Preface

The book is devoted to the study of approximate solutions of common fixed point problems and convex feasibility problems in the presence of computational errors. A convex feasibility problem seeks to find a point which belongs to the intersection of a given finite family of subsets of a Hilbert space. This problem is a special case of a common fixed point problem which examines how to find a common fixed point of a finite family of self-mappings of a Hilbert space. The study of these problems has recently become a rapidly growing area of research. This is not only due to theoretical achievements in this area but also because of numerous applications to engineering and, in particular, to computed tomography and radiation therapy planning.

We present a number of results on the convergence behavior of algorithms, which are known as important tools for solving convex feasibility problems and common fixed point problems. According to the results known in the literature, these algorithms should converge to a solution. In this book, we study these algorithms taking into account computational errors which always present in practice. In this case, the convergence to a solution does not take place.

Moreover, we show that our algorithms generate a good approximate solution, if computational errors are bounded from above by a small positive constant. Clearly, in practice, it is sufficient to find a good approximate solution instead of constructing a minimizing sequence. On the other hand, practice, computations induce numerical errors, and if one uses methods in order to solve minimization problems, these methods usually provide only approximate solutions of the problems. Our main goal is, for a known computational error, to find out what an approximate solution can be obtained and how many iterates one needs for this.

The monograph contains twelve chapters. Chapter 1 is an introduction. In Chap. 2, we study dynamic string-averaging methods for common fixed point problems in a Hilbert space. Its results are a generalization of the results for the convex feasibility problems obtained in our recent paper in the journal *Journal of Nonlinear and Convex Analysis*. In Chap. 3, using iterative methods, we study common fixed point problems in metric spaces. In Chap. 4, approximate solutions of these problems are obtained by dynamic string-averaging methods in normed

spaces. Dynamic string methods, for common fixed point problems in a metric space, are introduced and studied in Chap. 5. Common fixed point problems, in the spaces with distances of the Bregman type, are analyzed in Chap. 6. The results of Chaps. 3–6 are new. Chapter 7 is devoted to the study of the convergence of an abstract version of the algorithm which is called in the literature as component-averaged row projections or CARP. In Chap. 8, which is based on our recent paper published in the journal *Nonlinear Analysis*, we study a proximal algorithm for finding a common zero of a family of maximal monotone operators. In Chap. 9, we extend the results of Chap. 8 for a dynamic string-averaging version of the proximal algorithm. The results of Chaps. 7 and 9 are new. In Chaps. 10–12, subgradient projection algorithms for convex feasibility problems are studied for finite and infinite Hilbert spaces. The results of these chapters concerning iterative methods were obtained in our recent papers published in the *Journal on Optimization Theory and Applications* and in the *Journal of Approximation Theory*, while the results on their dynamic string-averaging versions are new.

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