

Chapter 2

The Two-Dimensional Electron System

At the heart of this thesis are the properties of high-quality two-dimensional electron systems when being exposed to strong magnetic fields and low temperatures. In our case, the 2DES is hosted inside of a GaAs/AlGaAs heterostructure—a system well known for its excellent quality. Details on the sample structure are provided in the first section of this chapter. The remaining sections summarize the basic properties of a 2DES in a perpendicular magnetic field and lay the foundation for the physics discussed in subsequent chapters. Of particular importance is here the integer and fractional quantum Hall effect. Special emphasis is put on the density-modulated phases in the quantum Hall regime as well as the peculiar physics of the second Landau level. We restrict ourselves to the situation in GaAs/AlGaAs heterostructures. The following sections are based on introductory textbooks [1, 2] as well as Ph.D. theses [3–8].

2.1 Realizing a 2DES in a GaAs/AlGaAs Heterostructure

The confinement of electrons into two dimensions can be realized in a variety of different systems, such as semiconductor heterostructures, graphene and topological insulators. The samples investigated in the course of this thesis employ a GaAs/AlGaAs heterostructure to form a 2DES of exceptional quality. Up to date, it is the superior platform to study quantum Hall physics, showing a large number of integer and fractional quantum Hall states (Sects. 2.3 and 2.4) as well as different density-modulated phases (Sect. 2.5). The samples were fabricated by molecular beam epitaxy (MBE), a method which provides precise control over the vertical layer growth and excellent crystalline purity. Figure 2.1 shows the basic sample structure. The key features are described below. Further details about the samples used in the following chapters can be found in Appendix A. The wafers for fabricating these

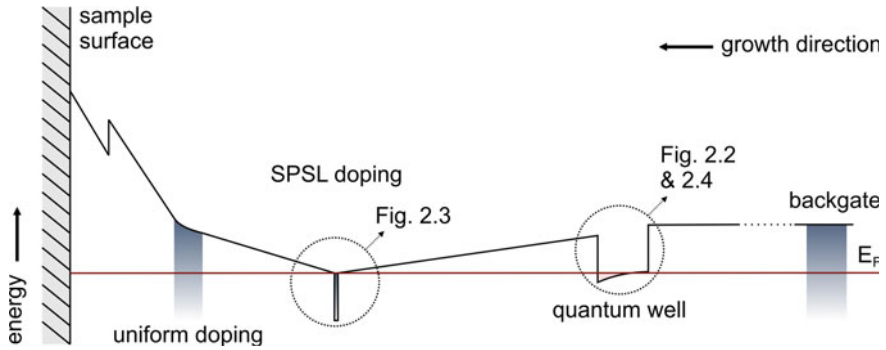


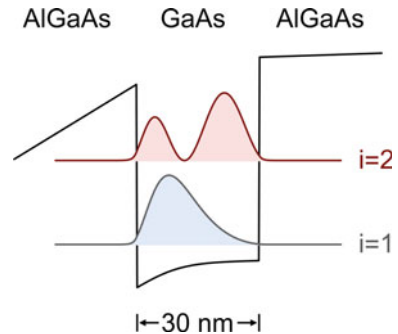
Fig. 2.1 Schematic of the conduction band profile along the growth direction in a typical GaAs/AlGaAs quantum well structure (not drawn to scale). Modulation doping is used to populate the QW with electrons. The backgate is realized by incorporating a highly doped layer underneath the QW

samples of excellent quality were developed and kindly supplied by Dr. Vladimir Umansky (Weizmann Institute of Science, Israel).

