1995a, p. 79).

[172] *The expressions of the second kind are the specific metalinguistic terms of a structural-descriptive character, which are briefly described here. With the help of both kinds of the terms it is possible to define all other metatheoretic concepts of the structural-descriptive kind.*

Another important auxiliary expression is:

the sequences R and S differ in at most the k th place: $\leftrightarrow \forall l (l \neq k \rightarrow R_l = S_l)$.

Translational Remarks

In the English translation we read that belonging to the second kind of the expressions are "names of concrete signs **or** expressions of the language of the calculus of classes", whereas Tarski explicitly connects the two components with an "**and of**". In Polish [p. 40] "nazwy konkretnych znaków **i** wyrażeń języka algebry klas, **nazwy klas i ciągów takich wyrażeń oraz zachodzących między nimi strukturalnych relacji**" which means the same as "names of concrete signs **and of** expressions of the language of the calculus of classes, **names of classes and of sequences of such expressions and of structural relations existing between them**". Furthermore, as it can easily be seen, the boldfaced part of this list is missing from the original translation. Only in 1997 it has been noticed by M. Schirn and added as a footnote to the later editions, see Tarski (2006g, p. 172). The German version provides us with the complete translation "Namen von konkreten Zeichen und Ausdrücken der Sprache des Klassenkalküls, Namen von Klassen, von Folgen solcher Ausdrücke und von zwischen ihnen bestehenden strukturellen Relationen." [p. 287].

After listing of the terms, Tarski notes that he hopes that the meaning of these terms will be clear "thanks to the **comments and examples**", in Polish [p. 40] "dzięki **uwagom i przykładom**", and in German "dank den [...] **Bemerkungen und Beispielen**" [p. 288].

Later, Tarski holds that with the help of these terms we can define all other concepts of the **metascience** of a structural-descriptive kind, not of the metalanguage, as we read in the English translation. In Polish “Przy pomocy tych terminów (i ew. wyrażen ogólnologicznych) można zdefiniować wszystkie inne pojęcia **metanauki** o charakterze strukturalnoopisowym” [p. 40]. The German translation is correct in this matter “Mit Hilfe dieser Termini (und eventuell der allgemeinlogischen Termini) kann man alle anderen Begriffe der **Metawissenschaft** von strukturell-deskriptivem Charakter definieren” [p. 288]. Although, the discussed concepts are presented in the metalanguage, they belong to the metascience, as we read in Polish and German.

The sentence ending with a reference to pp. 156 and 157, should be translated as “In particular, as it is easy to see, for every simple or composite expression of the language under investigation it is possible to construct in the metalanguage a certain individual name of this expression – **of the same type as** the structural-descriptive names of colloquial language”. In Polish [p. 40] “W szczególności, jak łatwo się zorientować, dla każdego prostego lub złożonego wyrażenia języka, stanowiącego przedmiot rozważań, daje się skonstruować w metajęzyku pewna nazwa indywidualna tego wyrażenia – **tego samego typu co** nazwy strukturalnoopisowe języka potocznego.” The expression in bold-face type “of the same type” is a much more specific and accurate translation, than “similar”. The German translation is accurate [p. 288] “Insbesondere lässt sich, wie leicht zu ersehen ist, für jeden einfachen oder zusammengesetzten Ausdruck der Sprache, die Gegenstand der Untersuchung ist, in der Metasprache ein individueller Namen dieses Ausdrucks **von demselben Typus wie** die strukturell-deskriptiven Namen der Umgangssprache konstruieren.”

[173] *After shortly listing the symbols of the variables of the metalanguage, the axiom system of the metascience is presented. Here also, a distinction is made between the general logical axioms and the specific axioms of the metascience. Following are the Axioms 1–5, which all belong to the second kind.*

Tarski is the first one to present the metascience in the form of an axiomatized system (as he himself remarks in footnote 3 of this page).

Before defining the crucial concepts Tarski presents a short list of necessary axioms. They enable him to present the defined concepts in an univocal manner.²¹

To apply the concepts of any currently used language univocally it has to be presupposed that its usage can be determined unequivocally. In reflecting on that language we determine possibilities and limits of such determination of concepts as well as, according to such limitations, the concepts themselves.

Concepts that turn out to be definable in the corpus of accepted statements of the language used are defined on the level of reflection according to the respective delimitations. Those concepts which are not definable in this way have their univocal usage by some axiom system which does not essentially contain the defined concepts and whose logical consequences are all the statements which describe the rule-based usage of these undefined concepts.²

The aim of developing axiom system is twofold³: The one purpose of doing this is to establish some general structure which is satisfied by many systems of objects, having thus a very general realm of application; for example, the axioms of group theory describing some

²¹For further discussion on Tarski's axioms see for example Essler (1999).

restricted aspects of rational as well as real numbers. The other purpose of developing axiom system is to describe some set of objects with respect to its fundamental aspects uniquely, i.e. unequivocally; the Peano axioms for natural numbers and Tarski axioms for real numbers are instances of that purpose. (Essler 1999, p. 149)

Translational Remarks

Tarski turns now to the “axiom system of the **metascience**” and not to that of ‘metalanguage’ or ‘Metasprache’, as we read in the inaccurate English and German translations “Wir wenden uns dem Axiomensystem der **Metasprache** zu” [p. 288]. In Polish “Przechodząc do listy aksjomatów **metanauki**” [p. 41]. Later on, we read that, corresponding to the two categories of expressions in the metalanguage, the list of axioms contains two kinds of expressions: *general logical axioms* and *specific axioms of the metascience*, again not of the *metalanguage* as both translators tell us. In German “spezifischen Axiome der **Metasprache**” [p. 289]. In Polish “*aksjomaty specyficzne metanauki*” [p. 41].

Tarski uses the term ‘**zdania**’ [p. 41], in German ‘**Aussagen**’ [p. 289], which mean the same as ‘**sentences**’ when listing the axioms of the second kind. ‘Statements’ is an inaccurate translation, see also [1.2.3].

Also the English translation of AXIOM 2 is not accurate enough. In Polish [p. 42] we read “AKSJOMAT 2. v_k jest wyrażeniem wtedy i tylko wtedy, gdy k jest liczbą naturalną różną od 0; v_k jest różne od wyrażeń ng , sm , qu , i in oraz **od każdego z wyrażeń** v_l , jeśli tylko $k \neq l$.” It means the same as “AXIOM 2. v_k is an expression if and only if k is a natural number distinct from 0; v_k is distinct from ng , sm , un , in and also from **every expression** v_l if $k \neq l$.” The German version provides us with a correct translation “AXIOM 2. v_k ist ein Ausdruck dann und nur dann, wenn k eine von 0 verschiedene natürliche Zahl ist; v_k ist von den Ausdrücken ng , sm , al , und auch, wenn $k \neq l$, von **jedem der Ausdrücken** v_l verschieden” [p. 289].

[174] Tarski notices that it is possible to prove that the above axiom system is *categorical*. Also certain existential consequences following from the character of the axiom system are noticed.

An axiom system is *categorical* if and only if it has the following three features; (1) it is independent, i.e., it is impossible to prove any of the axioms on the basis of the other axioms; (2) it is complete, i.e., every problem stated in terms of the theory of which the axiom system is the basis, can be answered by deductive inferences from these axioms; (3) it is consistent.²²

In the Polish original, as well as in the German translation, the footnote regarding the term ‘categorical’ is extended compared to the English translation. It concerns the possible interpretations of the term ‘categorical’. Under one such interpretation Tarski’s axiom system of the metascience would have to be expended by two additional axioms. Tarski holds that these axioms would not be of any essential importance, however. The missing part could be translated as follows: “In the interpretation

²²Tarski does not define the property of being categorical himself but refers to works by Veblen (1904) and (cf. Fraenkel (1928), pp. 334–354) Tarski did consider the problem of categoricity, however, together with the problem of completeness of concepts in Tarski (2006f).

of the term ‘categorical’, certain minor discrepancies occur. Without going into detail, I will remark that under one of the possible interpretations, in order to prove that the system is categorical it would be necessary to supplement the given axiom system of the metascience with two additional axioms. In these axioms, which possess no greater significance, a certain conception of expressions as classes would be manifested (cf. footnote 1, p. 156): the first axiom would state that any two expressions are disjunct classes, i.e., have no common elements; in the second one the number of elements of each expression would be determined – in one way or the other.”

In footnote 2, in order to avoid the supposition of infinitely many expressions, Tarski suggests to consider all physical objects as expressions.²³ Tarski decides to interpret expressions as types, however, and in Sect. 4, he introduces the notion of *semantical category*, comparing its role in the construction of formal deductive sciences to that of a type in Russell’s and Whitehead’s *Principia Mathematica*.²⁴

Translational Remarks

Tarski’s axiom system is categorical and thereby provides a sufficient basis for the construction of the **metascience**, once more not of the metalanguage. In Polish “Można by okazać, że powyższy układ aksjomatów jest kategoriyczny; daje nam to w pewnej mierze gwarancje, że stanowi on dostateczną podstawę dla ugruntowania **metanauki**” [pp. 42–43]. This discrepancy originated in the German translation where we read that “Man könnte nachweisen, dass obiges Axiomensystem kategorisch ist; dieser Umstand garantiert uns in einem gewissen Grade, dass es ein genügendes Fundament für den Aufbau der **Metasprache** bildet” [p. 290].

[175] *Before defining the concepts which establish the calculus of classes as a formalized deductive science, Tarski introduces a series of auxiliary symbols. In the first definition he defines the term ‘inclusion’. The second definition regards the term ‘negation’. In the third and fourth definition Tarski defines the notions of the ‘logical sum’ of expressions and of a finite n-termed sequence of expressions respectively.*

We have already mentioned some relevant literature regarding the notion of *definability* as used by Tarski. To mention another important issue regarding this notion, we will quote from Tarski’s article on “Some methodological investigations on the definability of concepts”, which is based on his speech given to the Warsaw Section of the Polish Mathematical Society on 17th December 1926. There, he held that the concept of definability, together with other semantical concepts, has to be related to sentences in order to make sense at all.

It is not difficult to see why the concept of definability, as well as all derived concepts, must be related to a set of sentences: there is no sense in discussing whether a term can be defined by means of other terms before the meaning of those terms has been established, and on the basis of a deductive theory we can establish the meaning of a term which has not previously been defined only by describing the sentences in which the term occurs and which we accept as true. (299 (Tarski 2006f) fn.1)

²³For a discussion on Tarski and physicalism see e.g. Field (1972), McDowell (1978), Rojszczak (1999).

²⁴Cf. Tarski (2006g), pp. 215–227.

Since the above quoted article appeared in German only in 1935, we can assume that Tarski still held the view presented here at the time of writing his monograph on truth. Patterson points out that also in this article Tarski's commitment to intuitionistic formalism is clearly visible. Tarski's interpretation of it differs from Leśniewski's, however, who strictly demanded a compositional account, whereas in Tarski's work the intuitive meaning of an expression depends also on the set of theorems in the entire deductive theory.²⁵

The entire article is concerned with "the definability of concepts". This means, I take it, that a definition determines the content of the concept expressed by its *definiendum*, and the only possible determination could be intuitionistic formalism identity: a term defined by another (simple or complex) shares its intuitive meaning with it. What should strike us here is that it follows that the intuitive meanings of terms aren't determined compositionally: a complex symbol's meaning isn't entirely determined by the meanings of its parts and how they are syntactically put together, it depends also on the intuitive meanings of other symbols for which possible definitions equating them and it are provable in the theory. We see here how very different the Intuitionistic Formalist account of meaning is from the referential semantics to which Tarski's own semantic work gave rise. (Patterson 2012, p. 74)

Here are some examples of expressions defined in the above mentioned definitions.

Definition	Metalanguage	Object language	Present notation
ad Def. 1	$\iota_{1,1}$	Ix_1x_1	$x_1 \subseteq x_1$
	$\iota_{2,3}$	$Ix_{11}x_{111}$	$x_2 \subseteq x_3$
ad Def. 2	$\overline{\iota_{1,2}}$	NIx_1x_{11}	$\neg(x_1 \subseteq x_2)$
	$\overline{\iota_{3,1}}$	$NIx_{111}x_1$	$\neg(x_3 \subseteq x_1)$
ad Def. 3	$\iota_{1,3} + \iota_{3,1}$	$AIx_1x_{111}Ix_{111}x_1$	$x_1 \subseteq x_3 \vee x_3 \subseteq x_1$
	$\iota_{1,2} + \iota_{3,1}$	$NAIx_1x_{11}Ix_{111}x_1$	$\neg(x_1 \subseteq x_2 \vee x_3 \subseteq x_1)$
ad Def. 4	$\iota_{1,1} + (\iota_{1,2} + (\iota_{1,3} + \overline{\iota_{1,3}}))$	$AIx_1x_1AIx_1x_{11}AIx_1x_{111}NIx_1x_{111}$	$x_1 \subseteq x_1 \vee (x_1 \subseteq x_2 \vee (x_1 \subseteq x_3 \vee \neg(x_1 \subseteq x_3)))$

Tarski notes that definition 4 is a recursive definition which, as Frege and Dedekind have proven, can be transformed into an equivalent normal, i.e., explicit definition. Recursive definitions have a less complicated logical structure and a clearer content, however, therefore Tarski does not attempt to avoid them in the further course of his investigations.

A discrepancy worth mentioning occurred within both translations of definitions 1–4 (and the same with regard to definitions 5–7 and 9 on the next page). The translator of the revised English version used the definite article before the defined auxiliary terms. In the first English version (1956) as well as in the German translation, however, the article is always indefinite. This difficulty may have been anticipated here, since the Polish language does not possess articles of any kind. Nevertheless, the editor of the revised English version has shown more expertise and consistency on the topic throughout the translation. The definite article is always used here when

²⁵Cf. Patterson (2012), p. 75.

defining functions. It is perhaps clearer on the next page where the article changes from definite to indefinite, see also [1.2.6].

[176] *On this page Tarski introduces further definitions of the fundamental operations by the means of which he arrives at the crucial concept of a sentence. In Def. 5 the logical product is defined. Definitions 6–9 define the universal and existential quantifications.*

Here are some examples of the defined expressions: the logical product, the universal quantification and the existential quantification.

Definition	Metalanguage	Object language	Present notation
ad Def. 5	$\iota_{1,2} \cdot \iota_{1,3}$	$NANIx_ix_{ii}NIX_ix_{iii}$	$\neg(\neg(x_1 \subseteq x_2) \vee \neg(x_1 \subseteq x_3))$, or $(x_1 \subseteq x_2) \wedge (x_1 \subseteq x_3)$
ad Def. 6	$\cap_1 \iota_{1,1}$	$\Pi x_i Ix_ix_i$	$\forall x_1 (x_1 \subseteq x_1)$
ad Def. 7	$\cap_1 \cap_2 \iota_{1,2}$	$\Pi x_i \Pi x_{ii} Ix_ix_{ii}$	$\forall x_1 \forall x_2 (x_1 \subseteq x_2)$
ad Def. 8	$\cap_1 \cap_2 (\iota_{1,2} + \iota_{2,1})$	$\Pi x_i \Pi x_{ii} AIX_ix_{ii} Ix_{iii}x_i$	$\forall x_1 \forall x_2 ((x_1 \subseteq x_2) \vee (x_2 \subseteq x_1))$
ad Def. 9	$\cup_1 \iota_{1,1}$	$N\Pi x_i NIX_ix_i$	$\neg\forall x_1 \neg(x_1 \subseteq x_1)$, or $\exists x_1 (x_1 \subseteq x_1)$
	$\cap_1 \cup_2 \iota_{2,1}$	$\Pi x_i N\Pi x_{ii} NIX_{iii}x_i$	$\forall x_1 \neg\forall x_2 \neg(x_2 \subseteq x_1)$, or $\forall x_1 \exists x_2 (x_2 \subseteq x_1)$

Here, in all of these definitions, with the exception of Definition 8, the translator of the revised English version used the definite article before each defined term. In Definition 8, the article is indefinite. The reason for this is that it concerns a universal quantification of an expression if there is *some* finite n -termed sequence p of natural numbers, hence it is not a function, as opposed to the previous definitions, see also [1.2.6].

[177] *A sentential function is defined in Def. 10. The footnote provides an explanation of a different type of a recursive definition, represented by Def. 10. It is also stated how to transform Def. 10 into an equivalent inductive definition and into an equivalent “normal” definition.*

In Definition 10 Tarski defines *sentential function*, which is important in so far that we can obtain the concept of a *sentence* as a special case of this notion. Tarski also provides the following examples of sentential functions:

Metalanguage	Object language	Present notation
$\iota_{1,2}$	IX_ix_{ii}	$x_1 \subseteq x_2$
$\overline{\iota_{1,3}}$	NIX_ix_{iii}	$\neg(x_1 \subseteq x_3)$
$\iota_{1,3} + \iota_{3,1}$	$AIX_ix_{iii} Ix_{iii}x_i$	$(x_1 \subseteq x_3) \vee (x_3 \subseteq x_1)$
$\cap_1 \overline{\iota_{1,2}}$	$\Pi x_i NIX_ix_{ii}$	$\forall x_1 \neg(x_1 \subseteq x_2)$

and the following examples of expressions which are not sentential functions:

Metalanguage	Object language	Present notation
ι	I	$\subseteq$
ι_1	Ix_1	$x_1 \subseteq$
$\iota_{1,3}+$	AIx_1x_{111}	$(x_1 \subseteq x_3) \vee$
$\cap \iota_{1,3}$	ΠIx_1x_{111}	$\forall (x_1 \subseteq x_3)$

Translational Remarks

In the footnote following Definition 10, Tarski notes that the above definition is a recursive definition of yet another different kind from Definition 4. He explains the differences and emphasizes the possible methodological objections.

A small, but perhaps not a trivial discrepancy within both translations, has occurred in this footnote. In Polish Tarski writes “Należy zaznaczyć, że definicje rekurencyjne typu def. 10 budzą o wiele poważniejsze wątpliwości natury metodologicznej niż zwykłe definicje indukcyjne: w przeciwstawieniu bowiem do tych ostatnich nie zawsze dają się **one** przekształcić na równoważne im definicje normalne” [p. 47]. In the English translation we read that: “It should be emphasized that recursive definitions of the type of Def. 10 are open to much more serious methodological objections than the usual inductive definitions, since in contrast to the latter, **statements** of this type do not always admit of a transformation into equivalent normal definitions”. This is a correct and accurate translation, except for the term ‘statements’ which was translated from the German version where we have **Aussagen** [p. 294] in this place. Tarski writes in Polish “nie zawsze dają się **one** przekształcić na równoważne im definicje normalne” which can be literally translated as “**they** cannot always be transformed into equivalent normal definitions”. The boldfaced term ‘one’ means the same as ‘they’ and in this context refers to the recursive definitions of the type of Definition 10. Both translations may cause confusion, since both terms, ‘statements’ and ‘Aussagen’, have previously been used with different meanings than that in this footnote, see also [1.2.3].

A further point to be remarked concerns the passage in which we read that “every expression has **a finite length**”. In Polish we read that “każde wyrażenie składa się **ze skończonej liczby znaków**” [p. 47], which means the same as “every expression has **a finite number of signs**”. In German we read that “jeder Ausdruck **aus einer endlichen Anzahl von Zeichen besteht**” [p. 294].

[178] A free variable is defined in Def 11. Definition 12 presents the crucial concept of a sentence. Later, there are some examples of the expressions which are sentences, and of functions which cannot be regarded as sentences. At the bottom of the page Tarski explains that the axioms (or primitive sentences) of his calculus of classes are of two different kinds. The first kind of axioms, viz. the axioms of the sentential calculus, serve as logical basis; the second kind of axioms are so-called “Eigenaxiome” of the calculus of classes. For the sentential calculus, Tarski chooses a system that was originally presented in Principia Mathematica and then modified by Hilbert and Ackermann. In its modified form it consists of the four axioms listed below.

Here are some of Tarski’s examples of functions which are not sentences, because they contain the free variable v_1 .

Metalanguage	Object language	Present notation
$\iota_{1,1}$	Ix_1x_1	$x_1 \subseteq x_1$
$\cap_2 \overline{\iota_{1,2}}$	$\Pi x_{11} N I x_{11} x_{11}$	$\forall x_2 \neg (x_1 \subseteq x_2)$
$\iota_{1,1} + \cap_1 \cup_2 \iota_{2,1}$	$A I x_1 x_1 \Pi x_{11} N \Pi x_{11} N I x_{11} x_{11}$	$(x_1 \subseteq x_1) \vee (\forall x_1 \exists x_2 (x_2 \subseteq x_1))$

Sentences, on the other hand, are sentential functions which contain no free variables, as shown in these examples:

$\cap_1 \iota_{1,1}$	$\Pi x_1 I x_1 x_1$	$\forall x_1 (x_1 \subseteq x_1)$
$\cap_1 \cap_2 \iota_{1,2}$	$\Pi x_1 \Pi x_{11} I x_{11} x_{11}$	$\forall x_1 \forall x_2 (x_1 \subseteq x_2)$
$\cap_1 \cup_2 \iota_{1,2}$	$\Pi x_1 N \Pi x_{11} N I x_{11} x_{11}$	$\forall x_1 \exists x_2 (x_1 \subseteq x_2)$
$\cap_1 (\iota_{1,1} + \cap_1 \cup_2 \iota_{2,1})$	$\Pi x_1 A I x_1 x_1 \Pi x_{11} N \Pi x_{11} N I x_{11} x_{11}$	$\forall x_1 ((x_1 \subseteq x_1) \vee (\forall x_1 \exists x_2 (x_2 \subseteq x_1)))$

Definition 12 is essential for Tarski's investigations, since he choose sentences as the primary truth bearers and thus, defines truth for sentences only. The sentences are fully interpreted, thus meaningful.

These are the four axioms which serve Tarski as a starting point for the description of the axioms of the sentential calculus in his theory of classes:

Object language of a sentential calculus	Present notation
$ANAppp$	$\neg(p \vee p) \vee p$
$ANpApq$	$\neg p \vee (p \vee q)$
$ANApqAqp$	$\neg(p \vee q) \vee (q \vee p)$
$ANANpqANArpArq$	$\neg(\neg p \vee q) \vee (\neg(r \vee p) \vee (r \vee q))$

Since the object language of Tarski's calculus of classes does not contain sentential variables ('p', 'q', 'r'), Tarski cannot use these four formulas in the form they are presented here as axioms of his calculus. Instead of listing single axioms of the sentential calculus he uses axiom schemata, i.e., he describes the axioms of the first type as universal closures of formulas resulting from the four formulas mentioned above by replacing sentential variables with sentential functions.

Translational Remarks

A couple of typing errors within the example expressions occurred in the English translation. The example of the function which is not a sentence should be negated; i.e. instead of $\cap_2 \iota_{1,2}$ it should be $\cap_2 \overline{\iota_{1,2}}$ as in Polish [p. 48] and German [p. 295].

In the footnote, Tarski speaks of the Ths. of Sect. 3, which express characteristics and properties of true sentences. The mentioned properties are, in Polish [p. 48] "**ważne z intuicyjnego punktu widzenia**"—"important from an intuitive point of view", see also [1.2.1].

[179] *First, Tarski presents some examples of axioms of the sentential calculus in the object language of his calculus of classes. Then he explains how he has chosen the "Eigenaxiome" of his calculus of classes, whose only undefined constant is the inclusion sign. Finally, he presents his formal definition of an axiom (of the calculus of classes).*

All sentences which are universal quantifications of sentential functions of one of the following forms are axioms:

- (β1) m-l: $\cap_1 \iota_{1,1}$
o-l: $\Pi x_1 I x_1 x_1$
p.n.: $\forall x_1 (x_1 \subseteq x_1)$
- (β2) m-l: $\cap_1 \cap_2 \cap_3 (\iota_{1,2} + \iota_{2,3} + \iota_{1,3})$
o-l: $\Pi x_1 \Pi x_{11} \Pi x_{111} ANI x_1 x_{11} ANI x_{11} x_{111} I x_1 x_{111}$
p.n.: $\forall x_1 \forall x_2 \forall x_3 (\neg(x_1 \subseteq x_2) \vee (\neg(x_2 \subseteq x_3) \vee (x_1 \subseteq x_3)))$
- (β3) m-l: $\cap_1 \cap_2 \cup_3 (\iota_{1,3} \cdot \iota_{2,3} \cdot \cap_4 (\iota_{1,4} + \iota_{2,4} + \iota_{3,4}))$
o-l: $\Pi x_1 \Pi x_{11} N \Pi x_{111} N N A N I x_1 x_{111} N N A N I x_{11} x_{111} N \Pi x_{1111} A N I x_1 x_{1111} A N I x_{11} x_{1111} I x_{111} x_{1111}$
p.n.: $\forall x_1 \forall x_2 \exists x_3 ((x_1 \subseteq x_3) \wedge ((x_2 \subseteq x_3) \wedge \forall x_4 (\neg(x_1 \subseteq x_4) \vee (\neg(x_2 \subseteq x_4) \vee (x_3 \subseteq x_4))))))$
- (β4) m-l: $\cap_1 \cap_2 \cup_3 (\iota_{3,1} \cdot \iota_{3,2} \cdot \cap_4 (\iota_{4,1} + \iota_{4,2} + \iota_{4,3}))$
o-l: $\Pi x_1 \Pi x_{11} N \Pi x_{111} N N A N I x_{111} x_1 N N A N I x_{111} x_{11} N \Pi x_{1111} A N I x_{1111} x_1 A N I x_{1111} x_{11} I x_{1111} x_{1111}$
p.n.: $\forall x_1 \forall x_2 \exists x_3 ((x_3 \subseteq x_1) \wedge ((x_3 \subseteq x_2) \wedge \forall x_4 (\neg(x_4 \subseteq x_1) \vee (\neg(x_4 \subseteq x_2) \vee (x_4 \subseteq x_3))))))$
- (β5) m-l: $\cap_1 \cap_2 (\cap_3 \cap_4 ((\iota_{3,1} + \iota_{3,2} + \iota_{3,4}) \cdot (\iota_{1,3} + \iota_{2,3} + \iota_{4,3})) \cdot \cap_5 (\iota_{5,1} + \cup_6 (\iota_{6,1} \cdot \iota_{6,2} \cdot \iota_{6,5})))$
o-l: $\Pi x_1 N \Pi x_{11} N A N \Pi x_{111} \Pi x_{1111} N A N A N I x_{1111} x_1 A N I x_{111} x_{11} I x_{111} x_{1111} N A N I x_{11} x_{111} A N I x_{11} x_{111} I x_{1111} x_{1111}$
p.n.: $\forall x_1 \exists x_2 (\forall x_3 \forall x_4 ((\neg(x_3 \subseteq x_1) \vee (\neg(x_3 \subseteq x_2) \vee (x_3 \subseteq x_4))) \wedge (\neg(x_1 \subseteq x_3) \vee (\neg(x_2 \subseteq x_3) \vee (x_4 \subseteq x_5)))) \wedge \forall x_5 ((x_5 \subseteq x_1) \vee \exists x_6 (\neg(x_6 \subseteq x_1) \wedge (x_6 \subseteq x_2) \wedge (x_6 \subseteq x_5))))))$

which suggests his preferred reading. I have followed this pattern of structuring the formulas in the object language and placing the parentheses in today's notation in my reconstruction of axioms $(\beta 3)$ – $(\beta 5)$. However, since the parentheses setting in regard to disjunctions and conjunctions is completely irrelevant, the content of the axioms remains the same, no matter the reading.

Translational Remarks

In the German version an error occurred in Tarski's example of an axiom of type $(\alpha 1)$. Instead of " $AN A \Pi x_i I x_i x_i \Pi x_i I x_i x_i \Pi x_i I x_i x_i$ " as in Polish [p. 49] and English [p. 179], the boldfaced part of the sentence: " $\Pi x_i I x_i x_i$ " is missing [p. 295].

A discrepancy regarding axiom $(\beta 5)$ occurred between different versions of Tarski's monograph. In the Polish original (Tarski 1933, p. 31), as well as in the first English edition and in the German translations (Tarski 1935, p. 296), in the boldfaced part of axiom $(\beta 5)$ we read " $\cap_5(\iota_{5,2} + \cup_6(\iota_{6,1} \cdot \overline{\iota_{6,2}} \cdot \iota_{6,5}))$ ". The version used here is taken from the Polish edition (Tarski 1933).

