

Mixing and Diffusion in Regular/Fractal Grid Turbulence

Yasuhiko Sakai, Koji Nagata, Hiroki Suzuki and Yasumasa Ito

Abstract Scalar mixing in turbulent flows is widely observed in nature as well as in industrial plants. In this chapter, we deal with three topics related to mixing and diffusion in grid-generated turbulence. The first topic (shown in Sect. 1) is experimental research on an axisymmetric CO₂ jet issuing into free-stream turbulent flows generated by a square-mesh biplane round-rod grid (referred to as a regular grid in Sect. 1) and a square fractal grid. The CO₂ jet issues from a small pipe located in the decaying region of these grid turbulences. A composite probe consisting of two concentration-sensitive I-type hot-wire sensors is used. For both flows, the mesh Reynolds number in the free stream is 6,000, and the jet Reynolds number based on the relative velocity between the free stream and the exit velocity of the jet is 5,000. The Taylor Reynolds numbers are about 100 and 35 for the square fractal grid and the regular grid, respectively. The results show that the half-widths of the mean velocity and concentration of the jets increase more rapidly, and the root mean square velocity and concentration in the axial direction decay more slowly for stronger free-stream turbulence. The second topic (shown in Sect. 2) is the development of a mixing layer of a high-Schmidt-number passive scalar in turbulent flows generated by a square-mesh biplane square-bar grid (referred to as a regular grid in Sect. 2) and a square fractal grid with the same mesh Reynolds number of 2500. A uniform passive scalar (Rhodamine B) is supplied only from the lower stream; therefore, scalar mixing layers with an initial step profile develop downstream of the grids. Particle image velocimetry and planar laser-induced fluorescence are used to investigate the

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velocity and concentration statistics. It is reconfirmed that the square fractal grid produces a higher turbulence intensity than the regular grid. The eddy diffusivity of the mass in the square fractal grid turbulence is approximately 4.2 times larger than that in the regular grid turbulence. The third topic (shown in Sect. 3) is direct numerical simulation of the mixing layer developed in grid turbulence. The simulations include the heat transfer in turbulent flows generated by four types of grid: (a) a square-mesh biplane square-bar grid (referred to as a regular grid in Sect. 3), (b) a square-mesh single-plane square-bar grid, (c) a composite grid consisting of parallel square-bars and (d) a square fractal grid. Two fluids at different temperatures are provided separately in the upper and lower streams upstream of the grids, generating a thermal mixing layer behind the grid. For grid (a), simulations with two different Prandtl numbers of 0.71 and 7.1, corresponding to air and water flows, respectively, are performed. The results show that the typical grid turbulence and thermal mixing layer can be simulated downstream of the grids, and a larger vertical turbulent heat flux is observed when the Prandtl number is large. Next the mixing layers in regular and square fractal grid turbulences are compared. In particular, the effects of the ratio of the largest to the smallest bar for the fractal grid, t_r , are investigated. The results show that turbulent mixing is enhanced to a greater extent in fractal grid turbulence than in regular grid turbulence, especially at large t_r . In Sect. 4, the conclusion and future prospects are presented.

1 Effects of Free-Stream Turbulence on Axisymmetric CO₂ Jet Diffusion Fields

1.1 Introduction

Turbulent diffusion of gases issuing into turbulent flows is observed in many industrial and environmental flows. Gas diffusion from airplane exhaust into atmospheric turbulence is one example. Therefore, it is very important to investigate the effect of surrounding free-stream turbulence on the turbulent jet diffusion fields. Turbulent diffusion of gases in axisymmetric jets issuing into a quiescent or laminar co-flow has been investigated (Way and Libby 1971; Panchapakesan and Lumley 1993; Brown and Rebollo 1972). However, few studies have examined the diffusion fields of turbulent jets issuing into turbulent flows. In this section, therefore, we will show the experimentally obtained characteristics of axisymmetric turbulent CO₂ jet diffusion fields when jets issue into two types of turbulent flows. Free-stream turbulent flows were generated in a wind tunnel using a classical biplane round-rod grid (hereafter referred to as a regular grid in this section) and a square fractal grid. The Taylor Reynolds numbers were approximately 35 and 100 for the regular and fractal grid turbulence, respectively. First, the streamwise and vertical instantaneous velocities in grid turbulence were measured using hot-wire anemometry with an X-type hot-wire probe. Then, the instantaneous axial velocity and CO₂ volume fraction concentra-

tion in the jet diffusion fields were simultaneously measured using a composite probe consisting of two concentration-sensitive I-type hot-wire sensors (Sakai et al. 2001). The effects of free-stream turbulence on jet diffusion fields are discussed.

1.2 Experiments

1.2.1 Experimental Apparatus

Figure 1 shows a schematic of the experimental apparatus. The wind tunnel, which is identical to that used in our previous study (Sakai et al. 2001), consists of a fan, diffuser, honeycomb, mesh, contraction section, test section and exhaust chamber. The cross section of the test section is $300 \times 300 \text{ mm}^2$. Regular and square fractal grids (Hurst and Vassilicos 2007; Seoud and Vassilicos 2007; Mazellier and Vassilicos 2010; Suzuki et al. 2010a; Nagata et al. 2008a) were used in this study to generate

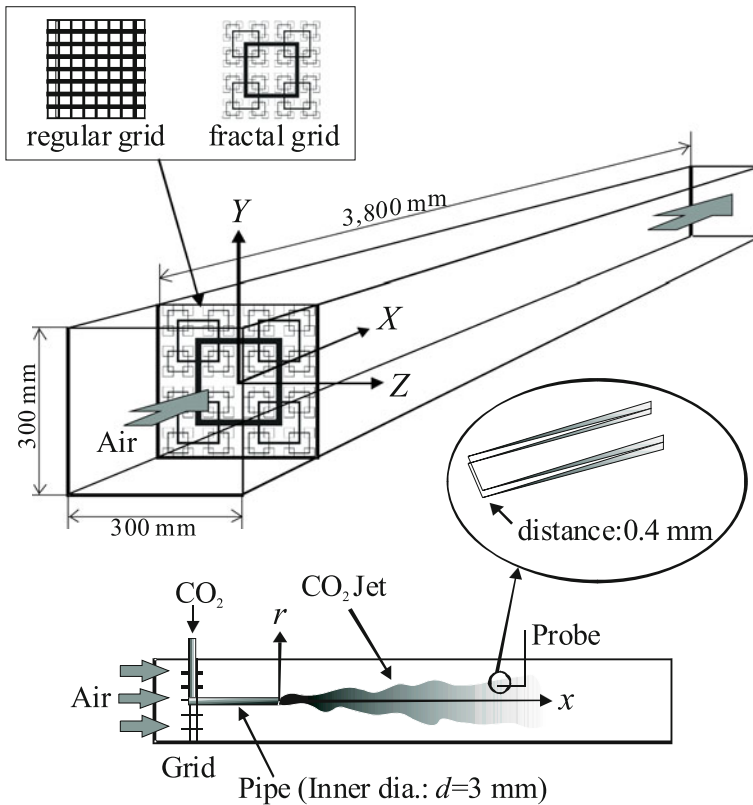


Fig. 1 Schematic of experimental apparatus and coordinate systems

Table 1 Grid parameters

Grid	D_f	N	σ	t_r	M_{eff} (mm)
Regular	2.0	1	0.36	1	15
Fractal	2.0	4	0.25	13.0	18.77

free-stream turbulences with different turbulence intensities and length scales. The grid parameters are listed in Table 1. Here D_f is the fractal dimension, N is the number of fractal iterations, σ is the solidity (the ratio between the blocking area of the grid and the area of the tunnel's test section), t_r is the thickness ratio of the largest to the smallest bar and M_{eff} is the effective mesh size, defined as follows.

$$M_{eff} = \left(\frac{4T^2}{P_M} \right) \sqrt{1 - \sigma}, \quad (1)$$

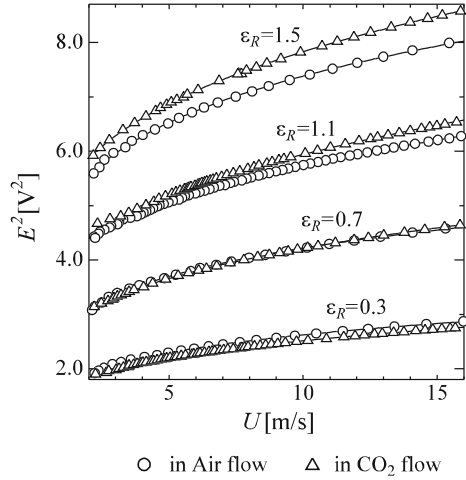
where T^2 is the cross-sectional area of the wind tunnel, and P_M is the fractal perimeter length of the grid. Note that for the regular grid, M_{eff} reduces to a mesh size M (Hurst and Vassilicos 2007). The turbulence-generating grid is installed, 200 mm downstream of the entrance to the test section. We set the Cartesian coordinates (X, Y, Z) in the streamwise, vertical and spanwise directions, respectively, with the origin at the centre of the turbulence-generating grids, as the coordinate system for measuring grid turbulence.

For the fractal grid, CO₂ gas is introduced into the centre of the grid through a pipe with an inner diameter of 3 mm and an outer diameter of 4 mm from a gas tank. For the regular grid, the centre vertical rod is replaced by a pipe with the same diameter as the grid bar, through which CO₂ gas is introduced. Then these pipes are connected to another straight pipe set horizontally in the downstream direction from the centre of the grids. CO₂ gas issues from the exit of this straight pipe into the grid turbulent flow. The exit of the issuing pipe is located 150 mm ($10M_{eff}$) and 1,126 mm ($60M_{eff}$) downstream of the regular and fractal grids, respectively, where the grid turbulences are decaying. The inner diameter of the issuing pipe d is 3 mm for both grids and the outer diameters are 3.2 and 4 mm for the regular and fractal grids, respectively. We set the cylindrical coordinates (x, r) as the coordinate system for measuring the jets. The origin in this case is set at the pipe exit.

1.2.2 Dual Hot-Wire Probe and Simultaneous Measurement of Instantaneous Velocity and Concentration

The output voltage E of the hot-wire probe varies according to the overheat ratio (OHR) ε_R and CO₂ concentration (Sakai et al. 2001; Chassaing 1977) as shown in Fig. 2. The OHR depends on the resistance of the wire R_W and is defined as

Fig. 2 Relation between square of output voltage E^2 and U for various OHR values



$$\varepsilon_R = (R_W - R_0)/R_0, \quad (2)$$

where R_0 is the resistance of the wire at the ambient fluid temperature. It is possible to simultaneously measure the axial velocity and concentration of the CO_2 -air mixture using this dependency (Sakai et al. 2001; Chassaing 1977, 1979). The dual hot-wire probe (Sakai et al. 2001) illustrated in Fig. 1 consists of platinum wires (each having a $5 \mu\text{m}$ diameter and 1 mm sensor length). The platinum wire probe can be operated at a high OHR because of its resistance to oxidation. The distance between the two wires is 0.4 mm. We carefully examined the set up to ensure that there is no physical or electrical interference between the wires at this separation. The OHR (ε_R) was set to 0.2 and 1.2 for low- and high-temperature wires, respectively. Calibrations were performed inside the test section of the wind tunnel using a nozzle (inner diameter 10 mm), as shown schematically in Fig. 3. The nozzle has a small hole (diameter 0.7 mm) to measure its internal static pressure. Before calibrating the probe, we

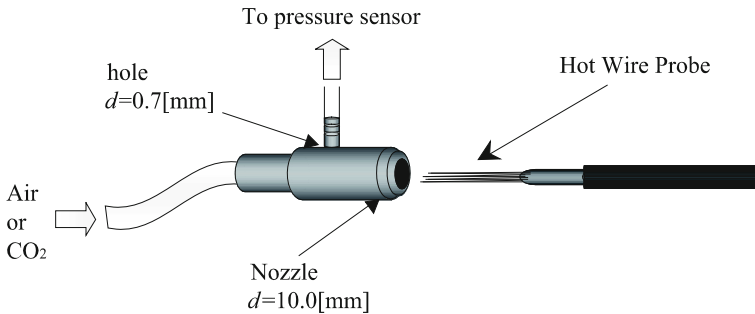


Fig. 3 Schematic of calibration nozzle

obtained the relationship between the static pressure inside the nozzle and the exit velocity. CO₂-air mixtures with various volumetric concentrations were introduced from the high-pressure gas tank into the nozzle through the pressure control valve, flow meter and rubber tube. The probe was placed inside the potential core of the jet, and the output voltages of the probe and the pressure inside the nozzle were recorded.

1.2.3 Experimental Conditions

Two experiments were performed in which CO₂ jets were issued into regular and fractal grid turbulences. The mesh and jet Reynolds numbers were $Re_M = M_{eff} U_0 / \nu_{air} = 6,000$ and $Re_J = (U_J - U_0) d / \nu_{CO_2} = 5,000$, respectively, for both cases, where U_0 is the mean velocity in the free stream, U_J is the mean velocity at the nozzle exit, d is the inner diameter of the CO₂-issuing pipe and ν_{air} and ν_{CO_2} are the kinematic viscosity of the air and CO₂, respectively. The grid turbulence was measured at the centreline of the test section ($Y = Z = 0$) and along the vertical (Y) direction at three streamwise locations ($X/M_{eff} = M_{eff} U_0 / \nu_{air} = 60, 100$, and 150 for fractal grid turbulence) using I- and X-type hot-wire probes (each having a $5 \mu\text{m}$ diameter and 1 mm sensor length). The CO₂ jet diffusion fields were measured at the centreline of the test section ($r = 0$) and along the radial (r) direction at $x/d = 15, 30$, and 45 using the dual hot-wire probe. The sampling frequency and number were 10 kHz and $262,144$ (approximately 26 s), respectively. These values were sufficient to obtain reliable statistics.

1.3 Results and Discussion

1.3.1 Characteristics of Grid-Generated Turbulence

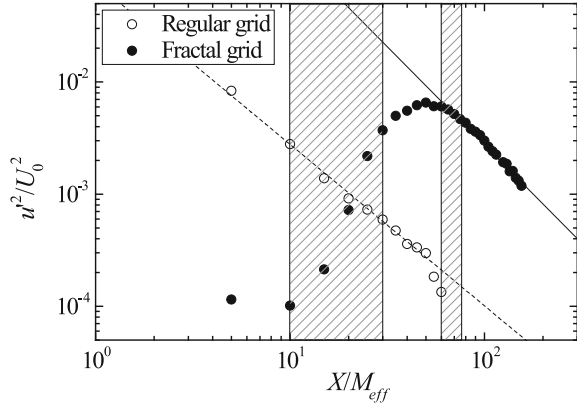
Turbulence Intensities, Homogeneity and Isotropy

Figure 4 shows the streamwise variation of the turbulence intensity of the streamwise velocity fluctuations u'^2 normalised by U_0^2 . The hatched regions in Fig. 4 show the streamwise regions, where the jet diffusion fields were measured (i.e. $x/d = 0-100$). It is found that u'^2 decays at $X/M_{eff} > 5$ in regular grid turbulence. On the other hand, u'^2 has a peak at approximately $X/M_{eff} = 50$ and decays in the region $X/M_{eff} > 50$ in fractal grid turbulence. Here, we assume the following simple power decay law for u'^2 :

$$\frac{u'^2}{U_0^2} = A \left(\frac{X}{M_{eff}} \right)^{-n}, \quad (3)$$

where n is the decay exponent, and A is some constant. For regular grid turbulence at $5 \leq X/M_{eff} \leq 50$, the decay exponent is $n = 1.44$, and the constant is $A = 0.0766$. This value of n is within the range of exponents observed in previous experiments

Fig. 4 Streamwise variation of streamwise turbulence intensity



(Sreenivasan et al. 1980; Warhaft 1984; Komori and Nagata 1996; Nakamura et al. 1987). On the other hand, for fractal grid turbulence at $60 \leq x/M_{eff} \leq 150$, we obtained $n = 1.75$ and $A = 8.70$. From Fig. 4, we find that the magnitudes of the turbulence intensity in the decay region in both flows are of the same order, but the decay rate is larger in fractal grid turbulence than in regular grid turbulence. Large decay exponents have also been confirmed by previous experiments (Hurst and Vassilicos 2007; Seoud and Vassilicos 2007; Mazellier and Vassilicos 2010; Nagata et al. 2013) and direct numerical simulation (DNS) (Zhou et al. 2014). Note that n varies according to the streamwise location of the virtual origin X_0 . Here we simply set $X_0 = 0$.

Figure 5 shows the streamwise variation of the ratio of the streamwise root mean square (rms) velocity u' to the vertical rms velocity v' . The ratio is found to be $u'/v' = 1.0$ – 1.15 in the decaying regions of regular and fractal grid turbulence. Thus, the isotropy of fractal grid turbulence is similar to that of regular grid turbulence (Comte-Bellot and Corrsin 1966).

Fig. 5 Streamwise variation of ratio of rms velocities

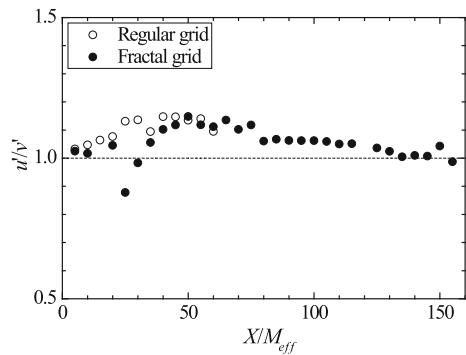


Fig. 6 Vertical profiles of streamwise turbulence intensity in fractal grid turbulence

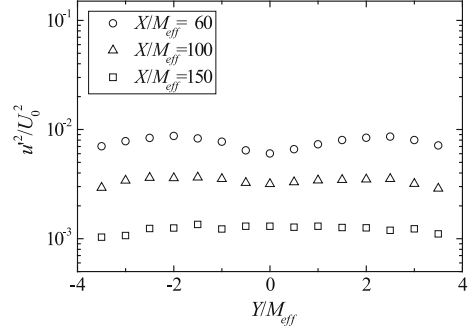


Figure 6 shows vertical profiles of the turbulence intensity u^2/U_0^2 in fractal grid turbulence. Although the turbulence intensity is not perfectly uniform at $X/M_{eff} = 60$, it becomes almost constant along the vertical direction in the far downstream region of $X/M_{eff} = 100$ and 150. Note that we have confirmed that the CO₂ release pipe does not affect the homogeneity and isotropy of the turbulence. We have also confirmed that it does not affect the skewness and flatness factors of the streamwise velocity fluctuations. The homogeneity and isotropy of fractal grid turbulence were also confirmed by previous experiments (Hurst and Vassilicos 2007; Seoud and Vassilicos 2007; Mazellier and Vassilicos 2010) and DNS (Nagata et al. 2008a).