The Quantum Well

The 2DES is located inside of a quantum well (QW) structure, which is formed by a thin layer of GaAs (typically 10–50 nm) sandwiched between AlGaAs barriers (Fig. 2.2). Because GaAs has a lower conduction band energy than AlGaAs, the movement of electrons is restricted to the GaAs layer provided that their energy is lower than the potential barrier of the QW. In order to fulfill this requirement, the QW width w , the density of electrons n_e as well as their temperature must be chosen accordingly. The confinement in the vertical direction (z -axis) leads to discrete values of the z -component k_z of the wavevector. In the case of a QW with infinitely high barriers, these values are $k_z = i\pi/w$ ($i = 1, 2, 3, \dots$). Consequently, the energy associated with the movement of electrons in the growth direction is quantized to

Fig. 2.2 Probability density of the two lowest wavefunctions in a 30 nm-wide GaAs/AlGaAs quantum well



$$E_z = \frac{\hbar^2 k_z^2}{2m^*} = \frac{\hbar^2 \pi^2 i^2}{2m^* w^2}, \quad (2.1)$$

where m^* is the effective mass of the conduction band electrons. Therefore, decreasing the QW width results in a larger spacing between the different energy subbands. In the more realistic case of a QW with finite barriers, the wavefunction may extend partially into the barriers, which increases the effective width of the QW. Figure 2.2 shows the probability density of the two wavefunctions which are lowest in energy for the case of a 30 nm-wide QW. The calculations were done by solving the Schrödinger and Poisson equations self-consistently using the software *nextnano++* [9]. The samples studied in this thesis are designed in such a way that only the lowest subband is occupied. In this case, all electrons have the same k_z component, and the electron system can be considered two-dimensional despite the finite extent of the wavefunction along the z -direction (Fig. 2.2). For a 2DES with parabolic energy dispersion and non-interacting electrons, the number of states per unit area and energy can be calculated according to

$$\text{DOS} = \frac{m^*}{\pi \hbar^2}. \quad (2.2)$$

Thus, the density of states (DOS) of a two-dimensional electron system with parabolic dispersion depends merely on the effective electron mass and is in particular independent of the electron energy.

The Doping Scheme

Intrinsic GaAs (as well as AlGaAs) turns insulating at low temperatures. Therefore, dopants must be integrated into the crystal structure to populate the QW with electrons. To achieve a high-quality 2DES, it is essential how the doping is implemented. The dopants should affect the movement of the electrons in the QW as little as possible. The obvious approach, i.e. to dope directly the GaAs layer, is unfavorable because it degrades the mobility of the electrons due to scattering from ionized donors. Instead, we use the concept of modulation doping, which separates the doping layer from the QW by a distance of typically 50–100 nm [11]. Despite the spatial separation, electrons from the donor atoms will accumulate in the QW because of its (initially) lower-lying unoccupied energy levels (Fig. 2.1). Once the equilibrium state has been reached, the QW is filled with electrons up to the Fermi energy E_F .¹

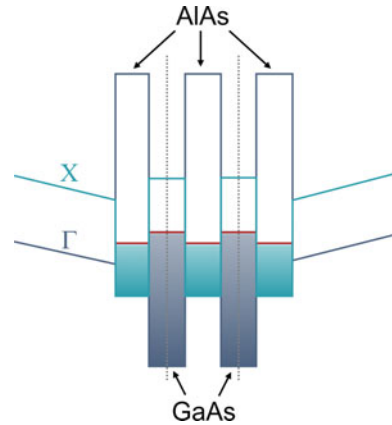
To realize a high-mobility 2DES, a large density of electrons is desirable because it mitigates the effect of disorder in the QW. The main sources of disorder in our heterostructures are residual impurities in the QW and the smooth disorder potential created by the remote ionized donors randomly distributed in the doping layer [10, 12]. The more electrons are located in the QW the better this disorder landscape is screened. The electron density in the QW can be adjusted by changing the distance of the doping layer from the QW. This distance is referred to as the spacer thickness. However, decreasing the spacer thickness increases the strength of the electrostatic

¹More precisely, the electrochemical potential should be used instead of the Fermi energy at finite temperatures. We neglect this subtlety throughout the thesis in view of the low sample temperatures.