[180] *Here, Def. 14 explicates what it means that an expression is obtained from a sentential function by substituting one free variable for another one. The term 'defined' here serves as a prequel for the next two definitions.*

Tarski gives the following examples illustrating Definition 14:

Metalanguage	Object language	Present notation
$\iota_{1,1}$	$I x_i x_i$	$x_1 \subseteq x_1$
$\cap_3(\iota_{3,1} + \iota_{1,3})$	$\Pi x_{iii} A I x_{iii} x_{ii} I x_i x_{iii}$	$\forall x_3((x_3 \subseteq x_1) \vee (x_1 \subseteq x_3))$
$\iota_{1,3} + \cap_2 \iota_{2,3}$	$A I x_i x_{iii} \Pi x_{ii} I x_i x_{iii}$	$(x_1 \subseteq x_3) \vee \forall x_2(x_2 \subseteq x_3)$

These are obtained from the following functions, by substituting v_1 (' x_i ' or ' x_1 ', respectively) for v_2 (' x_{ii} ' or ' x_2 ', respectively):

$\iota_{2,2}$	$I x_{ii} x_{ii}$	$x_2 \subseteq x_2$
$\cap_3(\iota_{3,2} + \iota_{2,3})$	$\Pi x_{iii} A I x_{iii} x_{ii} I x_{ii} x_{iii}$	$\forall x_3((x_3 \subseteq x_2) \vee (x_2 \subseteq x_3))$
$\iota_{2,3} + \cap_2 \iota_{2,3}$	$A I x_{ii} x_{iii} \Pi x_{ii} I x_{ii} x_{iii}$	$(x_2 \subseteq x_3) \vee \forall x_2(x_2 \subseteq x_3)$

However, the expressions:

$\cap_1 \iota_{1,3}$	$\Pi x_i I x_i x_{iii}$	$\forall x_1(x_1 \subseteq x_3)$
$\cap_1 \iota_{1,1}$	$\Pi x_i I x_i x_i$	$\forall x_1(x_1 \subseteq x_1)$
$\cap_1 \cap_3(\iota_{3,1} + \iota_{1,3})$	$\Pi x_i \Pi x_{iii} A I x_{iii} x_i I x_i x_{iii}$	$\forall x_1 \forall x_3((x_3 \subseteq x_1) \vee (x_1 \subseteq x_3))$

cannot be obtained by substitution from the functions:

$\cap_2 \iota_{2,3}$	$\Pi x_{ii} I x_{ii} x_{iii}$	$\forall x_2(x_2 \subseteq x_3)$
$\cap_2 \iota_{2,1}$	$\Pi x_{ii} I x_{ii} x_i$	$\forall x_2(x_2 \subseteq x_1)$
$\cap_2 \cap_3(\iota_{3,2} + \iota_{2,3})$	$\Pi x_{ii} \Pi x_{iii} A I x_{iii} x_{ii} I x_{ii} x_{iii}$	$\forall x_2 \forall x_3((x_3 \subseteq x_2) \vee (x_2 \subseteq x_3))$

because the variable v_2 (' x_{ii} ' or ' x_2 ', respectively) is bound.

[181] *In preparation for the definition of the concept of consequence (Def. 16), Tarski presents an informal explication: A sentence x is a consequence of a class X of sentences iff $x \in X$ or x can be obtained from members of X by applying an arbitrary number of times the following operations: substitution, detachment, insertion of the universal quantifier or deletion of the universal quantifier. Before the definition of a consequence is presented, the auxiliary concept of consequence of the n th degree is defined in Def. 15.*

Here, Tarski points to the work of Łukasiewicz (1929) in which the latter lists five such operations. In addition to the four operations mentioned by Tarski, Łukasiewicz names the *rule of replacement* which enables us to use the definition:

$$(D) \ Np = Cp\Pi pp.$$

This rule allows us in each theorem of the deductive system to substitute the right hand side of any definition, in particular of (D), respectively its substitutions, by the left hand side of the definition, respectively its substitutions.²⁶

Translational Remarks

Before proceeding to the actual definition of the concept of a *consequence*, Tarski defines an auxiliary concept of *consequence of the n th degree*. In clause (γ) of definition 15 the translator of the German, as well as the translator of the English edition twice uses the definite article (“**the** universal quantification” and “**die** Generalisation”) where the indefinite article would be the correct choice. (In the corresponding clause (β) of the “direct” definition as given in footnote 1 on page 182, the indefinite article is used in both places, in the German as well as in the English edition.) In the new English edition which is used here this mistake has not been corrected (in contrast to the improvement concerning pp.175 f.). See also [1.2.6] and the commentary to [175].

[182] *Finally the concept of a consequence is defined in Def. 16, and that of a provable sentence in Def. 17. In the footnote a direct way of obtaining the concept of consequence without the help of the notion of consequence of the n th degree is presented.*

In definition 16 Tarski defines what it means to say that *a sentence x is a consequence of a class X of sentences*. Thereby, he uses the term ‘consequence’ in a purely syntactical sense and not in the semantic sense of his famous definition of a *logical consequence*.²⁷

Since the article “On the concept of logical consequence” met with almost as much reservation and open criticism as “The concept of truth in formalized languages”, however, and because Tarski frequently refers to his earlier article on the concept of truth there, we shall mention its central points shortly.

In the article “On the concept of logical consequence” (Tarski 2006e) which is based on the talk given at the famous International Congress of Scientific Philosophy in Paris in 1935, Tarski points to Carnap as “the first to attempt to formulate a precise definition of the proper concept of consequence” (Tarski 2006e, p. 413) meaning, of course, the formulation given in *Logische Syntax der Sprache*, 1934. Tarski (Tarski 2006e, p. 414) summarises Carnap’s definition as follows:

²⁶Cf. Łukasiewicz (1929, p. 163).

²⁷There is a considerable amount of literature regarding Tarski’s definition of *consequence*, however, it concentrates mostly on his definition of *logical consequence* as presented in the article “On the concept of logical consequence” (Tarski 2006e). For detailed discussions on this topic see for instance (Etchemendy 1990; Gómez-Torrente 1996; Patterson 2012; Ray 1996; Sher 1991).

The sentence X follows logically from the sentences of the class K if and only if the class consisting of all the sentences of K and of the negation of X is contradictory.

It is apparent from the above formulation, that the concept ‘contradictory’ (‘kontradiktorisch’) was crucial for Carnap. Actually, as crucial as the concept ‘analytic’ (‘analytisch’), the role which, for Tarski, was played by the concept ‘true’.

Carnap’s definition led Tarski to the following formulations:

Certain considerations of an intuitive nature will form our starting-points. Consider any class *K* of sentences and a sentence *X* which follows from the sentences of this class. From an intuitive standpoint it can never happen that both the class *K* consists only of true sentences and the sentence *X* is false. Moreover, since we are concerned here with the concept logical, i.e. *formal*, consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, this relation cannot be influenced in any way by empirical knowledge, and in particular by knowledge of the objects spoken about in the sentence *X* or the sentences of the class *K*. The consequence relation cannot be destroyed by replacing the designations of the objects referred to in these sentences by the designations of any other objects. The two circumstances just indicated, which seem to be very characteristic and essential for the proper concept of consequence, may be jointly expressed in the following statement:

(*F*) *If, in the sentences of the class K and in the sentence X, the constants—apart from the purely logical constants—are replaced by any other constants (like signs being everywhere replaced by like signs), and if we denote the class of sentences thus obtained from K by ‘K’’, and the sentence obtained from X by ‘X’’, then the sentence X’ must be true provided only that all sentences of the class K’ are true.* (Tarski 2006e, pp. 415–415)

Tarski notes that this condition is necessary but not sufficient, since the designations of all possible objects are not present in the investigated language, or in any other language for that matter. Earlier in this article Tarski made a reference to his work “On the concept of truth in formalized languages” stating that “only the methods which have been developed in recent years for the establishment of scientific semantics, and concepts defined with their aid, allow us to present these ideas in an exact form” (Tarski 2006e, p. 414). He was referring especially to the concepts of truth and satisfaction. Also here, the solution to the problem is presented by semantics. Tarski provides a sketch of the concept of *satisfaction*, which goes along the same lines as the concept presented in his seminal work on truth. By the means of this concept, Tarski defines the concept of a *model* or *realization of the class L of sentences* as a sequence of objects which satisfies every sentential function of the given language. In terms of the concepts of satisfaction and of a model, Tarski defines the concept of *logical consequence* as follows:

The sentence X follows logically from the sentence of the class K if and only if every model of the class K is also a model of the sentence X. (Tarski 2006e, p. 417)

Now, returning to the present article, to say that *x* is a *consequence* of *X* in the sense of definition 16 is expressed in today’s terminology usually as ‘*x* is *derivable* from *X*’ which is defined as follows:

x is derivable from *X* if and only if there is a derivation of *x* from *X*, and a *derivation* then is usually defined as follows²⁸:

²⁸The formalization of this and of the following definitions, theorems and lemmata has its origin in a seminar: Alfred Tarski, *Wahrheitsbegriff* and is an intellectual property of Edgar Morscher, who

Y is a *derivation of x from X* if and only if:

1. x is a sentence ($x \in S$); and
2. X is a class of sentences ($X \subseteq S$); and
3. there is at least one $y_1, y_2, \dots, y_n$, such that:
 - a. $y_1, y_2, \dots, y_n$ are sentences (i.e., $y_1 \in S, y_2 \in S, \dots, y_n \in S$); and
 - b. $Y = \langle y_1, y_2, \dots, y_n \rangle$; and
 - c. every term y_i ($1 \leq i \leq n$) of Y satisfies one of the following conditions:
 - i. (α) $y_i \in X$, or
 - ii. (β) y_i results from one or two sentences preceding y_i in Y by one of the basic rules of inference (corresponding to the four operations Tarski mentions on p.181).

Translational Remarks

The problematic translation of Def. 17 must be noticed here. In the three discussed versions we have:

Def. 17. x jest **teżą (zdaniem uznanym)** [p. 54]

x ist ein **beweisbarer (oder anerkannter) Satz** [p. 299]

x is a **provable (accepted) sentence** or a **theorem** [p. 182]

In Def. 17 Tarski defines a *theorem of the calculus of classes*. However, the terms:

TWIERDZENIE 1,2,...[p. 73 ff.]

SATZ 1,2,...[p. 316 ff.]

THEOREM 1,2,...[p. 197 ff.]

refer to the *metatheorems of the calculus of classes*.

The Polish term ‘twierdzenie’ could be translated as ‘theorem’, but it is confusing, since ‘theorem’ serves also as a translation of ‘teza’ in Definition 17; see also [1.2.3] and the commentary to [197].

[183] *Carrying on the exposition from the previous page, Tarski shows that ‘ $\forall x_1(\neg(x_1 \subseteq x_1) \vee x_1 \subseteq x_1)$ ’ is a provable sentence. Later, he discusses the consequences of eliminating certain existential assumptions from the axioms of the metatheory. If we eliminate given existential assumptions from these axioms, it might turn out impossible to establish the provability of certain sentences even after having assumed their existence.*

Translational Remarks

In the English translation at the beginning of this page we read that “**Translating** the proof of this theorem...”. However, in the Polish original Tarski writes “**transponując** dowód tej tezy...” [p. 54], which could be translated as “**transposing** the proof of this theorem”, as it has been done in German; “Indem wir den Beweis dieses Lehrsatzes **transponieren**...” [p. 300].

(Footnote 28 continued)

held the mentioned seminar at the University of Salzburg in 2007. He kindly offered that I use his formalizations in my dissertation, on which this publication is based. The translation into English and the further dissemination of the formulas is my own responsibility.

‘Transposing’ comes from the latin verb ‘transpono’ which can mean ‘to translate’ but it also means ‘to take something from one place over to another’. It seems that Tarski used this verb precisely because of this second meaning. He did not use the Polish verb meaning simply ‘to translate’. It remains unclear, why the translator of the English version did not use the verb ‘transpose’, since it would be a direct translation of the Polish and German sentences.

[184] *A possibility of constructing a definition of theorem which is equivalent to Def. 17, but which rejects its existential assumptions is considered. A sketch of a solution to this problem is presented. It includes a new concept of accepted numbers, which is defined as “‘consequences’ of the class of all ‘primitive’ numbers”. The proof will still require a different existential assumption, however, i.e., that there are sufficiently many natural numbers.*

Translational Remarks

Tarski writes “definicja 17 nie chwyta już wszystkich **intuicji**” [p. 55], which means the same as “definition 17 would no longer embrace all the **intuitions**”. In German we read similarly as in English that it regards all the properties “die Def. 17 [...] nicht mehr alle **Eigenschaften** erfasst” [p. 301], see also [1.2.1].

Later, on the same page, Tarski talks about picking out “from the **totality of numbers**”, in Polish “spośród **ogółu liczb**” [p. 56], which has been correctly translated into German as “aus der **Gesamtheit aller Zahlen**” [p. 301]. This discrepancy does not, however, influence the content.

At the bottom of the same page, Tarski mentions an “existential assumption” which should be accompanied by an adjective ‘weaker’; as it is in Polish “**słabszego jednak**” [p. 56], and in German “**wenn auch schwächere**” [p. 302].

[185] *After some concluding remarks concerning the problematic assumptions of Def. 17, further notions are defined: in Def. 18 a deductive system, in Def. 19 a consistent class of sentences, and in Def. 20 a complete class of sentences. An explication of what it means for two sentences to be equivalent is presented in Def. 21.*

Here are Definitions 18–21 presented in today’s notation.²⁹

Def. 18. X is a *deductive system* if and only if X is a class of sentences ($X \subseteq S$) and is deductively closed ($Cn(X) \subseteq S$), i.e., if and only if all elements of X are sentences, and all consequences of X are elements of X , i.e., iff $\forall x(x \in X \rightarrow x \in S)$ and $\forall x(X \vdash x \rightarrow x \in X)$.

We call a class of sentences X *deductively closed* iff all consequences of X are elements of X , i.e., iff $\forall x(X \vdash x \rightarrow x \in X)$.

A theory or, as Tarski says, a deductive system is a deductively closed class of sentences.

Def. 19. X is a *consistent* class of sentences if and only if
 $X \subseteq S$ and $\forall x(x \in S \rightarrow (x \notin Cn(X) \text{ or } \bar{x} \notin Cn(X)))$, i.e.,
 iff $X \subseteq S$ and $\forall x(x \in S \rightarrow (X \not\vdash x \text{ or } X \not\vdash \bar{x}))$.

²⁹Morscher (2007).

Def. 20. X is a *complete* class of sentences if and only if

$X \subseteq S$ and $\forall x(x \in S \rightarrow (x \in Cn(X) \text{ or } \bar{x} \in Cn(X)))$, i.e., iff $X \subseteq S$ and $\forall x(x \in S \rightarrow (X \vdash x \text{ or } X \vdash \bar{x}))$.

Def. 21. x and y are *equivalent* with respect to X if and only if $x \in S$, and $y \in S$, and $X \subseteq S$, and $\bar{x} + y \in Cn(X)$, and $\bar{y} + x \in Cn(X)$, i.e., $x \in S$, and $y \in S$, and $X \subseteq S$, and $X \vdash x \rightarrow y$, and $X \vdash y \rightarrow x$.

2.4 Section 3. The Concept of True Sentence in the Language of the Calculus of Classes

[186] *In Sect. 3 the definition of “true sentence of the language of the calculus of classes” is constructed. Def. 17 from page 182, presents the term provable sentence, but does not provide the definition of a true sentence. Since the principle of excluded middle is not valid in the domain of provable sentences, whereas it is indisputable for the set of true sentences, the definition of truth must extend onto sentences which are not provable.*

It is important to notice that the title of this chapter, whether in English, in German (“Der Begriff der wahren Aussage in der Sprache des Klassenkalküls”), or in Polish (“Pojęcie zdania prawdziwego w języku algebry klas”), could be understood in two different ways:

1. the concept (of a true sentence in the language of the calculus of classes), or
2. (the concept of a true sentence) in the language of the calculus of classes.

It is clear from the previous chapters and from Sect. 3 that (1) is the correct interpretation. Nevertheless, the possible misinterpretation could easily be avoided if the title was “The concept of a true sentence *of* the language of the calculus of classes”.

Translational Remarks

In this chapter, Tarski deals with the main task of his article, i.e., the construction of the definition of **true sentence**. It is important here, in the formal context, to use a precise formulation, i.e. “definition of **true sentence**”, as we read in Polish “definicja 17 przedstawia zarazem definicję **zdania prawdziwego**” [p. 58], and also in German “die Def. 17 zugleich eine Definition der **wahren Aussage** ist” [p. 303], see also the commentary to [168].

Further, in the same paragraph, Tarski writes in Polish “żadna **zgodna z intuicją** definicja zdania prawdziwego nie powinna pociągać za sobą konsekwencji sprzecznych z zasadą wyłączonego środka [...]” [p. 58] – “no definition of a true sentence which is **in agreement with intuition** should have any consequences which contradict the principle of the excluded middle.” The English translation, which states that “no definition of true sentence which is **in agreement with the ordinary usage of language** should have any consequences which contradict the principle of the excluded middle”, originated in the German version where we read “keine

mit dem Sprachgebrauch übereinstimmende Definition der wahren Aussage darf dem Prinzip des ausgeschlossenen Dritten widersprechende Konsequenzen nach sich ziehen;” [pp. 303–304], see also [1.2.1].

There is a printing error in the German version that has not been carried over to the English translation. Namely, in the footnote 40, Tarski (1933), pp. 38–39 and Tarski (1995c), p. 59, Tarski names the sentence which should be included among theorems, i.e., “ $\cap_1(\cap_2\iota_{1,2} + \cup_2(\iota_{2,1} \cdot \cap_3(\cap_4\iota_{3,4} + \overline{\iota_{3,2}} + \iota_{2,3})))$ ”. In most German versions the mentioned footnote sentence is incorrect, and so for example in the version used here we read “ $\cap_1(\cap_2\iota_{1,2} + \cap_2(\iota_{2,1} + \cap_3(\cap_4\iota_{3,4} + \overline{\iota_{3,2}} + \iota_{2,3})))$ ” [p. 304].

[187] *In the prelude to the famous CONVENTION T, Tarski explicates on the idea of constructing a semantical definition of a true sentence. Finally, CONVENTION T is introduced.*

Translational Remarks

Tarski reminds us that “As we know from Sect. 2, to every sentence of the language of the calculus of classes there corresponds in the metalanguage not only an **individual name** of this sentence of the structural-descriptive kind, but also a sentence having the same meaning.” The adjective ‘individual’, here in boldface, has been left out in the English version. In Polish “Jak już wiemy z Sect. 2, każdemu zdaniu, należącemu do języka algebry klas, odpowiada w metajęzyku z jednej strony pewna **nazwa jednostkowa** tego zdania typu strukturalnoopisowego, z drugiej strony – pewne zdanie równoznaczne ze zdaniem danym;” [p. 59]. Here, the German translation is accurate “Wie wir schon aus Sect. 2 wissen, entspricht in der Metasprache jeder Aussage, die zur Sprache des Klassenkalküls gehört, einerseits ein **individueller Name** dieser Aussage von strukturell-deskriptivem Typus, andererseits eine mit der gegebenen Aussage gleich bedeutende Aussage;” [pp. 304–305].

The Polish sentence “Wszystkie uzyskane na tej drodze **tezy**, np. [...], należą, rzecz jasna, do metajęzyka i wyjaśniają w sposób precyzyjny i **zgodny z intuicją** znaczenie występujących w nich zwrotów kształtu “*x jest zdaniem prawdziwym*”.” [p. 60], has been translated into English as “All **sentences** obtained in this way, e.g. [...], naturally belong to the metalanguage and explain in a precise way, **in accordance with linguistic usage**, the meaning of the phrases of the form ‘*x is a true sentence*’ which occur in them.” The first of the phrases written in bold is actually an improvement of the Polish original and of the German translation. It should be noted that the Polish term ‘tezy’, as used in Def. 17 which defines a “provable (accepted) sentence” is not what is meant here. Only the English translation avoids this ambiguity, see also [1.2.3]. However, the second in bold printed phrase is an inaccurate translation since it should read here “**in accordance with intuition**”, see also [1.2.1]. In German this sentence reads in the following way “Alle auf diesem Weg gewonnenen **Sätze**, z. B. [...], gehören selbstverständlich zur Metasprache und erklären in präziser und **mit dem Sprachgebrauch übereinstimmender Weise** die Bedeutung der in ihnen auftretenden Redewendungen von der Form “*x ist eine wahre Aussage*”.” [p. 305]. The translation of the first boldfaced term is ambiguous, and the second one is not really surprising.

The above stated **postulates**, shortly before the famous CONVENTION **T** is introduced, in Polish “postulaty” [p. 60] and in German “Postulate” [p. 305] are of course meant to be in plural, not in singular as in the English version.

[188] *Convention T is presented here. Tarski mentions the circumstances under which condition (β) would not be essential. Later, Tarski sketches the method of constructing a definition of a true sentence in case the investigated language contained only a finite number of sentences. This method is inapplicable to any language with an infinite number of sentences, however, hence also to the language of the calculus of classes.*

Convention **T** is essential for Tarski’s definition of a true sentence. He wants his definition to “do justice to the intuitions which adhere to the *classical Aristotelian conception of truth*.”³⁰ Convention **T** states the conditions for a formally correct and (materially) adequate definition of *true sentence* in the metalanguage.

Since the translation of the famous Convention **T** has led to much discussion and disagreement among scholars, it is perhaps, not too redundant to make a few remarks on this topic here also. In Polish Tarski writes:

*Umowa P. Poprawną formalnie definicję symbolu “Vr”, sformułowaną w terminach metajęzyka, nazywać będziemy **trafną definicją prawdy**, o ile pociąga ona za sobą następujące konsekwencje:*

(α) wszystkie zdania, dające się uzyskać z wyrażenia “ $x \in Vr$ ” wtedy i tylko wtedy, gdy p ” przez zastąpienie symbolu “ x ” nazwą strukturalnoopisową dowolnego zdania rozważanego języka, zaś symbolu “ p ” – wyrażeniem, stanowiącym przekład tego zdania na metajęzyk;

(β) zdanie “dla dowolnego x – jeśli $x \in Vr$, to $x \in S$ ” (lub in. sł. “ $Vr \subset S$ ”). (Tarski 1933, p. 40)

‘Umowa’ means the same as ‘agreement’, which in this case became ‘convention’, however, Tarski did not write in Polish ‘konwencja’. I suppose, one could say that the term ‘konwencja’ implies a broader application of the agreement, whereas ‘umowa’ could hold between a smaller group of people. This could be attributed to the relatively small group of specialists familiar with Tarski’s methods and techniques at that time. Nonetheless, this point does not, essentially, influence the content. On the other hand, the terms “**poprawną formalnie**” and “**trafną definicją prawdy**” could be translated as “formally correct (or right)” and “adequate definition of truth”. We see that the English translation is correct and adequate. It should be noted, however, that the mentioned terms led both translators, Blaustein and Woodger, to much confusion throughout the article. See also [1.2.1], [1.2.2], as well as Hodges (2008, p. 117) and Patterson (2012, pp. 125–6).

Convention **T** captures the intuitive meaning of the concept of truth, which is in accord with the correspondence theory, at the same time subjecting it to a formal apparatus – a deductive theory.

The genius of Convention **T** is that incorporating the T-sentences into the metatheory forces the interpretation of “ $\in Tr$ ” to accord with the semantic definition, while leaving the definition itself unstated. This, I suppose, is the connection between the semantical definition and

³⁰Tarski (1944, p. 342).

the T-sentences, the two elements of Polish thought about truth that Tarski brings together. To put it another way, if the T-sentences are theorems, “ $\in Tr$ ” expresses the irreducibly semantic content of the concept of truth, the content expressed in the “semantical definition”, and if the T-sentences are theorems because the metatheory has been extended by a certain definition, we thereby force a theory with only mathematical primitives to express a semantic concept, thereby meeting the eliminative goal stated at [Tarski, 1983a, 154] that undefined semantic terms will no appear within the metatheory.” (Patterson 2012, p. 135)

Using Convention **T** as a basis, Tarski formulated in 1944, what he called an “*equivalence of the form (T)*”. It became known as the *equivalence scheme*.

“Let us consider an arbitrary sentence; we shall replace it by the letter ‘ p ’. We form the name of this sentence and we replace it by another letter, say ‘ X ’. We ask now what is the logical relation between the two sentences “ X is true” and ‘ p ’. It is clear that from the point of view of our basic conception of truth these sentences are equivalent. In other words, the following equivalence holds:

(T) X is true, if and only if, p .

We shall call any such equivalence (with ‘ p ’ replaced by any sentence of the language to which the word “true” refers, and ‘ X ’ replaced by a name of this sentence) an “*equivalence of the form (T)*”.”³¹

Convention **T** is of crucial importance for Tarski’s article, but also for the future theories of truth. The renowned *equivalence scheme*, which plays an essential role in Convention **T**, has ever since been used by philosophers and logicians to formulate their theories of truth. It must, however, be noted that Convention **T** is not identical with an *equivalence of the form (T)*. Tarski emphasizes that neither Convention **T** itself, nor any of the instances of the equivalence of the form **(T)** can be regarded as a definition of truth. It can be only said that every equivalence of the form **(T)** obtained in the described manner may be considered a partial definition of truth, explaining wherein the truth of this one individual sentence consists.³²

Certain philosophers,³³ however, misinterpreted Tarski and hold that all that can be meaningfully said about truth can be said by means of the equivalence scheme. Tarski believes the matter to be much more complicated. If the investigated language contained only a finite number of sentences, and if we could enumerate all these sentences, then the construction of a correct definition of truth would not be a problem. Since this is not the case, however, since languages in general contain infinitely many sentences, the definition constructed according to the above scheme would also have to consist of infinitely many words. Such sentences cannot be formulated either in the metalanguage or in any other language.³⁴ Hence, Tarski introduces the notion of *satisfaction of a given sentential function by given objects*, in this case by a given

³¹Tarski (1944, p. 344).

³²Cf. Tarski (1944, p. 344).

³³E.g. deflationists like Field or Horwich.

³⁴An important exception are the *infinitary languages* whose formulas are identified as infinite sets. They however, had not been investigated until the late 50’s for example in Scott and Tarski (1958) “The sentential calculus with infinitely long expressions”. See also Bell (2012).

class of individuals. The way Tarski explains the notion of satisfaction reflects the natural generalization of the method used for the concept of truth.³⁵

Translational Remarks

Except for the issue mentioned earlier, the translator of the English version presents a flawless translation of the famous Convention **T**, especially considering the difficult part regarding the choice of articles before the nouns. In the German translation quotation marks, which were to indicate the symbol ‘*x*’, are omitted [last line of p. 305]. Fortunately, the English translation is satisfactory in this respect.

In the footnote at the bottom of this page, a printing error occurred, this time only in the English translation. Tarski writes that “after a minor modification of its formulation the convention itself would then become a normal definition belonging to the **meta-metatheory**”, and not simply to ‘meta-theory’. In Polish “przy nieznacznej modyfikacji wyślowienia umowa sama zyskałaby wówczas charakter normalnej definicji z zakresu **meta-metanauki**” [p. 61]. Also the German translator speaks correctly of the “Meta-Metawissenschaft” [p. 306].

Another discrepancy occurred in the English translation. Here, we read that “**Whenever** a language contains infinitely many sentences [...]” [p. 188]. In Polish we read “Tak jednak w istocie nie jest, zdań w języku jest nieskończenie wiele [...]” [pp. 61–62], which could be translated as “But this is not the case, there are infinitely many sentences in the language [...]”. The German translation is accurate “So aber verhält es sich tatsächlich nicht; es gibt in der Sprache unendlich viele Aus-sagen, [...]” [p. 306]. Tarski means that the investigated language contains infinitely many sentences. The English translation, however, widens the scope of languages which would have to face similar difficulties as the investigated language.

[189] *The idea of constructing a recursive definition of a true sentence is considered shortly. Among the sentences of the language, we distinguish quite elementary expressions from more or less complex expressions. First, all the operations by which simpler sentences are combined into more composite ones would have to be given. Then, the way in which truth or falsity of composite sentences depends on the truth or falsity of the simpler ones contained in them would have to be determined. As follows from Defs. 10–12 of Sect. 2, however, in general composite sentences are not simply compounds of simpler sentences. Hence, there is no method which would allow us to define truth directly. Therefore, the notion of satisfaction is introduced. It can be defined recursively and is essential in constructing of the definition of a true sentence.*

Translational Remarks

The adjectives ‘simple’ and ‘composite’ are each used three times on this page. In the English text [lines 7–10, 18–19], only one is in a comparative form, whereas in the Polish original and in the German translation all six are written in a comparative form. In Polish [p. 62] “Chodziłoby więc o to, by wskazać wszystkie operacje, przy pomocy

³⁵ Patterson (2012, pp. 124–6) presents a thorough analysis of Convention **T**, including its historical and philosophical origins, also commenting on the problematic translations.

których zdania **prostsze** łączy się w **bardziej złożone**, i ustalić, jak prawdziwość bądź fałszywość zdań **bardziej złożonych** zależy od prawdziwości bądź fałszywości wchodzących w ich skład zdań **prostszych**” and “pobieżna nawet analiza definicji 10–12 z Sect. 2 wykazuje, iż w ogólnych przypadku zdania **bardziej złożone** nie są bynajmniej połączeniami zdań **prostszych**”; and in German [p. 307] “Es würde sich also darum handeln, sämtliche Operationen anzugeben, mit deren Hilfe **einfachere** Aussagen zu **zusammengesetzteren** vereinigt werden, und festzustellen, in welcher Weise die Wahrheit bzw. Falschheit der **zusammengesetzteren** Aussagen von der Wahrheit bzw. Falschheit der in ihnen enthaltenen **einfacheren** Aussagen abhängt” and “eine auch nur oberfachliche Analyse der Defs. 10–12 des Sect. 2 beweist, dass im allgemeinen Fall die **zusammengesetzteren** Aussagen keineswegs Verbindungen **einfacherer** Aussagen sind”, all six adjectives are in a comparative form. It is important to notice the difference, since the sentences which are contained in the more composite ones have to only be *simpler* and not entirely simple. The same holds for the *more composite* sentences. It suffices that they are more composite than the simpler ones. Moreover, the one adjective which also occurs in the English text in a comparative form is not consistent with the rest of the sentence.

In the last paragraph of this page, we read in Polish [p. 63] “Najprostszy z **intuicyjnego punktu widzenia** [...]” – “The simplest from the **intuitive point of view** [...]”, see also [1.2.1].

[190] *Examples clarifying the meaning of the notion of satisfaction are presented. First, sentential functions containing only one free variable are considered.*

What does it mean that an object a **satisfies** the sentential function x ?³⁶

Let $x(v_i)$ be a sentential function with one free variable v_i ; then we say:

(S_1) a satisfies $x(v_i)$ if and only if p

where for ‘ p ’ we have the sentential function we get by substituting every free occurrence of v_i in $x(v_i)$ with ‘ a ’ (or with a different name of a), and in place of ‘ $x(v_i)$ ’ we have an individual name of $x(v_i)$.