Length Scales, Spectra and Taylor Reynolds Numbers

Figure 7 shows the streamwise variation of the longitudinal integral length scale L_{uX} normalised by M_{eff} obtained by integrating the autocorrelation function of u . L_{uX} increases in the decaying region of regular grid turbulence, whereas it is approximately constant in the decaying region of fractal grid turbulence. This result is consistent with that of previous experiments (Hurst and Vassilicos 2007; Seoud and Vassilicos 2007; Mazellier and Vassilicos 2010) and DNS (Nagata et al. 2008a).

Fig. 7 Streamwise variation of integral length scale

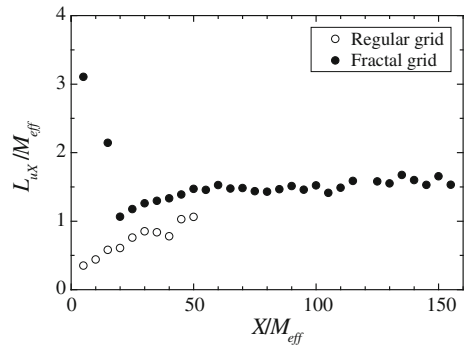


Fig. 8 Streamwise variation of Taylor microscale

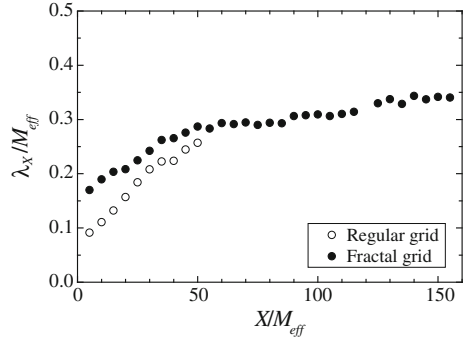


Figure 7 also shows that L_{uX} is much larger in the decaying region of fractal grid turbulence than in the decaying region of regular grid turbulence.

Figure 8 shows the streamwise variation of the Taylor microscale normalised by M_{eff} , obtained using

$$\lambda_x^2 = \frac{u'^2}{\langle (\partial u / \partial X)^2 \rangle} \quad (4)$$

in combination with the Taylor's frozen turbulence approximation for the streamwise spatial derivative. Figure 8 shows that the rate of increase is much slower in the decaying region of fractal grid turbulence than in that of regular grid turbulence. Previous experiments (Seoud and Vassilicos 2007) reported that λ is approximately constant in the decaying region of fractal grid turbulence. The present data lie between the classical result ($\lambda \propto x^{1/2}$) and the constancy observed in Seoud and Vassilicos (2007). Note that our previous DNS of fractal grid turbulence at $Re_M = 2,500$ showed the same tendency as the classical results (Nagata et al. 2008a). Thus, the constancy of λ could be a characteristic of fractal grid turbulence at higher Re_M . Mazellier and Vassilicos (2010) recently explained this effect using the solution of the spectral energy equation. The present result is qualitatively consistent with their analysis.

Figure 9 shows the power spectra of streamwise velocity fluctuations. No clear $-5/3$ power law region appears in the spectra of regular grid turbulence. On the other hand, the spectra of fractal grid turbulence show a region close to the $-5/3$ power law region. Thus, this result implies that fractal grid turbulence has a broader inertial subrange than regular grid turbulence for the same Re_M .

Figure 10 shows the streamwise variation of the turbulent Reynolds number $Re_\lambda = u' \lambda / \nu$. Re_λ is much larger in fractal grid turbulence than in regular grid turbulence, as suggested by the power spectra in Fig. 9 as well. Generation of high Reynolds number turbulence by fractal grids was first found by Hurst and Vassilicos (2007), and the present result verifies their work. For more details on fractal grid turbulence, see Hurst and Vassilicos (2007), Seoud and Vassilicos (2007), Mazellier and Vassilicos (2010), Nagata et al. (2008a, 2013).

Fig. 9 Power spectra of streamwise velocity fluctuations in **a** regular grid turbulence and **b** fractal grid turbulence

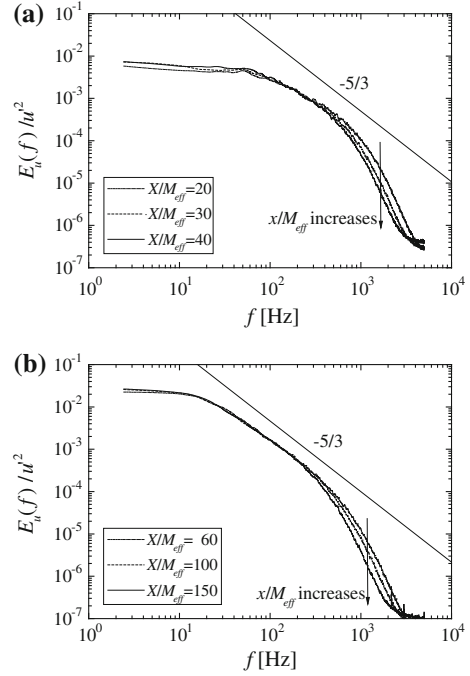
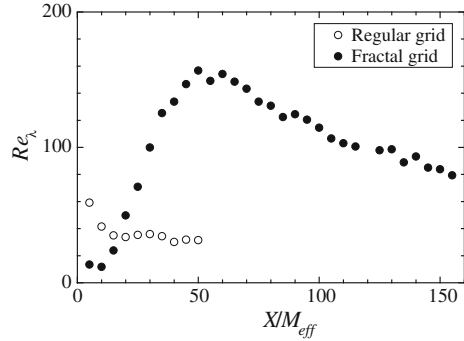


Fig. 10 Streamwise variation of turbulent Reynolds number



1.3.2 Characteristics of Turbulent Jet Diffusion Field

Mean Values

Figure 11 shows the axial variation of the mean velocity on the jet centreline, U_C . Here the abscissa is the axial distance x normalised by d , and the ordinate is the mean axial velocity relative to U_0 , $(U_C - U_0)$, normalised by the initial relative velocity at the exit of the pipe, $U_J - U_0$. The inverses of these quantities are also plotted. Figure 11 shows that in both cases, the inverse of the normalised mean velocity increases

Fig. 11 Axial variation of mean velocity

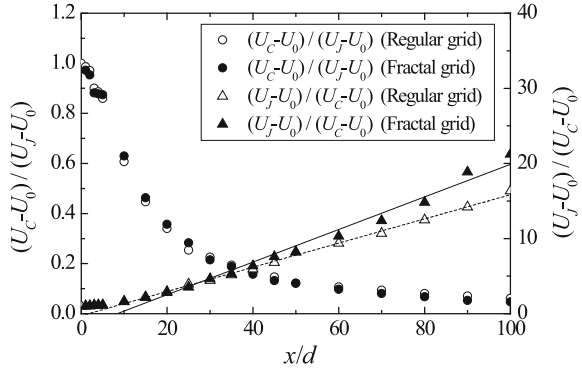
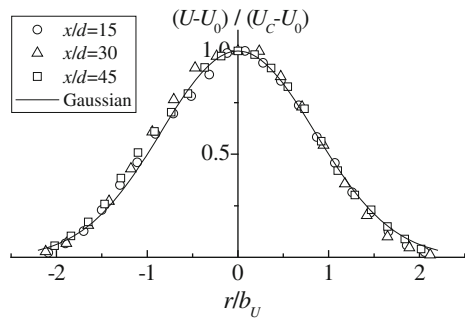


Fig. 12 Radial variation of mean velocity for the CO₂ jet in fractal grid turbulence



linearly in the downstream direction after $x/d = 10-20$. The rate of increase was found to be larger (i.e. decay is faster) in fractal grid turbulence than in regular grid turbulence. Figure 12 shows the radial profiles of the normalised mean velocity for fractal grid turbulence. The radial distance is normalised by the half-width of the radial profile of the mean velocity b_U . The result for regular grid turbulence was similar. As in the case where free-stream turbulence was absent, the radial profiles in this case also show self-similar Gaussian profiles.

Figure 13 shows the axial variation of the mean concentration C_C normalised by C_J . Its inverse is also plotted. As in the case of the mean velocity (Fig. 11), the inverse of the normalised mean concentration also increases linearly in the downstream direction after $x/d = 10-20$, and the rate of increase is larger (i.e. decay is faster) in fractal grid turbulence. Figure 14 shows the radial profiles of the normalised mean concentration for fractal grid turbulence. The radial distance is normalised by the half-width of the radial profile of the mean concentration b_C . The result for regular grid turbulence was similar to that for fractal grid turbulence. As in the case of no free-stream turbulence, the radial profiles here also show self-similar Gaussian shapes.

Figure 15 shows the axial variation of the half-width of the radial profiles of the mean velocity b_U and mean concentration b_C . Here b_U and b_C are normalised by d . The rates of increase of b_U and b_C are clearly larger in fractal grid turbulence than

Fig. 13 Axial variation of mean concentration

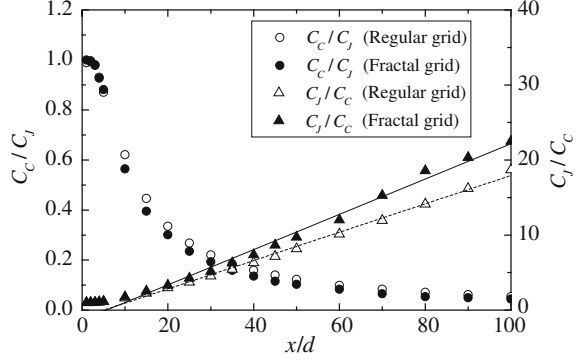
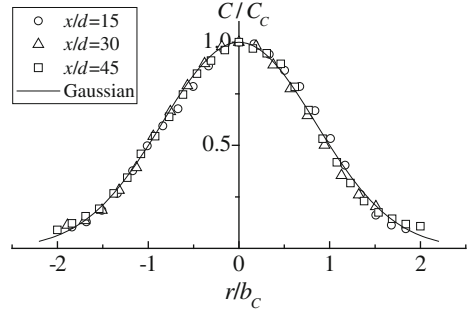


Fig. 14 Radial variation of mean concentration for the CO_2 jet in fractal grid turbulence



in regular grid turbulence. This result implies that the exchange of momentum and scalar between the jet and the ambient fluid is larger when the free-stream turbulence is stronger. Note that even in this case, the self-similar relation holds for the mean velocity and concentration fields (at the present turbulence level in the free stream), as shown in Figs. 12 and 14. When the radial profiles of the mean velocity and mean concentration are self-similar, the turbulent Schmidt number, $Sc_T = \nu_{Tr}/K_{Tr}$, can be estimated as $Sc_T = (b_U/b_C)^2$ (Hinze 1975; Sakai 2007), where ν_{Tr} and K_{Tr} are the eddy diffusivities of the momentum and concentration in the radial direction, respectively. From the data in Fig. 15, we obtain $Sc_T = 0.82$ for regular grid turbulence and $Sc_T = 0.67$ for fractal grid turbulence. The smaller value of Sc_T in fractal turbulence seems to be a noteworthy fact.

RMS Velocity

Figure 16 shows the axial variation of the rms velocity on the jet centreline, u'_C , normalised by the initial relative velocity $U_J - U_0$. Its inverse is also plotted. Figure 16 also shows that the inverse of the rms velocity increases linearly with x/d , as seen in jets without free-stream turbulence. The rate of increase is larger in regular grid turbulence than in fractal grid turbulence; i.e. the rms velocity decays more slowly

Fig. 15 Axial variation of half-width of mean velocity and mean concentration

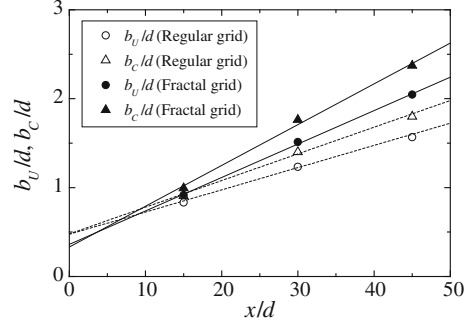
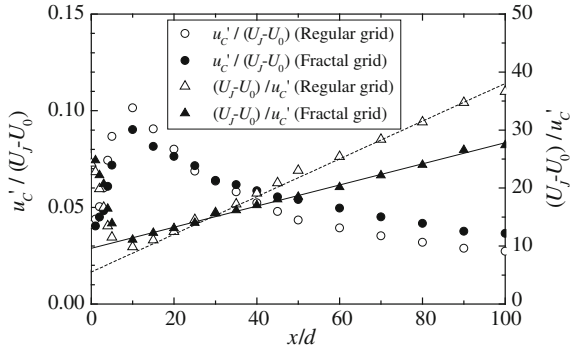


Fig. 16 Axial variation of normalised rms velocity and its inverse



in fractal grid turbulence. Figure 17 shows the axial variation of the relative intensity of the axial velocity fluctuations. The relative intensity is larger in fractal grid turbulence in the region $x/d > 35$. Figure 18 shows the radial variation of the axial rms velocity normalised by u'_c in fractal grid turbulence. The radial distance is normalised by b_U . The profiles are similar to those obtained in regular grid turbulence (Sakai et al. 2001), and they show a self-similar shape after $x/d = 15$.

Fig. 17 Axial variation of relative intensity of axial velocity fluctuations

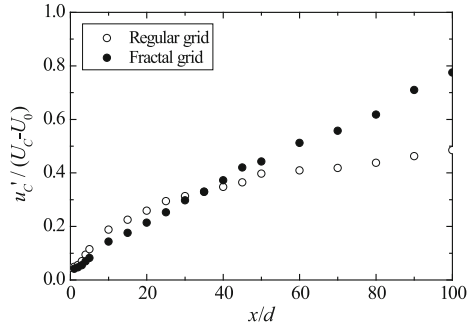
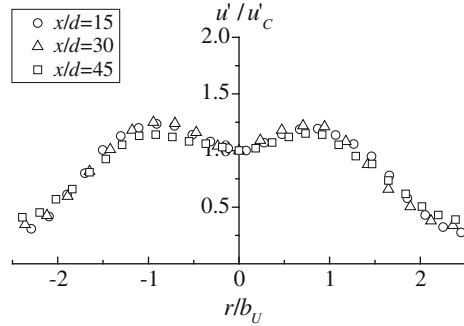


Fig. 18 Radial variation of axial rms velocity in fractal grid turbulence



RMS Concentration

Figure 19 shows the axial variation of the rms concentration on the jet centreline, c'_c , normalised by C_J . Its inverse is also plotted. A comparison of Fig. 19 with Fig. 16 with respect to the rms velocity reveals that the effect of free-stream turbulence on the rms concentration is similar to that on the rms velocity. Figure 20 shows the axial variation of the relative intensity of the concentration fluctuations. The relative intensity in fractal grid turbulence is larger in the region $x/d > 15$. Figure 21 shows the radial variation of the rms concentration normalised by C_C in fractal grid turbulence. The radial distance is normalised by b_C . Like the radial variation of the normalised rms velocity, the profiles here are similar to those in regular grid turbulence (Sakai et al. 2001). The profile at $x/d = 15$ does not collapse, but the profiles at $x/d = 30$ and 45 suggest that they are self-similar in this region.

Turbulent Mass Flux and Eddy Diffusivity of Mass

Figure 22 shows the axial variation of the axial turbulent mass flux normalised by the product of $U_J - U_0$ and C_J , $F = \langle uc \rangle / \{(U_J - U_0)C_J\}$. The axial variation of

Fig. 19 Axial variation of normalised rms concentration and its inverse

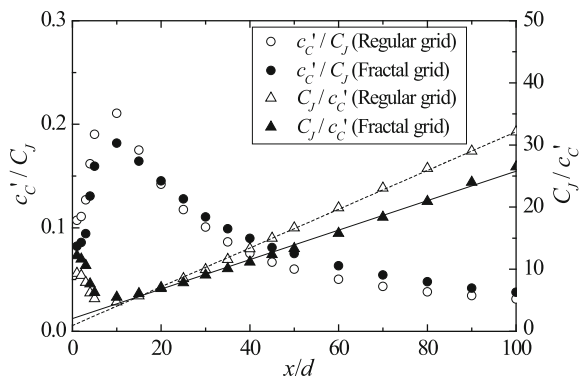


Fig. 20 Axial variation of relative intensity of concentration fluctuations

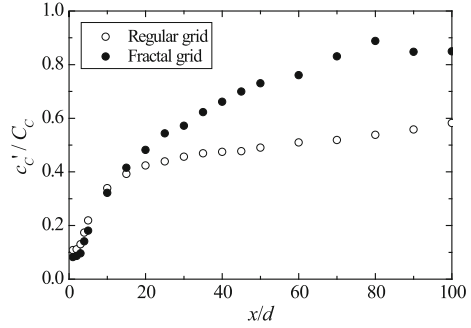
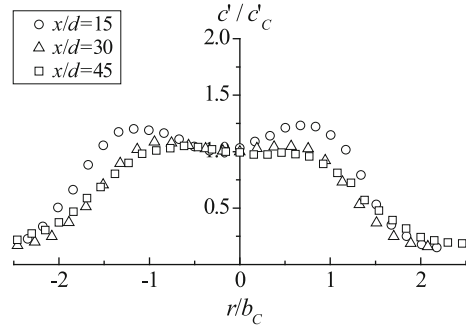


Fig. 21 Radial variation of rms concentration in fractal grid turbulence



$F^{-1/2}$ is also plotted. When the assumption of the gradient type of diffusion model for the axial turbulent scalar flux,

$$\langle uc \rangle = -K_{Tx} \frac{\partial C}{\partial x}, \quad (5)$$

is valid in the jet diffusion field, the axial variation of $F^{-1/2}$ is expected to be proportional to x/d (Chassaing 1979; Sakai et al. 1999). Here K_{Tx} denotes the eddy diffusivity of the concentration in the axial direction. Figure 22 shows that F is larger in fractal grid turbulence than in regular grid turbulence. This means that the turbulent axial mass transport is larger when fractal grid turbulence serves as the free stream. Figure 22 also shows that the axial variation of $F^{-1/2}$ is proportional to x/d in both cases, and therefore the concept of the gradient-type diffusion model holds in a jet diffusion field with free-stream turbulence under the present conditions. The eddy diffusivities calculated from Eq. (5) are $K_{Tx} = 3.6 \times 10^{-2} \text{ m}^2/\text{s}$ and $K_{Tx} = 1.3 \times 10^{-1} \text{ m}^2/\text{s}$ for regular and fractal grid turbulence, respectively, which implies that K_{Tx} is approximately three times larger for fractal grid turbulence.

