

Contents

1	Introduction	1
1.1	The Decadic System	1
1.2	Redundancy	3
1.3	Symbolic Spaces	4
1.4	The Extended Real Line $\overline{\mathbb{R}}$	5
1.5	Positional Systems for Bounded Intervals	8
1.6	Positional Systems for $\overline{\mathbb{R}}$	12
1.7	Continued Fractions	18
2	Symbolic Dynamics	25
2.1	Metric Spaces	25
2.2	The Cantor Space	30
2.3	Redundant Symbolic Extensions	33
2.4	Subshifts	38
2.5	Sofic Subshifts	40
2.6	Labelled Graphs	43
	References	46
3	Matrices and Transformations	47
3.1	Projective Geometry	47
3.2	The Extended Real Line	49
3.3	Intervals	51
3.4	Projective Metrics	52
3.5	Transformations	54
3.6	The Circle Derivation	57
3.7	Conjugated Transformations	58
3.8	Complex Transformations	62
3.9	Hyperbolic Geometry	64
3.10	Disc Transformations	67
3.11	Isometric Circles	70
3.12	Singular Transformations	75

3.13	Representing Sequences	77
3.14	General Continued Fractions	79
	References	81
4	Möbius Number Systems	83
4.1	Iterative Systems	83
4.2	Interval Number Systems	87
4.3	Sofic Expansion Subshifts.	95
4.4	Partition Number Systems.	99
4.5	Sofic Number Systems	103
4.6	The Contraction and Length Quotients	108
4.7	Polygonal Number Systems.	111
4.8	Discrete Groups.	114
	References	117
5	Arithmetical Algorithms.	119
5.1	Intervals	120
5.2	The Unary Algorithm.	125
5.3	The Branching Unary Algorithm	128
5.4	Bilinear Tensors	130
5.5	The Binary Algorithm	137
5.6	Polynomials	141
5.7	Rational Functions.	143
	References	145
6	Integer Vectors and Matrices	147
6.1	Determinant and Norm.	147
6.2	Rational Number Systems.	150
6.3	Modular Number Systems.	152
6.4	Finite State Transducers	155
6.5	Bimodular Systems	158
6.6	Binary Continued Fractions.	162
	References	164
7	Algebraic Number Fields	165
7.1	Polynomials with Rational Coefficients.	165
7.2	Extension Fields	166
7.3	Field Embeddings	172
7.4	Computable Ordered Fields.	175
7.5	Algebraic Integers	176
7.6	Pisot and Salem Numbers	179
7.7	Positional Number Systems.	180
7.8	Arithmetic in Positional Systems	188
	References	195

8 Transcendent Algorithms 197

8.1 Padé Approximants 197

8.2 Algebraic Tensors 205

8.3 The Transcendent Algorithm 209

8.4 Arithmetical Expressions 215

8.5 Iterative Algorithms 217

References 219

Index 221

Dynamics of Number Systems
Computation with Arbitrary Precision

Kurka, P.

2016, XI, 224 p. 74 illus., Hardcover

ISBN: 978-3-319-33366-3