Tarski’s Examples

a satisfies the sentential function ‘ α is white’ if and only if a is white; therefore snow satisfies the sentential function ‘ α is white’, because snow is white.

a satisfies the sentential function ‘ $\alpha + 2 = 3$ ’ if and only if $a + 2 = 3$; therefore 1 satisfies the sentential function ‘ $\alpha + 2 = 3$ ’, because $1 + 2 = 3$.

(In the two examples above I replaced Tarski’s ‘ x ’ with ‘ α ’ in order to avoid an ambiguity since ‘ x ’ is also used by Tarski as a variable for sentential functions.)

As soon as we apply the scheme S_1 formally to sentential functions of the language of the calculus of classes, all instances of S_1 must be formulated exclusively in terms

³⁶The following discussion on the notion of *satisfaction* is based on Morscher (2007). For further literature on this topic see Fine and McCarthy (1984), Betti (2008).

of the metalanguage. Instead of the sentential function itself described above, we have to insert its translation into the metalanguage for ‘ p ’ in S_1 .

Here is Tarski’s example illustrating this:

a satisfies $\cap_2 \iota_1 \iota_2$ (i.e., ‘ $\Pi x_{||} Ix_{||}x_{||}$ ’) if and only if for every class $b : a \subseteq b$.

Translational Remarks

According to the Polish original, the footnote on this page reads “Abstrahuję na razie od kwestii, związanych z tzw. kategorią semantyczną (lub typem) **zmiennej**; kwestiami tymi zajmę się w Sect.4.” [p. 63], which can be translated as “For now, I abstract from the problems connected with semantical categories (or types) **of the variables**; these problems will be discussed in Sect.4.” The German translation is complete “Ich abstrahiere vorläufig von den Fragen, die sich an die sog. semantische Kategorie (oder den Typus) **der Variablen** knüpfen; diese Probleme werde ich im Sect.4 besprechen” [p. 308].

[191] *Now, the examples of the concept of satisfaction being applied to a sentential function containing two distinct free variables are presented. Following are the examples of the concept of satisfaction applied to a sentential function containing an arbitrary number of free variables.*

Here are further expositions regarding the notion of satisfaction applied to sentential functions with two, and with arbitrarily many free variables.

$x(v_{i_1}, v_{i_2})$ is a sentential function with two free variables v_{i_1}, v_{i_2} ; we say that:

(S₂) a_1, a_2 satisfy $x(v_{i_1}, v_{i_2})$ if and only if p

where for ‘ p ’ we have the sentential function we get by substituting every free occurrence of v_{i_1} in $x(v_{i_1}, v_{i_2})$ with ‘ a_1 ’ (or with a different name of a_1) and every free occurrence of v_{i_2} in $x(v_{i_1}, v_{i_2})$ with ‘ a_2 ’ (or with a different name of a_2), and in place of ‘ $x(v_{i_1}, v_{i_2})$ ’ we have an individual name of $x(v_{i_1}, v_{i_2})$.

Here are some of Tarski’s examples of the application of the notion of satisfaction to a sentential function with two free variables:

a_1, a_2 satisfies the sentential function ‘ α_1 sees α_2 ’ if and only if a_1 sees a_2 .

a_1, a_2 satisfies the sentential function $\iota_{2,3}$ (i.e., ‘ $Ix_{||}x_{||}$ ’) if and only if $a_1 \subseteq a_2$.

[192] *The concept of satisfaction is discussed and some final remarks laying the ground for the definition of satisfaction are made. The reader’s attention is drawn to the operation of universal quantification in connection with the concept of satisfaction.*

The general case where x is a sentential function containing an arbitrary number of free variables, and $f = \langle f_1, f_2, \dots \rangle$ is an infinite sequence of objects, we say:

(S) f satisfies the sentential function x if and only if p

where for ‘ p ’ we have the sentential function we get from x by translating x into the metalanguage and by substituting every free variable v_i in x with a name of f_i , and in place of ‘ x ’ we have a (structural-descriptive) name of x in the metalanguage.

Tarki’s usage of the quotation marks in the marked passage is worth noticing. What we have to substitute “for all the free variables v_k, v_l , etc. occurring in s ” are of course not the symbols ‘ f_k ’ and ‘ f_l ’, but symbols for f_k and f_l . The quotation marks have been used identically in all three versions.

Translational Remarks

The passage beginning in the 9th line page [192] of the English version has been translated somewhat freely. In Polish we read “Mając daną funkcję zdaniową z zakresu algebry klas, zastępujemy w tym schemacie symbol “ x ” przez nazwę indywidualną (strukturalno-opisową) tej funkcji, wyrażoną w terminach metajęzyka, zaś symbol “ p ” przez wyrażenie, które uzyskujemy z rozważanej funkcji, przekładając ją na metajęzyk i równocześnie zastępując w niej wszystkie wolne v_k, v_l itd. odpowiednimi symbolami “ f_k ”, “ f_l ” itd.” [p. 66] If it were literally translated it would read “Having a given sentential function from the calculus of classes, we replace in the above scheme the symbol ‘ x ’ by an individual (structural-descriptive) name of this function expressed in the terms of the metalanguage; the symbol ‘ p ’ by the expression obtained from the considered function by translating it into the metalanguage; at the same time, replacing all the free variables v_k, v_l occurring in it by the corresponding symbols ‘ f_k ’, ‘ f_l ’.” The German translation is closer to the Polish original: “Haben wir eine Aussagefunktion aus dem Klassenkalkül, so ersetzen wir in diesem Schema das Symbol “ x ” durch einen individuellen (strukturell-deskriptiven), in der Metasprache formulierten Namen dieser Funktion, das “ p ” dagegen durch einen Ausdruck, den wir aus der betrachteten Funktion gewinnen, indem wir sie in die Metasprache übersetzen und zugleich in ihr alle freien Variablen v_k, v_l u.s.w. durch entsprechende Symbole “ f_k ”, “ f_l ” u.s.w. ersetzen” [p. 310].

In Polish, [p. 66] “Mając na względzie **treść intuicyjną** rozważanej operacji [...]”, which can be translated as “Considering the **intuitive content** of the discussed operation [...]”. It is clear, that the discrepancy originated in the German translation, [p. 311] “Indem wir den **Inhalt** der betrachteten Operation berücksichtigen [...]”, see also [1.2.1].

[193] *Definition 22 defining the concept of satisfaction is presented. Following are some examples of the application of the newly defined concept.*

The most important notion that brings Tarski to his definition of truth is the notion of satisfaction. It is introduced in Definition 22, and by the means of it, Tarski arrives at the definition of a true sentence. First, he gives a recursive definition of the notion of satisfaction and then immediately, in the footnote following the definition, presents an explicit definition of this notion.

Here are Tarski’s examples presented in today’s notation also.

The infinite sequence f satisfies the inclusion:

Metalanguage	Object language	Present notation
$\iota_{1,2}$	Ix_ix_{ii}	$x_1 \subseteq x_2$

if and only if $f_1 \subseteq f_2$.

The infinite sequence f satisfies the function:

$$\overline{\iota_{2,3}} + \overline{\iota_{3,2}} \quad ANIx_{ii}x_{iii}NIx_{iii}x_{ii} \quad \neg(x_2 \subseteq x_3) \vee \neg(x_3 \subseteq x_2)$$

if and only if $f_2 \neq f_3$.

Furthermore, the functions:

$$\begin{array}{lll} \cap_2 \iota_{1,2} & \Pi x_{ii}Ix_ix_{ii} & \forall x_2(x_1 \subseteq x_2) \\ \cap_2 \iota_{2,3} & \Pi x_{iii}Ix_{ii}x_{iii} & \forall x_2(x_2 \subseteq x_3) \end{array}$$

are satisfied by those, and only those, sequences f in which f_1 is the null class, and f_3 the class of all individuals.

And finally, every infinite sequence of classes satisfies the function:

$$\iota_{1,1} \quad Ix_ix_i \quad x_1 \subseteq x_1$$

and no infinite sequence of classes satisfies the function:

$$\iota_{1,2} \cdot \overline{\iota_{1,2}} \quad NANIx_ix_{ii}NNx_ix_{ii} \quad x_1 \subseteq x_2 \wedge \neg(x_1 \subseteq x_2)$$

The parallels between Tarski's notion of *satisfaction* and Carnap's *valuations* are, perhaps, worth noticing here. The notion of *valuation* introduced in Sect. 34c of *Logical Syntax of Language* plays a crucial role in Carnap's definition of the term 'analytic', actually a very similar role to that played by the notion of *satisfaction* in Tarski's definition of truth. In fact, Carnap's definition of 'analytic in Language II' can be understood, for certain languages, as a definition of 'true in Language II'. One possible reason why Carnap did not put forward a definition of truth, in spite of coming so close to defining truth in a manner very similar to Tarski: ' σ_1 is true in S', is that it would require an exposition of the meanings of the symbols occurring in the sentence of which truth is predicated, and that would go beyond Carnap's syntactical method.³⁷

Translational Remarks

In the footnote regarding Definition 22 in condition (β), we read that "there is a sentential function z such that $y = \bar{z}$ and the formula gRz does not hold". In the German version (Tarski 1986) used here [p. 311], the negation sign over z has been

³⁷Cf. Wagner (2009), p. 26.

left out. In the first German version from 1935 the negation signs have been printed correctly.

[194] *Tarski explains why the concept of satisfaction is crucial for the concept of a true sentence. In the footnote the concepts of denotation, satisfaction, definability, and truth are considered and their relations to one another are also explained.*

Here and on the following pages, the definitions, the lemmas as well as the theorems stated by Tarski will be presented using the logical symbols commonly used today and the following abbreviations:

‘ $Sat(f, x)$ ’ stands for ‘ f satisfies x ’

‘ $Seq(f)$ ’ stands for ‘ f is an infinite sequence of classes’

‘ $SFunc(x)$ ’ stands for ‘ x is a sentential function’

‘ $\bar{g}_k f$ ’ stands for ‘ g differs from f at most regarding f_k ’

It should also be noted, that the domain of values of the variables ‘ f ’ and ‘ g ’ is the class of the infinite sequences of classes.

Now follows Definition 22 in an abbreviated version of this kind.

Def. 22.* $Sat(f, x) : \Leftrightarrow$

1. $Seq(f) \wedge$
2. $SFunc(x) \wedge$
3. $\exists k \exists l (x = \iota_{k,l} \wedge f_k \subseteq f_l) \vee$
 $\exists y \exists z (x = y + z \wedge (Sat(f, y) \vee Sat(f, z))) \vee$
 $\exists k \exists y (x = \cap_k y \wedge \forall g (Seq(g) \rightarrow (\bar{g}_k f \rightarrow Sat(g, y))))$

Translational Remarks

In the footnote of the English translation, we read that “the sentential function x **defines** the property P of classes”. In the context of this paper, a different term should be used, e.g., **specifies** or **determines**. In Polish Tarski wrote “funkcja zdaniowa x **wyznacza** własność klas W ” [p. 68], where ‘wyznacza’ can be translated as ‘determines’, or ‘specifies’. In German we read “die Aussagefunktion x die Eigenschaft E von Klassen dann und nur dann **bestimmt**” [p. 312].

[195] *Finally, the definition of true sentence is presented. Definition 23 is formally correct and materially adequate in the sense of convention T. In the footnote the idea of operating with finite sequences with a variable number of terms instead of with infinite sequences is considered.*

23.* $x \in Tr : \Leftrightarrow x \in S \wedge \forall f (Seq(f) \rightarrow Sat(f, x))$

It is important to remember Tarski’s commitment to *Intuitionistic Formalism* at this place. As we have mentioned before, and as others have thoroughly discussed,³⁸

³⁸Patterson (2012).

it was essential for Tarski that all of the defined notions, thus also the notions of *satisfaction*, and most of all, of *true sentence*, were in agreement with our intuitions. Presenting the “intuitively clear” notions by means of mathematical apparatus in order to assure their clarity, however, Tarski may have overestimated his readers at that time. As Patterson argues, referring to Definitions 22 and 23:

This is hardly something of which Tarski’s readers had “intuitive” knowledge beforehand and it can’t be found discussed in works on the “theory of knowledge” prior to Tarski’s work. It also, famously, ties the defined term to the specific features of Tarski’s “language of the calculus of classes”, yet surely it was no part of the concept of truth of which readers could have had intuitive knowledge that sentences of a certain first-order theory of the subset relation are true as a degenerate case of the satisfaction relations. Whatever this definitions is for, it can’t in any familiar sense be an “analysis” of the concept of truth. (Patterson 2012, p. 20)

[196] *Using Defs. 22 and 23, it is illustrated with concrete example sentences how it can be shown that these sentences are true. The presented procedure can easily be applied to every sentence of the considered language.*

After presenting the definition of a true sentence, Tarski illustrates the properties of this definition with concrete examples of true sentences:

Metalanguage	Object language	Present Notation
$\cap_1 \cup_2 \iota_{1,2}$	$\Pi x_i N \Pi x_{ii} N I x_i x_{ii}$	$\forall x_1 \exists x_2 (x_1 \subseteq x_2)$
$\cap_1 \cup_2 (\iota_{1,2} + \overline{\iota_{1,2}})$	$\Pi x_i N \Pi x_{ii} N A I x_i x_{ii} N I x_i x_{ii}$	$\forall x_1 \exists x_2 ((x_1 \subseteq x_2) \vee \neg(x_1 \subseteq x_2))$

[197] *Some more examples of the application of Def. 23 are presented. At the same time it must be emphasized that Def. 23 gives no general criterion for the truth of a sentence. Later, the first two “theorems” are presented. They state the principle of contradiction and the principle of excluded middle respectively.*

It must be noted that in the English translation the term ‘theorems’ refers here to the *metatheorems* of the calculus of classes, and not to the *theorems* as defined in Def. 17. In Polish Tarski writes ‘TWIERDZENIE’ which has been here translated in German as ‘SATZ’. In order to avoid additional confusion, I will here be using the English translation, see also [1.2.3] and the commentary to [182].

Here are the principles of contradiction (Theorem 1) and of excluded middle (Theorem 2) in today’s notation:

- T1 $\forall x (x \in S \rightarrow (x \notin Tr \vee \bar{x} \notin Tr))$
T2 $\forall x (x \in S \rightarrow (x \in Tr \vee \bar{x} \in Tr))$

Translational Remarks

Most likely a printing error occurred in the German translation of the boldfaced example, where it reads “ $\cup_1 \cup_2 (\iota_{1,2} + \overline{\iota_{1,2}})$ ” [p. 315], instead of, as in the correct English translation, “ $\cap_1 \cup_2 (\iota_{1,2} + \overline{\iota_{1,2}})$ ”. In the Polish original edition (Tarski 1933, p. 49), we have “ $\cap_1 \cap_2 (\iota_{1,2} + \overline{\iota_{1,2}})$ ”. None of these discrepancies influence the content, however, since all the examples are correct.

In the place of the second expression in bold, in Polish Tarski writes “prawa natury ogólniejszej” [p. 73], which could be translated as “laws of a more general nature”. In German we read “allgemeine Sätze” [p. 316].

[198] *Here, Theorems 3–6 and Lemmas A–D are presented.*

Additional abbreviations:

‘ $F(v, x)$ ’ stands for ‘ v is a free variable of x ’

‘ $x \in Ax$ ’ stands for ‘ x is an axiom’

The symbols ‘ $Seq(f)$ ’ and ‘ $Seq(g)$ ’ could naturally also be left out.

LA $\forall f \forall g \forall x ((Sat(f, x) \wedge Seq(g) \wedge \forall k (F(v_k, x) \rightarrow f_k = g_k)) \rightarrow Sat(g, x))$

LB $\forall x ((x \in S \wedge \exists f Sat(f, x)) \rightarrow \forall f (Seq(f) \rightarrow Sat(f, x)))$

LC $\forall x \forall y (y = \cap x \rightarrow (\forall f (Seq(f) \rightarrow Sat(f, x)) \leftrightarrow \forall f (Seq(f) \rightarrow Sat(f, y))))$

LD $\forall x (x \in Ax \rightarrow x \in Tr)$, i.e., $Ax \subseteq Tr$

T3 $\forall X (X \subseteq Tr \rightarrow Cn(X) \subseteq Tr)$, thus $Cn(Tr) \subseteq Tr$

T4a $Cn(Tr) \subseteq Tr \subseteq S$

T4a $Tr \subseteq S \wedge \forall x (x \notin Cn(Tr) \vee \bar{x} \notin Cn(Tr))$

T4b $Tr \subseteq S \wedge \forall x (x \in Cn(Tr) \vee \bar{x} \in Cn(Tr))$

T5 $\forall x (x \in Pr \rightarrow x \in Tr)$, i.e., $Pr \subseteq Tr$

T6 $\exists x (x \in Tr \wedge x \notin Pr)$, i.e., $\neg \forall x (x \in Tr \rightarrow x \in Pr)$, i.e., $Tr \not\subseteq Pr$

Translational Remarks

It is important to notice that in the revised English edition a mistake (or possibly a printing error) has been corrected. In the Polish original, as well as in the early German editions, we read that Lemma B in combination with Definitions 22 and 23 leads to Theorem 1, instead of correctly to Theorem 2: “Jako bezpośredni wniosek z tego lematu i z definicji 22 uzyskujemy lemat B, który w zestawieniu z definicjami 22 i 23 prowadzi już z łatwością do **twierdzenia 1** [...]” [p. 73], and in the German translation “Als unmittelbare Folgerung aus diesem Lemma und der Def. 12 erhalten wir das Lemma B, welches im Verein mit den Def. 22 und 23 nunmehr leicht zum **Satz 1** führt [...]” [p. 317].

[199] *Lemma E and Theorem 7 are stated. The concept of a sentence being valid or true in a domain a of individuals is explained. It played an essential role in the works of Hilbert.*

As already mentioned in the *Introduction* [1.2.2], both translators made a rather poor choice of words when translating the term a “**correct** sentence in an individual domain a ”. The English phrasing is a translation of the German term ‘richtig’, which has been used here instead of the term ‘gültig’. An accurate English translation is “sentence **valid** or true in a domain a of individuals”. I will be using the accurate translation for the rest of my commentary.

LE $\cap_1 \cap_2 \iota_{1,2} \notin Pr \wedge \overline{\cap_1 \cap_2 \iota_{1,2}} \notin Pr$, i.e., ‘ $x_1 \subseteq x_{11}$ ’ $\notin Pr \wedge \neg(x_1 \subseteq x_{11})$ ’ $\notin Pr$

$$\text{T7a } Pr \subseteq S \wedge \forall x(x \notin Cn(Pr) \vee \bar{x} \notin Cn(Pr))$$

$$\text{T7b } Pr \subseteq S \wedge \neg \forall x(x \in Cn(Pr) \vee \bar{x} \in Cn(Pr))$$

[200] *From the concept mentioned on the previous page, Tarski proceed to Def. 24 which states the conditions under which a given sequence satisfies a given sentential function in the domain a of individuals. Defs. 25 and Def. 26 respectively, state what it means to say of a sentence that it is valid (true) in the domain a of individuals and valid (true) in a domain with k elements.*

Additional abbreviations:

‘ $C(a)$ ’ stands for ‘ a is a class of individuals’

‘ $x \in Ct_a$ ’ stands for ‘ x is a sentence valid (true) in a ’

‘ $x \in Ct_k$ ’ stands for ‘ x is a sentence valid (true) in a domain with k elements’

‘ $x \in Ct$ ’ stands for ‘ x is a sentence valid (true) in every domain of individuals’

‘ $\mathcal{K}(a)$ ’ stands for ‘the cardinal number of a ’

‘ $Seq(f, a)$ ’ stands for ‘ f is an infinite sequence of subclasses of the class a ’

‘ $\mathcal{L}$ ’ stands for ‘the language of the calculus of classes’

$$\text{D24.* } Sat(f, x, a) : \leftrightarrow$$

1. $C(a) \wedge$
2. $Seq(f, a) \wedge$
3. $SFunc(x) \wedge$
4. $\exists k \exists l (x = \iota_{k,l} \wedge f_k \subseteq f_l) \vee$
 $\exists y (x = \bar{y} \wedge \neg Sat(f, y, a)) \vee$
 $\exists y \exists z (x = y + z \wedge (Sat(f, y, a) \vee Sat(f, z, a))) \vee$
 $\exists k \exists y (x = \cap_k y \wedge \forall g (Seq(g, a) \rightarrow (g_{\bar{k}} f \rightarrow Sat(g, y, a)))$

$$\text{D25.* } x \in Ct_a : \leftrightarrow x \in S \wedge \forall f (Seq(f, a) \rightarrow Sat(f, x, a))$$

$$\text{D26.* } x \in Ct_k : \leftrightarrow \exists a (C(a) \wedge k = \mathcal{K}(a) \wedge x \in Ct_a)$$

Translational Remarks

Before presenting Defs. 24–26, Tarski introduces important notions, in Polish [p. 76] “pojęcie *zdania słusznego w dziedzinie złożonej z k elementów* oraz *zdania słusznego w każdej dziedzinie indywiduów*”, which can be translated as “a notion of sentence valid in a domain with k elements, and of sentence valid in every domain of individuals”. In German we read “den Begriff der *in einem Individuenbereich mit k Elementen richtigen Aussage* und den Begriff der *in jedem Individuenbereich richtigen Aussage* einführen” [p. 319]. The Polish original, as well as the German translation speak here of a “**domain of individuals**” and not of an “**individual domain**”, as we read in the English translation. This inaccuracy is repeated in Defs. 24–27, Th. 8, Lemmas F and G, and throughout the text. At the same time, on page 202 of the English edition, we read in Theorem 8, and in Lemmas F and G about “classes of individuals”, which is the correct translation. At the end of Lemmas F and G we read about “individual domains” again, however, instead of about “domains

of individuals”. This is a very inaccurate and inconsistent translation of the English version. It may be also relevant to notice that in the third of these definitions, i.e. in Def. 26 of a *sentence valid in a domain with k elements*, Tarski does not specify the domain in question, neither as an “individual domain” (as the English translation has it), nor as an “Individuenbereich” (as in the German translation).

It is also noteworthy that the translator of the German, as well as the translator of the English version, uses a definite article before the term “individual domain” in Def. 25, but not in Def. 26, where the article is indefinite before this term. This choice of the article is quite arbitrary; the indefinite article is also preferable in Def. 25.

Another problematic translation concerns the expression “to **define** their meaning more closely”, which closes the first paragraph. Except for this bold-faced term, this sentence has been translated correctly in both the German and the English versions. Even the controversial and previously avoided expression “intuitive sense” has been kept in both translations. Where in the English version, we read “define”, however, Tarski writes in Polish “**precyzować**” [p. 76], which has been as such translated into German “**präzisieren**” [p. 319], which means the same as “to specify” or “to state more precisely”. We read the same inaccurate translation in the footnote at the bottom of this page. “To define” does not mean the same as “to state more precisely”, but “to state or set forth the meaning of (a word, an expression, etc.)”.

[201] *The Definitions 27–32 are presented. In Def. 27, it is explained what it means that a sentence is valid (true) in every domain of individuals. The page ends with a definition of the important concept of a quantitative sentence in Def. 32.*

D27.* $x \in Ct \Leftrightarrow \forall a(x \in Ct_a)$

D28. It is worth noticing that here, the symbol ‘ ε_k ’ does not denote a name of a certain sentential function (a formula) of $\mathcal{L}$. It becomes obvious, as soon as we try to build a quotation-mark name of ε_k ; we arrive namely at:

$$\varepsilon_k = 'NANN\Pi x_{k+1}Ix_kx_{k+1}N\Pi x_{k+1}AA\Pi x_{k+2}Ix_{k+1}x_{k+2}NIx_{k+1}x_kIx_kx_{k+1}'$$

The expression standing on the right hand side of the identity symbol is not a quotation-mark name of a sentential function of $\mathcal{L}$, since ‘ x_k ’, ‘ x_{k+1} ’ and ‘ x_{k+2} ’ are not free variables of $\mathcal{L}$ (the language of the calculus of classes). For every $k \in \mathbb{N}$, however, ε_k is a certain formula of $\mathcal{L}$. The symbol ‘ ε_k ’ denotes a set of formulas, namely $\{x \mid x \in \mathcal{S} \wedge \exists k(x = \varepsilon_k)\}$. The different formulas of the form ε_k differ from each other only regarding the free variables v_k occurring in them, e.g., in ε_1 , ‘ x_1 ’ is free, in ε_2 it is ‘ x_{11} ’, etc.

D29. ‘ α ’ is – in contrast to ‘ ε_k ’ – a name of a certain formula of $\mathcal{L}$, for which we can build a quotation-mark name:

$$\alpha = '\Pi x_{11}A\Pi x_{11}Ix_{11}x_{11}N\Pi x_{11}NNA\!N\!I\!x_{11}x_{11}NNA\!N\!N\Pi x_{111}Ix_{111}x_{111}N\Pi x_{111}AA\Pi x_{1111}Ix_{1111}x_{1111}N\!I\!x_{1111}x_{1111}Ix_{1111}x_{1111}'$$

When we translate α in the meta-language we arrive at:

$$\alpha^* = '\forall x_1(\forall x_2(x_1 \subseteq x_2) \vee \exists x_2((x_2 \subseteq x_1) \wedge \neg \forall x_3(x_2 \subseteq x_3) \wedge \forall x_3((\forall x_4(x_3 \subseteq x_4) \vee \neg(x_3 \subseteq x_2)) \vee x_2 \subseteq x_3)))'$$

α is a sentence which states that every non-null class includes a one-element class as a part.

D30. β_n states that there are at most n different one-element classes. Since every one-element class consists of one individual, and in turn, for each individual there is one one-element class, the definition states also that there are at most n different individuals.

D31. γ_n states that there are exactly n different one-element classes, and therefore exactly n individuals.

D32. x is a quantitative sentence iff there is a finite sequence p of n natural numbers such that the following holds: x is the logical sum of the sentential functions stating that there are exactly l individuals ($k \leq l \leq n$), or x is the negation of this sum.

[202] *The list of the results characteristic of the calculus of classes begins with Theorem 8 and with Lemmas F and G.*

Additional abbreviations:

' $a \approx b$ ' stands for ' a and b are equinumerous'

' $\mathbb{N}$ ' stands for 'the class of natural numbers'

' $\mathbb{K}$ ' stands for 'the class of cardinal numbers'

' $Inf(k)$ ' stands for ' k is infinite'

' $Fin(X)$ ' stands for ' X has a finite number of elements'

' $Complete(X)$ ' stands for ' X is a complete deductive system'

' $Consistent(X)$ ' stands for ' X is a consistent deductive system'

' $Quantitative(x)$ ' stands for ' x is a quantitative sentence'

' $Rel(R)$ ' stands for ' R is a relation'

' $Equi(x, y, X)$ ' stands for ' x and y are equivalent with respect to X '

T8 $\forall a \forall k ((C(a) \wedge \mathcal{H}(a) = k) \rightarrow \forall x (x \in Ct_a \leftrightarrow x \in Ct_k))$

LF For all a, b, R, f, g , and x , if:

1. $C(a) \wedge C(b) \wedge Rel(R) \wedge$
 $\alpha) \forall f' \forall g' (Rf'g' \rightarrow (Seq(f') \wedge \forall k (f'_k \subseteq a) \wedge Seq(g') \wedge \forall k (g'_k \subseteq b)))$; and
 $(\beta) \forall f' (Seq(f') \wedge \forall k (f'_k \subseteq a)) \rightarrow \exists g' Rf'g'$; and
 $(\gamma) \forall g' (Seq(g') \wedge \forall k (g'_k \subseteq b)) \rightarrow \exists f' Rf'g'$; and
 $(\delta) \forall f' \forall g' \forall f'' \forall g'' \forall k \forall l ((Rf'g' \wedge Rf''g'' \wedge k \in \mathbb{N} \wedge l \in \mathbb{N} \wedge k \neq 0 \wedge l \neq 0) \rightarrow$
 $(f'_k \subseteq f''_l \leftrightarrow g'_k \subseteq g''_l))$;
then:

2. $(Rfg \wedge Sat(f, x, a)) \rightarrow Sat(g, x, b)$.

LG $\forall a \forall b \forall x ((C(a) \wedge C(b) \wedge a \approx b \wedge x \in Ct_a) \rightarrow x \in Ct_b)$

Translational Remarks

Here, Tarski wrote in Polish "pewne wyniki bardziej **szczegółowej** natury" [p. 78], which can be translated as "some results of a more **detailed** nature", and which closes

the first paragraph of this page. In the German version we read “einige Resultate **von mehr spezieller Natur**” [p. 321], which explains the English translation.

In Polish, at the beginning of Lemma G we read that “Jeśli klasy indywiduów *a* i *b* są **równej mocy**” [p. 78], which means the same as “If the classes *a* and *b* of individuals **are equinumerous**”. The German translation is accurate here “Wenn die Klassen von Individuen *a* und *b* **gleichmächtig sind**” [p. 321]. Even though these two terms are equivalent, they are not synonymous.