Finally, Fig. 23 shows the co-spectra of u and c measured at the centreline of the jet in fractal grid turbulence, normalised by $\langle uc \rangle$. In the upstream region at $x/d = 20$, the co-spectrum exhibits power law decay with an exponent close to -2 . On the other hand, in the downstream region, the exponent is close to $-7/3$. Although the same

Fig. 22 Axial variation of the axial turbulent mass flux F and $F^{-1/2}$

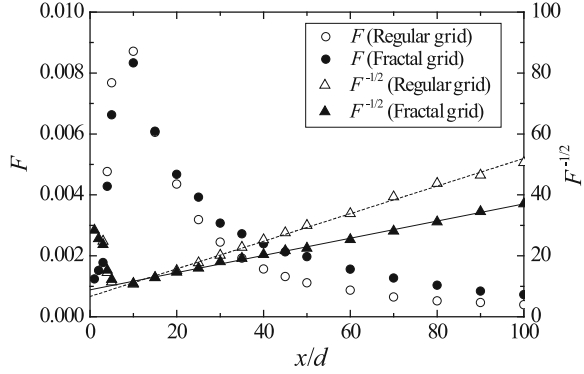
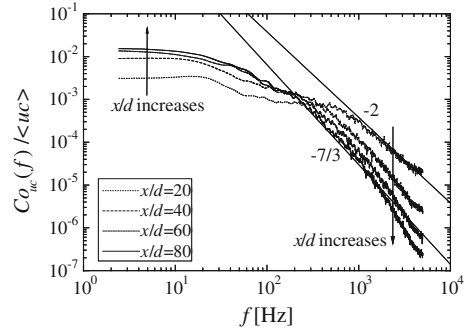


Fig. 23 Co-spectra of u and c in the jet diffusion field in fractal grid turbulence



tendency was seen for regular grid turbulence (not shown here), the slope is clearer for fractal grid turbulence. This is probably because the free-stream turbulence is more intense in fractal grid turbulence, and a clear $-5/3$ power region (i.e. an inertial subrange) was seen in the power spectra of u , as shown in Fig. 9.

1.4 Summary

The instantaneous axial velocity and volume fraction concentration were measured simultaneously in axisymmetric CO_2 turbulent jets issuing into regular and fractal grid turbulence to investigate the effects of free-stream turbulence on a CO_2 jet diffusion field. The main results are summarised as follows:

(1) The Taylor Reynolds number is much larger in fractal grid turbulence than in regular grid turbulence for the same mesh Reynolds number. Good homogeneity and isotropy were observed for both fractal and regular grid turbulence. These results concur with those of previous studies.

(2) The integral length scale of the streamwise velocity fluctuation u , L_{uX} , is much larger in fractal grid turbulence than in regular grid turbulence. L_{uX} increases in the decaying region of regular grid turbulence, whereas it is approximately constant in the decaying region of fractal grid turbulence. These results are in agreement with previous studies. The rate of increase of the Taylor microscale, λ , in the decaying region is much slower in fractal grid turbulence than in regular grid turbulence.

(3) The mean velocity and concentration on the axis of the jet decay faster in fractal grid turbulence (i.e. when the free-stream turbulence is stronger). This result implies that the exchange of momentum and scalar between the jet and the ambient fluid is larger when the free-stream turbulence is stronger. The radial profiles of the normalised mean velocity and concentration do not change under the influence of free-stream turbulence (at the present turbulence level), and they exhibit Gaussian shapes.

(4) The rms velocity and concentration in the axial direction in the jet diffusion field decay more slowly when the free-stream turbulence is stronger.

(5) The eddy diffusivity of mass in the axial direction in the jet diffusion field increases when the free-stream turbulence is stronger.

2 Development of Scalar Mixing Layer in Regular and Fractal Grid Turbulence

2.1 Introduction

Scalar mixing in turbulent flows is widely observed in nature as well as in industrial plants (Ulbrecht and Patterson 1985; Chate et al. 1999; Warhaft 2000; Dimotakis 2005). It is important to understand the mechanism of turbulent scalar transfer in order to increase the efficiency of industrial equipment (e.g. inline mixers, combustors and chemical reactors) or predict the diffusion of pollutants in nature.

Grid turbulence has commonly been used in fundamental research because its characteristics are close to those of isotropic decaying turbulence (Comte-Bellot and Corrsin 1966). Recently, turbulence generated by fractal grids consisting of self-similar objects (referred to in this section as fractal grid turbulence) has been experimentally investigated (Hurst and Vassilicos 2007; Seoud and Vassilicos 2007; Mazellier and Vassilicos 2010; Valente and Vassilicos 2011, 2012; Gomes-Fernandes et al. 2012, 2014; Nagata et al. 2013). Hurst and Vassilicos (2007) conducted their experiments with 21 different planar fractal grids for three types of fractal families (i.e. cross-type, I-type and square-type) and found characteristics that were very different from those of regular grid turbulence. In their experiments, the grid that generated the most ‘nonclassical’ or ‘nonequilibrium’ characteristics of decaying turbulence was called ‘a low blockage space-filling fractal square grid’. Therefore,

most subsequent studies on this topic concentrated on this type of fractal grid turbulence. The turbulence characteristics at moderately high Reynolds numbers in the downstream decay region relatively near the low blockage space-filling fractal square grid (i.e. in the region of $x_{peak} \leq x \leq x_*$, where x is the downstream distance from the grid, x_{peak} is the distance at which the turbulence intensity peaks and x_* is the wake-interaction length scale, defined as $x_* = L_0^2/t_0$, where L_0 and t_0 are the length and thickness of the biggest bar, respectively; see Fig. 24 for the coordinate system and grid geometry) are summarised as follows:

(1) With a blockage ratio σ as low as 25 %, fractal square grids produce higher turbulence intensities u_{rms}/U and turbulence Reynolds numbers $Re_\lambda = u_{rms}\lambda_x/\nu$ than do higher blockage regular grids for similar inlet velocities (Hurst and Vassilicos 2007). Here u_{rms} is the streamwise root mean square (rms) velocity; U is the mean velocity; λ_x is the Taylor microscale, defined as $\lambda_x = \sqrt{u^2}/\sqrt{(\partial u/\partial x)^2}$ ($-$ denotes the time average); and ν is the kinematic viscosity.

(2) The turbulence intensity along the centreline of the grid decays exponentially as follows:

$$\frac{u_{rms}^2}{U^2} = A \exp\left\{-B\left(\frac{x}{x_*}\right)\right\}, \quad (6)$$

where A and B are constants (Hurst and Vassilicos 2007; Mazellier and Vassilicos 2010). Note that Valente and Vassilicos (2011) showed later that the turbulence intensity obeys a power decay law when regions further downstream are considered ($0.6x_* \leq x \leq 1.5x_*$).

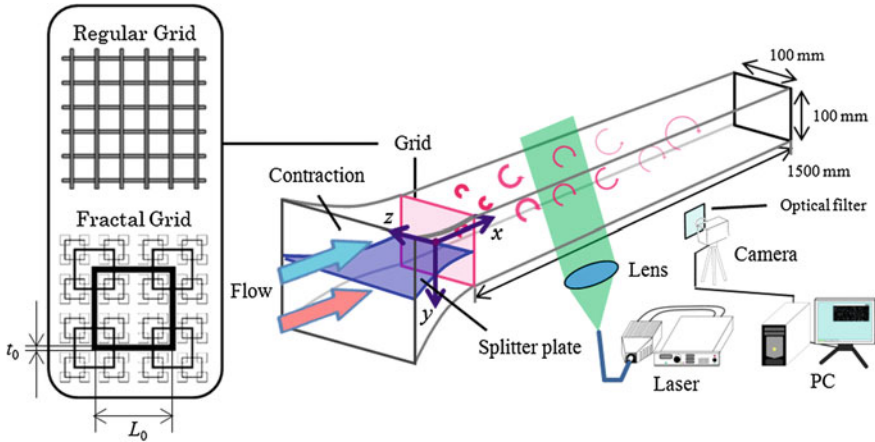


Fig. 24 Schematic of experimental apparatus and turbulence-generating grids

(3) The longitudinal integral length scale L_u , the Taylor microscale λ_x and their ratio L_u/λ_x are almost constant in the downstream direction and do not depend on the turbulence Reynolds number Re_λ . These scales are determined by the initial conditions, and L_u/λ_x is scaled as $L_u/\lambda_x \sim \text{Re}_0^{1/2}$, where $\text{Re}_0 = U_0 t_0/\nu$, and U_0 is the cross-sectionally averaged mean velocity (Mazellier and Vassilicos 2010; Valente and Vassilicos 2011).

(4) The dimensionless dissipation rate, $C_1 = \varepsilon L_u/u_{rms}^3$, is not constant, but it obeys the nonclassical scaling law $C_1 \propto \text{Re}_0^{1/2}/\text{Re}_\lambda$. In this case, the energy dissipation rate ε is proportional to u_{rms}^2 (not u_{rms}^3) as $\varepsilon = C_2 U_0 u_{rms}^2/x_*$, where C_2 is a constant of $O(1)$ (Mazellier and Vassilicos 2010). [Later Valente and Vassilicos (2012) showed that by introducing the global inlet Reynolds number Re_M , the same scaling law is also available for regular grid turbulence, i.e. $\varepsilon \sim \text{Re}_M/\text{Re}_L \sim \text{Re}_M^{1/2}/\text{Re}_\lambda$, which leads to $\varepsilon \sim U_0 u_{rms}^2 M/L_u^2$, where $\text{Re}_M = U_0 M/\nu$, M is a length scale characteristic of the grid, and $\text{Re}_L = u_{rms} L_u/\nu$.]

Among these characteristics, (1) and (4) seem to be particularly important in terms of applications. Mazellier and Vassilicos (2010) pointed out that based on characteristics (1) and (4) ‘it is actually possible to generate an intense turbulence with reduced dissipation and even design the level of this dissipation’. Therefore, we can expect that fractal grid turbulence is useful for designing energy-efficient mixers and lean premixed combustion gas turbines.

Because the above characteristics of fractal grid turbulence do not obey the Taylor–Richardson–Kolmogorov phenomenology (Davidson 2004; Vassilicos 2015), research on the origin of this nonclassical behaviour and internal turbulence structure is also very important to the field of turbulence physics. Therefore, direct numerical simulation (DNS) studies of fractal square grid turbulence have been conducted (Nagata et al. 2008a; Suzuki et al. 2010a, 2013a; Laizet et al. 2010; Laizet and Vassilicos 2011a; Laizet et al. 2013). Visualisations of these simulations showed that the vorticity field generated using fractal grids is more intermittent or clustered than that generated using regular grids (Nagata et al. 2008a; Suzuki et al. 2010a; Laizet and Vassilicos 2011a). Laizet et al. (2013) suggested the existence of a wide range of wavenumbers, wherein the interscale energy flux is dependent on the inlet/initial Reynolds number, which is negative, and is independent of the wavenumber but not necessarily of the viscosity. Furthermore, Laizet et al. (2013) performed a DNS of fractal square grid turbulence of $\text{Re}_M = 1,950$; the number of fractal iterations was $N = 4$, and $\text{Re}_M = U_0 M_{eff}/\nu$, a Reynolds number based on the effective mesh size M_{eff} defined by Eq. (1) (see Sect. 1.2). They investigated the downstream variation of the scaling law of the second-order structure function and the Q – R diagram (Q and R are the second and third invariants of the velocity gradient tensor, respectively) and found that there is a wide $2/3$ power law range in the near-field region $0 < x/x_* < 0.16$ even though the Q – R diagram has not yet adopted its tear-drop shape. More recently, Suzuki et al. (2013a) investigated fractal square grid turbulence of $\text{Re}_M = 2,500$ with $N = 4$. They found that both the turbulence kinetic energy, $k = (\overline{u^2} + \overline{v^2} + \overline{w^2})/2$ (where u , v , and w are the streamwise, vertical and spanwise fluctuation velocities, respectively), and the rms pressure fluctuation p_{rms} obey a

power law (not an exponential law) in the far downstream region, and their power indices are independent of the solidity σ . They also confirmed the constancy of L_u and considered why k decays according to the power law with a constant L_u on the basis of the self-preserving single-scale turbulence theory of George and Wang (2009).

DNS of the scalar diffusion fields in fractal grid turbulence has been attempted, but the simulations to date have been limited to cases of a low Prandtl (Pr) or Schmidt (Sc) number [$\text{Pr (or Sc)} \leq 0.71$] (Nagata et al. 2008b; Suzuki et al. 2010b; Laizet and Vassilicos 2011b, 2015). With regard to experiments with a high-Sc-number scalar field, Suzuki et al. (2010c, 2012a, 2013b) investigated the shear-free passive scalar mixing layer in turbulence generated by regular and fractal square grids using particle image velocimetry (PIV) together with planar laser-induced fluorescence (PLIF). They observed significant mixing and scalar transfer enhancement for the fractal square grid. However, their measurements were made only at very limited cross sections; consequently, the development of turbulence statistics in the scalar mixing layers could not be clarified sufficiently. We think that acquisition of more detailed data is necessary for the application of fractal grid turbulence to industrial equipment.

In this section, we present the results of extended velocity and concentration field measurements for a high-Sc-number scalar mixing layer in regular and fractal square grids using the same experimental apparatus as that used by Suzuki et al. (2010c, 2012a, 2013b); we investigate the mixing effects due to fractal grid turbulence quantitatively by comparing the spatial development of various turbulence statistics of the velocity and concentration fields in regular and fractal grid turbulence.

2.2 *Experimental Apparatus and Conditions*

2.2.1 *Water Channel, Grids and Experimental Conditions*

Figure 24 (right panel) shows a schematic diagram of the experimental apparatus and measurement system. The apparatus is a rectangular water channel made of polymethylmethacrylate; its length is 1.5 m, and its cross section is $0.1 \times 0.1 \text{ m}^2$. The grid is installed at the entrance of the test section. Figure 24 (left panel) shows schematic diagrams of the regular and fractal square grids. The grid parameters are listed in Table 2. Here D_f is the fractal dimension, t_r is the thickness ratio of the largest to the smallest bar and M_{eff} is the effective mesh size defined by Eq. (1) (see Sect. 1.2). D_f is chosen as $D_f = 2.0$ (space filling) here because space-filling grids are found to return the best homogeneity (Seoud and Vassilicos 2007). Re_M is adjusted to 2,500 for both grids. A uniform passive scalar (Rhodamine B) is supplied only from the lower stream, so scalar mixing layers with an initial step profile develop downstream of the grids. The Schmidt number Sc is approximately 2,100 (Culbertson et al. 2002).

Table 2 Flow conditions and grid parameters

Grid	N	D_f	t_r	σ	L_0 (mm)	t_0 (mm)	x_* (mm)	U_0 (mm)	M_{eff} (mm)	Re_M
Regular	1	2.0	1.0	0.36	10	2.0	50	0.286	10	2,500
Fractal	4	2.0	9.76	0.36	53.1	4.9	575	0.504	5.68	2,500

2.2.2 Particle Image Velocimetry

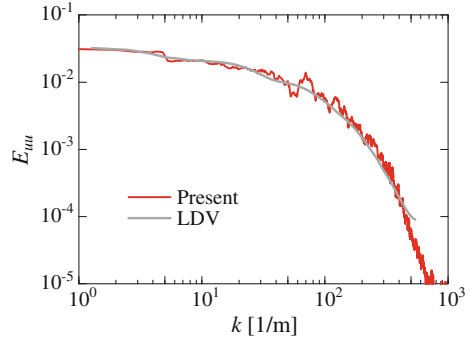
A continuous-wave laser operating at 532 nm (Omicron, FKLA-1000) is used as the excitation light source. The thickness of the laser sheet is about 1 mm. The mean diameter of the seeded particles is 50 μm , and their specific gravity is 1.03. In this measurement, more than seven particles are arranged in an interrogation window. The visualised flow field is captured using a high-speed video camera (Ametek Phantom V210) with $1,280 \times 800$ pixel resolution. The measured images are converted into 12-bit digital codes. The measurement area is $7.5 \times 30 \text{ mm}^2$ (128×512 pixels) in the x and y directions. The sampling frequency is 2,000 Hz, and a total of 2,048 images are captured. PIV analysis is carried out using a PIV software application developed in our laboratory. Fourier interpolation of the brightness distribution is introduced to improve the accuracy of small-scale velocity fluctuations by increasing the number of pixels by four times (Suzuki et al. 2012b). Then we apply a recursive local correlation method (Hart 2000) and a window offset method (Westerweel et al. 1997) to estimate the pixel-level velocity field. The window offset method enables us to decrease the level of uncertainty added to the exact velocity field. Furthermore, a gradient method (Sugii et al. 2000) is used for subpixel analysis. The spatial resolution of the estimated velocity field is $0.47 \times 0.47 \text{ mm}^2$ in the x and y directions (which is estimated by a final interrogation window of size 32×32 pixels with 50% overlap). The PIV measurement conditions are summarised in Table 3 with the PLIF conditions, where $L_x \times L_y$ indicates the measurement area, $N_x \times N_y$ is the number of measurement points (the number of velocity vectors for PIV and the number of pixels for PLIF), L_x/N_x ($=L_y/N_y$) is the spatial resolution, and N_s is the number of samples.

To confirm the measurement accuracy of the present PIV analysis, we compared the power spectra of the grid turbulence with that obtained using a laser Doppler velocimeter (LDV) (DANTEC 55X Modular system) with a polarisation beam splitter (55X24), 40 MHz Bragg cell, and beam expander; the laser was a 5 mW He–Ne

Table 3 Measuring area and spatial resolution

Method	$L_x \times L_y$ (mm^2)	$N_x \times N_y$	L_x/N_x ($=L_y/N_y$) (mm)	N_s
PIV	7.5×30	16×64	0.47	2,048
PLIF	10×100	$300 \times 3,000$	0.03	1,024

Fig. 25 Accuracy check of the present data processing algorithm by comparing the present data with the LDV result



laser (Spectra Physics 106-1; wavelength $\lambda = 632.8$ nm). The result at $x/M_{eff} = 20$ for regular grid turbulence is shown in Fig. 25. The present PIV result is found to be in good agreement with the result obtained using the LDV, which confirms the accuracy of the present PIV data processing algorithm.

