
Preface

At the origin of the Italian School of Algebraic Geometry, the figure of Corrado Segre (Saluzzo 1863–Torino 1924), celebrated for the excellence of his geometric investigations and his exemplary style of scholar, still offers today an enduring model of scientific education to new generations and an outstanding scientific legacy to contemporary geometry.

Corrado Segre played the role of leader of the above-mentioned School in the decades around the beginning of the twentieth century, for scientific, historical, and biographical reasons. The great British geometer Henry Frederick Baker affirmed that “He could probably be said to be the father of the wonderful, Italian School which has achieved so much in the birational theory of algebraical loci.”¹

The times were favorable for several reasons. Corrado Segre, as well as his students and scientific companions, belonged to the first generations growing up in the new, unified Italy. These scholars and scientists could be described, in some sense, as builders of the nation. They were adding the moral and concrete task of building new scientific institutions for a new, modernized country, to their own scientific interest.

In a more general sense, Segre’s School, though rooted within a specific discipline of the domain of science, was nevertheless an open and inclusive community of persons. Through scientific exchange and debate, they were interacting on a larger series of cultural issues: amongst themselves and both with society and the rest of the world. Certainly, in this period the ideas of science, education, and progress had many opportunities to meld; indeed, the model of the Italian School of Algebraic Geometry is a fine example of the consequences of this melding.

Federigo Enriques, a former disciple of Segre, was also certainly aware of this atmosphere and of the significance of Segre’s leadership. In 1938, while speaking about mathematical schools and the progress and evolution in mathematics, he

¹Baker, Henry Frederick, Corrado Segre, *Journal of the London Mathematical Society*, 1 (1926): 263–271, on p. 269.

probably referred back to his personal experiences and memories. His words are the most appropriate to understand the mood and soul of Segre's School:

Actually the progress of mathematics doesn't depend exclusively on the efforts of individual research, but also on the relationship between researchers and the cultural environment from which they originate. In order to correctly understand what history teaches us, it is necessary to underline the importance of the school in forming the mathematician [...]. The experiences and inspirations, together with unsuccessful attempts or glimpsed results and problems, as well as different types of research criteria formulated for practical purposes can only be communicated verbally in the intimacy of conversations between colleagues and friends or even better between master and pupil. The pupil continues the master's thoughts and ideas even after he has more or less knowingly reworked them into a new form. [...] Schools have a tendency to grow beyond their original conception and at that point the student will be influenced by the new and different ideas which nurture him. The development of mathematical schools [...] gains new life passing from one country to another, almost as if the spirit of the world could participate on a larger scale in this collective work.²

The School led by Segre flourished so much as to be directly associated with his name. In the same decades, Corrado Segre was a world-renowned master of geometrical sciences and author of fundamental achievements in the study of algebraic varieties.

On the occasion of the 150th anniversary of his birth, the Academy of Sciences, the University and the Polytechnic of Turin, in collaboration with several other scientific institutions organised the international conference *Homage to Corrado Segre (1863–1924)* and a series of initiatives to commemorate Corrado Segre and to reconstruct in a unified view the different aspects of Segre's scientific legacy.³

As a consequence, the conference brought together scholars in different fields, mainly from history of mathematics and algebraic geometry.

This volume recollects the refereed contributions of most of the participants in the conference and a few more invited papers, and naturally relies on two sections, reflecting the historical and the geometrical character of the international meeting.

²Enriques, Federigo, *Le matematiche nella storia e nella cultura*, Bologna: Zanichelli 1938, pp. 180–181: *Invero i progressi delle matematiche non dipendono soltanto dallo sforzo della ricerca individuale, sì anche dai rapporti dei ricercatori fra loro e coll'ambiente di cultura da cui traggono origine. Per bene comprendere questo insegnamento della storia, conviene rilevare l'importanza che ha nella formazione del matematico la scuola [...] Le esperienze e le suggestioni che si legano a tentativi non riusciti o a risultati e problemi appena intravisti, tanti criterii di ricerca che non sono formulati in maniera astratta, si comunicano soltanto a voce nell'intimità delle conversazioni fra colleghi ed amici o meglio fra maestro e scolaro. Lo scolaro riprende e continua il pensiero del maestro anche quando più o meno consapevolmente lo ricrea in una nuova forma [...] La scuola tende ad allargarsi al di fuori del proprio ambiente di origine, ed allora l'influenza sullo scolaro viene a comporsi con altri motivi diversi che la fecondano [...] Lo sviluppo delle scuole matematiche [...] si ravviva passando da una nazione ad un'altra, quasi a far partecipare più largamente all'opera comune lo spirito del mondo.*

³The Conference was held in Turin from November 28 to 30, 2013. See: <http://ricerca.mat.uniroma3.it/GVA/Segre150/segre150.html>.

The title of the volume contains the words *from classical to modern algebraic geometry*. They put in evidence the extraordinary influence of Segre and his importance today. This is generally visible in all the contributions, offered by the main specialists on subjects related to the life and work of Corrado Segre.

Historians propose to reconstruct how Segre's leadership became recognised in Italy and abroad taking also in account a great number of unpublished and unknown documents.