[203] *Theorems 9–12 as well as Lemmas H and I are presented. They all, with the exception of T11, employ the notion of a cardinal number in order to determine the concept of a sentence being valid in a domain of individuals.*

- T9 $\forall k(k \in K \rightarrow (Complete(Ct_k) \wedge Consistent(Ct_k)))$
 T10a $\forall k(k \in K \rightarrow Pr \subseteq Ct_k)$
 T10b $\forall k(k \in K \rightarrow \neg(Ct_k \subseteq Pr))$
 T11 $\forall k \forall X((k \in \mathbb{N} \wedge X = Ax \cup \{\alpha, \gamma_k\}) \rightarrow Ct_k = Cn(X))$
 T12 $\forall k \forall X((k \in K \wedge Inf(k) \wedge X = Ax \cup \{\alpha\} \cup \{x \mid \exists l(l \in \mathbb{N} \wedge x = \bar{\gamma}_l)\}) \rightarrow Ct_k = Cn(X))$
 LH $\forall k(k \in K \rightarrow \alpha \in Ct_k)$
 LIa $\forall k \forall l((k \in \mathbb{N} \wedge l \in K \wedge l \neq k) \rightarrow (\gamma_k \in Ct_k \wedge \gamma_k \notin Ct_l))$
 LIb $\forall k \forall l((k \in \mathbb{N} \wedge l \in K \wedge l \neq k) \rightarrow (\bar{\gamma}_k \notin Ct_k \wedge \bar{\gamma}_k \in Ct_l))$

[204] *At the beginning of the page, Lemma K is stated. The Theorems 13–16 presented later are crucial for the concept of a sentence being valid in a domain with *k* elements.*

- LK $\forall x \forall X((x \in S \wedge X = Ax \cup \{\alpha\}) \rightarrow \exists y(y \in S \wedge Equi(x, y, X) \wedge (Quantitative(y) \vee y \in Pr \vee \bar{y} \in Pr)))$
 T13 $\forall k \forall X((k \in K \wedge Inf(k) \wedge X \subseteq S \wedge Fin(X) \wedge \forall x(x \in X \rightarrow x \notin Ax)) \rightarrow \neg(Ct_k = Cn(X)))$
 T14a $\forall k \forall l((k \in \mathbb{N} \wedge l \in K \wedge l \neq k) \rightarrow \neg(Ct_k \subseteq Ct_l))$
 T14b $\forall k \forall l((k \in \mathbb{N} \wedge l \in K \wedge l \neq k) \rightarrow \neg(Ct_l \subseteq Ct_k))$
 T15 $\forall k \forall l((k \in K \wedge Inf(k) \wedge l \in K \wedge Inf(l)) \rightarrow Ct_k = Ct_l)$
 T16 $\forall k \forall x((k \in K \wedge Inf(k) \wedge x \in Ct_k) \rightarrow \exists l(l \in \mathbb{N} \wedge x \in Ct_l))$

Translational Remarks

The last sentence beginning on this page, reads in Polish as follows “Wobec twierdzeń 14–16 (bądź lematu I) dla każdej liczby naturalnej *k* istnieje takie zdanie, które jest słuszne w dziedzinie złożonej z *k* elementów, a nie jest słuszne w żadnej dziedzinie innej mocy;” [p. 80]. It has been translated correctly into German and into English, except for the bold-faced term “**every**”, which does not figure in the original, but only in both translations “Gemäß den Sätzen 14–16 (bzw. Lemma I) gibt es für jede natürliche Zahl *k* eine solche Aussage, welche in **jedem** Bereiche mit *k* Elementen und in keinem Bereiche von anderer Mächtigkeit richtig ist;” [p. 323]. Even though, Tarski was not explicit here, it follows from Lemma G that if *x* is a sentence valid in a domain *a* of individuals, and if a domain *b* of individuals has the same cardinal

number as the domain a , then x is valid also in b , and therefore in **every** domain with the same number of elements.

[205] *Theorem 17 is presented. When it is combined with Ths. 11 and 12 it gives a structural description of all complete deductive systems containing all the axioms and the sentence α .*

T17a $\forall X((X \subseteq S \wedge Ax \subset X \wedge \alpha \in X \wedge \text{Consistent}(X)) \rightarrow \exists k(k \in K \wedge X \subseteq Ct_k))$

T17b $\forall X((X \subseteq S \wedge Ax \subseteq X \wedge \alpha \in X \wedge \text{Consistent}(X)) \wedge \text{Complete}(X)) \rightarrow \exists k(k \in K \wedge X = Ct_k))$

The article “Some methodological investigations on the definability of concepts” is based on a talk Tarski gave in 1934. The results concerning the problem of completeness had, however, already been presented by Tarski to the session of the Logical Section of the Warsaw Philosophical Society on 15 June 1932.³⁹ It allows us to also conclude that between these dates, i.e. in 1933 when the Polish version of the monograph on truth appeared, these were the views Tarski held.

Patterson (2012) argues that the concepts presented in Tarski (2006f), especially in the section on the problem of completeness, were Tarski’s contribution to Intuitionistic Formalism.

Tarski attempts to establish some general results about the conditions under which a deductive theory (a) completely determines the concepts expressed by its non-logical vocabulary and (b) expresses *all* of the intuitive concepts of the domain from which these concepts are drawn—e.g., the conditions under which a theory expresses, for instance all geometrical concepts. (Patterson 2012, p. 81)

In the above mentioned article, Tarski introduces an auxiliary concept of a set of sentences being *essentially richer* than another *with respect to its terms*.

Let X and Y be any two sets of sentences. We shall say that the set Y is *essentially richer than the set X with respect to specific terms*, if (!) every sentence of the set X also belongs to the set Y (and therefore every specific term of X also occurs in the sentences of Y) and if (") in the sentences of Y there occur specific terms which are absent from the sentences of X and cannot be defined, even on the basis of the set Y , exclusively by means of those terms which occur in X .

If now there existed a set X of sentences for which it is impossible to construct an essentially richer set Y of sentences with respect to specific terms, then we should be inclined to say that the set X is complete with respect to its specific terms. It appears, however, that there are in general no such complete sets of sentences, apart from some trivial cases. (Tarski 2006f, p. 308)

Furthermore, Tarski introduces another concept essential for the concept of completeness, i.e., the concept of *categoricity*. He remarks only that “a set of sentences is called categorical if any two interpretations (realizations) of this set are isomorphic”

³⁹Tarski (2006f, p. 297, ftn.1).

(Tarski 2006f, p. 309).⁴⁰ For a specific definition of the term “categoricity” Tarski refers again to Veblen (1904). After these preliminaries Tarski states that

a set X of sentences is said to be *complete with respect to its specific terms* if it is impossible to construct a categorical set Y of sentences which is essentially richer than X with respect to its specific terms. In order to establish the incompleteness of a set of sentences it is from now onwards requisite to construct a set of sentences which is not only essentially richer but also categorical. (Tarski 2006f, p. 311)

“Categoricity” and “completeness” of a set (or a class) of sentences are two crucial notions which are included in Theorem 17 on the consistency of classes of sentences.⁴¹

Translational Remarks

In the 2nd footnote, Tarski speaks of the “**algebra klas**” [p. 81], which can be translated as “**algebra of classes**”, in German “**Klassenkalkül**” [p. 324]. He clarifies the difference in the footnote on page [207].

Furthermore, the last sentence of the English footnote was added only to the English translation, hence it does not occur in the Polish and German versions.

[206] *Now the sentences which are valid in every individual domain are considered. Thus, Theorems 18–25, as well as Lemma L are presented.*

- T18 $\forall x(x \in Ct \leftrightarrow \forall k(k \in K \rightarrow x \in Ct_k))$
- T19 $\forall x(x \in Ct \leftrightarrow \forall k(k \in \mathbb{N} \rightarrow x \in Ct_k))$
- T20a $\forall k(k \in K \rightarrow Ct \subseteq Ct_k)$
- T20b $\forall k(k \in K \rightarrow \neg(Ct_k \subseteq Ct))$
- T21a *Consistent*(Ct)
- T21b $\neg$ *Complete*(Ct)
- T22a $Pr \subseteq Ct$
- T22b $\neg(Ct \subseteq Pr)$
- T23 $\forall x(Quantitative(x) \rightarrow x \notin Ct)$
- T24 $\forall X(X = Ax \cup \{\alpha\} \rightarrow Ct = Cn(X))$
- T25 $\forall x((x \in S \wedge x \notin Ct \wedge \bar{x} \notin Ct) \rightarrow \exists y(Quantitative(y) \wedge Equi(x, y, Ct)))$
- LLa $\alpha \in Ct$
- LLb $\alpha \notin Pr$

[207] *Theorems 26 and 27 are presented. The latter states that the class of sentences valid in every domain is an element of the class of true sentences, but not the other way around*

⁴⁰Patterson (2012, p. 82) notes that the quotation marks around the terms “interpretations” and “realizations” have been stripped in the English version compared to the German one. In Polish, there are no quotation marks around the term “interpretation” and instead of “realizations” Tarski writes “relacja R przekształca x' w x'' ” (Tarski 2006f, p. 133), which could be translated as “relation R transforms x' in x'' ”. This, however, is equivalent to the English translation.

⁴¹For a detailed discussion on categoricity and completeness of terms in Tarski’s works see Patterson (2012, pp. 81–7), for a discussion on essential richness see Ray (2005).

T26a $\forall a(\forall b(b \subseteq a) \rightarrow \forall x(x \in Tr \leftrightarrow x \in Ct_a))$

T26b $\forall a\forall k(\mathcal{K}(a) = k \rightarrow Tr = Ct_k)$

T27a $Ct \subseteq Tr$

T27b $\neg(Tr \subseteq Ct)$

Translational Remarks

Theorem 26 reads in Polish as follows “Twierdzenie 26. *Jeśli a jest klasą wszystkich indywiduów, to na to, by $x \in Vr$, **potrzeba i wystarcza**, by x było zdaniem słusznym w dziedzinie a; jeśli więc mocą klasy a jest liczba kardynalna k, to $Vr = Rk$.*” [p. 83]. This can be translated as “Theorem 26. *If a is the class of all individuals, then in order for $x \in Tr$ it is **necessary and sufficient** that x is a sentence valid in the domain a; thus if k is the cardinal number of the class a, then $Tr = Ct_k$.*” This inaccuracy had already occurred in the German translation, where we read “Satz 26. *Ist a die Klasse aller Individuen, so gilt $x \in Vr$ **dann und nur dann, wenn** x eine im Be-reiche a richtige Aussage ist; wenn also die Kardinalzahl k die Mächtigkeit der Klasse a ist, so ist $Wr = Rt_k$* ” [pp. 325–326].

The difference in the translations is interesting inasmuch as Tarski decided to write “necessary and sufficient”, whereas both translations use “if and only if”. In other places where Tarski writes “necessary and sufficient” or “if and only if” both translations are accurate.

[208] *The final Theorem of Sect. 3 is presented. It could be considered a definition of a true sentence, however this accidental circumstance is owed to the special characteristics of the calculus of classes. Later, Tarski summarises the positive results regarding the construction of a formally correct and materially adequate definition of a true sentence for the language of the calculus of classes*

T28 $\forall x(x \in Tr \leftrightarrow x \in Cn(Ax \cup \{\alpha\} \cup \{y \mid \exists l(l \in \mathbb{N} \wedge y = \bar{\gamma}_l)\}))$

2.5 Section 4. The Concept of True Sentence in Languages of Finite Order

[209] *The same procedure as used for the investigation of the language of the calculus of classes is now applied to other formalized languages. Here, it is used to present the concept of a true sentence in regard to the languages of finite order. Therefore, the goal remains the same.*

At the very beginning, Tarski emphasizes that the sketched generality of this method considers exclusively languages of the same structure as those which are known today, since even these languages are very different from one another. These differences are of rather minor importance, however, which enables Tarski to draw a general sketch of this method.

Translational Remarks

In Polish [pp. 84–85] and in German [p. 327], ‘method’ (of construction) is always used in singular, not in plural as we read in English, first at the very beginning of the paragraph and then again in the 6th line of the new chapter.

[210] *The languages which are to be considered differ from one another in some aspects, therefore the following investigation will be rather general. At the beginning, Tarski constructs a corresponding metalanguage, starting with the three groups of primitive expressions.*

The second footnote on this page has been significantly shortened in the English version. The full version gives an insight into the complex relationship between Tarski and Leśniewski, and thus it contributes to the understanding of the content of the remaining text. The German translations [pp. 328–329] provides us with the complete version of it.

Um die folgenden Ausführungen in eine ganz präzise, konkrete und dabei genügend allgemeine Form zu kleiden, würde es genügen, als Gegenstand der Untersuchungen die Sprache irgend eines vollständigen System der mathematischen Logik zu wählen. Eine solche Sprache kann nämlich als “universale” Sprache betrachtet werden, und zwar in dem Sinne, dass alle anderen formalisierten Sprachen – wenn man von Unterschieden “kalligraphischer” Natur absieht – entweder Bruchstücke von ihr sind oder sich aus jener Sprache bzw. aus ihren Bruchstücken durch Hinzufügung dieser oder jener Konstanten gewinnen lassen, wobei semantische Kategorien der betreffenden Konstanten [...] schon durch gewisse Ausdrücke der gegebenen Sprache repräsentiert sind; die Anwesenheit oder Abwesenheit derartige Konstanten übt, wie wir uns überzeugen werden, nur einen minimalen Einfluss auf die Lösung des uns interessierenden Problems aus. (Tarski 1935, pp. 328–329)

This part has been translated into English without major discrepancies. In English, it ends with the following sentence:

As such a language we could choose the language of the general theory of sets which will be discussed in Sect. 5, and which might be enriched by means of variables representing the names of two- and of many-termed relations (or arbitrary semantical categories). (Tarski 2006g, p. 210)

In Polish, and also in German, the footnote does not end with this one sentence, instead we read the following.

Nichtsdestoweniger konnte ich mich hier nicht entschliessen die Untersuchungen in der erwähnten Richtung zu konkretisieren, und zwar aus folgendem Grunde. Das einzige mir bekannte vollständige System der mathematischen Logik, dessen Formalisierung – im Gegensatz z.b. zum System Whitehead-Russell – keine Einwände zulässt und vollkommene Präzision aufweist, ist das von Leśniewski begründete System, das bisher in seiner Gänze noch nicht veröffentlicht worden ist [...]. Leider scheint mir dieses System wegen gewisser spezifischer Eigentümlichkeiten ein überaus undankbares Objekt für methodologische und semantische Untersuchungen zu sein. Die Sprache dieses Systems ist nicht als etwas potentiell “Fergies” gedacht, sondern als etwas “Wachsendes”: es sind nicht im vorhinein alle Zeichen und Sprachformen vorgesehen, welche in dem Sätzen des Systems erscheinen können; dagegen sind genaue Regeln angegeben, welche in jedem Aufbaustadium des Systems seine sukzessive Bereicherung durch neue Ausdrücke und Formen ermöglichen; im Zusammenhang damit besitzen solche Termini wie “Aussage”, “Folgerung”, “beweisbarer Satz”,

“wahre Aussage” in Bezug auf das besprochene System keine absolute Bedeutung und müssen auf den jeweiligen aktuellen Zustand des Systems bezogen werden. Formal genommen würde es sogar schwer fallen, dieses System der allgemeinen, am Anfang des Sect. 2 gegebenen Charakterisierung der formalisierten deduktiven Wissenschaften unterzuordnen. Um unter diesen Umständen das System Leśniewski’s den Bedürfnissen der vorliegenden Untersuchungen anzupassen, müsste es einer recht gründlichen Umarbeitung unterzogen werden, was jedoch den Rahmen dieser Arbeit vollständig sprengen würde. (Tarski 1935, pp. 328–330)

In the missing part, Tarski gives his reasons for rejecting the option of taking a ‘universal’ (in Polish and in German placed within quotation marks) language of mathematical logic as an object of our investigations. These reasons are that the only complete system of mathematical logic known to him, the formalization of which – in contrast to the systems of Whitehead and Russell 1925 – is flawless and perfectly precise is the system of Leśniewski, which had not been completely published then (cf. Leśniewski 1929 and 1930). Unfortunately, because of its very specific characteristics the system is highly unsuitable for any methodological and semantical investigations. The language of this system is not designed as something potentially ‘finished’, but rather as something ‘growing’. The signs and the linguistic forms of this language are not given, instead the rules of this system are stated. They let us add new expressions and forms to the language. Hence, such terms as ‘sentence’, ‘consequence’, ‘theorem’, and ‘true sentence’ have no fixed meaning and have to be updated to the current state of the system.

The above quoted footnote has given ground to much philosophical debate regarding Tarski and Leśniewski’s relationship, both personal and philosophical. As we know, Leśniewski’s framework which Tarski chose for the original, Polish version of this monograph, was rejected by him in *Nachwort* written for the German translation. As we see, already in this footnote, written in the Polish original, Tarski begins to abandon his mentor’s methods, and he does it in a decisive manner.

Überaus undankbares Objekt für methodologische und semantische Untersuchungen—these are strong words, especially when applied to the cherished system of your *Doktorvater*. On what, if not methodological and semantical investigations pertaining to his system, had Leśniewski given his best energies for more than a decade? Even someone with an ego of smaller size than that of St. Leśniewski might take umbrage at these words. What their effect on his would have been in 1933, when the Polish version appeared, I can only begin to guess. If this behaviour, when faced with criticism from Waław Sierpiński a couple of years earlier, more about which below, is anything to go by, Leśniewski’s reaction to Tarski’s very public apostasy, in a major work that was bound to attract attention in large measure, will have been nothing short of utter outrage. (Sundholm 2003, p. 119)

As Sundholm notes, Tarski’s work on the concept of truth contributed to his abandonment of Leśniewski’s framework, which, a couple of years earlier, Tarski was so convinced of.⁴²

⁴²Cf. Sundholm (2003).

Translational Remarks

As Tarski mentions on the previous page, the differences between the languages investigated here are of a mere ‘calligraphical’ nature. In the original, as well as in both translations, the word ‘calligraphical’ has been written within quotation marks, which suggests that it is not meant literally, but rather in a figurative sense. On this page in the English translation, however, the word ‘calligraphy’ has been written without quotation marks. The same is true of the expressions ‘linearly ordered’ and “‘universal’ language” (2nd footnote), which are also meant to be placed within quotation marks, as they are in Polish [p. 86] and in German [p. 328].

In the above discussed footnote, regarding the terms between the quotation marks, in Polish [p. 87], Tarski writes that these terms “nie posiadają znaczenia **bezwzględnego**”, which could be translated as “have no **absolute** meaning”, in the sense of “**fixed meaning**”. In German, “keine **absolute** Bedeutung” [329].

[211] *Tarski emphasizes here how indispensable in the metalanguage the full axiom system is. Corresponding to the three groups of primitive expression are the three groups of axioms of this system, which are introduced.*

Translational Remarks

Here, in the footnote Tarski, names the conditions which the investigated sciences have to fulfil. These are of an “intuitive nature”, as we read in Polish [p. 88] “**natury intuicyjnej**”. The translator of the German version obviously felt that Tarski might not be understood quite clearly if the difference between an intuitive and formal nature is not made clear, hence the translation “**nicht formaler, sondern inhaltlicher Natur**” [p. 330]. This translation has been carried over onto the English version “**of an intuitive not a formal nature**”, see also [1.2.1].

Still in this footnote, in regard to the rules of inference, Tarski writes that, if needed, they may be **transposed** from the science to the metascience, in Polish original (Tarski 1933 [p. 63]) “regóły wnioskowania, które wolno nam w razie potrzeby **przetransponować** z nauki do metanauki”, which means the same as to “**transpose**”, has been accurately translated into German as “**transponieren**” [p. 330], not in English though where we read **transfer**, see also the commentary to page [183]. In (Tarski 1995c, p. 88) we read “przetransportować”, however, which could suggest that the two terms are seen as equivalent.

[212] *Here, sentential functions are distinguished from sentences. Furthermore, some expressions of the language are marked as constants, which usually are finite in number, and variables, of which there usually are infinitely many.*

Translational Remarks

In the English translation we read about “our next task” [line 6], in Polish it is “pierwsze zadanie” [p. 89], which can be translated as the “first task”, as has been done in the German translation “zunächst die Aufgabe” [p. 331].

As Tarski points out at the beginning of this chapter, his investigation will consider more languages, not just one [line 9], therefore he writes in plural “badanych

języków” [p. 89] as has accurately been translated into German “**der untersuchten Sprachen**” [p. 331].

In Polish [p. 89] we read that “obok nich napotyamy niekiedy i inne znaki, związane z **indywidualnym charakterem** rozważanego języka” which could be translated as “in addition to these, we sometimes find different signs which are connected with the **individual character** of the investigated language.” In German [p. 331] we read “daneben begegnen wir manchmal auch anderen Zeichen, welche mit dem **individuellen Charakter** der betrachteten Sprache zusammenhängen”. Instead of the used term ‘peculiarities’ the word ‘**character**’ would be a much better choice here.

The additional signs “**denote in an intuitive interpretation** concrete individuals, classes, or relations”, as we read in Polish “**oznaczające w intuicyjnej interpretacji** konkretne indywidua, klasy lub relacje” [p. 89]. In German we read that “**in inhaltlicher Deutung** konkrete Individuen, Klassen oder Relationen **bezeichnen**” [p. 331].

A more accurate translation of the Polish sentence “Dokładny opis kształtu tych wyróżnionych wyrażeń i ustalenie ich intuicyjnego sensu są ściśle zależne od **specyficznych własności** rozważanego języka” [p. 90] would be “the exact description of the form of these distinguished expressions and determining their intuitive sense depends upon the **specific properties** of the investigated language”. Also the German translation is not very accurate “Die genaue Beschreibung der Gestalt dieser Aussagefunktionen und die Bestimmung ihres inhaltlichen Sinnes sind von den **speziellen Eigentümlichkeiten** der betrachteten Sprache abhängig” [p. 332]. Also here, the word ‘peculiarities’ is a rather poor choice.

In the first footnote, Tarski mentions other categories of constants and variables which can occur in other languages, which however, are not considered in the present article. Among these expression are so called name-forming functors which, in combination with variables, form composite expressions “which **are or represent** the names of individuals, classes and relations”. This is the correct translation of Polish “**będące nazwami lub reprezentujące** nazwy indywiduów, klas i relacji” [p. 89]. Thereby name-forming functors of colloquial language have been left out. This mistake originated in the German translation where we read “durch die Namen von Individuen, Klassen und Relationen **vertreten werden**” [p. 331].

[213] *Now, the complexities of constants are considered in more detail. Their first sign is called a (sentence forming) functor of the given primitive sentential function and the remaining signs are called arguments. Later, fundamental operations on expressions are introduced.*

Translational Remarks

In Polish, the 3rd sentence of the footnote reads as follows “Z różnych natomiast względów, w które nie będę tu bliżej wchodził, rozróżnianie tych dwóch kategorii wyrażeń **w odniesieniu do języków sformalizowanych** nie jest, zdaniem moim, niezbędne ani celowe” [p. 90]. It can be translated as “For different reasons, which I shall not discuss in detail here, distinguishing between these two categories of expressions **with respect to formalized languages**, in my opinion, is neither necessary nor

purposive.”. And the German version “Dagegen scheint mir aus vielen Gründen, auf die ich nicht näher eingehen werde, die Unterscheidung dieser beiden Kategorien von Ausdrücken (d.i. aussagebildender Funktoren und Namen von Klassen bzw. Relationen) **in Bezug auf formalisierte Sprachen** keineswegs notwendig oder zweckmässig” [p. 332]. The part printed in bold has been left out in the English version.

[214] *Free and bound variables are distinguished as usual, and sentences are defined as sentential functions without free variables. Then other concepts are defined, namely the concepts axiom, consequence, and theorem. Their definitions, for the most part, follow the patterns of Sect. 2. Before we begin with the main task, i.e., the construction of a correct definition of true sentence, another essential notion is introduced, namely the notion of a satisfaction of a sentential function by a sequence of objects.*

[215] *Due to the differences between the various languages, the definition of the concept of satisfaction meets with serious obstacles. Therefore, it is necessary to introduce yet another concept, namely the concept of a semantical category.*

Here, Tarski introduces the notion of *semantical* (or *meaning*) *category*, in Polish *kategorii semantycznej* (lub *znaczeniowej*) [p. 93], and also in German *semantische* (oder *Bedeutungs-*) *Kategorie* [p. 334]. Working within his interpretation of the simple theory of types (STT), the concept of *semantical category* was of crucial importance for Tarski’s investigations. The concept first used by Husserl, was introduced into formal sciences by Leśniewski. There are clear parallels between Tarski’s theory of semantical categories and Russell and Whitehead’s theory of types, although Tarski emphasizes that from the formal point of view his theory more closely resembles Chwistek’s simplified theory of types, and is even an extension of Carnap’s *Typentheorie* presented in *Abriss der Logistik*.⁴³ Furthermore, it is interesting to notice how convinced Tarski was at the time of writing this essay in Polish (1933) that

the theory of semantical categories penetrates so deeply into our fundamental intuitions regarding the meaningfulness of expressions, that it is scarcely possible to imagine a scientific language in which the sentences have a clear intuitive meaning, but the structure of which cannot be brought into harmony with the above theory. (Tarski 2006g, p. 215).

Perhaps, because of this fact, Tarski suggests that both expressions, ‘*semantical*’ or ‘*meaning category*’ are suitable for this theory.

Only two years later, as the German translation appeared, Tarski no longer defended this view. In the German version from 1935, Tarski wrote an additional *Nachwort* stating his new point of view (Tarski 1935, p. 133), as we also read in *Postscript* to the English version [p. 268].

There is, naturally, a debate regarding Tarski’s choice of a logical framework. Loeb⁴⁴ holds that Tarski was always working with a type-theoretical framework,

⁴³Cf. Tarski (2006g), p. 215.

⁴⁴Loeb (2014).

even in the postscript. Sundholm⁴⁵ describes the circumstances which made Tarski change the framework from type-theoretical in the main text to set-theoretical in the postscript. Feferman⁴⁶ argues that considering Tarski's mathematical background, we can state that he was always working in set theory. I am not in the least qualified to render a judgement in this debate, however, in the course of this commentary, it will not be entirely possible to overlook my preferred reading on this matter.

Translational Remarks

Right after referring to Whitehead's and Russell's *Principia Mathematica*, Tarski speaks of “**intuitive content**”, in Polish [p. 93] – “**treść intuicyjną**”, and in German we read “**Inhalt**” [p. 335], as in the English version, see also [1.2.1].

[216] *The necessary and sufficient conditions for two expressions to belong to the same semantical category are listed and simple examples of semantical categories are provided. Furthermore, the first principle of the theory of semantical categories is introduced.*

As Tarski points out, the concept of *semantical category* as it is used by him does not differ much from Carnap's theory of levels, but is rather an extension of it. Tarski's approximate formulation of the conditions for two expressions to *belong to the same semantical category* are very similar to Carnap's main principle of type theory.

“Die Hauptregel der Typentheorie lautet nun: die Werte eines bestimmten Argumentes einer bestimmten Aussagenfunktion können nur Gegenstände vom gleichen Typus sein. Daraus folgt: alle Elemente einer bestimmten Klasse müssen vom gleichen Typus sein, ebenso alle Vorderglieder einer bestimmten Relation, ebenso alle Hinterglieder einer bestimmten Relation (es können aber Vorder- und Hinterglieder derselben Relation von verschiedenem Typus sein), allgemein alle Glieder der gleichen Stelle einer bestimmten Relation.”⁴⁷

The different types must always be strictly separated from one another. This means that the elements of a class cannot be of different types. Carnap's *Verbot der Stufenvermischung* forbears the occurrence of antinomies.

Tarski does not present us with a definition of the notion of semantical category, but instead he explains that

two expressions *belong to the same semantical category* if (1) there is a sentential function which contains one of these expressions, and if (2) no sentential function which contains one of these expressions ceases to be a sentential function if this expression is replaced in it by the other. (Tarski 2006g, p. 216)

Later, Tarski deals with the problem of deciding whether two given expressions belong to the same semantical category and introduces a rule which he calls the *first principle of the theory of semantical categories*.

It is important to notice that what Tarski and Leśniewski called ‘semantical category’ is today (and already by Adjukiewicz) known as a ‘syntactical category’.

⁴⁵Sundholm (2003).

⁴⁶Feferman (2002).

⁴⁷Carnap (1929, p. 31).

Translational Remarks

The Polish sentence “Jako najprostsze przykłady kategorii semantycznych, **spotykanych w różnych znanych językach** [...]” [p. 94] should be translated as “As the simplest examples of semantical categories **met in different known languages** [...]”. It has been accurately translated into German “Als einfachste Beispiele der semantischen Kategorien, **die man in verschiedenen bekannten Sprachen antrifft** [...]” [p. 336]. The bold-typed part has been left out of the English translation.

Later, we read in Polish, “**Z intuicyjnego punktu widzenia** odpowiedź jest niewątpliwa: na to by dwa wyrażenia należały do tej samej kategorii semantycznej, wystarcza, by istniała **choć jedna** funkcja, która by zawierała jedno z tych wyrażen i pozostawała **funkcją zdaniową** po zastąpieniu wyrażenia tego przez drugie” [p. 95]. This can be translated as follows, “**From an intuitive point of view**, the answer leaves no doubt: in order that two expressions shall belong to the same semantical category, it suffices if there exists **at least one** function which contains one of these expressions and which remains a **sentential function** when this expression is replaced by the other.” “From the standpoint of the ordinary usage of language” is a misleading translation. In German we read “**Will man sich an den üblichen Sprachgebrauch anlehnen**, so erscheint die zweite Eventualität viel natürlicher: damit zwei Ausdrücke zu derselben semantischen Kategorie gehören, genügt es, wenn es **nur eine** Funktion gibt, die einen dieser Ausdrücke enthält und die nach der Ersetzung dieses Ausdrucks durch den anderen eine **Aussagefunktion** bleibt” [p. 336], see also [1.2.1].