We also confirmed the accuracy of the current processing algorithm using both a PIV standard image (Okamoto et al. 2000) and quasiparticle-image analysis (Keane and Adrian 1992; Suzuki et al. 2012b).

2.2.3 Planar Laser-Induced Fluorescence

The excitation light source is the same continuous-wave laser operating at 532 nm as that used for the PIV measurements. The thickness of the laser sheet is about 0.5 mm. A single-lens reflex camera (Nikon D700) is used to capture the PLIF images. The solid-state image-sensing device of the camera is 36.0×23.9 mm² in size and has a resolution of $4,256 \times 2,832$ pixels. The captured images are converted into 14-bit digital codes (RAW images). The measurement area is 10×100 mm² ($300 \times 3,000$ pixels) in the x and y directions. A Nikkor 50 mm F1.2S lens is used, and the F number is set to 2.0. Using an optical filter (Asahi Spectra shortpass filter), we cut off the excitation light at wavelengths greater than 450 nm. The ISO level is set to 400, and the shutter speed is 1/1,000 s. We developed an automatic measurement system and acquired a total of 1,024 images for statistical calculations. The measurement area and resolution are summarised in Table 3. PLIF analysis is carried out using a data processing method developed in our laboratory (Suzuki et al. 2010c, 2013b). Using this method, we can eliminate errors caused by (i) nonuniformity of the fluorescence dye concentration, (ii) the time dependence of the fluorescence quantum yield and (iii) temporal changes in the incident laser intensity and the effect of laser attenuation owing to dye absorption. The validity of the PLIF data processing method was confirmed by comparing the first- and second-order moments of the concentration fields with data obtained using the time-resolved single-point laser-induced fluorescence technique (Suzuki et al. 2010c, 2013b).

2.3 Results and Discussion

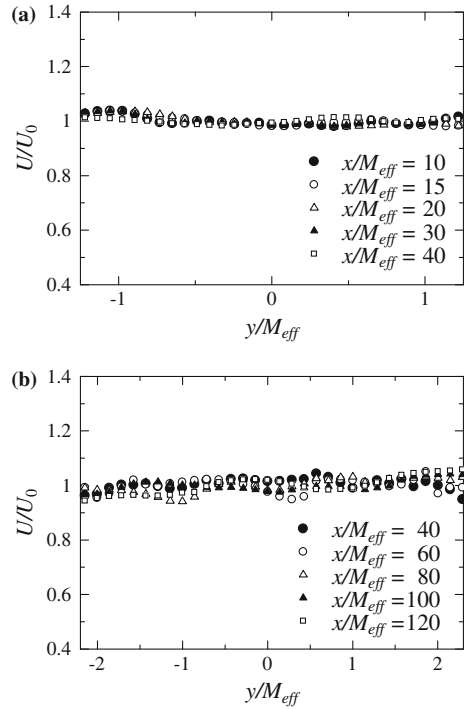
2.3.1 PIV Results

To elucidate the development of the scalar mixing layer in regular and fractal grid turbulence, the fundamental characteristics of each velocity field should be understood. In this subsection, we present in detail the results of PIV measurements.

Mean Velocity Field

Figure 26a, b show the downstream variation of the vertical (y -direction) profile of the mean velocity U in the regions $10 \leq x/M_{eff} \leq 40$ ($2.0 \leq x/x_* \leq 8.0$) for regular grid turbulence and $40 \leq x/M_{eff} \leq 120$ ($0.395 \leq x/x_* \leq 1.19$) for fractal grid turbulence, respectively. Here, the ordinate is normalised by the cross-sectionally averaged mean velocity U_0 , and the abscissa represents the y coordinate normalised by M_{eff} . The mean velocity has an almost uniform profile for both regular and fractal grid turbulence in the measurement region.

Fig. 26 Downstream variation of vertical profiles of mean velocity. **a** Regular grid turbulence. **b** Fractal grid turbulence



Velocity Fluctuation Field

The downstream variation of the vertical profile of the streamwise rms velocity u_{rms} and the vertical rms velocity v_{rms} are shown in Fig. 27a, b, respectively. Here the rms velocities u_{rms} and v_{rms} are normalised by U_0 , and the y coordinate is normalised by M_{eff} . Although there is some scattering of the data, the vertical profiles of u_{rms} and v_{rms} at each downstream location are almost uniform in the measurement range, and the magnitude of these profiles decreases in the downstream direction. In addition, u_{rms}/U_0 for regular grid turbulence at $x/M_{eff} = 10$ ($x = 100$ mm) is about 5%. In contrast, u_{rms}/U_0 for fractal grid turbulence at $x/M_{eff} = 40$ ($x = 227$ mm) is around 10% which is much higher than that for regular grid turbulence even though the measurement position was further downstream. Furthermore, for both types of turbulence, u_{rms} is higher than v_{rms} at all the measurement points, and their ratios (the anisotropy of the turbulent rms velocity) are $u_{rms}/v_{rms} = 1.15$ – 1.18 for regular grid turbulence and $u_{rms}/v_{rms} = 1.1$ – 1.25 for fractal grid turbulence. Note that anisotropy at this level has been recorded for the common regular grid turbulence (Comte-Bellot and Corrsin 1966).

Figure 28 shows the downstream variation of the turbulence relative intensity, u_{rms}^2/U^2 , along the centreline of the regular and fractal grids. In this figure, the ordinate is normalised by U , the abscissa is normalised by M_{eff} and both coordinates are on a logarithmic scale. The error bars in the figure show the range of data scat-

Fig. 27 Vertical profiles of the rms velocity u_{rms} (in the x direction) and the rms velocity v_{rms} (in the y direction). **a** Regular grid turbulence. **b** Fractal grid turbulence

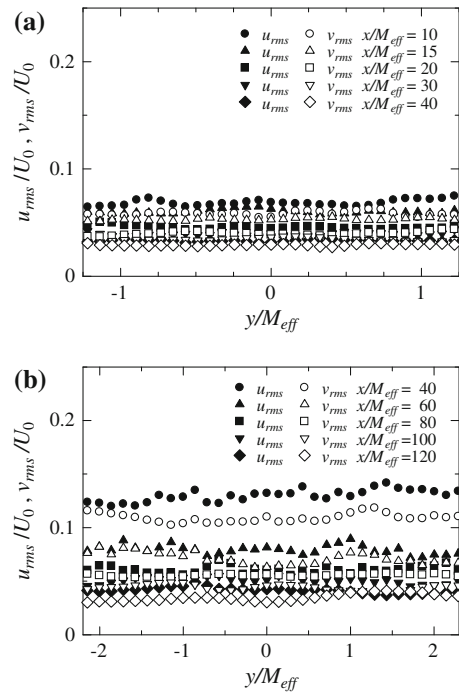
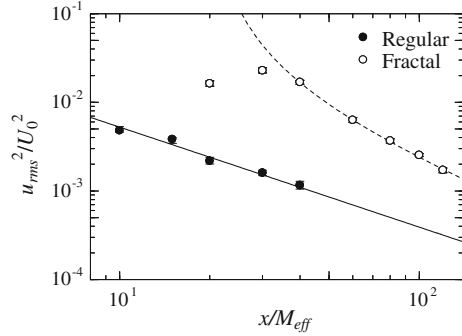


Fig. 28 Downstream variation of turbulent fluctuation intensity



tering for several measurements. The range of data scattering is -10.1 – 11.0 % (with a standard deviation of 4.7 %) for regular grid turbulence and -6.4 – 7.2 % (with a standard deviation of 4.6 %) for fractal grid turbulence. Although the Reynolds numbers are the same, fractal grid turbulence has a much higher turbulence intensity than regular grid turbulence, and fractal grid turbulence peaks at around $x/M_{eff} = 30$. These characteristics are in good qualitative agreement with past reports (Hurst and Vassilicos 2007; Seoud and Vassilicos 2007; Mazellier and Vassilicos 2010; Valente and Vassilicos 2011; Gomes-Fernandes et al. 2012; Nagata et al. 2013).

The functional form of the turbulence intensity decay is usually assumed to follow a power law as follows:

$$\frac{u_{rms}^2}{U^2} = a \left(\frac{x}{M_{eff}} - \frac{x_0}{M_{eff}} \right)^{-n}, \quad (7)$$

where a is a constant, x_0 is the location of the virtual origin, and n is the decay exponent. In this study, x_0 is estimated as follows. For regular grid turbulence, we use the linear fitting method for λ_x^2 (Hurst and Vassilicos 2007; Valente and Vassilicos 2011). For homogeneous (isotropic) turbulent decaying flows, advection balances dissipation. Using $\lambda_x^2 = 15\nu\overline{u^2}/\varepsilon$ in conjunction with the advection dissipation balance characteristic of homogeneous isotropic turbulence ($-(3/2)U\partial\overline{u^2}/\partial x = \varepsilon$) and assuming power-law decay [Eq. (7)], we obtain $\lambda_x^2 \sim \nu(x - x_0)/U$. Therefore, from the following linear fitting of λ_x^2 ,

$$\left(\frac{\lambda_x}{M_{eff}} \right)^2 = b \left(\frac{x}{M_{eff}} - \frac{x_0}{M_{eff}} \right), \quad (8)$$

x_0 can be obtained. For fractal grid turbulence, the nonlinear least-square regression method (Mohamed and LaRue 1990; Krogstad and Davidson 2010) is applied to the decay region of u_{rms} ($40 \leq x/M_{eff}$). Taking the logarithm of Eq. (7), we obtain the following expression:

Table 4 Values of decay parameters

	n	a	x_0/M_{eff}
Regular	1.21	0.071	-0.196
Fractal	1.40	1.11	20

$$\ln\left(\frac{u_{rms}^2}{U^2}\right) = \ln(a) - n \ln\left(\frac{x}{M_{eff}} - \frac{x_0}{M_{eff}}\right). \quad (9)$$

We search for x_0 to obtain a fit with the smallest variance between the data and Eq. (9); i.e. if $f(x)$ is the right-hand side of Eq. (9), we search for the minimum of

$$\sigma^2 = \frac{1}{(N-1)} \sum_{i=1}^N \{d_i(x_i) - f(x_i)\}^2, \quad (10)$$

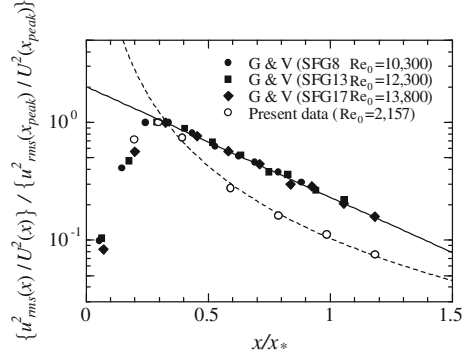
where N is the number of sampling data items, and $d_i(x_i)$ is a set of data points. The values of n , a and x_0/M_{eff} for regular and fractal grid turbulence are summarised in Table 4.

The solid and broken lines in Fig. 28 are determined as described above for regular and fractal grid turbulence, respectively. From these results, we notice that the value $n = 1.21$ for the present regular grid turbulence is close to the value of the Saffman turbulence, $n = 6/5 = 1.2$; in contrast, for fractal grid turbulence, we obtain $n = 1.4$, which is higher than the value for regular grid turbulence.

Hurst and Vassilicos (2007) and Mazellier and Vassilicos (2010) showed that the turbulence intensity u_{rms}^2 along the centreline of a fractal square grid obeys an exponential decay law [see Eq. (6)]. To examine this characteristic, we replotted the present data for fractal grid turbulence shown in Fig. 28 by converting the abscissa from a logarithmic scale to a linear scale. The results are shown in Fig. 29, where x is normalised by x_* , and the ordinate represents the relative intensity u_{rms}^2/U^2 normalised by the value at its peak location, $u_{rms}^2(x_{peak})/U^2(x_{peak})$. In Fig. 29, the data obtained by Gomes-Fernandes et al. (2012) for fractal square grids of $N = 4$ with three different thickness ratios, $t_r = 8.5, 13$ and 17 (which are referred to as SFG8, SFG13 and SFG17, respectively, in their paper) are also plotted. Furthermore, in the figure, we show the values of the Reynolds number Re_0 . All the data of Gomes-Fernandes et al. (2012) are found to collapse very well on the straight line. This implies that their data obey the exponential decay law. However, we cannot find any similar behaviour in the present data; therefore, the exponential decay law does not hold in the present fractal grid turbulence. With regard to the decay law for fractal square grid turbulence, Mazellier and Vassilicos (2010) proposed the following functional form, which is consistent with both power-law decay and the exponential decay law proposed by George and Wang (2009):

$$u_{rms}^2 = u_{rms}^2(x'_0) \left\{ 1 + \frac{4\nu\hat{a}|\hat{c}|}{L_u^2(x'_0)U_0}(x - x'_0) \right\}^{(1-\hat{c})/2\hat{c}}, \quad (11)$$

Fig. 29 Linear-log plot of downstream evolution of rms value of turbulent fluctuation. *Solid line* $\{u_{rms}(x)/U(x)\}^2 / \{u_{rms}(x_{peak})/U(x_{peak})\}^2 = A' \exp \{-B(x/x_*)\}$ with $A' = 2.01$, $B = 2.16$. *Broken line* Eq. (7) with $n = 1.4$, $a = 1.11$ and $x_0/M_{eff} = 20$



where $\hat{a} > 0$ and $\hat{c} < 0$ are non-dimensional constants that depend on Re_0 and the boundary/inlet/initial condition, and x'_0 is some downstream location that differs from x_0 . Under the limit $Re_{L_u} = L_u U_0 / \nu \rightarrow \infty$, u_{rms}^2 approaches exponential decay asymptotically, but for a small Re_{L_u} , it follows power-law decay. Notably, Eqs. (7) and (11) are equivalent with $n = (\hat{c} - 1)/2\hat{c}$ and $x_0 = x'_0 - L_u^2(x'_0)U_0/(4\nu\hat{a}|\hat{c}|)$. The Reynolds number in the present experiment is considerably smaller than those in the experiments by Gomes-Fernandes et al. (2012). Therefore, it seems natural to consider that the present data obey power-law decay rather than the exponential decay law.

Length Scales and Turbulence Reynolds Number

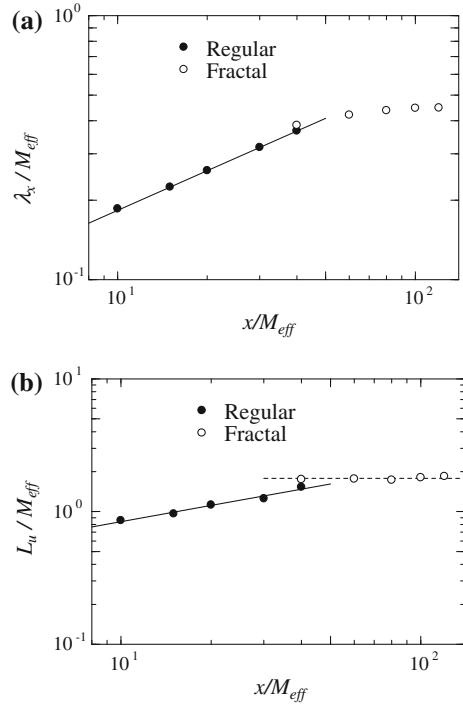
Figure 30a shows the downstream evolution of the Taylor microscale λ_x calculated as $\lambda_x = u_{rms} / \sqrt{(\partial u / \partial x)^2}$ along the centreline ($y/M_{eff} = 0$ and $z/M_{eff} = 0$) for both regular and fractal grid turbulence. Here both the abscissa and the ordinate are expressed on a logarithmic scale. Because the evolution of λ_x for regular grid turbulence is approximated by Eq. (8), it shows a linear increase with a slope of 1/2 [the solid line represents Eq. (8)]. In contrast, the λ_x value for fractal grid turbulence tends to increase slightly in the downstream direction and approaches a constant value. This tendency can also be seen in wind tunnel experiments when the Reynolds number is small (Mazellier and Vassilicos 2010; Nagata et al. 2013).

Figure 30b shows the downstream evolution of the integral length scale L_u , which is obtained by integrating the longitudinal autocorrelation velocity coefficient. The evolution of L_u for regular grid turbulence is usually assumed to obey the following power law:

$$\frac{L_u}{M_{eff}} = D \left(\frac{x}{M_{eff}} - \frac{x_0}{M_{eff}} \right)^q. \quad (12)$$

For the present data, we obtain $D = 0.329$ and $q = 0.407$ by the least-squares method. For Saffman turbulence, it is known that $L_u \sim x^{2/5}$ (Davidson 2004); there-

Fig. 30 Downstream variation of Taylor microscale λ_x and integral length scale L_u . **a** Taylor microscale λ_x . *Solid line* indicates Eq. (8). **b** Integral length scale L_u . *Broken line* shows $L_u = M_{eff} = 1.78$

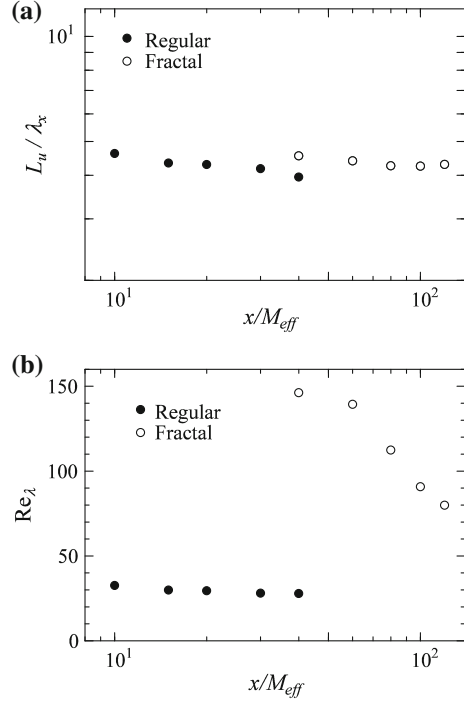


fore, the present result is close to that for Saffman-type turbulence. In contrast, the value of L_u for fractal grid turbulence is almost constant in the downstream direction. This is consistent with past reports (Hurst and Vassilicos 2007; Seoud and Vassilicos 2007; Mazellier and Vassilicos 2010; Nagata et al. 2013).