The first essay by Alberto Conte and Livia Giacardi offers a picture frame for the following papers. The authors, running through the 36 years (1888–1924) of teaching higher geometry in Turin, show how Segre's courses were a veritable forge for future researchers. The forty handwritten notebooks of his university lectures and other unpublished sources allow them to understand how he stimulated and closely interacted with his Italian and foreign students, and to identify the most salient features of his scientific leadership. Erika Luciano and Silvia Roero in their paper, relying on a very rich documentation, illustrate the complex dynamic of scientific exchanges with the international mathematical community, as well as some aspects of the scientific and personal biography of Segre, related to his institutional, political, and editorial role. The remaining essays are dedicated to a thorough analysis of less studied aspects of Segre's work relating to three different stages of his life. David Rowe shows how line geometry was an excellent starting point for both Segre and Italian algebraic geometry, concentrating his attention on two of Segre's papers dating back to the beginning of the eighties. Paola Gario focuses on the relationship between Segre, Guido Castelnuovo, and Federico Enriques, referring to the period (1887–1897) of their collaboration on the problem of the resolution of singularities of algebraic surfaces, without overlooking the interpersonal dynamics emerging from their rich correspondence. Finally, Aldo Brigaglia shows the genesis and the historical and scientific relevance of Segre's important works concerning the complex projective geometry and comments on how a genuine recognition of it arrived only later, with Julian Coolidge's work, and above all that by Elie Cartan.

The section is completed by the biographical timeline of Segre livened up by quotations and enriched by the portraits of the mathematician at different ages.

The section dedicated to contributions from the field of algebraic geometry confirms the continuity and the presence of the research themes considered by Corrado Segre. Classical algebraic geometry is sometimes used today as a name for a large active area of research within algebraic geometry. This area appears to be connected in a more direct way to the language, themes, and problems (often concerning concrete examples or special projective varieties), which were familiar to algebraic geometers of Segre's times. The above-mentioned contributions largely fit in this area. The picture emerging from them enlightens a very interesting series of nice geometric problems and new results. Several subjects of classical flavor are touched by this picture, with the use of modern techniques and new methods. All the contributions have a correspondence to Segre's work. We can partially summarize them as follows.

Hyperquadrics are considered by Laura Costa, Maria Rosa Mirò-Roig, and Joan Pons-Llopis in order to generalize and study, on an odd dimensional hyperquadric, instanton bundles, and their families. A special attention is paid to three-dimensional quadrics and to Hooft bundles on them. The paper by Luca Chiantini and Duccio Sacchi aims to introduce a notion of Hilbert function for subvarieties of Segre products, that is, products of projective spaces. This notion is more sophisticated than the natural one, defined via Segre embedding, and appears to be a starting point for a new theory and advances in the study of the complexity of a general tensor, with applications to several fields. Nodal cubic threefolds with isolated singularities were classified by Segre. In particular, the cubic threefold with maximal number of nodes bears his name as the Segre primal. Igor Dolgachev takes up the study of six nodal cubic threefolds, which is a case of special beauty and interest. The split surface of lines of a six nodal cubic threefold is described in all details. The Segre primal and its ubiquity in geometry are also revisited. The classical, and modern, subject of algebraic surfaces of general type and their moduli is well represented by the paper of Margarida Mendes Lopes and Rita Pardini. In it, some famous surfaces appear, namely Enriques surfaces and Godeaux surfaces. In particular, the authors construct the family of those Enriques surfaces which are quotient of a Godeaux surface by an involution, proving several properties related to this construction. As mentioned, algebraic curves and their linear series are very well present in this volume, due to the lecture notes of Corrado Segre reproduced here. Moreover, a paper by Edoardo Sernesi offers to the reader a self-contained and very interesting reconstruction of the proof of Riemann–Roch theorem for curves, which is essentially due to Castelnuovo and reflects ideas and observations of both Segre and Castelnuovo. Line geometry is a further very important theme where Corrado Segre played a leading role. This theme is taken up by Emilia Mezzetti in the paper “Geometry of lines and degeneracy loci of morphisms of vector bundles.” The title reflects the modern point of view and the modern use of vector bundles techniques in view of several applications. Nevertheless, this paper is also an original survey where the deep connections between new methods and Segre’s ideas are pointed out. Finally, the Cremona group of birational automorphisms of a projective space is obviously present in this volume. On this subject, new and important progress was made very recently. Moreover, very interesting connections to other fields, for instance, in the different fields of complex dynamics and of algebraic statistics, were deepened. Two papers on the Cremona group complete the series of geometric contributions to this volume. They reflect very well some of the recent changes of the “state of the art” on this subject. One is the paper by Jeremy Blanc on the set of the algebraic elements of the Cremona group of any projective space, which is proved to be a non-closed countable union of closed sets. The other paper, due to Ciro Ciliberto, Maria Angelica Cueto, Massimiliano Mella, Kristian Ranestad, and Piotr Zwiernik, introduces an effective method, with applications, to linearize suitable rational varieties by a sequence of Cremona transformations.

Notably, papers from historical or from geometric sections often converge on the same geometric theme or question, offering different but complementary points of

view. The reader can profit of both. This appears to be an interesting feature of this book and an achievement of the goals of the conference *Homage to Corrado Segre*.

Furthermore, a third part of the volume is enriched by the anastatic print of the unpublished Segre's manuscript *Introduzione alla geometria sugli enti algebrici semplicemente infiniti*. This document, with an introduction by Alberto Conte, is one of the forty manuscripts which recollect the notes of Segre's courses during the academic years. It corresponds to the year 1890–1891. Interestingly, it appears as a preliminary version of the famous memoir on algebraic curves published by Segre in 1894.⁴

This volume is completed by the list of documents of the Segre Archives, due to Livia Giacardi, Erika Luciano, Chiara Pizzarelli and C. Silvia Roero. The bibliography of the works by Corrado Segre, including all the reports on the papers of his disciples and collaborators, and the list of the handwritten notebooks, closes the volume.

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⁴Segre, Corrado, *Introduzione alla geometria sopra un ente algebrico semplicemente infinito*, *Annali di Matematica pura ed applicata*, 2, 22 (1894a): 41–142.

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