[217] *The crucial role of the above defined principle is emphasized here. It is essential in the definitions of the concept of sentential function, and of the operation of substitution. Also, the law concerning the semantical categories of sentence-forming functors is closely connected with this principle.*

The *first principle of the theory of semantical categories*, at which Tarski arrived on the previous page, led him directly to formulating

a general law concerning the semantical categories of sentence-forming functors: the functors of two primitive sentential functions belong to the same category if and only if the number of arguments in the two functions is the same, and if any two arguments which occupy corresponding places in the two functions also belong to the same category. (Tarski 2006g, p. 217)

For the sake of clarity, we remind that the sentence-forming functors are signs representing sentential functions.

Translational Remarks

In the second sentence of the first footnote, the word which the translator should have used instead of “often” is “**sometimes**”, as it is in Polish “**niekiedy**” [p. 95]. It is not the translator of the English version that erred first, however, but the translator of the German version who wrote “**oft**” [p. 336].

In the 3rd line of this page, we have an inaccurate English translation: “in the **definition** of the concept of sentential function”, of the Polish expression “przy

precyzowaniu pojęcia funkcji zdaniowej” [p. 95], which means the same as “in specifying the concept of a sentential function”. The German translation is problematic in a different aspect (concerning the usage of the term ‘Begriff’), but the word in bold is translated correctly “bei der **Präzisierung** des Begriffs der Aussagefunktion” [p. 337], see also the commentary to [194].

[218] *Here, the semantical categories are classified, i.e., the concept order of a category is introduced. Following is the convention determining the meaning of this term. Another important reference to Carnap is made.*

In writing the original version of the article (1933), Tarski had in mind only formalized languages, the structure of which adheres to the theory of semantical categories. The notion of *order of semantical category* played a crucial role for the languages investigated in the Polish original.

We require a classification of the semantical categories; to every category a particular natural number is assigned called the *order of the category*. This order is also assigned to all expressions which belong to this category. The meaning of this term can be determined recursively. For this purpose we adopt the following convention (in which we have in mind only those languages which we shall deal with here and we take account only of the semantical categories of the variables): (1) the 1st order is assigned only to the names of individuals and the variables representing them; (2) among expressions of the $n + 1$ th order, where n is any natural number, we include the functors of all those primitive functions all of whose arguments are of at most the n th order, where at least one of them must be of exactly the n th order. Thanks to the above convention all expressions which belong to a given semantical category have the same order assigned to them, which is therefore called the order of that category. (Tarski 2006g, p. 218)

Thus, the 1st order includes only the names of individuals and the variables representing them. To the 2nd order belong the names of classes of individuals and the names of two-, three-, and many-termed relations between individuals. The $(n + 1) - nt$ order is assigned to the functors of all primitive functions, all of whose arguments are of at most the n th order (at least one of them is exactly of the n th order). It is important to notice that the orders of the variables occurring in a language determine the order of this language.⁴⁸

Tarski makes yet another reference to Carnap. Carnap’s definition regards the *order* of the objects, which is determined by its type/level. Except for this, it is analogous to Tarski’s.

“Unter der Stufe eines Gegenstandes verstehen wir eine bestimmte Zahl, die durch den Typus des Gegenstandes bestimmt ist; zu einer Stufe können verschiedene Typen gehören.”(Carnap 1929, p. 31–32)

The concept *order of the category* is essential for the further investigations carried out in Sects. 4 and 5. What is more, it also plays a crucial role in *Postscript*, published first in the German edition in 1935. It is noteworthy that in Sect. 4, the 1st order is assigned only to the names of individuals and to the variables representing them.

⁴⁸Cf. Gruber (2015).

Two years later (in the *Nachwort* of the German edition in 1935), however, they were assigned order 0.⁴⁹

Translational Remarks

In the second footnote on this page, in the English translation, there is a part missing, which we find in Polish and in German. It is the last part of the condition (2) which should be translated as “but are not themselves expressions of the *n*th **or of a lower order**” (the missing part is written in boldface). The Polish phrasing is “a przy tym same nie są wyrażeniami *n*-tego **ani też niższego rzędu**.” [p. 97]. We find the accurate version in German “aber dabei selbst keine Ausdrücke *n*ter **oder niedrigerer Ordnung sind**” [p. 338].

[219] *In order to classify the sentential functions of the language, the concept semantical type is presented. Another way of using the term ‘semantical category’ is explained, i.e., as applying it not to the expressions of the language, but to the objects these expressions denote.*

Furthermore, Tarski defines the notion of *semantical type* which depends on the number of free variables of a given semantical category, i.e. if the number of free variables of every semantical category in two functions is the same, then these functions are of the same semantical type.⁵⁰

[220] *Four kinds of languages are distinguished. The division criterion is the multitude of semantical categories, i.e., whether the expressions and especially the variables belong to a finite or an infinite number of semantical categories. In the case of an infinite number of categories, a distinction is made between those which are bounded from above and those which are not.*

Depending on the multiplicity of the semantical categories appearing in the language, on whether the variables of the language belong to a finite or an infinite number of categories and, in the latter case whether the orders of these categories are bounded above or not, Tarski distinguishes 4 kinds of languages:

1. languages in which all variables belong to one and the same semantical category (e.g. calculus of classes, sentential calculus $+ \forall, \exists$)
2. languages in which the number of categories in which the variables are included is greater than 1, but finite (the variables are bounded above), (e.g. language of the logic of two-termed relations)
3. languages in which the variables belong to infinitely many different categories, but the order of these variables does not exceed a previously given natural number *n* (the variables are bounded above), (e.g. language of the logic of many-termed relations)

⁴⁹For further discussion regarding Tarski’s usage of the notion of *order* see e.g. Coffa (1987), de Rouilhan (1998), Loeb (2014), Patterson (2012).

⁵⁰Cf. Tarski (2006), p. 219.

4. languages which contain variables of arbitrarily high order (the variables are not bounded above), (e.g. language of the general theory of classes)

The languages of the first three kinds, in which the variables are bounded above, are *languages of finite order*, in contrast to the languages of the fourth kind, in which the variables are *not* bounded above, are the *languages of infinite order*.

It is essential to remember that in writing the original version of the article (1933) Tarski had in mind only formalized languages, the structure of which adheres to the theory of semantical categories.⁵¹

Translational Remarks

In Polish we read “przy konstrukcji poprawnej definicji **zdania prawdziwego**” [p. 99], which means the same as “in the construction of a correct definition of a **true sentence**”, as the translator of the German version wrote “bei der Konstruktion einer korrekten Definition der **wahren Aussage**” [p. 340], see also [1.2.3] and the commentary to [168].

[221] *Here, the languages of the 1st kind, which have the simplest logical structure, are discussed in more detail. A typical example of the language of the 1st kind is the language of the calculus of classes. The language of the ordinary sentential calculus enlarged by the addition of universal and existential quantifiers is presented as another, particularly simple example of the languages of the 1st kind.*

Defining truth for the languages of the 1st kind did not present much difficulties for Tarski. By means of the concept of *satisfaction* of a sentential function by a sequence of objects, introduced in Sect.3, he was able to define the concept of *true sentence* for the language of the calculus of classes. Thus, since we are considering sentences, i.e. sentential functions with no free variables, every infinite sequence of classes must satisfy a given sentence if it is to be true. One of the examples given by Tarski is: every infinite sequence of classes satisfies the function $x_1 \subseteq x_1$, hence $\forall x_1 (x_1 \subseteq x_1)$ is a true sentence.

Translational Remarks

The method presented in Sect. 3 can be applied to other languages of the 1st order. Certain small adaptations may be necessary, e.g. to operate “ciągami indywiduów lub relacji, zależnie od **intuicyjnej interpretacji i kategorii** semantycznej występujących w języku zmiennych” [pp. 100–101], which should be translated as “with sequences of individuals or relations depending on the **intuitive interpretation** and the semantical **category** of the variables occurring in the language”. The German translation is also not very accurate “mit Folgen von Individuen oder Relationen – je nach der **inhaltlichen Interpretation** und semantischen **Kategorie** der in der Sprache auftretenden Variablen.” [p. 341], see also [1.2.1]. The English version is actually more accurate than the Polish and German ones inasmuch as the word ‘category’ comes in plural. In fact the variables do not have to belong to one and the same category.

⁵¹Cf. Gruber (2015).

It is perhaps worth mentioning that the Polish word “**kształt**” [p. 101], which can mean “shape” or “form”, and has been translated into German as “**Gestalt**” [p. 342], has been here translated as “**structure**”. This translation could perhaps be accepted, if the translator stuck to it throughout the paper. This unfortunately is not the case. In (Tarski 2006g, p. 223), the same word, has been translated as “**form**”, which actually is a better translation.

[222] *After some concluding remarks regarding the languages of the 1st kind, Tarski proceeds to the languages of the 2nd kind and chooses as an example the language of the logic of two-termed relations. He presents constants and variables of this language.*

Translational Remarks

In the sentence beginning in the fourth line from the bottom, in Polish we read “**W intuicyjnej interpretacji** zmienne pierwszego rzędu reprezentują nazwy indywiduów, zaś zmienne drugiego rzędu – nazwy relacji dwuczłonowych między indywiduami; **z intuicyjnego** zatem, jak zresztą, zgodnie z dalszym opisem języka, i z formalnego punktu widzenia znaki “ v_k ” i “ V_k ” należą **odpowiednio** do dwóch różnych kategorii semantycznych.” [pp. 102–103]. It could be translated as “**In the intuitive interpretation** the variables of the 1st order represent names of individuals, the variables of the 2nd order – names of two-termed relations between individuals; **from the intuitive**, as is in agreement with the further description of the language, and from the formal point of view, the signs ‘ v_k ’ and ‘ V_k ’ belong, **respectively**, to two distinct semantical categories”. This sentence ends on the next page in the English version. Here is the German translation of this sentence, which is also not very accurate “**Bei inhaltlicher Deutung** repräsentieren die Variablen 1^{ter} Ordnung Namen von Individuen, die Variablen 2^{ter} Ordnung Namen von zweigliedrigen Relationen zwischen Individuen; **von inhaltlichem** und übrigens – in Übereinstimmung mit der weiteren Beschreibung der Sprache – auch von formalem Gesichtspunkt aus gehören also die Zeichen “ v_k ” und “ V_k ” **beziehungsweise** zu zwei verschiedenen semantischen Kategorien.” [p. 343]. For further commentary see also [1.2.1].

[223] *Here, the language of the logic of two-termed relations is considered in more detail. First, all the signs and expressions appearing in this language are listed. It is also important to notice that all the crucial definitions of this language are analogous to those of Sect. 2. An example of this language follows at the bottom of the page.*

In the languages of the 2nd order, the variables belong to two distinct semantical categories. The sign ‘ v_k ’ denotes the variables of the 1st order which represent names of individuals. The symbol ‘ V_k ’ denotes the variables of the 2nd order representing names of two-termed relations between individuals. The definitions employed in this language are analogous to the definitions introduced in Sect. 2. Since in this language we are dealing with variables of two distinct semantical categories, however, we also have to consider two operations of generalization or particularization, respectively for the variables of the 1st and of the 2nd order. The same regards the operation of substitution, of which there will also now be two.

According to Tarski, all sentences which are universal quantifications of sentential functions of the following form:

$$\cup_k'' \cap_l' \cap_m' (\rho_{k,l,m}, \cdot y + \overline{\rho_{k,l,m}} \cdot \bar{y})$$

where k , l , and m are natural numbers with $l \neq m$ and y is an arbitrary sentential function in which the variable V_k is not free, are axioms. This is an example of an axiom of this type:

$$\forall X_2 \exists X_1 \forall x_2 \forall x_3 ((X_1 x_2 x_3 \wedge \exists x_4 (X_2 x_4 x_4)) \vee (\neg X_1 x_2 x_3 \wedge \neg \exists x_4 (X_2 x_4 x_4))).$$

[224] *It turns out that the concept of satisfaction, which was defined in Sect. 3, is highly ambiguous, from the semantical point of view. In the light of the consideration made in Sect. 4, it is clear that there is not just one concept of satisfaction, but infinitely many concepts belonging to different semantical categories.*

Translational Remarks

The expression “correct definition” used in the first line, deserves some more attention. It happens repeatedly throughout Tarski’s article, as also on this page, that the Polish “**poprawna i trafna definicja**” [p. 104] is translated simply as “correct definition”. In the German version we read “eine **korrekte und richtige Definition**” [p. 344], which is also confusing. Polish “poprawna” can be translated as “correct” and “trafna” means the same as “adequate”, therefore the accurate translation should be “a **correct and adequate definition**”. Moreover, it may be wondered why there is only *one* term in the English version, where in Polish and German there are *two*, see also [1.2.2].

In Polish, Tarski speaks of extending our “**intuicyjną znajomość**” [p. 104], which means the same as “**intuitive understanding**”. In German it is the “**inhaltliche Kenntnis**” [p. 344], however, “**intuitive Verständnis**” would have been a better translation, see also [1.2.1].

In the previous chapter Tarski spoke of a sentential function being satisfied by several objects. In this respect, the concept of satisfaction was rather ambiguous, since in fact, we were dealing with several concepts of satisfaction corresponding to several distinct semantical categories. By exact examination of the examples of Sect. 3 it becomes clear that “between the free variables of the sentential function and the objects which satisfy **this function** there exists a strict semantical correlation [...]” [p. 224]. Note that in Polish the satisfied function is in singular “między zmiennymi wolnymi funkcji zdaniowej a spełniającymi **tę funkcję** przedmiotami zachodzi ścisła odpowiedniość semantyczna [...]” [pp. 104–105]. It is also the case in the German version “zwischen den freien Variablen der Aussagefunktion und den **diese Funktion** erfüllenden Gegenständen eine strenge semantische Zuordnung besteht [...]” [p. 345].

As Tarski later points out, if we are investigating a language in which the variables belong to at least two different semantical categories, then “it does not suffice to restrict consideration to only a single category of objects **while speaking of functions of this language being satisfied by objects**”. This is a more accurate translation of the

Polish “**mówiąc o spełnianiu funkcji tego języka przez przedmioty**, nie możemy się ograniczyć do rozważania przedmiotów jednej tylko kategorii” [p. 105]. This inaccurate translation originated in the German translation “so kann man sich nicht darauf beschränken, **bei der Erörterung des Begriffs des Erfülltseins** nur eine einzige Kategorie von Gegenständen zu betrachten” [p. 345].

In Polish the word “only” does not come up in the sentence between the brackets (line 6 from the bottom), it reads “ostatnia dziedzina składa się, jak i poprzednio, wyłącznie z funkcji zdaniowych” [p. 105] and can be translated as “the last domain consists as before of exclusively sentential functions”. The additional term had already occurred in the German translation “**nur** der letzte Bereich besteht wie vorher ausschliesslich aus Aussagefunktionen” [p. 345].

[225] *The category of the concept of satisfaction depends on two circumstances, i.e., the number and the categories of the free variables occurring in the sentential functions to which the concept of satisfaction relates. These remarks are followed by some examples.*

The semantical category of each single concept of satisfaction depends on the semantical type of the sentential function to which the concept of satisfaction is applied. There always are two semantically distinct concepts of satisfactions corresponding to functions belonging to two distinct semantical types. Tarski provides some examples, i.e. the function

$$\cap'_2 \cap'_3 (\overline{\rho_{1,2,3}} + \rho_{1,3,2})$$

which in the present notation is written as:

$$\forall x_2 \forall x_3 (\neg X_1 x_2 x_3 \vee X_1 x_3 x_2)$$

is satisfied only by symmetrical relations.

[226] *The concept of satisfaction in its new form is a two-termed relation, whose domain consists of sequences and whose codomain consists of sentential functions. Since the language of the logic of relations contains variables of two different semantical categories, the method applied as in Sect. 3 proves useless here. This conclusion is followed by some examples.*

Translational Remarks

In the Polish sentence we read “pojęcie to okazało się o tyle ogólniejsze od **poprzednich**, że – z intuicyjnego punktu widzenia – “objęło” **je wszystkie jako szczególne wypadki** [...]” [p. 106]. It corresponds to the English translation of the sentence beginning in line 3 of this page, and it would be better translated as “this concept happened to be sufficiently more general than the **previous ones** – intuitively speaking – to “include” **them all as special cases** [...]”. In Polish and in German the terms in bold are used in plural. Also, the term “include” is placed within quotation marks, which suggests that it is not to be understood literally, but rather figuratively.

In German we read “es erwies sich, dass dieser Begriff insofern allgemeiner als **die vorhergehenden** ist, als er – anschaulich gesprochen – **sie alle als Spezialfälle** “umfasst”?[...]” [p. 347].

Later on, we read on this page that since “the language of the logic of relations contains variables of two different semantical categories, we must likewise use **at least** two categories of sequences in our investigations.” The part in bold has been added as a translation of the Polish term “**przynajmniej**” which occurs in this place in Tarski’s original paper. Also in German [p. 347] we read “wenigstens” in this place.

It is perhaps clear that the ambiguity, of which we read at the very bottom of this page and which regards the concept of satisfaction, is a “**semantical ambiguity**”. In English it is not explicit, however, as it is in Polish “**wieloznaczności semantycznej**” [p. 107] and in German “**semantischen Mehrdeutigkeit**” [p. 348].

[227] *A new interpretation of the concept of satisfaction, which deprives it of its ambiguity, is sought for. Two new methods enabling this are introduced, namely the method of many rowed sequences and the method of semantical unification of the variables. The first of the two methods is explained in more detail.*

Defining truth for the languages of the 2nd, the 3rd, and the 4th kind turned out to be a more complicated task, hence Tarski introduced two new methods, which he called the *method of many-rowed sequences* and the *method of semantical unification*.

The first method requires that we should treat satisfaction not as a two-termed, but as a three-termed relation which holds between sequences of individuals, between sequences of two-termed relations and between sentential functions. We use the following mode of expression: ‘the sequence f of individuals and the sequence F of relations together satisfy the sentential function x . (Tarski 2006g, p. 217)

Tarski used this method in order to define truth for the second kind of languages. As an example Tarski states that

the sequence f of individuals and the sequence F of relations together satisfy the function $\rho_{1,2,3}$ if and only if the individual f_2 stands in the relation F_1 to the individual f_3 . (Tarski 2006g, p. 227)

We can also say that the function $\forall x_2 \forall x_3 (\neg X_{1x_2x_3} \vee X_{1x_3x_2})$ is satisfied only by symmetrical relations.

Translational Remarks

In regard to the second paragraph of this page, we can remark that the **method** (in Polish [p. 107], and in German [p. 348] used in singular) introduced in Sect. 3 can also be applied to languages investigated in Sect. 4, however it has to be significantly modified. It is possible to free the concept of satisfaction of its ambiguity and at the same time give it “such general character, that it “**included**” as **special cases** all of the original concepts of satisfaction”. This is an accurate translation of this part of the Polish sentence “tak ogólny charakter, że “**obejmuje**” jako **szczególne przypadki** wszystkie pojęcia spełniania w ich pierwotnym ujęciu.” [p. 108]. It is curious that this phrase has been translated so inaccurately now, when it had already

appeared in the text on the previous page, and received a correct translation there. This time, the error occurred in the German translation, and was simply copied onto the English one; “einen so allgemeinen Charakter annimmt, dass er **alle Spezialfälle** des ursprünglichen Begriffs des Erfülltseins “umfasst”.” [p. 348].

Further, the content of the discussed term, namely ‘satisfaction’ is for Tarski **intuitive**, as in Polish [p. 108] we read “Treść **intuicyjną**”, see also [1.2.1].

In the same sentence, it is perhaps obvious that the **function** which is being satisfied by the sequence f of individuals and the sequence F of relations is a “**sentential function**”. It is made explicit, however, in Polish “funkcję zdaniową” [p. 108], and in German “Aussagefunktion” [p. 348].

Another part which has also been left out of the English translation regards the next sentence, particularly the general definition to be formulated. In Polish we read “Przy formułowaniu ogólnej definicji **rozważanego zwrotu**[...]” [p. 108], and we find a more or less accurate German translation “Um **für diese Wendung** eine allgemeine Definition zu formulieren [...]” [p. 348]. It means the same as “In order to formulate a general definition **of the discussed notion**”. An attentive reader certainly realizes that this sentence regards the concept of satisfaction. Tarski’s paper is extremely demanding, which he was aware of, and hence was very specific and detailed in Polish leaving no blanks to be filled out, however, even by the most attentive readers.

[228] *Here, the method of semantical unification of the variables is considered in detail. It is shown how every sentence about individuals can be transformed into an equivalent sentence about relations. This is crucial for the present investigations of the language of the logic of relations. The intuitive interpretation of the expressions of this language is changed, while its formal structure is left intact.*

Tarski introduces the method of semantical unification which, as he will later show, can be successfully applied to both the languages of the 2nd and the 3rd kind.

Translational Remarks

In introducing the method of semantical unification of the variables in regard to the language of the logic of relation, the intuitive interpretation of the expressions of this language is changed, whereas their formal structure remains the same. As we read in the last paragraph, “All constants will retain their previous meaning, whilst all variables both of the 1st and 2nd order are from now on to represent names of **two-termed relations between individuals**”. This is the complete translation of the Polish sentence “Mianowicie znaki stałe zachowują swe dawne znaczenie, natomiast wszystkie znaki zmienne zarówno pierwszego, jak i drugiego rzędu reprezentują odąd nazwy **relacji dwuczłonowych między indywiduami**” [pp. 109–110]. The inaccuracy occurred in the German version, where this sentence reads as follows “Alle Konstanten sollen dabei ihre frühere Bedeutung behalten, während alle Variablen sowohl 1ter wie 2ter Ordnung von nun an Namen **zweigliedriger Relationen** vertreten sollen” [p. 350].

In the last sentence of this page, Tarski writes that “**intuicyjnie** jest niemal oczywiste” [p. 110], which means the same as “**intuitively** it is almost evident”. In the German translation we read “**Inhaltlich** ist es fast evident” [p. 350], see also [1.2.1].

[229] *The new intuitive interpretation of the expressions of the language enables the application of the method used for the languages of the 1st kind and also for the language now considered. Certain complications of a technical nature arise, it is shown, however, how they can be overcome. Later, it is explained how the two introduced methods can be applied to all languages of the 2nd kind.*

Translational Remarks

The considered language can be investigated by exactly the same **method** as the languages of the 1st kind (line 6). In Polish [p. 110], but also in German [p. 350], we have a singular, not a plural of this noun.

An attentive reader has most definitely noticed that not *semantical*, but **sentential functions** are meant here (2nd sentence from the bottom). As we read in Polish “**funkcjami zdaniowymi**” [p. 111], and in German “**Aussagefunktionen**” [p. 351].

A minor supplementation of the last sentence in the footnote is required. Namely, in Polish we read that “**zdania zawierające tego rodzaju zmienne dają się bowiem całkowicie wyeliminować z zakresu rozważań**” [p. 111]. It can be translated as “sentences which contain such variables can be **completely eliminated from the area of consideration**”. Also, this inaccuracy originated in the German translation, where we read “Aussagen, die derartige Variable enthalten, kann man nämlich in der Weise **ausschalten**, [...]” [p. 351].

[230] *It is explained how the method of semantical unification of the variables is to be applied. A new essential concept is introduced, namely the concept of the unifying category. The new concept is described in detail and some examples are provided.*

Here, Tarski emphasizes the essential part played by the concept of the *unifying category*, which is understood as

that semantical category in which all the variables of the language studied can be interpreted.
(Tarski 2006g, p. 230)

The method of semantical unification requires that a category unifying all the variables of the languages be introduced, which itself cannot be of lower order than any of the variables of the language. Consequently, sequences of the terms of this category and the relation of satisfaction holding between these sequences and the corresponding sentential functions must be of higher order than all the variables of the language.⁵²

Translational Remarks

In the second paragraph, Tarski emphasizes how important the choice of the *unifying category* in applying the method of semantical unification of the variables is. There is only one requirement which the unifying category has to fulfil. The

⁵²Cf. Gruber (2015).

way it has been translated into English, however, the requirement sounds somewhat awkward and ambiguous and probably made a lot of readers wonder what the “effective objects” are. In Polish, the requirement is the following: “tego, by wszystkim przedmiotom każdej kategorii semantycznej, reprezentowanej przez zmienne danego języka, można było przyporządkować **“efektywnie”** przedmioty tej wybranej kategorii i to w sposób jedno-jednoznaczny (tj. tak, by różnym przedmiotom odpowiadały różne)” [p. 111]. It can be translated as “that with all objects of every semantical category which is represented by the variables of the given language, objects of this chosen category could ‘effectively’ be correlated in a one-one fashion (i.e., so that to different objects, different ones correspond)”. The German translation is accurate “dass man nämlich allen Gegenständen jeder semantischen Kategorie, die durch die Variablen der gegebenen Sprache repräsentiert ist, **“effektiv”** Gegenstände der gewählten Kategorie zuordnen kann, und zwar in eindeutiger Weise (d.h. so, dass verschiedenen Gegenständen verschiedene entsprechen)” [p. 351].

The second discrepancy within the English translation is rather minor. Regarding the choice of the unifying category, Tarski writes that it cannot be always made from the categories present in the particular language. In Polish we read that “zmienne **rozważanego języka**” [p. 112], which means the same as “the variables of the **considered language**”, and has been accurately translated into German “die Variablen der **betrachteten Sprache**” [p. 351].

Later, Tarski adds two remarks, in the first one we read that “the unifying category cannot be of lower order than **any one** category among those occurring in the language”, which has been translated from German “die vereinheitlichende Kategorie kann nicht niedrigerer Ordnung sein als **irgend eine** Kategorie der in der Sprache vorkommenden Kategorien” [p. 352]. In Polish we read “kategoria ujednastajniająca nie może być niższego rzędu od **żadnej z** kategorii, reprezentowanych w języku” [p. 112]. An accurate translation of the original is that “the unifying category cannot be of lower order than **any of** the categories represented in the language”. It avoids possible ambiguity and misinterpretations.

[231] *The essential advantages of the method of semantical unification of the variables over the method of many-rowed sequences are mentioned shortly. Then, the languages of the 3rd kind are considered, choosing as an example the language of the logic of many-termed relations.*

Translational Remarks

In the first paragraph, it is perhaps clear that the essential advantages consider here the “method of **semantical** unification of the variables”. Nevertheless, the adjective “semantical” should not have been left out of the English translation, since it is a part of the name of the method. Hence, it reads in Polish “metody ujednastajniania **semantycznego** zmiennych” [p. 112]. The German translation is just as inaccurate as the English one here, however, and it reads “der Methode der Vereinheitlichung der Variablen” [p. 352].

[232] *The description of the language of the logic of many-termed relations is completed. Then, it is shown how to conceive the concept of satisfaction and to construe the definition of truth for the considered language. The method of many-rowed sequences cannot be applied here.*

Translational Remarks

The Polish original, and the German translation, are more precise when referring to the operations of quantification. Where, in the English translation we read about the operations of “quantification”, the two previous versions speak clearly of the “generalization”. In Polish this sentence begins as follows “Jako operacje **generalizowania** wprowadzamy **generalizowanie**...” [p. 114], and in German “Als Operationen des **Generalisierens** führen wir das **Generalisieren** ...” [p. 354]. Therefore, in the English translation one should read about the “operation of **universal quantification**”, see also [1.2.4].

[233] *It is shown how the method of semantical unification of the variables can successfully be applied to the language of the logic of many-termed relations. Furthermore, it is shown how to partially unify the semantical categories of the variables of the considered language.*

Translational Remarks

A minor inaccuracy occurred in the English translation. In Polish and in German, before giving the example we read “**w szczególności**” [p. 115] and “**insbesondere**” [p. 355] respectively, which can mean “in particular”. It may be that this has no great influence on the general understanding of this sentence, however, on the other hand, it may point out to the particularly good example of applying the method of semantical unification of the variables.

We have already encountered the discrepancy which now occurs in the last paragraph of this page (see the commentary to [159]). The Polish phrasing “**dawne znaczenie**” [p. 116], and the German “die frühere **Bedeutung**” [p. 355] can be translated as “the previous **meaning**”, which is what Tarski meant here. The bold-faced words in Polish and in German have yet another denotation, however, namely they can mean “significance”, which is the unfortunate option the translator chose.

Another, this time not so minor, discrepancy within the English translation occurred in the footnote at the bottom of this page. Here, we read in Polish “mianowicie jako klasy tych wszystkich klas, które są **równej** mocy z pewną klasą daną” [p. 116] which can be translated as “namely as the classes of all those classes which are **equinumerous** with a given class”. In German we read “nämlich als die Klassen aller jener Klassen, die mit einer gegebenen Klasse **gleichmächtig** sind” [p. 355], see also p. 129. The bracketed part regarding *Principia Mathematica* has been added later and appears only in the English version.

[234] *Introducing the phrase ‘the sequence f of individuals and the sequence F , whose terms form classes of finite sequences of individuals, together satisfy the given sentential function’ makes it possible to use the method of many-rowed sequences.*

First, a one-one correlation between the variables V_k^1 and terms of the sequence F must be set up. This is followed by some examples and the conclusion.

Translational Remarks

For the sake of clarity, in the first sentence we should read the “**sequences of individuals**”, as it is in Polish “**ciągów indywiduów**” [p. 116] and in German “**Folgen von Individuen**” [p. 356].

In the next sentence, the presence of the word ‘still’ is rather surprising. The Polish version of this part of this sentence does not contain the possible translation of this word. Instead we read that “zmienne należą odtąd do dwóch tylko różnych kategorii semantycznych” [p. 116], which can be translated as “the variables belong from now on to only two different semantical categories”. In German we read “gehören die Variablen von nun an nur mehr zu zwei verschiedenen semantischen Kategorien” [p. 356].