The downstream evolution of the ratio of the integral length scale to the Taylor microscale, L_u/λ_x , is shown in Fig. 31a. The data for both regular and fractal grid turbulences show values at the same level that tend to decrease in the downstream direction. However, the decrease in L_u/λ_x for fractal grid turbulence is small, and the ratio tends to approach a constant value in the downstream region. A similar decreasing tendency for fractal grid turbulence was observed in wind tunnel experiments when the Reynolds numbers were small (Mazellier and Vassilicos 2010; Nagata et al. 2013).

Figure 31b shows the downstream variation of the turbulence Reynolds number Re_λ . Re_λ is much higher for fractal grid turbulence than for regular grid turbulence. Further, Re_λ for regular grid turbulence shows little change in the downstream direction. In contrast, Re_λ decreases rapidly in the downstream direction for fractal grid turbulence. This tendency is consistent with past reports (Hurst and Vassilicos 2007; Seoud and Vassilicos 2007; Mazellier and Vassilicos 2010; Valente and Vassilicos 2011; Gomes-Fernandes et al. 2012; Nagata et al. 2013).

Fig. 31 Downstream variation of **a** L_u/λ_x and **b** turbulence Reynolds number Re_λ



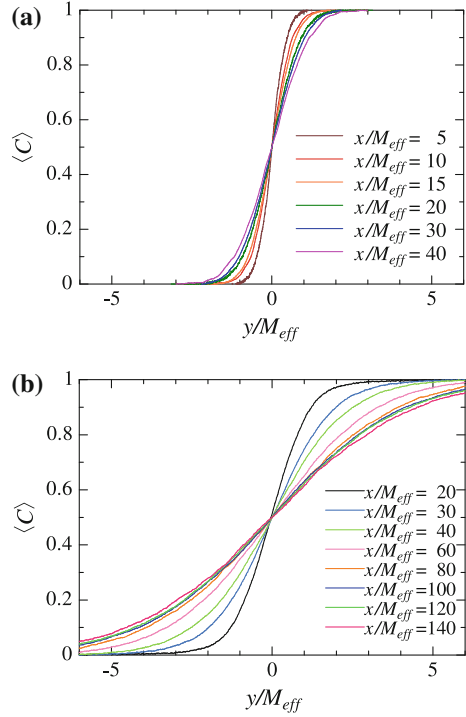
As described above, the characteristics of regular and fractal grid turbulences in this study are almost in accordance with past experimental reports; therefore, we could realise the ‘standard’ regular and fractal square grid turbulence suitable for investigating scalar mixing phenomena.

2.3.2 PLIF Results

Mean Scalar Profile and Its Downstream Evolution

Figure 32a, b show the downstream evolution of the vertical profile of the mean scalar $\langle C \rangle$ in the centre of the water channel (on the $z/M_{eff} = 0$ plane) for regular and fractal grid turbulence, respectively. Here, C is the instantaneous concentration normalised by the initial concentration C_0 , and $\langle \rangle$ denotes the ensemble average. The vertical distance y is normalised by M_{eff} . From Fig. 32a, the slope of the mean scalar for the regular grid becomes gentle in the downstream direction, and the extent of the profile grows with increasing downstream distance. Thus, the typical scalar mixing layer is realised in this experiment. In contrast, from Fig. 32b, we can see that the rate of change in the slope of the mean scalar for the fractal grid is relatively high, and the extent of the profile is wider than that of the regular grid. Therefore,

Fig. 32 Downstream evolution of vertical profile of mean scalar. **a** Regular grid turbulence. **b** Fractal grid turbulence



it is possible to say that mixing by the fractal grid is more enhanced than that by the regular grid. Later we estimate the turbulent diffusivity quantitatively from the growth rate of the width of the mean scalar profile.

Next we investigate the self-similarity of the mean scalar profiles. Figure 33a, b show the evolution of the vertical profiles of the mean scalar $\langle C \rangle$ replotted using y/h_m (y normalised by the half-width h_m) along the abscissa for regular and fractal grid turbulence, respectively. Here the half-width h_m is defined as the width of y for which the value of $\langle C \rangle$ is between 0.25 and 0.75 (i.e. $0.25 \leq \langle C \rangle \leq 0.75$). The data collapse very well for both grid turbulence types. This implies that the mean scalar profiles for both grids are self-similar in the measurement region. Furthermore, these profiles agree very well with the theoretical prediction [Eq. (14), the broken line in the figure], which is discussed later in this section.

To investigate the rate of evolution of the mixing layer, it is useful to see the change in the half-width of the mean scalar profile in the downstream direction, as shown in Fig. 34. In this figure, the abscissa and ordinate axes are normalised by M_{eff} . The non-dimensional half-width h_m/M_{eff} for both grids increases monotonically in the downstream direction, but its value is higher for the fractal grid than for the regular grid.

Here we consider the relation between the downstream change in h_m and the turbulent diffusivity. When the molecular diffusivity and turbulent diffusion in the

Fig. 33 Downstream evolution of vertical profiles of mean scalar as a function of y/h_m . **a** Regular grid turbulence. **b** Fractal grid turbulence

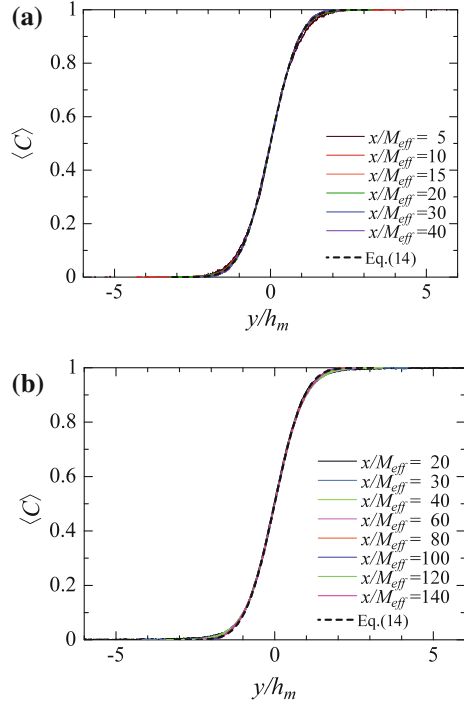
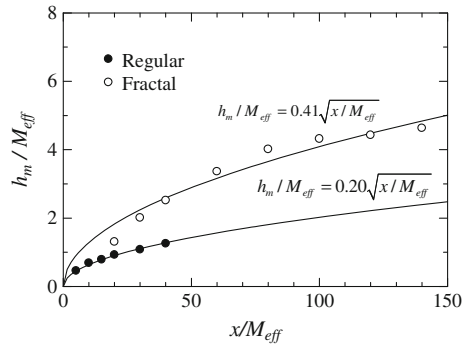


Fig. 34 Downstream evolution of half-width of vertical profile of mean scalar $\langle C \rangle$ for regular grid turbulence and fractal grid turbulence



x and z directions are neglected, the evolution equation of the mean scalar $\langle C \rangle$ for statistically stationary turbulence is given as follows (Libby 1975):

$$U \frac{\partial \langle C \rangle}{\partial x} = K_y \frac{\partial^2 \langle C \rangle}{\partial y^2}, \quad (13)$$

where K_y is the turbulent diffusivity in the y direction, and U is the mean velocity. These values are assumed to be constant in the flow field. The initial and bound-

any conditions are $\langle C \rangle(0, y > 0) = 1$, $\langle C \rangle(0, y < 0) = 0$, $\langle C \rangle(x, y \rightarrow \infty) = 1$, and $\langle C \rangle(x, y \rightarrow -\infty) = 0$. The solution of Eq. (13) is given as follows (Libby 1975):

$$\langle C \rangle(x, y) = \frac{1}{2} + \frac{1}{2} \operatorname{erf} \left\{ \frac{y}{2\sqrt{K_y x/U}} \right\}, \quad (14)$$

where the second term includes an error function defined as $\operatorname{erf}(x) = (2/\sqrt{\pi}) \int_0^x e^{-\xi^2} d\xi$. Then h_m is given as follows:

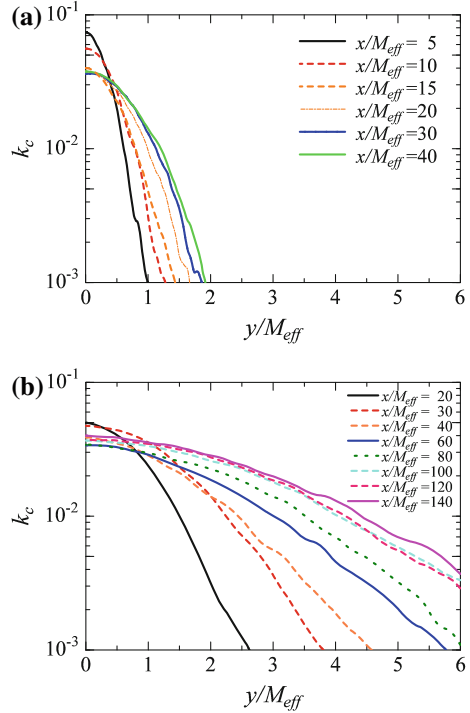
$$\frac{h_m}{M_{eff}} = k \sqrt{\frac{x}{M_{eff}}}, \quad (15)$$

where k is a constant obtained numerically; it is related to K_y as $k = 1.91\sqrt{K_y/(M_{eff}U)}$. [Note that the broken lines in Fig. 33a, b are drawn using Eqs. (14) and (15)]. The solid lines in Fig. 34 are curves drawn using Eq. (15). By least-squares approximation, we obtain $k = 0.20$ for the regular grid and $k = 0.41$ for the fractal grid. We find that the turbulent diffusivity for fractal grid turbulence is $(0.41/0.20)^2 = 4.20$ times larger than that for regular grid turbulence. Notably, for fractal grid turbulence, some deviation from the approximated line is seen near the grid ($x/M_{eff} \leq 40$, which corresponds to $x/x_* \leq 0.4$). This may be ascribed to the fact that the turbulence near the fractal grid would be under development. Consequently, the inhomogeneity of the mean and rms velocity fields is strong, and K_y and U are not constant in this region.

Scalar Variance Profile and Its Downstream Evolution

Figure 35a, b show the downstream evolution of the vertical profile of the scalar variance $k_c = \langle c^2 \rangle/2$ at the centre of the water channel (on the $z/M_{eff} = 0$ plane) for regular and fractal grid turbulence, respectively. Here c is the concentration fluctuation normalised by C_0 , and the figures show only half of the profile for $y/M_{eff} \geq 0$ because of its symmetry about the plane corresponding to $y/M_{eff} = 0$. Figure 35a shows that for regular grid turbulence, k_c on the centreline ($y/M_{eff} = 0$ and $z/M_{eff} = 0$) decreases in the downstream direction but remains almost constant in the downstream region of $x/M_{eff} \geq 20$. In contrast, in the region away from the centreline, k_c increases, and its profile expands in the y direction. Figure 35b shows that k_c for fractal grid turbulence on the centreline decreases in the downstream direction, as in the case of regular grid turbulence. However, its value is smaller than the value for regular grid turbulence, and further downstream of $x/M_{eff} = 60$, it is constant or increases slightly. This behaviour is discussed later in relation to the downstream evolution of the time scale of the scalar fluctuation. Furthermore, in the region away from the centreline, k_c grows in the downstream direction and its growth rate is much higher for fractal grid turbulence than for regular grid turbulence.

Fig. 35 Downstream evolution of vertical profile of scalar fluctuation variance.
a Regular grid turbulence.
b Fractal grid turbulence



We now consider the similarity of the profiles of the scalar variance. Libby (1975) derived an equation for the evolution of the scalar variance in the thermal mixing layer downstream of a partially heated regular grid and considered the functional form of its solution. However, the form of the solution predicted by Libby (1975) contained a term that depends on the downstream location x ; i.e. the profile of the scalar variance does not show exact self-similarity. Later, Yoon and Warhaft (1991) and Huq and Britter (1995) performed experiments on the scalar mixing layer in regular grid turbulence under boundary conditions similar to that of the present study (Yoon and Warhaft 1991: thermal mixing layer using a wind tunnel, Huq and Britter 1995: saline mixing layer using a water channel). They found that the scalar variance in the far downstream region shows a self-similar Gaussian profile. Therefore, we also check whether the scalar variance in our experiment becomes self-similar. Figure 36a, b show the downstream variation of the vertical profiles of the scalar fluctuation variance k_c normalised by its maximum value $k_{c\max}$ using the abscissa scaled by the half-width h_f for regular and fractal grid turbulence, respectively. Here, $h_f = y|_{k_c=k_{c\max}/2}$ (the value of the y coordinate at which the value of k_c is half that of $k_{c\max}$). The profiles for both the regular and fractal grids show very good similarity, and the profiles are in excellent agreement with the Gaussian distribution given by the broken line in the figures. Note that the Gaussian distribution is given by $k_c/k_{c\max} = \exp\{-(\ln 2)(y/h_f)^2\}$. From this result, we can expect that

Fig. 36 Vertical profile of scalar fluctuation variance using the abscissa scaled by the half-width h_f . **a** Regular grid turbulence. **b** Fractal grid turbulence

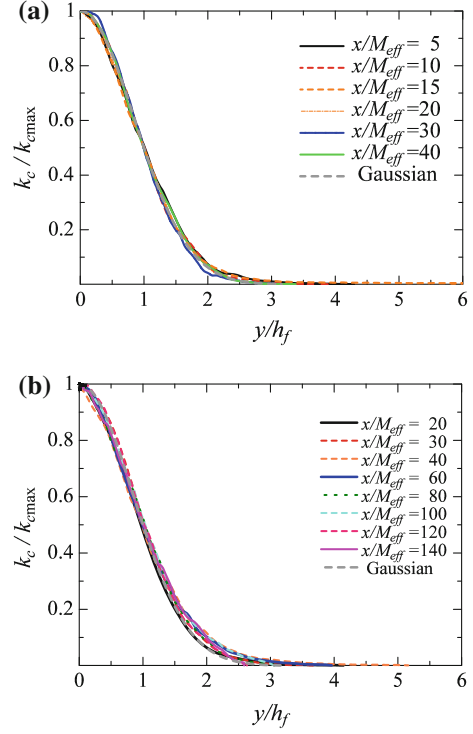
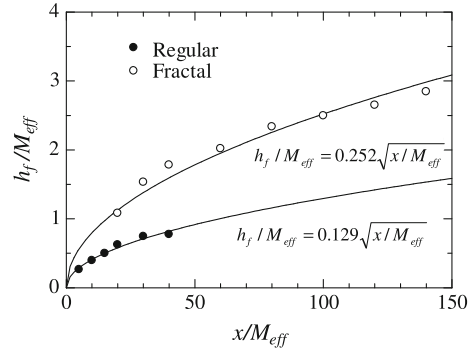


Fig. 37 Downstream evolution of half-width of vertical profile of scalar variance k_c for regular grid turbulence and fractal grid turbulence



the Gaussian distribution becomes the self-similar solution for the evolution equation of the scalar variance. In fact, through a careful similarity analysis, it is possible to show the existence of the self-similar Gaussian solution for a particular condition. We intend to present this similarity analysis in a future report.

Figure 37 shows the downstream variation of the half-width h_f of the scalar variance. The half-width increases in the downstream direction for both regular and fractal grid turbulence, and the rate of increase slows down further downstream.

Apparently, the half-width for fractal grid turbulence is much larger than that for regular grid turbulence. The solid lines are the approximation curves and are expressed as follows:

$$\frac{h_f}{M_{eff}} = k_f \sqrt{\frac{x}{M_{eff}}}, \quad (16)$$

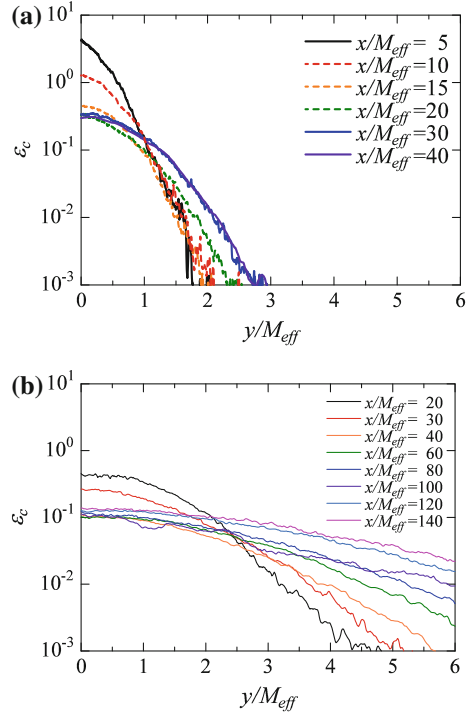
where $k_f = 0.129$ for regular grid turbulence, and $k_f = 0.252$ for fractal grid turbulence; the value for fractal grid turbulence is 1.95 times larger than that for regular grid turbulence.

Downstream Evolution of Vertical Profile of Scalar Dissipation

The downstream evolution of the vertical profile of the mean scalar dissipation ε_c at the centre of the water channel (on the $z/M_{eff} = 0$ plane) for regular and fractal grid turbulence are shown in Fig. 38a, b, respectively. Here ε_c is calculated as follows:

$$\varepsilon_c = \frac{1}{\text{Re}_M \text{Sc}} \left\{ 2 \left\langle \left(\frac{\partial c}{\partial \hat{x}} \right)^2 \right\rangle + \left\langle \left(\frac{\partial c}{\partial \hat{y}} \right)^2 \right\rangle \right\}, \quad (17)$$

Fig. 38 Downstream evolution of vertical profile of mean scalar dissipation.
a Regular grid turbulence.
b Fractal grid turbulence



where $\hat{x} = x/M_{eff}$, $\hat{y} = y/M_{eff}$, c is the concentration fluctuation normalised by C_0 and $\langle (\partial c / \partial \hat{x})^2 \rangle$ is assumed to be equal to $\langle (\partial c / \partial \hat{z})^2 \rangle$. The change in ε_c for regular grid turbulence (Fig. 38a) reveals that the value on the centreline (the line of $y/M_{eff} = 0$, $z/M_{eff} = 0$) declines, and the width of the profile increases in the downstream direction. However, in the downstream region of $x/M_{eff} \geq 20$, the value on the centreline remains almost constant. The value and profile for fractal grid turbulence also show similar changes; i.e. until $x/M_{eff} = 40$, the value of ε_c on the centreline decreases and the width of profile increases in the downstream direction, but in the more downstream region of $x/M_{eff} \geq 40$, the value on the centreline remains almost constant, whereas the profile width increases. Additionally, it is apparent that the rate of increase is considerably higher for fractal grid turbulence than for regular grid turbulence.

Downstream Evolution of Scalar Variance, Scalar Dissipation and Time Scale of Scalar Fluctuation Along the Centreline

Figure 39a, b show the downstream evolutions of k_c and ε_c , respectively, along the centreline (the line of $y/M_{eff} = 0$, $z/M_{eff} = 0$) for both regular and fractal grid turbulence. Note that the abscissa is the downstream distance divided by U_0 , which corresponds to the advection time from the grid. The error bars show the range of data scattering for several measurements. The range of scattering for k_c is -3.5 – 3.7 % for regular grid turbulence and -4.7 – 7.0 % for fractal grid turbulence. The range of scattering for ε_c is -5.1 – 5.2 % for regular grid turbulence and -7.9 – 7.2 % for fractal grid turbulence. Figure 39a shows that for a short advection time (in the downstream region near the grid), k_c is smaller for fractal grid turbulence than for regular grid turbulence. However, for longer advection times, $0.7 \text{ s} < x/U_0$ ($20 \leq x/M_{eff}$ for the regular grid, $60 \leq x/M_{eff}$ for the fractal grid), the values of k_c for both fractal and regular grid turbulence are almost equal. In contrast, Fig. 39b shows that ε_c for both regular and fractal grid turbulence decays in the region just downstream of the grids and then approaches a constant value. In addition, we also notice that the value of ε_c for fractal grid turbulence is approximately 1/3 lower than that for regular grid turbulence. This fact indicates that small-scale mixing is more pronounced for fractal grid turbulence than for regular grid turbulence; i.e. smearing of the concentration progresses to a greater extent in fractal grid turbulence.

Figure 40 shows the downstream evolution of the turbulent time scale for the concentration fluctuation $\tau_c = k_c/\varepsilon_c$ along the centreline of the mixing layer. The error bars show the maximum error estimated from the measurement errors of k_c and ε_c . The value of τ_c for both regular and fractal grid turbulence increases quickly (almost linearly) for a short advection time ($x/U_0 < 0.5 \text{ s}$) from the grid and then approaches a constant value. The rate of increase of τ_c is considerably higher for fractal grid turbulence than for regular grid turbulence, and the value of τ_c for fractal grid turbulence is almost thrice that for regular grid turbulence.

Here we consider the relation between the downstream variation of k_c and the time scale τ_c to understand the characteristics of scalar mixing. The evolution equation

Fig. 39 Downstream evolution of scalar variance and scalar dissipation on the centreline of the mixing layer. **a** Scalar variance k_c . **b** Scalar dissipation ε_c

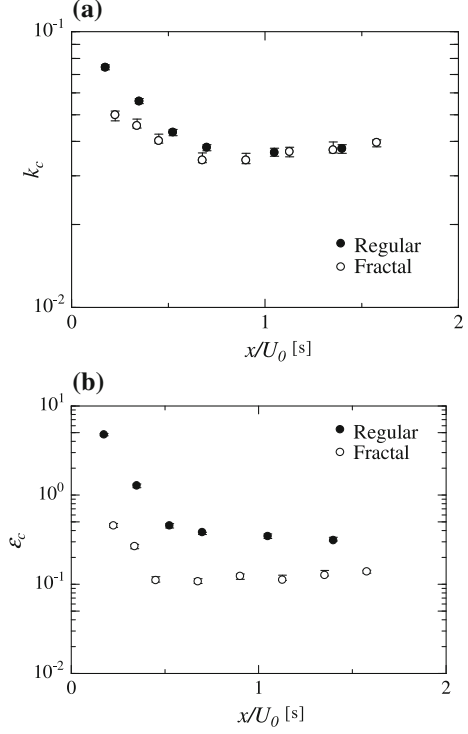
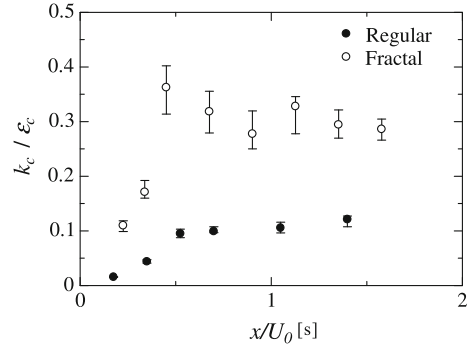


Fig. 40 Downstream evolution of scalar fluctuation time scale $\tau_c = k_c/\varepsilon_c$ along the centreline of the mixing layer



of the scalar variance k_c , assuming that the inhomogeneity of the scalar field in the spanwise (z) direction is negligible, is given in non-dimensional form as follows (Libby 1975; Huq and Britter 1995):

$$\hat{U} \frac{\partial k_c}{\partial \hat{x}} = -\langle c \hat{v} \rangle \frac{\partial \langle C \rangle}{\partial \hat{y}} - \frac{\partial}{\partial \hat{y}} \left(\left\langle \frac{1}{2} c^2 \hat{v} \right\rangle \right) + \frac{1}{\text{Re}_M \text{Sc}} \frac{\partial}{\partial \hat{y}} \left(\frac{1}{2} \langle c^2 \rangle \right) - \varepsilon_c, \quad (18)$$

where $\hat{U} = U/U_0$, and $\hat{v}$ is the non-dimensional vertical (y-direction) velocity fluctuation, $\hat{v} = v/U_0$. The left-hand side of Eq. (18) is the advection term. On the right-hand side of Eq. (18), the first term is the production term, the second term is the turbulent diffusion term, the third term is the molecular diffusion term, and the last term is the dissipation term. Considering the centreline of the mixing layer (the line of $\hat{y} = 0, \hat{z} = 0$), Eq. (18) can be rewritten as follows:

$$\hat{U}(\hat{x}) \frac{dk_c}{d\hat{x}} = F_c(\hat{x}) - \varepsilon_c(\hat{x}), \quad (19)$$

where $F_c(\hat{x})$ is the summation of the first, second and third terms of Eq. (18) on the centreline. When we set the ratio of $F_c(\hat{x})$ to $\varepsilon_c(\hat{x})$ as $\alpha(\hat{x}) = F_c(\hat{x})/\varepsilon_c(\hat{x})$ and use the time scale τ_c , Eq. (19) can be written as follows:

$$\frac{dk_c(\hat{x})}{d\hat{x}} = \frac{(\alpha(\hat{x}) - 1) k_c(\hat{x})}{\hat{U}(\hat{x}) \tau_c(\hat{x})}. \quad (20)$$

Now we assume that $k_c(\hat{x})$ and $\tau_c(\hat{x})$ obey a simple power law expressed as $k_c = E_1 \hat{x}^{-n}$, $\tau_c = E_2 \hat{x}^m$, where E_1 and E_2 are constants. Then we obtain $\alpha(\hat{x}) = 1 - n E_2 \hat{U} \hat{x}^{m-1}$ and $\varepsilon_c(\hat{x}) = (E_1/E_2) \hat{x}^{-(n+m)}$. Figure 40 shows that τ_c increases in proportion to x in the region near the grids. In contrast, k_c decreases in the downstream direction, as shown in Fig. 39a. Therefore, $m \sim 1$ and $n \geq 0$ in the region near the grid, and then $\alpha(\hat{x}) < 1$. This situation implies that $F_c(\hat{x})$ (the summation of the production, turbulent diffusion and molecular diffusion terms) is less than $\varepsilon_c(\hat{x})$ (the dissipation term). In contrast, Figs. 39a and 40 show that in regions further downstream, τ_c and k_c are almost constant. This is true when $n = 0$ and $m = 0$. Then, ε_c does not change in the downstream direction, and a type of equilibrium state ($\alpha(\hat{x}) = 1.0$) in which $F_c(\hat{x})$ balances ε_c is realised. It can be expected that such an equilibrium state is the condition for the vertical profile of the scalar variance k_c to become self-similar in the downstream direction.

Fractal Dimension of Iso-Scalar Set

The fractal dimension is useful as a measure of the complexity of an object (Sreenivasan 1991; Dimotakis and Catrakis 1999). Here the fractal dimension of the iso-scalar (level) set is investigated using the box-counting method. In this method, we cover the object with boxes of size $\delta \times \delta$ without overlapping and count the number of boxes $\hat{N}(\delta)$ necessary to cover the entire object. If the following power-law relation between $\hat{N}(\delta)$ and δ holds, its (negative) exponent $\hat{D}_f$ is identified as the fractal dimension:

$$\hat{N}(\delta) = k_D \delta^{-\hat{D}_f}, \quad (21)$$

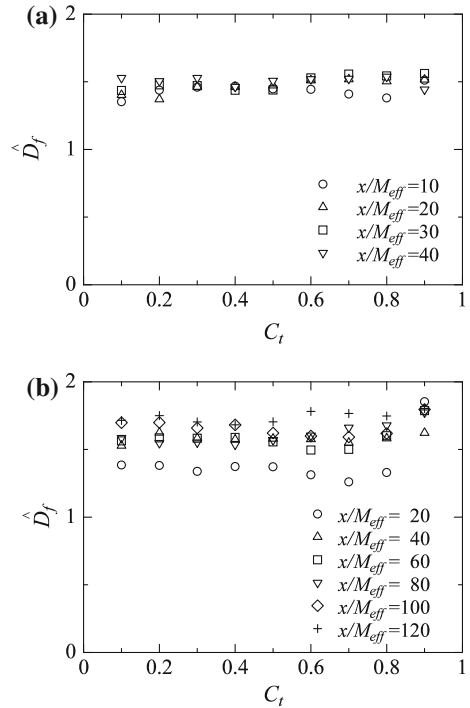
where k_D is a constant. Accordingly, we obtain $\hat{D}_f = -d \left\{ \ln \hat{N}(\delta) \right\} / d(\ln \delta)$; thus, $\hat{D}_f$ can be calculated by the least-squares method. With regard to the range of the box size δ , we choose the maximum and minimum sizes of the box to be $t_m \leq \delta \leq h_m$, where t_m is the thickness of the laser sheet. Note that although the figures are omitted here because of limited space, it is confirmed that, for all the measurement data treated here, in the range of $t_m \leq \delta \leq h_m$ the relation between $\hat{N}(\delta)$ and δ obeys well the power law given by Eq. (21). Figure 41a shows the distribution of $\hat{D}_f$ for various threshold levels C_t of the non-dimensional concentration C at different downstream locations of $10 \leq x/M_{eff} \leq 40$ on the centreline of the mixing layer in regular grid turbulence. $\hat{D}_f$ shows a flat distribution at each downstream location and remains almost unchanged in the downstream direction. Here we take the average of $\hat{D}_f$ over all the measurement data shown in Fig. 41a, and we obtain $\hat{D}_f = 1.48$.

Figure 41b shows the downstream variation of the distribution of $\hat{D}_f$ for various C_t levels in the region of $20 \leq x/M_{eff} \leq 120$ on the centreline of the mixing layer in fractal grid turbulence. In this case as well, we do not find any significant dependence of $\hat{D}_f$ on C_t . However, $\hat{D}_f$ increases monotonically in the downstream direction and reaches $\hat{D}_f = 1.75$ at the most downstream measurement location of $x/M_{eff} = 120$, which is considerably higher than $\hat{D}_f = 1.48$ for regular grid turbulence. Figures 38 and 39 show that k_c and ε_c for fractal grid turbulence remain almost constant in the

Fig. 41 Dependence of the fractal dimension $\hat{D}_f$ on the threshold level C_t and its downstream variation.

a Regular grid turbulence.

b Fractal grid turbulence



downstream region of $60 \leq x/M_{eff}$, which corresponds to $0.7s \leq x/U_0$. However, the fractal dimension increases even in this region in the downstream direction. This seems to be a remarkable characteristic of scalar mixing in fractal grid turbulence.

2.4 Summary

The development of the turbulent mixing layer of a high-Schmidt-number passive scalar in regular and fractal grid turbulence was investigated experimentally by PIV and PLIF using a water channel. The conclusions obtained are summarised as follows:

(1) It is confirmed that fractal grid turbulence shows stronger turbulence intensity and a higher turbulence Reynolds number than regular grid turbulence does for the same mesh Reynolds number. The decay exponent of the turbulence intensity is $n = 1.21$ for regular grid turbulence and $n = 1.40$ for fractal grid turbulence. The value of n for regular grid turbulence is close to the value for Saffman turbulence, $n = 6/5$. The decay rate is higher for fractal grid turbulence than for regular grid turbulence in the present measurement region.

(2) Diffusion is stronger in fractal grid turbulence than in regular grid turbulence. The eddy diffusivity of mass in fractal grid turbulence is approximately 4.2 times larger than that in regular grid turbulence.

(3) In the downstream region near the grid (for a short advection time from the grid), the scalar variance k_c for the fractal grid is smaller than that for the regular grid. In contrast, for longer advection times, $0.7s \leq x/U_0$ ($20 \leq x/M_{eff}$ for the regular grid, $60 \leq x/M_{eff}$ for the fractal grid), the values of k_c for both the fractal and regular grids are almost equal.

(4) Scalar dissipation is lower in fractal grid turbulence than in regular grid turbulence.

(5) The scalar fluctuation time scale τ_c for both regular and fractal grid turbulence increases in proportion to the advection time in the region near the grid and then approaches a constant value. For longer advection times ($0.5s < x/U_0$), τ_c for fractal grid turbulence is almost thrice that for regular grid turbulence.

(6) The fractal dimension of the iso-scalar set $\hat{D}_f$ on the centreline of the mixing layer was investigated. $\hat{D}_f$ for both regular and fractal grid turbulence does not depend on the threshold level C_t . For regular grid turbulence, $\hat{D}_f$ is almost constant and it does not change in the downstream direction. The average of $\hat{D}_f$ over all the present data is $\hat{D}_f = 1.48$. In contrast, $\hat{D}_f$ for fractal grid turbulence increases monotonically in the downstream direction, and its value reaches $\hat{D}_f = 1.75$, which is considerably higher than that for regular grid turbulence, at the most downstream location in this study.

3 Direct Numerical Simulation of Mixing Layer in Grid-Generated Turbulence

3.1 Introduction

Direct numerical simulation (DNS) of turbulence is a powerful tool for detailed investigation of a three-dimensional turbulent flow field, although the applications of DNS are currently restricted to moderate Reynolds numbers owing to limitations in computer resources. The numerical methods for DNS of turbulent flows are broadly categorised as spectral and finite difference methods according to their numerical method (the finite element method is also used for coupling problems of fluid–structure interaction, but this method is beyond the scope of this chapter). The spectral method is highly accurate; however, owing to the numerical procedure involved, its application is limited to simple domains such as a cubic domain. On the other hand, the finite difference method can be applied to complex geometries, although it is generally less accurate than the spectral method. However, with recent developments in fully conservative higher order finite difference schemes (Morinishi et al. 1998), in the higher order compact scheme originally developed for compressible flows (Lele 1992), and in the immersed boundary method for handling complex wall geometries (Fadlun et al. 2000; Ikeno and Kajishima 2007), DNS with spectral-like accuracy can be conducted around complex geometries with scalar transfer.

In this section, we describe the numerical results of a DNS of the scalar mixing layer in grid-generated turbulence (GGT).

3.2 Simulations of Four Types of Grid-Generated Turbulence with Scalar Transfer

3.2.1 Computational Domain, Grids and Numerical Method

Figure 42 shows a schematic of the computational domain and the details of the turbulence-generating grids. The turbulence-generating grid, on which the velocity components are set to zero, is located 20 mm downstream from the entrance. Four types of grid are numerically constructed: (a) a square-mesh biplane grid, (b) a square-mesh single-plane grid, (c) a composite grid consisting of parallel square-bars with different mesh sizes (Veeravalli and Warhaft 1989) and (d) a fractal grid (Seoud and Vassilicos 2007; Hurst and Vassilicos 2007; Mazellier and Vassilicos 2010; Valente and Vassilicos 2011; Gomes-Fernandes et al. 2012, 2014; Nagata et al. 2013). The mesh size M and thickness of the square rod d are listed in Table 5. The computational domains are $L_x \times L_y \times L_z = 1,152 \times 80 \times 80 \text{ mm}^3$ for (a) and (b), $560 \times 80 \times 40 \text{ mm}^3$ for (c), and $1,152 \times 160 \times 160 \text{ mm}^3$ for (d). The grid numbers are $1,280 \times 256 \times 256$ for (a) and (b), $1,280 \times 256 \times 128$ for (c), and $1,280 \times 448 \times 448$

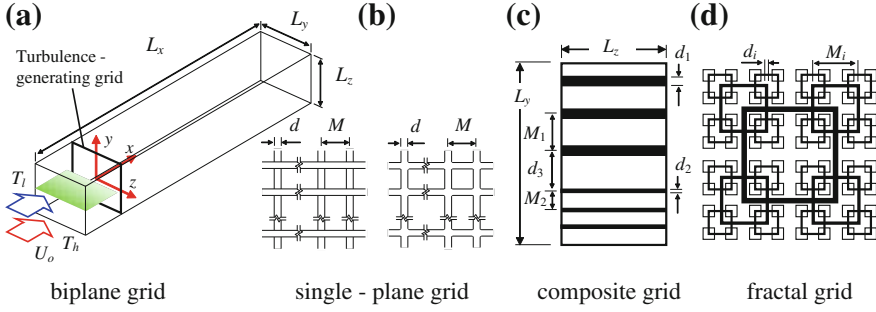


Fig. 42 Computational domain and details of the turbulence-generating grids. **a** Biplane grid. **b** Single—plane grid. **c** Composite grid. **d** Fractal grid

for (d). The spatial resolution is comparable to the Kolmogorov scale η . A cold fluid of temperature T_L and a hot fluid of temperature T_H are supplied from the upper and lower streams, respectively; consequently, thermal mixing layers with an initial step profile develop downstream of the grids. The temperature differences between the two layers are set to be small, so the temperature is treated as a passive scalar. Uniform mean flow is given at the entrance, and the viscous–convective outflow condition is applied at the exit. Periodic boundary conditions are applied in the y and z directions for (a), (b) and (d). For grid (c), a periodic boundary condition is applied only in the z direction, and the nonslip condition is applied in the y direction because of the inhomogeneity in the y direction.