While introducing the method of many-rowed sequences, another error occurred in the English translation. It is clear that it is the “**sequences f and F** ” which satisfy the sentential function, and not “**functions f and F** ” as we read in the English version. Furthermore, a crucial part has been left out.

In Polish we read that “podobnie funkcję $\rho_{k,m,n}$ spełniają łącznie **te i tylko te** ciągi f i F , które sprawdzają warunek ...” [p. 117]. It can be translated as “similarly, the function $\rho_{k,m,n}$ is satisfied together by **those and only those** sequences f and F which satisfy the following condition...”. Also in German we read “in analoger Weise erfüllen die Funktion $\rho_{k,m,n}$ gemeinsam **jene und nur jene** Folgen f und F , die folgender Bedingung genügen ...” [p. 356].

In the footnote of the Polish text [p. 117], Sierpiński 1928, pp. 43–44 is given as an example. In the German translation the better known book by Fraenkel is mentioned. Tarski makes an explicit comment on this in a letter to Kazimierz Twardowski

instead of the book by Sierpiński I can quote in my work a book by Fraenkel (I prefer to quote books by Polish authors, but I cannot be exempted from quoting Fraenkel. [Translation M.G.] Letter L. 115/35 archived in Polskie Towarzystwo Filozoficzne, Poznań.

[235] *The fact that a one-one correlation can hold between any individuals and certain classes of finite sequences plays an essential role when also applying the method of semantical unification of the variables. After modifying the intuitive interpretation of the variables of the 1st and the 2nd order, they all now belong to the same semantical category. Later, it is explained how the method of semantical unification of the variables can be applied to the investigation of any language of the 3rd kind.*

Translational Remarks

Again, it may be clear that in the first sentence Tarski speaks of applying the “method of semantical unification of the variables”, however the adjective “semantical” should not have been left out of the English translation, since it is a part of the name of the method. Hence, we read in Polish “metodę ujednostajniania **semantycznego** zmiennych” [p. 117]. Also here, the German translation is just as inaccurate as the English one and we read there “der Methode der Vereinheitlichung der Variablen” [p. 357].

Several discrepancies occurred in the second paragraph. The Polish sentence “Większą nieco trudność może **jedynie** nastrećcać ustalenie kategorii ujenostajniającej” [p. 118] should rather be translated in the following way “**Only** the determination of the unifying category may be a bit more difficult”. In German we read “Eine etwas grössere Schwierigkeit kann **nur** die Festsetzung der vereinheitlichenden Kategorie bieten” [p. 357].

Furthermore, for the sake of clarity, the second half of this paragraph will be quoted as a whole and then translated: “co więcej, w przeciwstawieniu do tamtych języków, **nie zawsze nawet** można dokonać wyboru spośród kategorii tych samych rzędów co kategorie języka, Nietrudno natomiast okazać, że, jeśli rząd zmiennych języka nie przekracza liczby n , to każda kategoria rzędu $n + 2$ służyć może jako kategoria ujednostajniająca jeśli zaś $n > 2$, to kategorię taką znaleźć można już w obrębie rzędu $n + 1$ ” [p. 118]. The following translation is as literal as possible, therefore it may seem awkward sometimes: “What is more, in contrast to those languages it is **not even always** possible to make the choice among the categories of the same orders as the categories of the language. It is, however, not difficult to show that if the order of the variables of the language does not exceed the number n , then every category of the order $n + 2$ may serve as the unifying category; if, however, $n > 2$ then such a category can be found already in the range of the order $n + 1$.” A quick look at the German translation explains the origin of some of those discrepancies, it is, however, closer to the original than the English version: “im Gegensatz zu jenen Sprachen kann man hier **nicht einmal immer** die Wahl unter den Kategorien einer jener Ordnungen treffen, die in der Sprache vertreten sind. Diese Schwierigkeit ist übrigens nicht wesentlich und betrifft ausschliesslich die Sprachen niedrigster Ordnung: es lässt sich nachweisen, dass für jene Sprachen, in denen die Ordnung der Variablen eine gegebene Zahl n nicht überschreitet, wobei $n > 3$ ist, als vereinheitlichende Kategorie eine beliebige Kategorie n ter Ordnung dienen kann” [pp. 357–358].

[236] *The results of the investigations are summarised. The most important result is that the definition of a true sentence is a correct definition of truth in the sense of convention **T**. Also, it can be proven that all axioms of the science under investigation are true. Furthermore, the principle of contradiction and the principle of excluded middle follow from this definition as well as other important theorems. Finally, it is stated that there is a general method for proof of the consistency of various sciences, for which we can construct a definition of truth.*

Translational Remarks

At the beginning of this page, while listing the consequences following from the constructed definition, in Polish [p. 119] we read “*definicja zdania prawdziwego jest **trafną** definicją prawdy w sensie umowy **P** z Sect. 3*” which should be translated as “*the definition of a true sentence is an **adequate** definition of truth in the sense of convention **T** of Sect. 3.*”, not ‘correct’. In the German version we read: “*zunächst ist die Definition der wahren Aussage eine **richtige** Definition der Wahrheit im Sinne der Konvention **W** aus Sect. 3*” [p. 358], see also [1.2.2].

Later in the same sentence, we read in Polish that “obejmuje ona jako szczególne przypadki wszelkie definicje cząstkowe, opisane w warunku (α) tej umowy a wyjaśniające w sposób **precyzyjny i intuicyjnie trafny** sens zwrotów typu “ x jest zdaniem prawdziwym”, which has been correctly translated, except for the part “it embraces, as special cases, all partial definitions which were described in condition (α) of this convention and which elucidate **in a precise and intuitively (materially) adequate** way the sense of expressions of ‘ x is a true sentence’”. The German version is also not very accurate: “sie umfasst als Spezialfälle sämtliche Teildefinitionen, die in der Bedingung (α) dieser Konvention beschrieben wurden und die in **präziser und sachlich richtiger** Weise den Sinn der Wendungen vom Typus” x ist eine wahre Aussage “erläutern” [p. 358]. In the next sentence, Tarski wrote “**wspomniane tezy**” [p. 119] which means the same as “**mentioned theorems**”, but it has been translated as “**partial definitions mentioned**” after the German translation “**erwähnten Teildefinitionen**” [p. 358].

Just for the sake of clarity, in the last sentence of this paragraph, in Polish we read “badanego **zdania**” [p. 119] which means the same as “the investigated sentence”. The word ‘sentence’ in Polish appears in singular, just as it does in German “untersuchten **Aussage**” [p. 358]. Of course, the mentioned definition allows us to determine the truth or falsity of many sentences, not just one. Nevertheless, the point is that it has to be done for each sentence separately.

The following discrepancy is worthy of notice. In the second sentence from the bottom, Polish “dla **każdej** nauki” [p. 120] means the same as “of **every** science” and not “of **various** sciences” as we read in English, in German it is “für **jede** Wissenschaft” [p. 539]. It is possible that this is a deliberate correction of the Polish and German texts, however. Not every science, for which a definition of truth can be constructed, can a proof of consistency be produced.

The footnote at the bottom of this page appeared only in the English translation. Another footnote, however, is missing in the English translation. Namely, after the Polish part “odpowiednie reguły samej nauki” [p. 119] which has been accurately translated as “the corresponding rules of the science itself” (lines 17–18), and after German “die entsprechenden Regeln der Wissenschaft selbst” [p. 359] there is a footnote sending us back to the footnote 61 in Polish and 57 in German. In the English translation it is the 1st footnote on page 211.

[237] *The importance of a general method for proof of the consistency of various deductive sciences is emphasised once again. Moreover, if a class of provable sentences is consistent and complete it can be easily shown that it coincides with the class of true sentences. Such a situation results in a new definition of truth of a purely structural character and very different from the original semantical definition.*

Translational Remarks

The expression of the 4th line “**such a general method of proof**” is not a very accurate translation of Polish “**ogólna metoda tego rodzaju dowodów**” [p. 120], which should rather be translated as “**a general method of such proofs**”. The German

version provides us here with an accurate translation “**eine allgemeine Methode derartiger Beweise**” [p. 359], see also [1.2.3].

It seems to be worthy of notice that later on in the same sentence Tarski writes that this method is applicable to an extensive “**kategorii nauk dedukcyjnych**” [p. 120] – “**category of deductive sciences**”. Also, in German we read “**Kategorie von deduktiven Wissenschaften**” [p. 359]. The translator of the English version notices rightly that the term “category” may lead to much confusion in this context and hence decided to translate it as “**range**”, which reflects Tarski’s intentions.

The first footnote on this page may cause some confusion. The translator decided to use the term “**assertion**”, where Tarski writes “**teza**” [p. 120], which has been previously and later on, for the most part, correctly translated as “**theorem**” or “**provable sentence**”. Perhaps, it has to do with the German translation where we read “**Behauptung**” [p. 359], although it has previously and later on been translated as “**beweisbaren Sätze**” [p. 359].

Later in the same footnote the translator chose to use the word “**hypotheses**” as a translation of Polish “**przesłanek**” [p. 120] which means the same as “**premisses**” and has, as a matter of fact, appeared as such at the beginning of this page. The German translation is not accurate here either, since we read “**Voraussetzungen**” [p. 359] instead of “**Prämissen**”, see also [1.2.3].

The last footnote on this page is a new addition to the English translation.

[238] *It was stated that when the class of provable sentences, in addition to being consistent, is also complete, then it coincides with the class of true sentences. These two concepts identified lead to a new structural definition of truth. There is, however, no general method of construction of such a definition.*

Translational Remarks

The footnote beginning on the previous page and taking up a major part of this page has been rewritten by Tarski for the English version. The differences are, however, rather minor and do not influence the meaning of the main text.

[239] *Whenever it is possible to define the notions of satisfaction and of a true sentence it is also possible to specify two other concepts, namely satisfaction and sentence being valid in a given domain of individuals. The general concept of a sentence being valid in a given domain of individuals plays an essential role in the investigations of mathematical logic.*

Translational Remarks

A discrepancy which we dealt with before, see the commentary to [194], also occurs here (line 5). Namely, the English word “define” is supposed to be a translation of Polish “**sprecyzować**” [p. 123] and of German “**präzisieren**” [p. 361]. It is clear that an accurate translation is to “**specify**”.

Another already known inconsistency occurs in the same sentence and it regards the Polish expression “**zдания słusznego (prawdziwego)**” [p. 123], which has been

translated into German as “**der richtigen (wahren) Aussage**” [p. 361], and in English as simply a “**correct sentence**”, see also [1.2.2].

The last sentence beginning on this page considers the concept of a ‘sentence **valid** in every individual domain’, in Polish ‘zдания **słusznego** w każdej dziedzinie indywiduów’ [p. 124], and in an inaccurate German translation ‘in jedem Individuen-beriche **richtigen** Aussage’ [p. 362], see again [1.2.2].

[240] *Here, special attention is paid to the concept of a sentence valid in every domain of individuals. An attempt to transform the system of provable sentences of every investigated science into a complete one meets with serious difficulties. These are outlined later on the page.*

Translational Remarks

Tarski holds that the concept of a *sentence valid in every domain of individuals* deserves special consideration (line 3). In the next Polish sentence we read “Jest ono co do zakresu czymś pośrednim między pojęciem tezy a pojęciem zdania prawdziwego: **klasa wszystkich zdań słuszných w każdej dziedzinie** obejmuje wszystkie tezy i składa się wyłącznie ze zdań prawdziwych (twierdzenia 22 i 27)” [p. 124]. It means the same as “In its extension it stands midway between the concept of a theorem and that of a true sentence; **the class of all sentences valid in every domain** embraces all theorems and consists exclusively of true sentences (Ths. 22 and 27)”. The German translation is also incomplete: “Seinem Umfang nach steht er in der Mitte zwischen dem Begriff des beweisbaren Satzes und jenem der wahren Aussage: **die Klasse der in jedem Bereiche richtigen Aussagen** umfasst alle beweisbaren Sätze und besteht ausschliesslich aus wahren Aussagen (Satz 22 und 27)” [p. 362]. See also [1.2.2].

Again, perhaps it is clear that we are speaking of “the system of provable sentences of **every investigated** science”, nonetheless, Tarski made it explicit in Polish “**każdej badanej nauki**” [p. 124]. This time the German translation is correct: “**jeder untersuchten Wissenschaft**” [p. 362].

Still in the same sentence, the question: “**how many individuals are there?**” should be placed within quotation marks, as it is in Polish “**ile jest wszystkich indywiduów?**” [p. 124]. Another question is whether the problematic verb “exists” should be used here. A more accurate translation of the Polish version would be literal, as the one suggested above.

A minor inaccuracy regards the term “definition” used in the English version in the last line of this page. In Polish we read more generally “**strukturalną charakterystykę**” [p. 125] meaning the same as “**structural characteristic**”, as well as in German “eine strukturelle **Charakteristik**” [p. 362].

[241] *Sect. 4 ends with an emphasis on the importance of the concept of a sentence valid in every domain of individuals as a basis for the investigations leading to the formulation of the definition of a true sentence. In Sect. 5 the problem of the concept of a true sentence in the languages of infinite order is considered. As an example the language of the general theory of classes is chosen.*

2.6 Section 5. The Concept of True Sentence in Languages of Infinite Order

[242] *The language of the general theory of classes is given as an example of the languages of infinite order. The constants and the variables of the language of the general theory of classes are introduced, as well as the primitive sentential functions. In the footnote, the investigated language is compared to the languages which Whitehead and Russell as well as Leśniewski worked with.*

Translational Remarks

In the language of the general theory of classes, we are able to formulate every idea which can be expressed “**in the complete** language of mathematical logic”. This is a correct translation of the Polish expression “**w kompletnym** języku logiki matematycznej” [p. 126]. The phrasing of the German translation here is rather surprising “**in der gesamten** Sprache der mathematischen Logik” [p. 364]. The Polish word “kompletny” can easily be translated as the English “complete” and the German “vollständig” as it has been done before, for example in Def. 20 “a complete class of sentences” in (Tarski 2006g, p. 185).

In the footnote we read about “setting up **the list** of axioms”, which should obviously be in singular, not in plural. In Polish it reads “przy układaniu **listy** aksjomatów” [p. 126], and also clearly in German “bei der Anlage **einer Axiomenliste**” [p. 364].

[243] *Continuing the discussion from the previous page, the sentential functions, including their quantifications, are described. Later, the axioms of the general theory of classes are divided into 4 groups. In the last paragraph, the discussion of the concept of satisfaction begins.*

Here are the axioms of the general theory of classes in today’s notation⁵³:

- (1) the axioms of the sentential calculus
- (2) pseudodefinitions, i.e. formulas which are universal closures of sentential functions of the following form:

$$\exists X_k^{n+1} \forall X_l^n \underbrace{((X_k^{n+1} X_l^n \wedge A) \vee (\neg X_k^{n+1} X_l^n \wedge \neg A))}_{\substack{X_k^{n+1} X_l^n \leftrightarrow A \\ X_l^n \in X_k^{n+1}}}$$

For the sake of clarity, we substitute, here and in the following formulas, the expressions of the form ‘XY’ with ‘Y ∈ X’:

$$\exists X_k^{n+1} \forall X_l^n (X_l^n \in X_k^{n+1} \leftrightarrow A),$$

where ‘A’ is a sentential function in which the variable ‘X_kⁿ⁺¹’ is not free.

⁵³Cf. Morscher (2007).

(3) the *laws of extensionality*

$$\begin{aligned} & \forall X_k^{p+2} \forall X_l^{p+1} \forall X_m^{p+1} (\exists X_n^p ((X_l^{p+1} X_n^p \wedge \neg X_m^{p+1} X_n^p) \vee \\ & \vee (\neg X_l^{p+1} X_n^p \wedge X_m^{p+1} X_n^p)) \vee (\neg X_k^{p+2} X_l^{p+1} \vee X_k^{p+2} X_m^{p+1})) \end{aligned}$$

If we substitute for $p = 1, k = 1, l = 2, m = 3, n = 4$ we arrive at:

$$\begin{aligned} & \forall X_1^3 \forall X_2^2 \forall X_3^2 (\exists X_4^1 ((X_2^2 X_4^1 \wedge \neg X_3^2 X_4^1) \vee (\neg X_2^2 X_4^1 \wedge X_3^2 X_4^1)) \\ & \quad \underbrace{\neg(X_2^2 X_4^1 \leftrightarrow X_3^2 X_4^1)}_{(X_1^3 X_2^2 \rightarrow X_1^3 X_3^2)} \\ & \quad \vee (\neg X_1^3 X_2^2 \vee X_1^3 X_3^2)) \\ & \forall X_1^3 \forall X_2^2 \forall X_3^2 (\exists X_4^1 \neg (X_4^1 \in X_2^2 \leftrightarrow X_4^1 \in X_3^2) \vee (X_2^2 \in X_1^3 \rightarrow X_3^2 \in X_1^3)) \\ & \forall X_1^3 \forall X_2^2 \forall X_3^2 (\neg \forall X_4^1 (X_4^1 \in X_2^2 \leftrightarrow X_4^1 \in X_3^2) \vee (X_2^2 \in X_1^3 \rightarrow X_3^2 \in X_1^3)) \\ & \forall X_1^3 \forall X_2^2 \forall X_3^2 (\forall X_4^1 (X_4^1 \in X_2^2 \leftrightarrow X_4^1 \in X_3^2) \rightarrow (X_2^2 \in X_1^3 \rightarrow X_3^2 \in X_1^3)) \\ & \forall X_2^2 \forall X_3^2 (\forall X_4^1 (X_4^1 \in X_2^2 \leftrightarrow X_4^1 \in X_3^2) \rightarrow \forall X_1^3 (X_2^2 \in X_1^3 \rightarrow X_3^2 \in X_1^3)) \\ & \forall X_2^2 \forall X_3^2 (\forall X_4^1 (X_4^1 \in X_2^2 \leftrightarrow X_4^1 \in X_3^2) \rightarrow X_2^2 = X_3^2) \end{aligned}$$

(4) the *axiom of infinity*

$$\begin{aligned} & \exists X_1^3 (\exists X_1^2 X_1^3 X_1^2 \wedge \forall X_1^2 (\neg X_1^3 X_1^2 \vee \exists X_2^2 (X_1^3 X_2^2 \wedge \\ & \wedge \forall X_1^1 (X_1^2 X_1^1 \vee \neg X_2^2 X_1^1) \wedge \exists X_1^1 (X_1^2 X_1^1 \wedge \neg X_2^2 X_1^1))) \\ & \exists X_1^3 (\exists X_1^2 X_1^3 X_1^2 \wedge \forall X_1^2 (X_1^3 X_1^2 \rightarrow \exists X_2^2 (X_1^3 X_2^2 \wedge \\ & \wedge \forall X_1^1 (X_1^2 X_1^1 \rightarrow X_2^2 X_1^1) \wedge \exists X_1^1 (X_1^2 X_1^1 \wedge \neg X_2^2 X_1^1))) \\ & \exists X_1^3 (\exists X_1^2 (X_1^2 \in X_1^3) \wedge \forall X_1^2 (X_1^2 \in X_1^3 \rightarrow \exists X_2^2 (X_1^2 \in X_1^3 \wedge \\ & \wedge \forall X_1^1 (X_1^1 \in X_2^2 \rightarrow X_1^1 \in X_1^2) \wedge \exists X_1^1 (X_1^1 \in X_1^2 \wedge X_1^1 \notin X_2^2))) \\ & \quad \underbrace{X_2^2 \subsetneq X_1^2}_{X_1^3 \subsetneq X_1^2} \\ & \exists X_1^3 (\exists X_1^2 (X_1^2 \in X_1^3) \wedge \forall X_1^2 (X_1^2 \in X_1^3 \rightarrow \exists X_2^2 (X_2^2 \in X_1^3 \wedge X_2^2 \subsetneq X_1^2))) \end{aligned}$$

Translational Remarks

Tarski discusses the general theory of classes in detail. In (1) of the list of sentences included as the axioms, in Polish we read “podstawienia aksjomatów rachunku zdań **i ich** generalizacje” [p. 127], which means the same as “substitutions of the axioms of the sentential calculus **and their** universal quantifications”. In the German translation we read “aus den Axiomen des Aussagenkalküls durch Einsetzung, **gegebenfalls auch** durch nachfolgende Generalisierung” [365]. Compared with the original, both translations present an improved version of the text, inasmuch as the universal quantification is not always needed in addition to substitution. The English edition is not precise enough when it comes to naming the operations, however. A correct version of this part of the sentence would be “substitutions of the axioms of the sentential calculus sometimes followed by their universal quantifications”.

The term ‘**quantifications**’ should be specified to ‘**universal quantifications**’. In Polish we have ‘**generalizacjami**’ [p. 127], and also in German ‘**Generalisationen**’ [p. 365].

It should be clear that the **consequences** of the axioms are also **provable sentences**. Nonetheless, Tarski chose to use exactly this term here “Przy wyprowadzaniu z aksjomatów innych **zdań uznanych**” [p. 128] which can be translated as “In deriving other **provable sentences** from the axioms”. The German translator had already decided to take a short cut here and wrote “Bei der Ableitung der **Folgerungen** aus den Axiomen” [p. 366].

Where in the English edition we have ‘**universal quantifier**’, we read in Polish [p. 128] simply ‘**kwantifikatora**’ – ‘**quantifier**’, and in German [p. 366] ‘**Allzeichen**’; the translations are improved versions of the original.

The English ‘to define’ serves as a translation of the Polish term “sprecyzować” [p. 129] which means – literally translated – “to specify” and in German “zu präzisieren” [p. 366], see the commentary to [194].

[244] *The concept of satisfaction is considered, however, neither the method of many-rowed sequences nor even the method of semantical unification of the variables are of any use in specifying this concept in regard to the languages of infinite order. Hence, the concept of satisfaction has to be applied in its original multiple formulations. This means that for every sentential function a separate concept of satisfaction must be defined.*

In the 4th kind of languages the variables are of arbitrarily high order, which means that there is an ‘infinite diversity’ of semantical categories in the language, which excludes the method of many-rowed sequences, and if we wanted to apply the method of semantical unification we would have to use expressions of infinite order, which were not available in the languages the structure of which adheres to the theory of semantical categories.⁵⁴ As Tarski notes,

In the language with which we are now dealing variables of arbitrarily high (finite) order occur: consequently in applying the method of unification it would be necessary to operate with expressions of ‘infinite order’. Yet neither the metalanguage which forms the basis of the present investigations, nor any other of the existing languages, contains such expressions. It is in fact not at all clear what intuitive meaning could be given to such expressions. (Tarski 2006g, p. 244)

From this quotation it is clear that Tarski was still committed to Leśniewski’s interpretation of type-theory – STT. Moreover, this line of argumentation gave rise to another philosophical debate, this time regarding Tarski’s stance regarding *universal language*.

Because the definition of *true sentence* should be given in a language of a higher order than that of the language under consideration, it follows that there is, so to speak, no language of the highest order: there would not be a universal language. (Loeb 2014, p. 2282)

⁵⁴Cf. Gruber (2015).

There is still no consensus among Tarski's readers on what kind of languages Tarski actually had in mind. The notion of *universal* language is not unambiguous, and has been interpreted in different ways.⁵⁵

Translational Remarks

When trying to define the concept of satisfaction for the language of the general theory of classes we meet with serious difficulties. Due to the infinite diversity of semantical categories represented in this language the method of many-rowed sequences is "excluded **a priori**". This is the term that Tarski used in Polish [p. 128], and we also read it in German [p. 366]. The English translation may be accurate, however it seems that especially in the philosophical context the phrase chosen by Tarski has a much deeper meaning.

Unfortunately, also the method of semantical unification is useless here, since the unifying category must be of higher order than the variables of the considered language. In the present context the term "**corresponding**" is redundant, and what is worse, since it has been used in similar contexts, but with a different meaning, it may be confusing now. It could easily be confused with the strict correspondence holding between every expression of the object language and the name of this expression in the metalanguage. In Polish we read "relacja spełniania, zachodząca między takimi ciągami a funkcjami zdaniowymi, muszą więc być wyższego rzędu od wszystkich tych zmiennych." [p. 129], which can be translated as "the relation of satisfaction holding between such sequences and sentential functions must thus be of a higher order than all those variables". The German translation is unambiguous "die Relation des Erfüllseins, die zwischen derartigen Folgen in **den entsprechenden** Aussagefunktionen besteht, müssen also von höherer Ordnung sein als all jene Variablen" [p. 366].

The expression "**define this meaning**" in the 4th line from the bottom, is also a bit confusing. In the Polish version we read that "w odniesieniu do jakiegokolwiek konkretnej funkcji zdaniowej **sens ten** potrafimy nawet **dokładnie sprecyzować**" [p. 129], and it means the same as "for any particular sentential function we can, in fact, **exactly specify this sense**". In Polish "sens ten", which means the same as "this sense", clearly refers to "sens intuicyjny" – "the intuitive sense" of the infinite number of the concepts of satisfaction mentioned earlier in this sentence. The translator of the German version still refused to translate "sens intuicyjny" accurately and wrote "inhaltliche Sinn". Nevertheless, his translation of the next sentence leaves no doubt as to the correct interpretation, "für jede beliebige konkrete Aussagefunktion können wir sogar **diesen Sinn genau präzisieren**" [p. 367], see also [1.2.1] and the commentary to [194].

[245] *Unfortunately, the idea of constructing definitions of each special concept of satisfaction as a certain specialization of the general concept also fails here. Using the recursive method renders the task impossible. The concept of satisfaction, how-*

⁵⁵For further reading on this topic see e.g., de Rouilhan (1998), Hintikka (1988), Mancosu (2010), Rodriguez-Consuegra (2005), Van Heijenoort (1967).

ever, plays a crucial role in defining the concept of truth, which allows to anticipate the difficulties for the main task.

Translational Remarks

Tarski names the reasons why it is impossible to construct a recursive definition of the concept of satisfaction in the investigated language. In the sentence with the problematic translation, he says that in order to construct arbitrary functions of a given type “we must use as a material, **sentential functions of all possible semantical types**”. This is an exact translation of the Polish “musimy się posługiwać jako materiałem **funkcjami zdaniowymi wszelkich możliwych typów semantycznych**” [p. 130]. Here, the German translation is flawless “**Aussagefunktionen von allen möglichen semantischen Typen** als Material verwenden” [p. 368].

[246] *Since no method of constructing a definition of truth is known which does not presuppose the concept of satisfaction, it follows that it is impossible to construct a correct and materially adequate definition of truth for the investigated language. After a brief inquiry into the nature of this failure, the basic result is restated.*

Translational Remarks

The goal of this article, as Tarski puts it forward himself, is to construct *a materially adequate and formally correct definition of the term ‘true sentence’*. This, however is not the phrasing Tarski uses in the Polish original. Here, in the first paragraph, where he states that at the present stage of our investigations we are unable to construct, in Polish “poprawnej i **zgodnej z intuicją** definicji zdania prawdziwego” [p. 131] which means the same as “correct definition of a true sentence that is **in accordance with intuition**”. The German translation is not very surprising here “korrekte und **sachlich zutreffende** Definition der wahren Aussage” [p. 368], see [1.2.1] and [1.2.2].

Later, Tarski ponders whether our failure is accidental and “**exclusively** caused by the defects of the currently used methods”, which is a translation of this Polish phrase “są spowodowane **wyłącznie** niedoskonałością stosowanych aktualnie metod” [p. 131]. In German we read “**etwa nur** an der Unvollkommenheit der tatsächlich angewandten Methoden liegt” [p. 369]. It makes a difference if our failure is caused “in some way” or “exclusively” by the defects of the methods used.

In order to answer this question we must first give it a “less **general** form”, as we read in Polish “mniej **ogólnikową** postać” [p. 132]. The German translation of this part is also not very accurate “eine weniger **unbestimmte** Form” [p. 369].

Furthermore, this page’s footnote appeared only in the English version.

[247] *After this negative conclusion an account of the positive aspects of the present investigations is given. To simplify these considerations: the meta-language is so constructed that the investigated language is a part of it. Then Theorem I is presented, followed by a sketch of its proof.*

Tarski's Theorem I:

(α) *In whatever way the symbol 'Tr', denoting a class of expressions, is defined in the metatheory, it will be possible to derive from it the negations of one of the sentences which were described in the condition (α) of the convention T; (β) assuming that the class of all provable sentences of the metatheory is consistent, it is impossible to construct an adequate definition of truth in the sense of convention T on the basis of the metatheory.* (Tarski 2006g, p. 247)

This theorem states that it is impossible to define a true sentence for a language if it is of infinite order. Its proof is based on Gödel's *indefinability theorem*.⁵⁶ This theorem should not be confused with Tarski's claim on undefinability of the truth predicate for an object language within this language itself. This claim is closed in Theorem I, but Theorem I also makes a further statement. As Coffa notes:

This result should not, of course, be confused with what is now known as "Tarski's theorem", the claim that the truth-predicate for OL is not definable *within* OL. That claim is, to be sure, contained in the proof of Theorem I, but the main point of this Theorem [...] was the thesis that one *cannot* define truth for an OL *even in its ML* when OL is of infinite order. The basic reason was that the theory of semantic categories determines that no meaningful languages is more powerful than the language of order ω . To ask for the definition of truth for an infinitary language is therefore to ask for the definition of truth in a language within itself and, by Tarski's theorem, no consistent language can do that. (Coffa 1987, p. 563)

This is one of the controversial points of Tarski's monograph, by far not the only one, which caused a very lively debate among the philosophers and logicians alike.⁵⁷

Translational Remarks

The fact that every expression of the investigated language is at the same time an expression of the metalanguage enables us in certain cases "to speak simply of the expressions of the language **themselves**". This is a correct translation of the Polish "mówić po prostu o **samych** wyrażeniach języka" [p. 133]. In German we read "einfach von den Ausdrücken der Sprache **selbst** zu sprechen" [p. 370], which is ambiguous, because '**selbst**' can refer to both the language itself, and to its expressions. This ambiguity does not exist in Polish. It should be clear from the context that we mean the 'expressions' of the language, however, and not the language itself.