The governing equations are the incompressible Navier–Stokes equations, the equation of continuity, and the transport equation for heat:

$$\partial \tilde{\mathbf{U}} / \partial t + (\tilde{\mathbf{U}} \cdot \nabla) \tilde{\mathbf{U}} = -\nabla P + \text{Re}^{-1} \nabla^2 \tilde{\mathbf{U}}, \quad \nabla \cdot \tilde{\mathbf{U}} = 0, \quad (22)$$

$$\partial \tilde{T} / \partial t + (\tilde{\mathbf{U}} \cdot \nabla) \tilde{T} = (\text{Re} \cdot \text{Pr})^{-1} \nabla^2 \tilde{T}, \quad (23)$$

where $\tilde{\mathbf{U}}$ and $\tilde{T}$ are the instantaneous velocity vector and temperature, respectively. In this subsection, the components of $\tilde{\mathbf{U}}$ are denoted as $\tilde{U}_i$, which is decomposed as $\tilde{U}_i = \langle U_i \rangle + u_i$, where $\langle \rangle$ denotes the average value. $\tilde{T}$ is also decomposed as $\tilde{T} = T + \theta$, where $T = \langle \tilde{T} \rangle$, and θ denotes the temperature fluctuation. The fractional-step method is used to solve the governing equations. The governing equations are discretised on the staggered mesh arrangement to construct the finite-difference formation. The consistent and conservative type of second- and fourth-order central difference schemes are used for the spatial derivatives in the streamwise (x) and transverse (y, z) directions, respectively. Time integrations are performed using the third-order Runge–Kutta method. The Poisson equation for the pressure is solved by the tridiagonal matrix algorithm method using the fast Fourier transform (FFT) in the y and z directions except for grid (c), where the FFT is used only in the z direction. The velocities are not necessarily defined on the surface of the turbulence-generating grids because of the staggered mesh arrangement. Therefore,

Table 5 Flow conditions

Run	Grid type	M (cm)	d (mm)	U_0 (m/s)	ν (m ² /s)	Re_M	Pr
1	Biplane	1.0	2.0	1.875	1.5×10^{-5}	1,250	0.71
2	Biplane	1.0	2.0	0.125	1.0×10^{-6}	1,250	7.1
3	Single-plane	1.0	2.0	1.875	1.5×10^{-5}	1,250	0.71
4	Composite	$M_1 = 3,$ $M_2 = 1$	$d_1 = 10$	2.5	1.5×10^{-5}	–	0.71
			$d_2 = 3$				
			$d_3 = 20$				
5	Fractal	$M_1 = 4,$ $M_2 = 2,$ $M_3 = 1,$ $M_4 = 0.5$	$d_1 = 2.66$	1.875	1.5×10^{-5}	1,250	0.71
			$d_2 = 1.56$				
			$d_3 = 0.63$				
			$d_4 = 0.31$				

the immersed boundary method with direct forcing (Fadlun et al. 2000) is used to satisfy the nonslip boundary conditions on the grid surface.

In most runs, the Prandtl number is set to 0.71, corresponding to an air flow. For grid (a), water flow at $Pr = 7.1$ is also simulated to investigate the effect of Pr on turbulent mixing. For this case, a finer mesh (the typical size is $100 \mu\text{m}$) is used in the y direction for the temperature calculation, as the smallest temperature scale is smaller at $Pr = 7.1$ than that at $Pr = 0.71$. Note that the smallest scale of the scalar (i.e. the Batchelor scale) is $\eta/Pr^{1/2}$, the value of which is approximately $100 \mu\text{m}$ in the present flows. The flow conditions are also listed in Table 5, where Re_M ($= U_0 M/\nu$) is the mesh Reynolds number, ν is the kinematic viscosity and U_0 is the inlet mean velocity. More details about the present code and its validation can be found in Nagata et al. (2010).

3.2.2 Results and Discussion

Figure 43 shows snapshots of the instantaneous flow fields near the grids. The turbulence-generating grids are constructed adequately in the present DNS, and the nonslip conditions of $\tilde{\mathbf{U}} = 0$ on the grid surface are satisfied. Note that in Fig. 43 the grids are visualised using the iso-surface of $\tilde{\mathbf{U}} = 0$. The iso-surfaces of $\tilde{\mathbf{U}}$, $\tilde{Q}$ [$\equiv \frac{1}{2}(W_{ij}W_{ij} - S_{ij}S_{ij})$; $S_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j)$; $W_{ij} = \frac{1}{2}(\partial_j u_i - \partial_i u_j)$] and $\tilde{T}$ imply that typical GGT and thermal mixing layers are simulated in the present DNS. Comparison of the magnitude of $\tilde{Q}$ in the fractal-generated turbulence with that in the conventional biplane grid (not shown) implies that the turbulence generated by the fractal grid is much stronger than that generated by the conventional grid at the same mesh Reynolds number. The result is consistent with previous experiments (Seoud

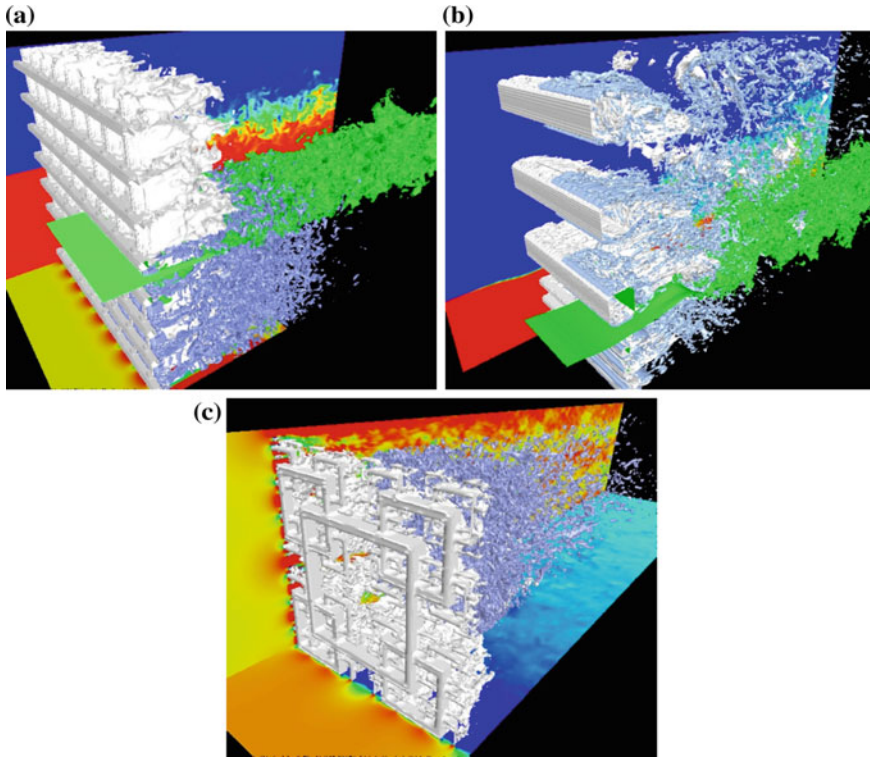


Fig. 43 Instantaneous flow fields near the grids: **a** biplane grid, **b** composite grid, **c** fractal grid. *White* iso-velocity surface of $\bar{U} = 0$; *purple* iso-surface of the second invariant Q ; *green* iso-temperature surface of $\bar{T} = 0.5$; *contour*: normalised temperature (*red* 1, *blue* 0). In **c**, the contours in the $x-y$ and $x-z$ planes show the velocity and pressure profiles, respectively

and Vassilicos 2007; Hurst and Vassilicos 2007; Mazellier and Vassilicos 2010; Valente and Vassilicos 2011, 2012; Gomes-Fernandes et al. 2012, 2014; Nagata et al. 2013).

Figure 44 shows the streamwise distributions of the turbulence intensities normalised by U_0^2 . Here we expressed u_i as $u_1 = u$, $u_2 = v$ and $u_3 = w$ and the subscript $_{rms}$ denotes the root mean square value. All the intensities have almost the same values for grid (a) and grid (b), and their decays follow a power law in the developed region with an exponent of -1.2 , which agrees well with previous experiments (Krogstad and Davidson 2010). Note that the anisotropy is slightly larger for the single-plane grid than for the biplane grid. The Reynolds stress (not shown) is almost zero in the entire region. The results imply that typical grid turbulence is generated for grid (a) and grid (b). For the composite grid (c), as expected, greater anisotropy is observed. Figure 45 shows the vertical distributions of the turbulence intensities for grid (c). In this case, the turbulence intensities are not homogeneous in the y direction, unlike cases (a) and (b), and a shearless mixing layer is generated around $y = 0$. The result agrees qualitatively with a previous measurement (Veeravalli and

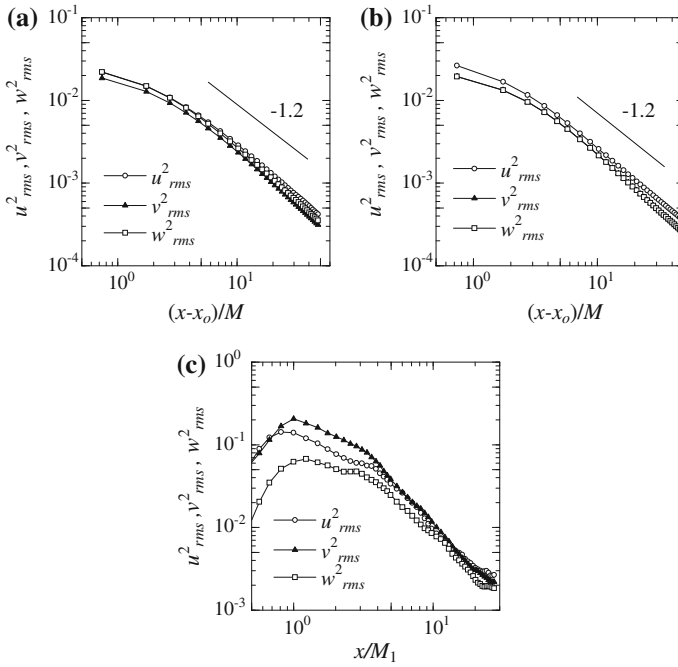


Fig. 44 Streamwise distributions of turbulence intensities at $y = z = 0$: **a** biplane grid, **b** single-plane grid, **c** composite grid. x_0 is the virtual origin

Warhaft 1989), although a larger computational domain and Reynolds number will be required to simulate the fully developed shearless mixing layer (Veeravalli and Warhaft 1989).

Figure 46 shows the streamwise distribution of the turbulence intensity of the pressure fluctuation p_{rms}^2 for the conventional biplane grid (a). It is shown that p_{rms}^2 decays linearly (on the log–log plot), and its decay exponent is -2.4 . Similar results are obtained for grid (b). Note that few statistics on the pressure fluctuation in GGT have been reported because of the difficulty of the measurement.

Figure 47 shows the vertical distributions of the mean temperature T and turbulence intensity of the temperature fluctuation θ_{rms}^2 for grid (a) and (c) (Fig. 47a, b, Run 1 and Fig. 47c, Run 4), normalised by $(T_H - T_L)$. In Fig. 47b, $y_{1/2}$ is the half-width of the θ_{rms}^2 profiles. The profiles in Fig. 47a, b suggest that a typical thermal mixing layer develops in the downstream region of grid (a). Similar results are obtained for grid (b). For grid (c), the vertical distributions of the mean temperature (not shown) and turbulence intensity of the temperature fluctuations (Fig. 47c) are asymmetric, like the turbulence intensity (Fig. 45), as the eddy scale or eddy diffusivity changes in the y direction (Veeravalli and Warhaft 1989).

Figure 48 shows the streamwise distributions of the vertical turbulent heat flux $< v\theta >$ for the biplane grid (Runs 1 and 2) normalised by $U_0(T_H - T_L)$. The decay

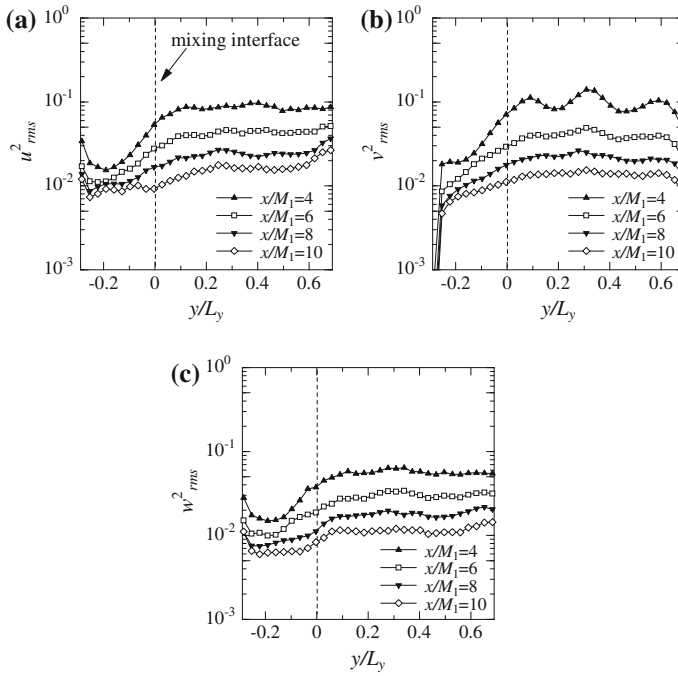
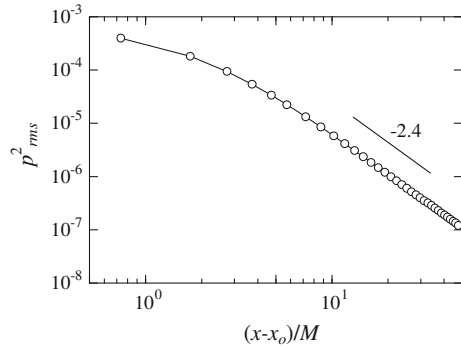


Fig. 45 Vertical distributions of turbulence intensities for grid (c)

Fig. 46 Streamwise distribution of turbulence intensities of pressure fluctuation at $y = z = 0$ for the conventional biplane grid (a)



of $\langle v\theta \rangle$ follows a power law in the developed region, as reported in previous studies (LaRue and Libby 1981; Huq and Britter 1995). In Fig. 48, the effect of Pr is clearly observed: $\langle v\theta \rangle$ at $Pr = 7.1$ is larger than that at $Pr = 0.71$. The result is consistent with our previous experiments (Komori and Nagata 1996). The effect of Pr can be seen more clearly when we look at the instantaneous temperature field (Fig. 49), which shows that finer temperature structure exists at $Pr = 7.1$, whereas it disappears at $Pr = 0.71$ because of the large smearing effect of molecular diffusion

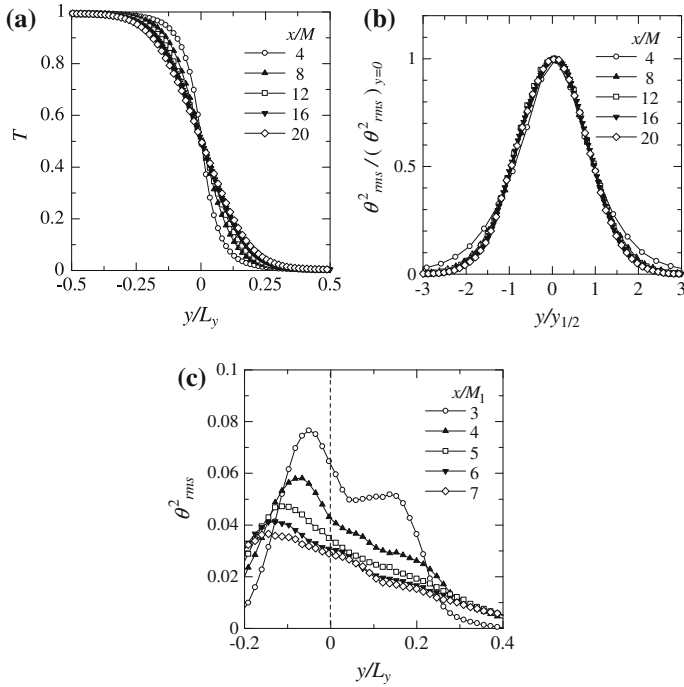
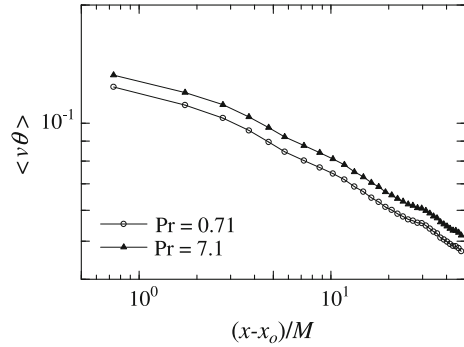


Fig. 47 **a** Mean temperature profile for grid (a) (Run 1), **b** vertical distribution of turbulence intensity of temperature fluctuation for grid (a) (Run 1), and **c** vertical distribution of turbulence intensity of temperature fluctuation for grid (c) (Run 4)

Fig. 48 Streamwise distributions of the vertical heat flux at $y = z = 0$ for the biplane grid (Runs 1 and 2)



(Komori and Nagata 1996). Note that the smaller Pr induces a larger correlation coefficient between v and θ .

Here GGT with heat transfer is investigated by DNS with high accuracy. The fundamental turbulence quantities show that typical grid-generated turbulent flow fields are simulated by the present DNS.

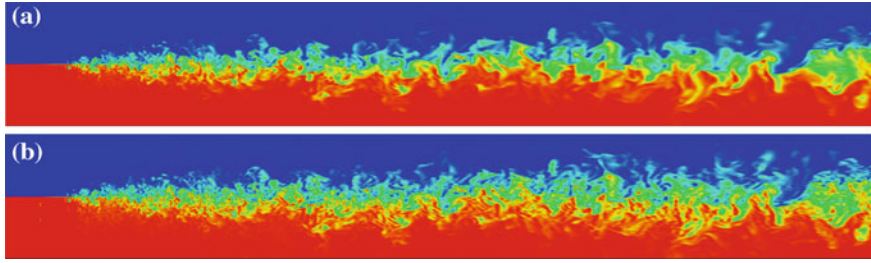


Fig. 49 Instantaneous temperature: **a** $Pr = 0.71$ (Run 1); **b** $Pr = 7.1$ (Run 2)

3.3 Comparison of Mixing Layer in Regular and Fractal Grid Turbulence

In this subsection, we compare the results of the turbulent mixing of heat ($Pr = 0.71$) in regular and fractal grid turbulence. For information on the flow field obtained by our DNS, refer to Nagata et al. (2008a, 2010) and Suzuki et al. (2010a). The instantaneous temperature fields, instantaneous fluctuating temperature fields and fundamental turbulence statistics are also presented.