[248] *The sketch of the proof of Theorem I is presented in two parts. (1) regards the interpretation of the metalanguage in the language, by which the metalanguage contains not only every sentence of the language, but also an individual name of that sentence or of an equivalent sentence. (2) deliberates on the possibility of avoiding the liar paradox in the metalanguage.*

Translational Remarks

It is clear that in the footnote we should be reading that "the antinomy of the liar actually approximates to the **antinomy of heterological expressions**". In Polish it reads

⁵⁶Gödel (2006, pp. 174–5).

⁵⁷For a thorough discussion regarding the problems of indefinability and inconsistency see, for example, Patterson (2012, pp. 163–188).

“antynomia kłamcy zbliża się istotnie do **antynomii wyrazów heterologicznych**” [p. 135]. The German translation is already inaccurate “die Antinomie des Lügners tatsächlich der **Antinomie des Ausdrucks ‘heterologisch’** nähert” [p. 371].

Also, the part of the footnote immediately following the discussed part has been added later to the English version, hence it does not appear in Polish or German.

[249] *The sketch of the proof of Theorem I is now presented in more detail.*

Tarski presents X_1^3 which is the class of all classes which have just one element, here it is in today's notation⁵⁸:

$$\forall X_1^2 (X_1^3 X_1^2 \wedge \exists X_1^1 \forall X_2^1 \forall X_2^2 (X_1^2 X_1^1 \wedge (\neg X_1^2 X_2^1 \vee \neg X_2^2 X_1^1 \vee X_2^2 X_2^1))) \vee \\ \vee \neg X_1^3 X_1^2 \wedge \forall X_1^1 \exists X_2^1 \exists X_2^2 (\neg X_1^2 X_1^1 \vee (X_1^2 X_2^1 \wedge X_2^2 X_1^1 \wedge \neg X_2^2 X_2^1)))$$

To avoid the lower indices ‘1’ and ‘2’, we replace ‘ X_1 ’ with ‘ X ’ and ‘ X_2 ’ with ‘ Y ’, additionally we use the symbol ‘ $\rightarrow$ ’ and replace ‘ YX ’ with ‘ $Y \in X$ ’:

$$\forall X^2 (X^2 \in X^3 \wedge \exists X^1 \forall Y^1 \forall Y^2 (X^1 \in X^2 \wedge (Y^1 \in X^2 \rightarrow (X^1 \in Y^2 \rightarrow Y^1 \in Y^2)))) \vee \\ \vee (X^2 \notin X^3 \wedge \forall X^1 \exists Y^1 \exists Y^2 (X^1 \in X^2 \rightarrow Y^1 \in X^2 \wedge X^1 \in Y^2 \wedge Y^1 \notin Y^2)))$$

Translational Remarks

In the last sentence of this page, regarding the one-one correspondence which can be set between the expressions of the language and the natural numbers, in Polish we read “Na zasadzie tej odpowiedniości każdej operacji and wyrażeniami można przyporządkować pewną operację and liczbami naturalnymi (o tych samych formalnych własnościach), każdej klasie wyrażeń – klasę liczb naturalnych itd.; dzięki temu metajęzyk zyskuje pewną **“interpretację”** w arytmetyce liczb naturalnych i, pośrednio, w języku ogólnej teorii klas” [p. 136]. The boldfaced term ‘interpretation’ should be placed within quotation marks, see also the commentary to [210]. This sentence could be translated as “With the help of this correlation we can correlate with every operation on expressions an operation on natural numbers (which possesses the same formal properties), with every class of expressions a class of natural numbers, and so on; thanks to the fact that the metalanguage receives an **‘interpretation’** in the arithmetic of the natural numbers and indirectly in the language of the general theory of classes”. The quotation marks are missing already in the German translation “Auf Grund dieser Zuordnung kann man jeder Operation an Ausdrücken eine Operation an natürlichen Zahlen (welche dieselben formalen Eigenschaften besitzt) zuordnen, jeder Klasse von Ausdrücken eine Klasse von natürlichen Zahlen, u. s. w.; demzufolge gewinnt die Metasprache eine **Interpretation** in der Arithmetik der natürlichen Zahlen und mittelbar in der Sprache der allgemeinen Klassentheorie” [p. 373].

⁵⁸Cf. Morscher (2007).

[250] *The proof of the first part of Theorem I is given. It is shown that given a formally correct and materially adequate definition of the symbol 'Tr' in the metalanguage, we obtain two contradicting sentences among its consequences.*

Translational Remarks

"Let us suppose **in particular** that we have defined the class *Tr* of sentences in the metalanguage" is an accurate translation of what Tarski writes in Polish; "ZałóŜmy **w szczególności**, Ŝe zdefiniowaliŝmy w metajęzyku klasę zdań *Vr*" [p. 136]. The German translation is accurate here: "Nehmen wir **insbesondere** an, dass wir in der Metasprache die Aussagenklasse *Wr* definiert haben" [p. 373].

Several discrepancies occur later in the text, where Tarski writes in Polish "dochodzimy do **tezy**" [p. 137], we read in English "we obtain a **sentence**" and in German "erhalten wir einen **Satz**" [p. 373]. The Polish term '**teza**' can be translated as 'thesis', 'provable sentence' or 'asserted sentence', as it has been done in Definition 17, the meaning of which is clearly quite different from that of 'sentence', see also [1.2.3].

Later in the same paragraph, Polish 'zdanie' [p.137], meaning 'sentence', has been translated as 'statement'. The German translation is consistent and we read 'Aussage' [p. 373], see again [1.2.3].

Also, Tarski decided to write the following sentence in italics "*istnieje takie n , iŝ $n=k$ i $\psi(k)$* " [p. 137]. The German translation is accurate here insofar as the phrase "*es gibt ein solches n , dass $n=k$ und $\psi(k)$* " is printed in italics [p. 374].

[251] *Now, the proof of part (β) of Theorem I is delivered. The assumption of consistency in this part of the theorem is of essential importance, still, it will never be possible to prove the consistency of the metatheory on the grounds of the meta-metatheory. The page ends with the remark that this metatheory is in fact the morphology of language.*

Translational Remarks

At the beginning of the second paragraph on this page, Tarski emphasizes the importance of the assumption of consistency for Theorem I. Namely, "gdyby klasa wszystkich tez metanauki była sprzeczna, kaŝda definicja w metanauce pociągałaby za sobą **jako** konsekwencje wszystkie moŝliwe zdania (gdyŝ wszystkie one byłyby tezami metanauki)" [p. 137]. It can be translated as follows "if the class of all provable sentences of the metascience were contradictory, then every definition in the metascience would have **as** its consequences all possible sentences (since they all would be provable sentences in the metascience)". In the German version we read "enthielte nämlich die Klasse aller beweisbaren Sätze der Metawissenschaft einen Widerspruch, so würde jede Definition in der Metawissenschaft alle überhaupt möglichen Aussagen (denn sie alle wären in der Metawissenschaft beweisbar) **nach sich ziehen**" [p. 374].

In the last paragraph, Tarski lists the structural-descriptive terms which belong to the meta-language that we use for our investigations. It is important to notice, which is clear from the Polish syntax, that **all of these terms are names of** certain expressions of the language and of structural properties of these expressions

and of structural relations between these expressions, as well as logical expressions. We read in Polish “**nazwy** wyrażeń języka, własności strukturalnych tych wyrażeń, relacji strukturalnych między wyrażeniami itd.” [p. 138]. The German translation is also clear in this matter “**Namen** von Ausdrücken der Sprache, von strukturellen Eigenschaften dieser Ausdrücke, von strukturellen Relationen zwischen Ausdrücken u.s.w.” [p. 375].

[252] *Since both the language studied and the deductive science carried out in this language are formalized, it is possible to reduce certain concepts, e.g. the concept of consequence, to other concepts belonging to the morphology of language. The semantics of language, a domain to which the essential concepts of satisfaction and truth belong, is discussed.*

It is, perhaps, the right place to give Tarski additional praise for his investigations into the semantical concepts. As he notes, other specialists in the study of language, and naturally mathematicians, avoided involving themselves in semantical investigation, and did not think much of these notions. As it has been observed, however

Tarski's primary aim in formulating his theory of truth was to make metamathematics a respectable mathematical enterprise.⁶ Indeed, Tarski's work on truth can be seen as the birth of model theory as a branch of mathematical logic, which is in turn a branch of mathematics. Nonetheless, Tarski's work on truth does show that where many mathematicians consider it positively harmful for their mathematical career to engage with philosophical questions, Tarski took the opposite view. And history has shown that this was a fruitful attitude to take. In philosophy, this has given rise to a discipline that is called 'formal theories of truth'. (Horsten 2015, p. 151)

Indeed, Tarski's work has had an enormous impact on the development of formal epistemology, especially on investigations into truth. It laid ground for all future theories of truth and even today, eighty year later, it is indispensable within mathematical logic and mathematical philosophy.

Translational Remarks

Tarski speaks of the characteristic feature of the semantical concepts, which is “that they express certain relations between the expressions of the language and the objects **‘about which these expressions speak’** ”. This translation does not essentially differ from the official English translation, except for the quotation marks, which have once again been left out in the English version, where Tarski clearly writes “że wyrażają one pewne zależności między wyrażeniami języka a przedmiotami, **“o których w tych wyrażeniach mowa”** ” [p. 139]. The German translation is accurate: “dass sie gewisse Abhängigkeit zwischen den Ausdrücken der Sprache und den Gegenständen, **“von denen in diesen Ausdrücken die Rede ist”**, zum Ausdruck bringen” [p. 376]. Also the **‘evil reputation’** of the semantical concepts should be placed within quotation marks, as it is in Polish “**złą sławą**” [p. 139] and in German “**üblen Rufes**” [376], see also the commentary to [210].

Further in the last paragraph, regarding the semantical concepts, Tarski writes that “**intuicyjne na pozór ich własności** prowadziły do paradoksów i antynomii”

[p. 139], which can be translated as “**their seemingly intuitive features** led to paradoxes and antinomies”. The German translation explains the English one “**die inhaltlich einleuchtend erschienen, haben zu Paradoxien und Antinomien geführt**” [p. 376], see also [1.2.1].

Because of the ‘evil reputation’ of the semantical concepts it seemed only natural to try to reduce them to **structural-descriptive ones**, “which had a clear and explicit content and evident features”. This is the translation of the missing part which we find in the original “o jasnej i wyraźnej treści i oczywistych własnościach” [p. 139]. Here, the German translation is accurate “mit klarem und deutlichem Inhalt und evidenten Eigenschaften” [p. 376].

In the footnote, the word ‘stage’ should be in singular, as it is in Polish ‘rozwoju’ [p. 139] and in German ‘Entwicklungsstadium’ [p. 375].

[253] *Although it is possible to formulate infinitely many partial definitions for every semantical concept, which exhaust all possibilities of the application of these concepts to concrete expressions, this does not lead to a general definition of these concepts, which would embrace them all as special cases and would form their infinite logical product.*

Translational Remarks

In the first sentence of this page, Tarski points out the fact that we have always been able to replace every phrase containing semantical terms by a phrase which is “**intuitively equivalent**”. This is an accurate translation of the Polish “**intuicyjnie równoważnym**” [p. 140]. The German translation is not surprising: “**inhaltlich äquivalente**” [p. 376], see also [1.2.1].

Again, we are missing the quotation marks in the English version: “which embraces them all as special cases and would form their ‘**infinite logical product**’”. In Polish we read “obejmującej je wszystkie jako szczególne przypadki, stanowiącej ich “**nieskończony iloczyn logiczny**”” [p. 140], and also in German “die sie alle als Spezialfälle umfassen und ihr “**unendliches logisches Produkt**” bilden würde” [p. 377], see also the commentary to [210].

The word ‘**actually**’ appearing in the footnote is a typical translation error. The Polish word ‘**aktualnie**’ and the German ‘**aktuell**’ may have the same root as their English version, however their meanings are very different from each other. Hence, the English translation should rather be ‘**currently**’ or ‘**at the present**’, which is exactly what is meant in Polish and German.

[254] *The methods successfully used in previous chapters for constructing a correct definition of a true sentence fail in regard to the ‘richer’ languages. Furthermore, an important methodological consequence of Theorem I is pointed out.*

Translational Remarks

At the end of the first paragraph of this page, two discrepancies occurred. The complete translation of the following passage should be “we were able to **show definitively in Theorem I**” as it is in Polish “zdołaliśmy **w twierdzeniu I** definitywnie

wykazać” [p. 142], and in German “wir konnten im Zusammenhang damit **in Satz I endgültig zeigen**” [p. 378]. It may be clear from the context what Tarski means, nevertheless he mentions it explicitly in the original.

In the next sentence we read that “the significance of the results reached reduces **just to this**”, which seems to trivialize the investigations a bit. In Polish Tarski writes “**Do tego właśnie** sprowadza się znaczenie uzyskanego wyniku” [p. 142]. It has been accurately translated into German, where we read “**Eben darauf** reduziert sich die Bedeutung des erzielten Ergebnisses” [p. 378]. The parts written in boldface would be better translated as “to this exactly”, which would express the emphasis which is clear in Polish and German.

Later, Tarski mentions the important methodological consequences of Theorem I. Where in Polish we read “zdań **intuicyjnie prawdziwych**” [p. 142], it should be translated as “**intuitively true** sentences, and not as in German “**inhaltlich wahren** Aussagen” [p. 378], see also [1.2.1].

In the footnote we read that a structural definition of truth “cannot be constructed **even for a bit richer** languages of finite order”, which is an accurate translation of the Polish “nie daje się skonstruować **nawet dla nieco bogatszych** języków skończonego rzędu” [p. 141], and of the German version “sich **sogar für einigermassen reichere** Sprachen endlicher Ordnung nicht konstruieren lässt” [p. 378]. Furthermore, the part of the footnote following the example of Hilbert-Ackerman’s ‘engere Funktionenkalkül’ has been newly added to the English version, and hence does not appear in Polish or German.

[255] *It is emphasized that on the basis of Th.I, the possibility of operating consistently and in agreement with intuition with semantical concepts cannot be excluded. A possible way of doing this is by means of an axiomatic method. As a consequence of the discussion in the previous section Theorem II is introduced.*

Translational Remarks

Here, Tarski draws our attention to the natural idea of “setting up semantics as a **separate** deductive science”. This is a more accurate translation of the Polish “ugruntowania semantyki jako **odrębnej** nauki dedukcyjnej” [p. 143]. The German translation (Tarski 1935) is already inaccurate “die Semantik als eine **besondere** deduktive Wissenschaft zu begründen” [p. 378]. In the later editions (Tarski 1986) the term ‘**besondere**’ has been simply omitted.

The translation of Theorem II has proven rather difficult. First, we have the already mentioned inaccuracy where we read “**among** its consequences” instead of “**as** its consequences”, which is the translation of “**jako** konsekwencje” [p. 143], and of “**als** Folgerungen” [p. 379]. Later, again instead of the two boldfaced occurrences of the term ‘sentence’, it should read ‘**provable sentence(s)**’, since in Polish it is ‘**tezy(ę)**’ [p. 143], and in German ‘**Satz**’ [p. 379], see also [1.2.3]. The second error is worth remembering because later in Theorem III we have the exact opposite translational problem.

[256] *Theorem II concerns only a fragment of the studied language, which obviously must be finite. From this it follows that single fragments of the theory of truth can be established as fragments of the metatheory. Later, Theorem III is presented, followed by a sketch of its proof.*

Translational Remarks

In the second paragraph, Tarski refers to the single fragments of the investigated theory which “can be established as **fragments** of the metatheory”. The boldfaced term should rather be translated as ‘sections’ or ‘branches’, which are better translations of the Polish ‘**działy**’ [p. 144] or of the German ‘**Teilgebiete**’ [p. 380].

In Theorem III, first Tarski writes in Polish ‘**wyraz** pierwotny’ [p. 144], which should be translated as ‘primitive **expression**’ or ‘primitive **term**’. The German translation is better here: ‘**Grundterminus**’ [p. 380]. Later in Th. III, Tarski writes ‘**zдания**’ [p. 144], which means the same as ‘**sentences**’. In German we have ‘**Sätze**’ [p. 380]. Here, the English and the German translations are, by way of an exception, more accurate than the original, since what Tarski really means are ‘provable sentences’.

In the proof of this theorem we should read that “**No finite number** of these axioms can lead to a contradiction”, which is a better translation of “**Żadna skończona liczba** tych aksjomatów nie może prowadzić do sprzeczności” [p. 144]. The German translation reads along the lines of the English one: “**Eine endliche Zahl** dieser Axiome kann [...] nicht zu einem Widerspruch führen” [p. 380]. It is perhaps worthy of notice that in Polish grammar the so-called ‘double negation’ is allowed, and quite often practiced for that matter, as a means of emphasis.

[257] *The proof of Theorem III is finished, only to notice its rather restricted power. A concrete example to illustrate the problem is provided.*

Translational Remarks

The translation of the first sentence beginning on this page is pretty sloppy. In Polish we read “**Z drugiej strony**, jeśli jakakolwiek **nieskończona** klasa zdań jest sprzeczna, to, jak łatwo okazać, sprzeczność musi tkwić **już** w pewnej skończonej części tej klasy” [p. 145]. Here is the accurate translation “**On the other hand**, if any **infinite** class of sentences is contradictory, then, as is easily shown, the contradiction must **already** appear in a finite part of this class.” The German translation already leaves much to be wished of “Wenn **irgend eine Klasse** von Aussagen einen Widerspruch enthält, so muss **andererseits** der Widerspruch – wie man leicht zeigen kann – **schon** in einem endlichen Teil dieser Klasse auftreten” [p. 380].

Later, Tarski speaks of substituting for the variable ‘*x*’ “**arbitrary** structural-descriptive names of sentences”. In Polish “**dowolne** nazwy strukturalnoopisowe zdań” [p. 145], and also correctly in German: “**beliebige** strukturell-deskriptive Namen von Aussagen” [p. 381].

The translation of the last sentence beginning on this page is just as sloppy as that of the first one. This is the Polish version: “Można by **np.** przyjąć jako nowe aksjomaty zasady sprzeczności i wyłączonego środka oraz **prawa**, zgodnie z którymi

konsekwencje zdań prawdziwych są **zawsze** zdaniem prawdziwym i wszystkie **tezy** badanej nauki należą również do zdań prawdziwych” [p. 146]. It should be translated as “We could **for example** take as new axioms the principles of contradiction and excluded middle, as well as those **principles** which assert that the consequences of true sentences are **always** true, and that all **provable sentences** of the science investigated are also true sentences”. The German translation is a bit better than the English one, but not flawless: “Als neue Axiome könnte man **z.B.** die Sätze annehmen, nach denen die Folgerungen aus wahren Aussagen **stets** wahr sind und auch alle **Grundsätze** der untersuchten Wissenschaft zu den wahren Aussagen gehören” [p. 381]. It should be noted that both translations of the last term in bold, which in this English edition appears on p.258, ‘**primitive sentences**’ and ‘**Grundsätze**’ are, however, actually an exceptional improvement of the original.

[258] *The possibility of extending Th. III to an enlarged axiom system is discussed. It is concluded that the axioms of the theory of truth together with the original axioms of the metatheory should constitute a categorical system, a requirement which fails to be satisfied.*

Translational Remarks

In Polish, the ‘**accidental character**’ of the enlargement of the axiom system is preceded by the expression “**w znacznej mierze**” [p. 146], which means the same as “**for the most part**”. It has been left out of the German translation as well, nevertheless it seems important to know to what extent the character of this enlargement is accidental.

Later, the Polish expression “**terminu** pierwotnego” [p. 147] should be translated as “**primitive term**”, not necessarily as “**primitive sign**”. The German translation is correct: “**Grundterminus**” [p. 382].

[259] *A successful solution to strengthening the theory of truth is presented. The rule of infinite induction is introduced as an additional rule of inference.*

Translational Remarks

The second part of the first sentence has been written in the Polish original in plural: “**zdań będących generalizacjami tych funkcji**” [p. 147] which can be translated as “**the sentences which are the generalizations of such functions**”. In the German translation we already have the singular “**der Aussage, die die Generalisation dieser Funktion ist**” [p. 383]. Also here, both translations are rather an improvement of the original.

Later in the same sentence, we should read “**imperfection and incompleteness**”, as we read in Polish “**niedoskonałości i niekompletności**” [p. 147], and in German “**Unvollkommenheit und Unvollständigkeit**” [p. 383].

In the last sentence, the term ‘rule’ calls for a certain supplement. Although this paragraph deals with the *rule of infinite induction*, it also mentions many other notions. Therefore, in Polish, Tarski writes clearly “**rozważanej reguły**” [p. 148], which has been accurately translated into German: “**der betrachteten Regel**”

[p. 384], and means the same as “the **considered** rule”.

[260] *The advantages of the rule of infinite induction over the standard rules of inference are emphasized. The new rule leads to a positive solution of many problems where the old rules failed. Furthermore, it is superior to any additional axioms.*

Translational Remarks

In the first sentence beginning on this page, Tarski speaks of the “non-finitist” nature of the rule of infinite induction and, in Polish, “infinistyczny” [p. 149] is written within quotation marks, as well as it is in German “infinistischen” [p. 384]. Not so in the English translation, see also the commentary to [210].

Right in the next sentence, where we read ‘sentences’ in English, Tarski wrote in Polish ‘tez’ [p. 149], which means the same as ‘provable sentences’. In the German translation we have ‘Sätze’ [p. 384], see also [1.2.3]. Still in the same sentence, in the English version, there are quotation marks missing, this time around the term ‘effectively’, see also the commentary to [210]. Later, we read in Polish “na gruncie skonstruowanych dla tych języków **definicji** prawdy” [p. 149], which can be translated as “on the grounds constructed for these languages’ **definitions** of truth”. We should notice that ‘definitions’ is written in plural, because for all of these languages a separate definition of truth has to be constructed. The German translation is also inaccurate “auf dem Boden der für diese Sprachen konstruierten **Definition** der Wahrheit” [p. 385].

In the first footnote the title of the report is quoted: in Polish [p. 148] *O niesprzeczności i zupełności nauk dedukcyjnych*, and in German [p. 385] *Über die Widerspruchsfreiheit und Vollständigkeit der deduktiven Wissenschaften*.

In the second footnote in Polish we read that “przyjmując rozważaną regułę w **metanauce**, a nie włączając jej do **nauki**, możemy wykazać, że klasa tez **nauki** jest niesprzeczna” [p. 149]. It should be translated as “if we adopt the considered rule in the **metascience** without including it in the **science**, we can prove that the class of provable sentences of the **science** is consistent”. These inaccuracies originated in the German translation “wenn wir diese Regel in der **Metasprache** annehmen, ohne sie der **Sprache** anzugliedern, beweisen, dass die Klasse der beweisbaren Sätze der **Wissenschaft** widerspruchsfrei ist” [p. 385].

[261] *If the rule of infinite induction is adopted in the metatheory, then the axiom system referred to by Th.III suffices for the development of the theory of truth. It is impossible to answer the question whether the theory of truth built in such a way remains without inner contradiction, however.*

Translational Remarks

Tarski speaks of not being able, for the present, to prove Th.III for the enlarged ‘**meta-science**’, and not for the ‘**metalanguage**’; in Polish ‘**metanauki**’ [p. 151]. And again, the inaccuracy comes from the German translation, where we read ‘**Metasprache**’ [p. 386].

The footnote is new and appears only in the English version.

[262] *It is summarized that the liar paradox cannot be directly reconstructed when the rule of infinite induction is adopted. Also, the results obtained for the general theory of classes can be applied to other languages of infinite order. The footnote deals with the problem of infinite inductive definitions.*

Translational Remarks

There is a minor discrepancy in the sentence referring to the axioms adopted in Tarski's theory of truth. In Polish we read "w przeciwstawieniu do języka potocznego, aksjomaty, które przyjmujemy w teorii prawdy, **noszą wyraźny charakter** częściowych definicji" [p. 151], which means the same as "in contrast to colloquial language, the axioms which we adopt in the theory of truth **bear a distinct character** of partial definitions". The discrepancy originated again in the German translation "Die in der Theorie der Wahrheit angenommenen Axiome **besitzen** nämlich hier, im Gegensatz zur Umgangssprache, **deutlich den Charakter** von Teildefinitionen" [p. 386]. This minor syntactical discrepancy does not influence the content of the sentence.

"Meaningful expressions", in the lines 2-3 from the bottom, is an awkward and confusing translation of Polish '**form znaczeniowych**' [p. 152], which should rather be translated as "**meaning forms**", as it is in German: "**Bedeutungsformen**" [p. 387].

[263] *The possibility of investigating whole classes of languages is considered. It is certain that the expression 'true sentence' will become ambiguous when it concerns more than one language. Moreover, the difficulties experienced so far are also expected to increase.*

Translational Remarks

The Polish phrasing "twierdzenie I zachowuje swój walor dla wszystkich języków rozważanej **kategorii**" [p. 153], means the same as "Theorem I retains its validity for all languages of the considered **category**". Here, in German we read "Satz I seine Geltung für alle Sprachen der betrachteten **Art** behält" [p. 387]. This German sentence has been translated as the first sentence beginning on this page. Both German and English versions avoid ambiguity which we have in Polish, where Tarski uses the term '**kategorii**', not relating to any particular semantical category, but to all languages of infinite order.

Where in English we read that we try to define the regarded expression "**within** the metalanguage", it would be better to write "**on the ground of** the metalanguage" which is a correct translation of "**na gruncie** metajęzyka" [p. 153]. The expressions are defined in the metascience by the means, or precisely on the ground of, the metalanguage. The German translation is just as inaccurate and misleading as the English one: "**innerhalb** der Metasprache" [p. 388].

In the next sentence we are dealing with a translation of the Polish term '**pierwotnych**' [p. 154], which has been alternating in this article between 'primitive' and 'fundamental', leading to confusion. An accurate translation here is '**primitive**'.

The correctness of the German translation may perhaps be subject to discussion, nevertheless it is at least consistent throughout the paper, and also here we read ‘**Grundaussdrücke**’ [p. 388], see also footnote ‡ on p. 212 of the English edition.

The expression “**zrelatywizowany charakter terminu**” [p. 154], of the next sentence, means the same as “**relativized character of the term**”. It has been accurately translated into German as “**relativisierten Charakters des betrachteten Terminus**” [p. 388].

[264] *The results obtained here can be extended to other semantical concepts by setting up a system of postulates containing partial definitions analogous to the sentences described in condition (α) of Convention T. Methods similar to those described in Sects. 3 and 4 enable the construction of the required definitions for other languages of finite order. Using Th.I as a basis it can be shown that no such definitions can be constructed for the languages of infinite order.*

Translational Remarks

Regarding the semantical concepts, Tarski writes that for each of them “a system of postulates can be set up which (1) contains partial definitions analogous to the **statements** described in condition (α) of the convention **T**”. As discussed in [1.2.3], the translation of the considered terms in boldface has been alternating within the translations of this paper. Here, Tarski writes “analogiczne do **zdań**” [p. 154] which means the same as “analogous to the **sentences**”. The translator of the German version was also inconsistent here: “**Sätzen** analog” [p. 389].

In the 3rd line from the bottom, it may not be clear from the English translation but “rząd **wszystkich** zmiennych” [p. 155] means the same as “the order of **all** variables”. In German we read correctly this time: “die Ordnung **alle** Variablen” [p. 389].

[265] *Once again the advantages of the rule of infinite induction in strengthening the metatheory are emphasized. In this case, however, it is not clear whether, and how, the consistency of the system can be proved. Sect. 6 begins with listing the results of the investigations. The three theses A-C summarize the results concerning the definition of a true sentence.*

Ever since the publication of Tarski’s masterpiece on the concept of truth, there have been lively philosophical debates concerning its contribution to the contemporary discussions on the topic of truth. Without going into detail, we will just emphasize Tarski’s accomplishments. First, however, we wish to distinguish clearly Tarski’s goal from what has often,⁵⁹ misleadingly, been held against him. Tarski’s definitions, and hence the results of this monograph, were never meant to constitute “a theory of truth”. Tarski’s goal was to construct, for a given formalized language, a formally correct definition of a true sentence, which was at the same time adequate with regard to its content. And that he did, at the same time providing an excellent starting point for a parallel analysis of other semantical notions. As Patterson points out very accurately, again

⁵⁹E.g. Putnam (1994), Davidson (1990).

Tarski's project was not to provide a "theory of truth" in the sense of a conceptual analysis or something like a metaphysical account of the nature of truth. It was to provide, in accord with Intuitionistic Formalism, (i) an account of the conditions under which a term of a deductive theory has a role that constrains it to express the concept of truth and (ii) the means of introducing terms with such roles, perhaps while meeting further desiderata—in particular that the terms be introduced via explicit definitions so as to guarantee relative consistency. Tarski's account of (i) is that " $\in Tr$ " expresses the concept of truth as applied to the object language if and only if all T-sentences formed with it are theorems. His account of (ii) is the method of recursion of satisfaction. (Patterson 2012, p. 150–1)

Tarski never claimed that any of these definitions, or his Convention **T** provided an analysis of the concept of truth or a "theory of truth". One objection to Tarski's account of truth is that it demands a new definition of truth each time a new notion is added to a language. Paradoxically, it provides an answer to the misinterpretation of Tarski's CTFL, which describes it as Tarski's theory of truth.