3.3.1 Turbulence-Generating Grids

Figure 50 shows a schematic diagram of the regular and fractal grids used here. M_i is the length of the bar, and d_i is the thickness at iteration i ($=0 - N - 1$). The grids are numerically generated using the immersed boundary method with a forcing term in the Navier–Stokes equations (Fadlun et al. 2000). The grid parameters are listed in Table 6. For more information about the fractal grids, refer to Hurst and Vassilicos (2007). On the basis of Hurst and Vassilicos’s results, we choose the fractal dimension

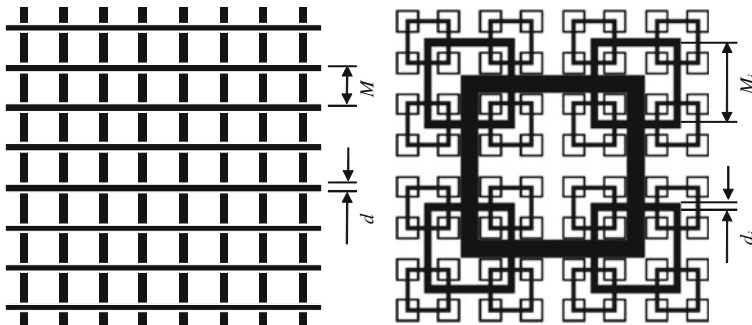


Fig. 50 Schematic of the regular grid (*left*) and square type fractal grid (*right*)

Table 6 Grid type and grid parameters

Run ID	Grid type	N	D_f	t_r	σ	Re_M
RGT	Regular grid	1	2.0	1.0	0.36	2,500
FGT1	Square fractal grid	4	2.0	5.0	0.36	2,500
FGT2	Square fractal grid	4	2.0	8.5	0.36	2,500

$D_f = 2.0$ in the present study. The grid solidity σ is maintained at 0.36 in order to compare the flow fields generated by fractal grids with those generated by the regular grid at the same blockage ratio. Typically, for a regular grid, $\sigma = 0.3 - 0.4$ (Comte-Bellot and Corrsin 1966). The thickness ratios $t_r (= d_0/d_{N-1})$ of the fractal grids are set at 5.0 and 8.5. The effective mesh size of the fractal grids, M_{eff} , is defined by Eq. (1) (see Sect. 1.2).

Computational Domain and Flow Conditions

The size of the computational domain normalised by M_{eff} and the number of grid mesh points are listed in Table 7. The turbulence-generating grids are numerically generated at $4M_{eff}$ downstream of the entrance to the computational domain. The inflow is given as a uniform flow. Periodic boundary conditions for the flow field are applied in the vertical and spanwise directions. The initial non-dimensional temperature is $\tilde{T} = 1$ and $\tilde{T} = 0$ in the upper and lower half-streams, respectively. Therefore, mixing layers of heat that initially have a step profile develop downstream of the grids. For the transverse boundary conditions for the temperature, a periodic boundary condition is applied in the spanwise direction and the adiabatic condition is applied in the vertical direction, where the temperature field is not uniform. At the exit of the computational domain, convective outflow conditions are applied to all physical quantities; in this case, the convection velocity is set at a value equal to the cross-sectionally averaged velocity.

The mesh Reynolds number $\text{Re}_M (= U_0 M_{eff} / \nu)$ is set to 2,500 (Huq and Britter 1995; Nagata and Komori 2001), where ν is the kinematic viscosity, and U_0 is the inlet mean velocity. The Prandtl number Pr is set to 0.71 considering heat transfer in

Table 7 Size of the computational domain and grid mesh points

Run ID	$L_x \times L_y \times L_z$	$N_x \times N_y \times N_z$
RGT	$64 \times 8 \times 8$	$768 \times 256 \times 256$
FGT1	$64 \times 16 \times 16$	$768 \times 256 \times 256$
FGT2	$64 \times 16 \times 16$	$768 \times 320 \times 320$

air flow. Note that it is impossible to perform DNS of high-Schmidt-number scalar fields because the smallest scale of the scalar field (i.e. the Batchelor scale), $\eta/\sqrt{Sc}$, is very small at a high-Schmidt number; here, η is the Kolmogorov scale (in this flow, $\eta \approx 200 \mu\text{m}$) and Sc is the Schmidt number [$Sc \approx 2, 100$ in the previous experiments (Suzuki et al. 2010c)].

3.3.2 Results and Discussion

Instantaneous Temperature Field

Figures 51 and 52 show the instantaneous temperature field $\tilde{T}$ and instantaneous fluctuating temperature field θ , respectively, for regular grid turbulence (Run RGT) and fractal grid turbulence (Runs FGT1 and FGT2) at $z/M_{eff} = 0$. The interval between the vertical grey lines in Figs. 51 and 52 corresponds to a distance of $10M_{eff}$. Figure 51 shows that the width of the mixing layer is considerably larger for fractal grid turbulence (Fig. 51b, c) than for regular grid turbulence (Fig. 51a). Thus, as confirmed in our experiment (Suzuki et al. 2010c and Sect. 3.2 in this chapter), for the same Re_M , turbulent mixing is enhanced to a greater extent for fractal grid turbulence than for regular grid turbulence. The fluctuating temperature fields (Fig. 52) also show

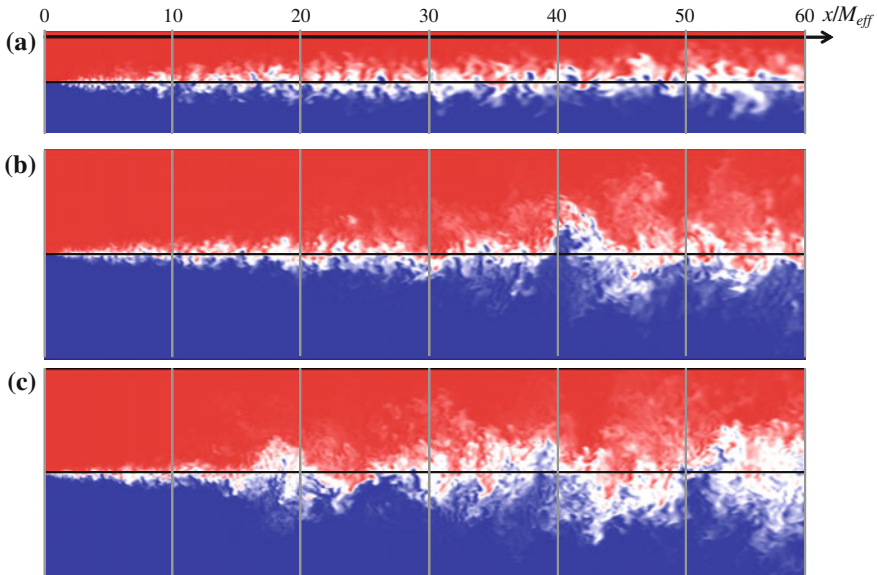


Fig. 51 Instantaneous temperature fields in **a** regular grid turbulence (Run RGT), **b** fractal grid turbulence at $t_r = 5.0$ (Run FGT1) and **c** fractal grid turbulence at $t_r = 8.5$ (Run FGT2). In all figures, red $\tilde{T} = 1$, white $\tilde{T} = 0.5$, blue $\tilde{T} = 0$

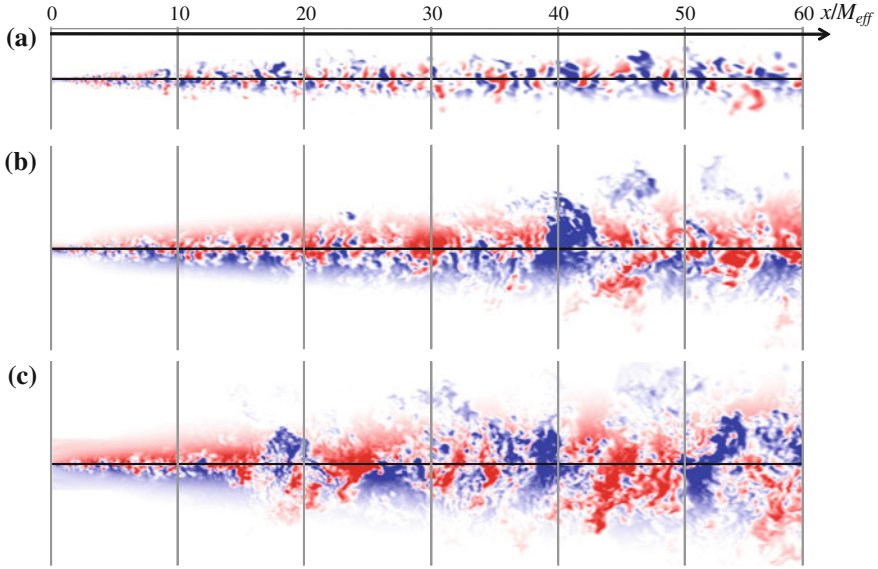


Fig. 52 Instantaneous fluctuating temperature fields in **a** regular grid turbulence (Run RGT), **b** fractal grid turbulence at $t_r = 5.0$ (Run FGT1) and **c** fractal grid turbulence at $t_r = 8.5$ (Run FGT2). In all figures, red $\theta = 0.3$, white $\theta = 0$, blue $\theta = -0.3$

that turbulent mixing is highly enhanced in fractal grid turbulence. Further, Figs. 51 and 52 suggest that small-scale scalar fluctuations exist in fractal grid turbulence.

3.3.3 Turbulence Statistics

Figure 53 shows the vertical profiles of the mean temperature T , temperature variance $k_\theta (= \frac{1}{2}\langle\theta^2\rangle)$ and turbulent heat flux $\langle -v\theta \rangle$ for regular grid turbulence (Run RGT) and fractal grid turbulence (Runs FGT1 and FGT2) at $x/M_{eff} = 40$, where v is the vertical velocity fluctuation. Figure 53a shows that $\partial T/\partial y$ at the centre of the mixing layer ($y/M_{eff} = 0$) is smaller for fractal grid turbulence (Runs FGT1 and FGT2) than for regular grid turbulence (Run RGT). The result suggests that turbulent mixing is more enhanced in fractal grid turbulence, and this is in accordance with the instantaneous temperature field shown in Fig. 51. Figure 53b, c show that the values of the half-widths of k_θ and $\langle -v\theta \rangle$ are larger in fractal grid turbulence (Runs FGT1 and FGT2) than in regular grid turbulence (Run RGT). In addition, $|\langle -v\theta \rangle|$ is larger for fractal grid turbulence than for regular grid turbulence over the entire range of y/M_{eff} . These trends are more apparent at $t_r = 8.5$ (Run FGT2) than at $t_r = 5.0$ (Run FGT1).

Figure 54 shows the streamwise profiles of the typical length of the scalar mixing layer, l_T , which is defined as the distance between the points at $T = 0.25$ and 0.75

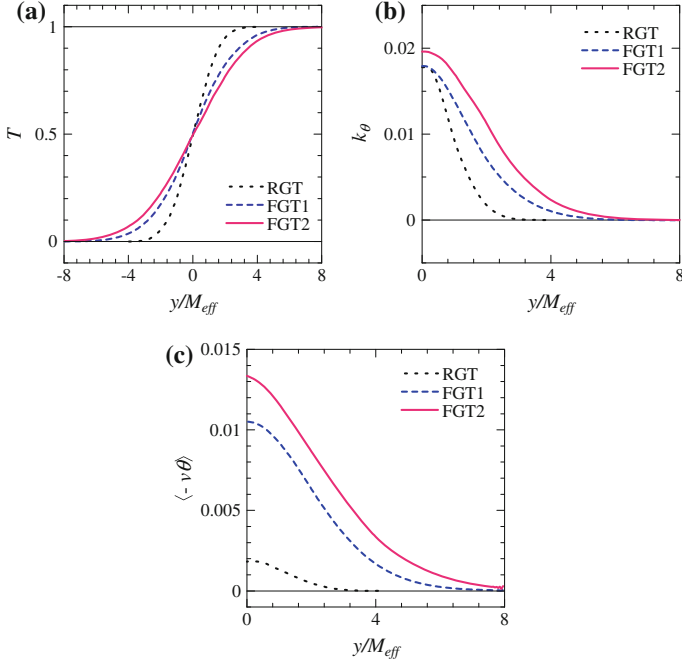


Fig. 53 Vertical profiles of **a** mean temperature, **b** temperature variance and **c** turbulent heat flux at $x/M_{eff} = 40$

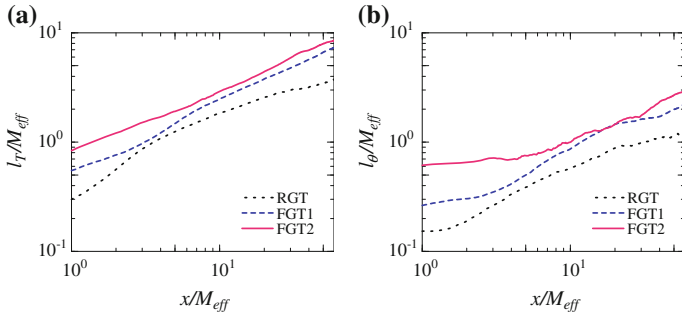


Fig. 54 Streamwise profiles of **a** l_T and **b** l_θ , normalised by M_{eff}

(Yoon and Warhaft 1991), and the half-width l_θ of the temperature variance k_θ , normalised by M_{eff} . It is confirmed that l_T/M_{eff} and l_θ/M_{eff} in regular grid turbulence are smaller than those in fractal grid turbulence over the entire range of x/M_{eff} . Note the differences between the results at $t_r = 5.0$ (Run FGT1) and $t_r = 8.5$ (Run FGT2): l_T/M_{eff} and l_θ/M_{eff} at $t_r = 8.5$ are larger than those at $t_r = 5.0$ over the entire range of x/M_{eff} . This difference is well pronounced in the upstream region.

These results show that turbulent mixing is enhanced to a greater extent in fractal grid turbulence than in regular grid turbulence even when Re_M is identical in both cases. Further, it is apparent that turbulent mixing is more enhanced at $t_r = 8.5$.

3.4 Summary

DNSs of GGT were performed. The simulations included the heat transfer in turbulent flows generated by four types of grid: (a) a square-mesh biplane square-bar grid (referred to as a regular grid in Sect. 3), (b) a square-mesh single-plane square-bar grid, (c) a composite grid consisting of parallel square bars and (d) a square fractal grid. In a flow with grid (c), two energy-containing turbulences interact in the absence of mean flow (Veeravalli and Warhaft 1989). The results show that typical GGT with heat transfer is simulated with high accuracy. Next, the turbulent scalar mixing layer developed in regular and fractal grid turbulence was compared: a biplane square grid (regular grid) and two square fractal grids with $t_r = 5.0$ and 8.5 were used for generating turbulence. The results show that turbulent mixing is enhanced to a greater extent in fractal grid turbulence than in regular grid turbulence, especially at $t_r = 8.5$. These results are in good agreement with those of our previous experiment performed at $Sc \approx 2100$ in a water channel (Suzuki et al. 2010c).

4 Conclusion and Future Prospects

In this chapter, we treated three topics: experiments on CO_2 gas jet diffusion in regular and fractal grid turbulence (Sect. 1), experiments on the scalar mixing layer in regular and fractal grid turbulence (Sect. 2), and simulations of the mixing layer in the turbulence generated by four types of grid, where the mixing layers in regular and fractal grid turbulence, in particular, are compared (Sect. 3). In Sect. 1, we obtained larger eddy diffusivity of CO_2 gas in the axial direction for fractal grid turbulence than for regular grid turbulence, and we also obtained a smaller turbulent Schmidt number in the radial direction for fractal grid turbulence than for regular grid turbulence. In these cases, the grid turbulence acts as an external (outer) disturbance on the jet diffusion. The external (outer) disturbances are expected to change the mechanism of transfer of momentum and scalar inside the jet through entrainment of the external (outer) disturbed flow to the inside of the jet. The effect of external (outer) disturbances seems to be closely related to the ‘dissimilarity’ of momentum and scalar transfer (Kong et al. 2001). One future work is to elucidate the relation between the external (outer) disturbances, the entrainment process and the dissimilarity of the momentum and scalar. The topic in Sect. 2 is closely related to practical application to a ‘fractal mixer’ with strong turbulence intensity and reduced energy dissipation, which has been suggested by Mazellier and Vassilicos (2010). From the results in Sect. 2, we could understand the characteristics of scalar mixing in fractal grid turbulence to some extent. However, experiments are currently limited to separate measurements of the

velocity and concentration. To investigate the mechanism of scalar mixing in fractal grid turbulence in more detail, we need to simultaneously measure the velocity and concentration and obtain data about the joint statistics between the velocity and the concentration such as the turbulent fluxes, e.g. $-\langle c\hat{v} \rangle$ and $-\langle c^2\hat{v} \rangle$. Laizet and Vassilicos (2012) recently suggested the fractal space-scale unfolding mechanism for energy-efficient turbulent mixing. The details of this mechanism will be elucidated by simultaneous measurement of the velocity and concentration and investigation of the joint statistics between the velocity and the concentration, in particular in the region near the grid. We leave this research as future work. In Sect. 3, we showed that direct numerical simulation is a very powerful tool by which various types of grid turbulence are simulated with high accuracy, and that the turbulent mixing is enhanced to a greater extent in fractal grid turbulence than in regular grid turbulence even when Re_M is the same in both cases. We also confirmed that a large value of t_r is effective for enhancing the mixing. Very recently, Laizet and Vassilicos (2015) investigated stirring and scalar transfer by grid-generated turbulence in the presence of a mean scalar gradient and showed that fractal grids enhance turbulent scalar fluxes and also greatly the streamwise growth of the fluctuating scalar variance in the region downstream of the peak position of turbulence intensity. They also showed that these phenomena originate in the ‘fractality’ of the grids. At present, however, all simulations of the scalar field are limited to low Schmidt number (0.1–0.4 in the paper of Laizet and Vassilicos (2015); 0.71 and 7.1 in this chapter). In the future, it is expected that the dependence of the Schmidt number and Reynolds number on turbulent mixing in fractal grid turbulence will be elucidated. Finally, we believe that the data presented in this chapter will be useful for investigating the feasibility of employing fractal grids as high-performance mixers in practical applications (Hurst and Vassilicos 2007).

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