It does follow from Tarski's way of proceeding that if a new term is added to a language the definition of truth needs to be altered. If the definition were intended as a conceptual analysis, this would be bizarre, just as commentators who assume that the definition is an analysis have taken it to be. But since the definition isn't a conceptual analysis, the point [is, MG] simply that recursion by satisfaction needs to begin from lexical base clauses. Is it, however, a failing of Tarski's account that it tells us nothing about how to extend a definition when the object language is extended with a new expression? The answer here is that Tarski's account, taken as a whole, tells us exactly as much that it should tell us, neither more nor less. What Tarski's account tells us is that the new definition for the extended object language has to be such that, when added to formal syntax (which, note, likewise needs to be extended in a way not indicated by the prior syntactic theory) the T-sentences for the extended language become theorems. What the account doesn't tell us is exactly how to do this. (Patterson 2012, p. 151)

As we have seen, Patterson emphasizes the important role Intuitionistic Formalism played in Tarski's work, at least until 1935.⁶⁰ As we will see in the commentary to *Postscript*, Tarski's philosophy developed radically away from Leśniewski's influence.

Translational Remarks

Also here, instead of '**metalinguage**' we should be reading the '**metascience**' twice, as we do in Polish '**metanauce**' and '**metanauki**' [p. 155]; the German translation where we read '**Metasprache**' twice [p. 389 and p. 390] deviates from the Polish original in the same way the English translation does.

Here in the second line, the Polish "**i w tej sytuacji**" [p. 155], which means the same as "**also in this case**", regards the languages of infinite order in general, therefore it is written in singular. In German however, we also have the plural expression "**auch in diesen Fällen**" [p. 390].

⁶⁰For a detailed discussion see Patterson (2012), Chap. 1.

2.7 Section 6. Summary

[266] *Here, the results are presented in a more general form, as they can be applied to other semantical concepts. Furthermore, it is emphasized that by acquiring the definition of a true sentence for deductive sciences of finite order, a general method for proving their consistency can be obtained.*

A crucial result of Tarski's work is that intuitively adequate definitions of the semantical notions of definability, denotation, or satisfaction can be reached in an analogous manner. Tarski had been looking for a precise theory of the semantical notions, for such a theory did not exist at the time. Semantical notions lacked a systematic analysis and they had not been defined in terms of already accepted concepts used in logical and mathematical systems. Moreover, there was no coherent axiomatic theory of the semantical notions either.

It is also important to remember that Tarski deliberately did not give a general formulation of the method of defining a true sentence. Instead, he chose a few languages as examples and presented his definition using them as basis for his investigations.⁶¹

[267] *Final remarks about the definition of a true sentence belonging to the theory of knowledge and about its value are made. Also, once more the advantages and the importance of the formalized languages are emphasized.*

Translational Remarks

As Tarski speaks of the “thankless task of a **reform** of this language” and how the everyday language “would still preserve its **naturalness**”, the two bodfaced terms should be placed within quotation marks, as they are in Polish [p. 158] and in German [p. 393].

2.8 Section 7. Postscript

[268] *The Postscript brings out a new perspective on this article, because now the languages whose structure cannot be brought into harmony with the theory of semantical categories are also to be considered.*

As it is known, *Postscript* and *Historical Notes* were written by Tarski later than the original Polish paper and added only to the German translation. What is less known, is that Tarski considered writing it already in March 1935, before the German version was published.⁶² Also, not many know that he wrote it in Polish and it was translated into German by Kazimierz Adjukiewicz. Although Adjukiewicz knew German better than Tarski and Tarski appreciated his offer of translating the postscript, he had some

⁶¹ Gómez-Torrente (2015).

⁶² See also Sect. 2.1.

reservations. In a letter to Twardowski from Paris, on the 28th of August, 1935, he wrote that

In many places I had doubts whether prof. Adjukiewicz chose for the translation the most suitable term or if he conveyed most accurately the original text. Obviously, with my knowledge of German I could not even think of making any essential stylistic changes myself. On the other hand though, since Dear Sir Professor wished to receive the manuscript back as soon as possible, I could not have sent it to any of the Viennese friends who were helping me with the previous corrections. Therefore, I contented myself with revising Nachwort in regard to its objective, unifying it with the preceding pages in regard to the terms, the symbols and partly the orthography, and I introduced a few minor changes. [Translation M.G.]⁶³

Tarski emphasized that his positions regarding certain points have radically changed since writing the original. The postscript was so important to him, that he was willing to leave out some earlier parts of the work, in order for the postscript to appear at the end of the German translation of his article. In a letter to Kazimierz Twardowski from Vienna written on the 9th of April, 1935 he makes his preferences explicit.

I'm sending at the same time "Nachwort"; it took, unfortunately more space than I expected. I will be very happy if it will be published with the whole paper; however, should the lack of space stand in a way, I would be willing to leave out some other part of my paper. [Translation M.G.]⁶⁴

As we have mentioned, the influence of Tarski's *Doktorvater* – Leśniewski, was rather substantial, at least at the beginning of Tarski's career.. While writing the original version before 1933, Tarski committed himself to working within Leśniewski's interpretation of STT based on the theory of semantical categories. This fact has significantly influenced the entire work and so its final results.

It seemed to me then that 'the theory of the semantical categories penetrates so deeply into our fundamental intuitions regarding the meaningfulness of expressions, that it is hardly possible to imagine a scientific language whose sentences possess a clear intuitive meaning but whose structure cannot be brought into harmony with the theory in question in one of its formulations' (cf. p. 215). Today I can no longer defend decisively the view I then took of this question. (Tarski 2006g, p. 268)

⁶³“W wielu miejscach miałem jednak wątpliwości, czy prof. Adjukiewicz obrał przy przekładzie najtrafniejszy zwrot lub też czy zupełnie wienie oddał tekst oryginału. Rzecz jasna, przy mojej znajomości niemieckiego nie mogłem nawet myśleć o wprowadzeniu na własną rękę jakichkolwiek istotniejszych zmian stylistycznych, Z drugiej strony, ponieważ Wielce Szanowny Pan Profesor życzył sobie bezzwłocznego zwrotu manuskryptu, nie mogłem go przesłać do nikogo z tych znanych wiedeńskich, którzy mi uprzednio przy korekcie pomagali, Dlatego też zadowolilem się skontrolowaniem Nachwort'u pod względem rzeczowym, uzgodnieniem go pod względem terminologicznym, symbolicznym i częściowo ortograficznym z poprzednimi arkuszami pracy oraz wprowadzeniem kilku mało istotnych zmian.” Letter L. 224/35 archived in Polskie Towarzystwo Filozoficzne, Poznań.

⁶⁴“Przesyłam równocześnie “Nachwort”; zajął on, niestety, więcej miejsca niż przypuszczałem. Bardzo się będę cieszył, jeśli zostanie on wydrukowany wraz z całą pracą; gdyby brak miejsca stał na przeszkodzie, gotów byłbym nawet zgodzić się na opuszczenie jakiegoś innego fragmentu pracy”. Letter L. 115/35 archived in Polskie Towarzystwo Filozoficzne, Poznań.

Two years later it seemed important for Tarski to also investigate the formalized languages for which the fundamental principles of the theory of semantical categories no longer hold. In the postscript Tarski abandoned STT and turned to a new framework. It has been interpreted by some (cf. Sundholm 2003 pp. 119–120) to be set theory, others hold that it is still type theory (Loeb 2014). We will take a look at a few possible interpretations of Tarski's choice of a framework for the postscript. The reader of this commentary will inevitably be left with an impression of my preferred interpretation however.⁶⁵

The structure of the languages now investigated exhibits the greatest possible analogy with the languages previously studied, except for the differences connected with the theory of semantical categories. Just as in Sects. 2 and 4 Tarski specifies the basic concepts for the newly investigated languages (primitive sentential function, axiom, consequence, provable theorem etc.).

Translational Remarks

Where in the English translation we read that the sentences of the scientific language possess a clear “**intuitive meaning**” we are most likely dealing with an improved version of the German “**inhaltlichen Sinn**” [p. 393]. See also [1.2.1].

Later, the German expression “**der fundamentalen Aussagenfunktion**” [p. 394] has been translated as “**primitive sentential function**” which is a bad choice since ‘primitive sentence’ serves as a synonym of ‘axiom’ in Definition 13. Tarski himself was unsatisfied with this translation and suggested to call these functions *fundamental* (or *elementary*); see footnote ⁺ on p. 212 of the commented English edition (2006).

[269] *For each of the newly considered languages the basic concepts are specified after the already introduced manner of the procedure from Sects. 2 and 4. The concept of the order of an expression plays once again an essential part.*

Here, the concept of *order* of an expression, introduced in Sect. 4, also plays an essential part, however, Tarski's change of the logical framework must be considered, since to the names of individuals and to the variables representing them Tarski assigns now order 0 (and not as before 1). It could be easily interpreted as a direct parallel between Tarski's *new* framework and Carnap's theory of levels.

By a **system of levels** in S , we understand an ordered series $\mathfrak{N}_1$ of non-empty classes of expressions which fulfil the six conditions given on p. 188. Since the number of the expressions of a language is, at the most, denumerably infinite, the number of classes of $\mathfrak{N}_1$ is likewise at the most denumerably infinite. These classes we call **levels**; let them be numbered with the finite—and, if necessary, also with the transfinite—ordinal numbers (of the second number-class): level 0 (or the zero level), level 1, 2, ..., ω , $\omega + 1$ We shall designate the expressions which belong to the classes of $\mathfrak{N}_1$ by ‘ $\mathfrak{S}_\alpha$ ’ [*Stufe*], and, specifically, those which belong to level α (where ‘ α ’ designates an ordinal number) by ‘ $^\alpha \mathfrak{S}_\alpha$ ’. (Carnap 1937, pp. 186–7)

⁶⁵The discussion in the postscript is based on an article I've been working on simultaneously to this monograph. It appeared in *Journal for the History of Analytical Philosophy*, Vol. 3, No. 10 (2015); cf. Gruber (2015).

For Carnap “The ⁰Σtu are called *individual expressions* and, as symbols, individual symbols.”⁶⁶

The order of a sentence-forming functor of a sentential function has been previously unambiguously determined by the orders of all arguments of this function, but now the principles of the theory of semantical categories no longer apply. The theory of levels, or simply Tarski’s new interpretation of the theory of types, allows both expressions of infinite order and predicates and functors that take arguments of variable order. Following Carnap’s theory of levels, also for Tarski *orders* could all be numbered by finite or transfinite ordinal numbers. The fact that the level of the arguments of a predicate is not fixed, but variable, allows Tarski to introduce the variables which ‘run through’ all orders. Therefore, as Tarski notes

...it may happen that one and the same sign plays the part of a functor in two or more sentential functions in which arguments occupying respectively the same places nevertheless belong to different orders. Thus in order to fix the order of any sign we must take into account the orders of all arguments in all sentential functions in which this sign is a sentence-forming functor. (Tarski 2006g, p. 269)

Translational Remarks

The German version informs us that “**das Hauptprinzip** der Theorie der semantischen Kategorien nunmehr nicht gilt” [p. 394]. “**The main principle**” (of the theory of the semantical categories which no longer holds) should be written in singular (as it is in German), not in plural as in the English translation. The same divergence occurs later in the Postscript [p. 272].

[270] *In connection with the classification of the signs of infinite order the concept of transfinite ordinal numbers is introduced. The symbol ‘ω’ is used to represent the smallest transfinite number, and hence the order of the language of the general theory of classes of Sect. 5.*

In order to classify the signs of infinite order Tarski employs the notion of *ordinal number* which is a generalization of the concept of natural number – the smallest ordinal numbers. Since this notion plays a central role in set theory, it has often been argued that Tarski was working within that framework in the postscript. Since for every infinite sequence of ordinal numbers there are numbers greater than every term of the sequence, there are also numbers which are greater than all natural numbers. These are *transfinite ordinal numbers*. In every non-empty class of ordinal numbers there is the smallest ordinal number, hence also the smallest transfinite number – denoted by the symbol ‘ω’. To the signs of infinite order which are functors of sentential functions containing exclusively arguments of finite order we assign the number ‘ω’ as their order (e.g. the language of the general theory of classes has the order ω). These explications are followed by a general recursive definition of order used by Tarski:

⁶⁶Carnap (1937, p. 188).

the order of a particular sign is the smallest ordinal number which is greater than the orders of all arguments in all sentential functions in which the given sign occurs as a sentence-forming functor.² (Tarski 2006g, p. 270)

It is important to notice that the footnote [2] takes us directly to the introduction of the system of levels in Carnap's *The Logical Syntax of Language* (LSL from now on). Tarski realized that in order to define truth for 'superior' languages, it was crucial that the variables in the languages investigated now were not of a definite order.

Ray (2005) presents an argument for an interpretation of the notion of order as used by Tarski. Ray notes that

a language might be of a higher order for either of two distinct reasons. In its original formulation the only way to have a *language* of higher order was to have *variables* of higher order. Call this limited notion *higher order in the narrow sense*. However, as a result of this extension of the notion of order to languages like the language of Zermelo set theory, it becomes possible to have a language of higher order but which does *not* have variables of higher type (nor any difference of grammatical form at all). This is because the order of the language in these cases is determined by the "order of all sets whose existence follows from the axioms adopted in the language".[2] Thus, under some circumstances the order of a language could be increased merely by the addition of an axiom. Call the notion which allows for this *higher order in the extended sense*. (Ray 2005, p. 436)

The definition of higher order languages in the extended sense, as presented by Ray, applies to the languages of set theory, e.g. the language of Zermelo set theory. For the language of Carnap's theory of levels, according to Ray's distinction, the notion of higher order in the narrow sense applies. Further, Ray points us in the direction of the (Tarski 1944) 'simplified' version of the manuscript on the concept of truth, where Tarski upheld this informal definition of *essential richness*. He held that in the construction of the required definition of truth using the recursive definition of satisfaction we need to

introduce into the meta-language variables of a higher logical type than those which occur in the object-language; or else to assume axiomatically in the meta-language the existence of classes that are more comprehensive than all those whose existence can be established in the object-language. (Tarski 1944, p. 353, fn. 16)

The second condition applies to the languages of set theory. If we allow for the interpretation that Tarski was working within Carnap's theory of levels, however, it becomes clear why he emphasized that we can always introduce into the metalanguage variables of higher order than all the variables of the object language. This means that the metalanguage can always be constructed in such a way as to become a language of higher order than the object language.

In particular it is always possible to construct the metalanguage in such a way that it contains variables of higher order than all the variables of the language studied. The metalanguage then becomes the language of higher order and thus one which is essentially richer in grammatical forms than the language we are investigating. This is a fact of the greatest importance from the point of view of the problems in which we are interested. For with this the distinction between languages of finite and infinite orders disappears – a distinction which was so prominent in Sects. 4 and 5 and was strongly expressed in the theses A and B formulated in the *Summary*. (Tarski 2006g, pp. 271–2)

This means that a construction of a formally correct and materially adequate definition of *true sentence* for languages of infinite order is now possible, as long as the metalanguage is of higher order than the object language. This bold statement has ever since been the source of a debate examining its readability.⁶⁷

Before we proceed with other relevant issues of the postscript, we could, perhaps, shortly notice how meticulous Tarski was regarding the translation of his text. In a letter to Kazimierz Twardowski from Paris on the 28th of August, 1935, Tarski writes

In regard to the issues, Dear Sir Professor mentions in his letter and regarding the text of the Nachwort, I could not, of course, decide myself which word “Nachfolger” or “Fortsetzer” fits better – maybe rather the latter one (others seem to me less accurate). The issue of using the terms “unendlich” and “transfinite” I resolved in such a way that I use exclusively (just like up to now in the work) the expression “Sprachen unendlicher Ordnung”; the word “transfinite” however, had to stay to indicate the ordinal numbers. [Translation M.G.]⁶⁸

Another concept which has been discussed and commented on very often is the concept of *essential richness* of a language.⁶⁹ Tarski actually defined this term in his article (Tarski 2006f), where he introduced it as an auxiliary concept to the problem of completeness.

Let X and Y be any two sets of sentences. We shall say that the set Y is *essentially richer than the set X with respect to specific terms*, if (1) every sentence of the set X also belongs to the set Y (and therefore every specific term of X also occurs in the sentences of Y) and if (‘’) in the sentences of Y there occur specific terms which are absent from the sentences of X and cannot be defined, even on the basis of the set Y , exclusively by means of those terms which occur in X .

If now there existed a set X of sentences for which it is impossible to construct an essentially richer set Y of sentences with respect to specific terms, then we should be inclined to say that the set X is complete with respect to its specific terms. It appears, however, that there are in general no such complete sets of sentences, apart from some trivial cases. (Tarski 2006f, p. 308)

[271] *In order to obtain languages which are superior to the previously discussed ones it is necessary to introduce into those new languages variables of transfinite order. This applies not only to the languages which are the objects of the investigations, but also to the metalanguages in which the investigations are carried out.*

Tarski knew that in order to define truth for ‘superior’ languages, the variables in the languages investigated were not of a definite order.

⁶⁷See for example Field (2008, pp. 23–41).

⁶⁸“Co do spraw, poruszonych w liście Wielce Szanownego Pana Profesora a dotyczącego tekstu “Nachwort’u”, to nie mogłem oczywiście sam rozstrzygnąć, który ze zwrotów “Nachfolger” czy “Fortsetzer” lepiej się nadaje – może raczej ten drugi (inne wydają się mniej odpowiednie). Sprawę użycia terminów “unendlich” i “transfinit” załatwiłem w ten sposób, że używam wyłącznie (tak jak i dotąd w pracy) “Sprachen unendlicher Ordnung”; słowo “transfinit” musiało jednak pozostać dla oznaczenia liczb porządkowych”. Letter L. 224/35 archived in Polskie Towarzystwo Filozoficzne, Poznań.

⁶⁹See e.g. Ray (2005).

we must introduce into the languages variables of indefinite order which, so to speak, ‘run through’ all possible orders, which can occur as functors or arguments in sentential functions without regard to the order of the remaining signs occurring in these functions, and which at the same time may be both functors and arguments in the same sentential functions. (Tarski 2006g, p. 271)

Admitting the expressions of transfinite order, Tarski allows for variables to be of indefinite order, which in turn means that variables can act as functors or arguments in sentential functions, or even in the same sentential function, at the same time disregarding the order of other signs in this function. It is essential to notice, as (Patterson 2012, p. 191) does, however, that using expressions of infinite order is not merely a matter of adding transfinite levels atop the hierarchy of STT, and thus being able to define truth for the general theory of classes in a languages which adheres to the principles of semantical category. Referring to Sundholm (2003, p. 118) Patterson concludes that Tarski now adheres

to the principle that the order of an expression is the least ordinal greater than any that specifies the order of any argument it takes, but there is no finite ordinal α such that ω is the least ordinal greater than α , the only way to get expressions of transfinite order is to have expressions that take arguments of *all* finite orders and hence to allow for variability in the order of the arguments that a functional expressions takes. (Patterson 2012, p. 192)

Following these elucidations is the often quoted footnote in which Tarski points to the similarity between the languages considered here and the languages of set theory. He holds that from the languages considered in the postscript it is *but a step* to languages of another kind.⁷⁰ The languages of another kind are the languages of set theory, such as presented by Zermelo and his successors. Tarski also explains the notion of order for the languages considered in this article. It has been argued⁷¹ that Tarski’s change of logical framework causes an ambiguity regarding the notion of order. The ambiguity, however, occurs only if we apply the method presented in the postscript to the languages of set theory. Since Tarski was not working within set theory but either within type theory, or possibly Carnap’s theory of levels, there is no ambiguity in the notion of order; both Tarski and Carnap apply the notion of order to the expressions of the language, hence it is a syntactical notion in both cases.⁷²

For the languages here discussed the concept of order by no means loses its importance; it no longer applies, however, to the expressions of the language, but either to the objects denoted by them or to the language as a whole. Individuals, i.e. objects which are not sets, we call objects of order 0; the order of an arbitrary set is the smallest ordinal number which is greater than the orders of all elements of this set; the order of the language is the smallest ordinal number which exceeds the order of all sets whose existence follows from the axioms adopted in the language. Our further exposition also applies without restriction to the languages which have just been discussed. (Tarski 2006g, p. 271, fn.1)

⁷⁰Cf. Tarski (2006g), p. 271, fn.1.

⁷¹Cf. Patterson (2012), p. 191.

⁷²For a detailed discussion on this topic see de Rouilhan (1998) and Loeb (2014). Loeb also presents an interesting argument on a possible interpretation of Tarski’s choice of a logical framework in the postscript.

Even though the postscript itself is not written within set theory, Tarski emphasizes in the last sentence of this footnote, that his expositions apply without restriction also to the languages of set theory. If we wanted to define truth for first-order set theory, we would have to do it in the language of second-order set theory. The statement in this footnote cannot be regarded as a radical change of framework by Tarski, since set theory was central to all of his earlier and later work in logic and mathematics. Moreover, at that time in Poland all work in mathematics and logic was done in set theory.⁷³

Perhaps, it is worth mentioning here that Tarski was reading a lot of works by his German speaking colleagues at the time the German edition was being translated, and at the time he was writing the postscript. He became friends with many members of the Vienna Circle during his stay in Vienna and he consulted with them regularly on the translation of his manuscript. In particular, he exchanged letters with Carnap asking for his advice in regard to the translation of certain problematic terms. In the course of this correspondence he received not only advice, but as we read in a post card to Kazimierz Twardowski from the 10th of May, 1934, the corrected version of Carnap's new book *Die logische Syntax der Sprache*. Having asked Carnap about the translation of the expression "Anführungszeichennamen" – "quotation-mark names", and not receiving much help in this case, Tarski received much more than he imagined.

I have already received an answer from Carnap, but unfortunately I have not found any reasonable advice there. Carnap only refrained from using in this situation which I'm concerned with, the word "Name" (= "Eigennamen") and he sent me the correction of his new book *Die logische Syntax der Sprache*, I suggest we adopt the terminology used there. [Translation M.G.]⁷⁴

It was important for Tarski that his masterpiece was easily understood by the international philosophical and logical community. Spending the few months in Vienna, made him aware of the differences between the way Polish and Austrian, in this case international, logicians worked. Just as he had to substitute his reference to a work by Sierpiński with a reference to a work by Fraenkel, he had to adopt an international terminology, and he soon realized the best source for this was Carnap's *Die logische Syntax der Sprache*.

Translational Remarks

Also here, the German version uses the singular, while the translator of the English version chose to use the plural: "wenn wir für **eine Sprache**, die derartige Variable enthält, die Einsetzungsregel formulieren und die von uns als Pseudodefinitionen bezeichnete Axiome beschreiben" [p. 397].

⁷³Cf. Feferman (2002, pp. 2–3).

⁷⁴"Otrzymałem już odpowiedź odd Carnapa, ale nie znalazłem w niej niestety żadnej rozsądnej rady. Carnap zastrzegł się tylko przeciw użyciu w tej sytuacji, o która mi chodzi, słowa "Name" (= "Eigennamen") i przysłał mi korektę swej nowej książki "Die logische Syntax der Sprache", proponując dostosować się do przyjętej tam terminologii". Letter L. 149/34 archived in Polskie Towarzystwo Filozoficzne, Poznań.

[272] *Since it is possible to construct a metalanguage of higher order than the investigated language, the construction of an adequate definition of truth for languages of infinite order presents no difficulty. Theorem I of Sect. 5 is still valid, if the order of the metalanguage does not exceed the order of the object language.*

Tarski emphasized that the essential move is the introduction of the variables of transfinite order not only to the investigated (object) language, but also to the metalanguage in which the investigations are carried out. This allows for the metalanguage to be constructed in such a way that it contains *variables of higher order* than the variables of the object language and thus, to become an *essentially richer* language. This essential richness of the metalanguage constitutes it as a language of higher order than the object language. In STT, in which Tarski was working in the Polish original, the order of each category determines the orders of all expressions belonging to this category, i.e. all expressions belonging to a given semantical category have the same order assigned to them – called the *order of this category*.⁷⁵ The theory of semantical categories worked only within the languages of finite order, however. In the postscript, Tarski turns to a different framework, possibly to Carnap's system of levels and thus, allows for the expressions to be of transfinite order, and more importantly for expressions which do not determine the orders of their arguments.⁷⁶ The fact that the metalanguage becomes a language of higher order than the studied language cancels the difference between languages of finite and infinite order. This means that a construction of a formally correct and materially adequate definition of truth for languages of infinite order is now possible, as long as the metalanguage is of higher order than the object language. With this statement Tarski rewrote the final results of his original paper. At the bottom of this page Tarski writes that the results presented in Th. I of Sect. 5 are still valid and can be extended to languages of any order.

It is impossible to give an adequate definition of truth for a language in which the arithmetic of the natural numbers can be constructed, if the order of the metalanguage in which the investigations are carried out does not exceed the order of the language investigated (cf. the relevant remarks on p. 253). (Tarski 2006g, p. 272)

Translational Remarks

It seems plausible to assume that the inconsistent translation from the previous paragraphs has been carried over onto the Postscript. In German we read that we are interested in the construction of “**einer richtigen und korrekten** Definition der Wahrheit für die Sprachen **endlicher** Ordnung” [p. 398], which brings out two discrepancies. First, Tarski probably means the concept of an adequate and correct definition of truth, see also [1.2.2]. Second, the definition concerns the languages of infinite order, as is correctly written in English, but not in German.

⁷⁵Cf. Tarski (2006g), p. 218.

⁷⁶Tarski (2006g, p. 270, ftn. 2).

[273] *Theses A and B are presented in their new formulation. Thesis C is left out, since it is of some value only when the order of the metalanguage is not higher than the order of the object language.*

Tarski makes his stance on the definability of truth explicit in the new theses.

A. *For every formalized language a formally correct and materially adequate definition of true sentence can be constructed in the metalanguage with help only of general logical expressions, of expressions of the language itself, and of terms from the morphology of language – but under the condition that the metalanguage possesses a higher order than the language which is the objet of investigations.*

B. *If the order of the metalanguage is at most equal to that of the language itself, such a definition cannot be constructed.* (Tarski 2006g, p. 273)

When we compare the new theses with the old ones, we notice at once that Tarski went down from 3 statements to only 2. The original statement C loses its importance, in the light of thesis A. The newly written thesis A states clearly that a formally correct and materially adequate definition of *true sentence* can be constructed for *every* finite, or infinite formalized language, as long as the metalanguage is of higher order than the object language. With this statement Tarski rewrote the final results of his original paper.

In defining truth for the languages of indefinite order, the essential step is the introduction of variables of transfinite order, not only to the object language, but also to the metalanguage. This allows for the construction of a higher order metalanguage which is essentially richer in grammatical forms than the language studied. This step cancels the distinction between the languages of finite and infinite order which yielded the negative conclusion in Sect. 5 of the original paper. Thus, as Tarski notes in retrospect

the setting up of a correct definition of truth for languages of infinite order would in principle be possible provided we had at our disposal in the metalanguage expressions of higher order than all the variables of the language investigated. The absence of such expressions in the metalanguage has rendered the extension of these methods of construction to languages of infinite order impossible. But now we are in a position to define the concept of truth for any language of finite or transfinite order, provided we take as the basis for our investigations a metalanguage of an order, which is at least greater by 1 than that of the language studied (an essential part is played here by the presence of variables of indefinite order in the metalanguage) (Tarski 2006g, p. 272)

It is important to inquire about the influence which allowed Tarski to come to these conclusions. We could naturally assume that Tarski arrived at the idea of transfinite types independently, for example through his own work on set theory. Even though this is not utterly impossible, Tarski would definitely have made an explicit statement on this, just as he did in *Historical Notes* and in a footnote on page 247 in regard to his and Gödel's results on the indefinability of truth. Since Tarski makes no such statement about the idea of using the variables of transfinite order, we must allow for the possibility that Carnap's theory of levels was the influence that helped Tarski arrive at the new theses. We know that Tarski read Carnap's *Logical Syntax of the Language* before it was published, in fact Tarski was among the scholars who

proof-read Carnap's monograph before it was published. In the Preface to the English edition Carnap explicitly thanks Tarski for his contribution.

The majority of these corrections and a number of further ones have been suggested by Dr. A. Tarski, others by J.C.C. McKinsey and W.V. Quine, to all of whom I am very much indebted for their most helpful criticisms. (Carnap 1937, p. xi)

An apt young scholar himself, Tarski understood quickly how valuable Carnap's theory of levels would prove for his definition of truth, and that it would enable him to reach positive results, where Leśniewski's framework did not work.

[274] *Theses A' and B' are presented as more general formulations of Theses A and B, which can now be extended to other semantical concepts. Also, certain parallels between the results of Tarski's and Gödel's investigations are emphasized.*

[275] *The application of Gödel's method of constructing undecidable sentences is outlined here. It runs parallel to the proof of Theorem I of Sect. 5, in which the symbol 'Tr' is now replaced by the symbol 'Pr', denoting the class of all provable sentences.*

Translational Remarks

In German the adjective describing the discussed definition is '**richtige**' [p. 402], whereas in English it is '**correct**', see also [1.2.2].

[276] *The results and the consequences of the application of Gödel's method are presented in connection with Tarski's paper. Also, further results for other semantical concepts are mentioned.*

[277] *In Historical Notes Tarski emphasizes the independence of his investigations from those of Gödel and points to the parallels of the two.*

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A Running Commentary with Consideration of the Polish Original and the German Translation

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