disorder in the QW due to the remote ionized donors and thus enhances the scattering rate of electrons in the QW. Alternatively, higher n_e can be realized by increasing the number of doping atoms. This handle is of course limited to the point when the energy level of the donors coincides with the Fermi level. Once this situation is reached, increasing the doping density will not raise the electron density any further. Nevertheless, such a strong doping is often intentionally applied as it ensures a more homogeneous electron density across the wafer. In our samples, the doping strength is even increased beyond this point. The idea behind such an over-doping is that the additional electrons will remain in the doping layer and partly screen the disorder potential of the ionized donors. This effect has been proven crucial to improve the sample quality and in particular the strength of the fractional quantum Hall states [13]. On the other hand, over-doping can create a conducting layer at the site of the dopants, which gives rise to parallel conduction in transport measurements and may impede the stable operation of topgates as they are commonly used in interferometer structures.

In our samples, this issue is partly solved by using the dedicated doping scheme depicted in Fig. 2.3 and described in detail in references [10, 14, 15]. It is based on a short-period superlattice (SPSL) of thin GaAs layers separated by AlAs layers. The Si-dopants are only placed inside of the GaAs layers. Each of the neighboring AlAs layers forms a barrier at the Γ -point, leading to a confinement of the electrons like in a narrow quantum well. The same is true for the AlAs layers at the X -point. The confinement raises the energy of the electrons and improves the charge transfer into the 2DES. Furthermore, the superlattice is designed such that the ground state energy in the AlAs layers at the X -point is lower than the one of the GaAs layer. Thus, the excess electrons present due to over-doping have the character of X -electrons in AlAs, which implies a higher effective mass compared to the Γ -electrons in GaAs. The main idea behind this doping scheme is that the heavier electrons in the AlAs layers do not contribute as strongly to parallel conduction as electrons in a GaAs layer but are still mobile enough to partly screen the disorder potential generated by the ionized

Fig. 2.3 Conduction band energy at the Γ - and X -point for a short-period superlattice doping scheme (based on [10]). The doping is placed in the center of the GaAs layers (dotted lines)

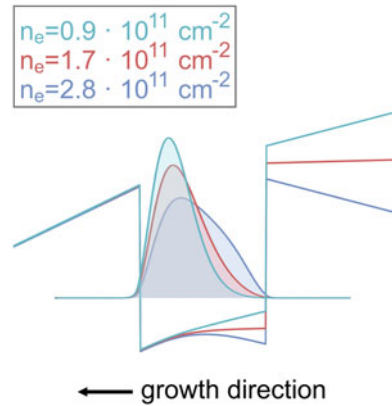


donors. Doping directly the AlAs layer is intentionally avoided because it leads to the creation of DX centers. These negatively charged donor states cause lattice distortions of the crystal when trapping an additional electron and create states deep within the band gap [1, 16]. As a consequence, the charge transfer efficiency to the 2DES is reduced. Nevertheless, DX center doping is commonly used in high mobility 2DES structures because it gives rise to an effect known as persistent photoconductivity: Illuminating the sample with infrared light at low temperatures can release the trapped charges and increase the carrier concentration in the 2DES. After interrupting the illumination, the charge density remains high because an energy barrier has to be overcome for re-trapping the electrons in the DX centers. Above the SPSL doping structure a second, uniform doping layer is placed to compensate the influence of the surface states. Separating these two doping regions is not essential but simplifies the search for optimal doping parameters.

The Backgate

In many cases, the previously introduced doping scheme is used symmetrically on either side of the quantum well to double the charge density. In most of our samples, however, modulation doping is applied only at the top and left out at the bottom. Instead, a highly conductive layer is incorporated during growth at a distance of about 800 nm from the QW. This layer, if separately contacted, can be used to electrostatically change the electron density in the QW by applying different gate voltages. This feature is an important advantage. In fact, many of the experiments described in the following chapters would not have been possible without in-situ and fast control over the electron density. When tuning the backgate voltage, not only n_e but also the shape of the electron wavefunction is affected. Figure 2.4 shows the shape of the wavefunction calculated for three different densities. In the present configuration, increasing the electron density obviously broadens the shape of the wavefunction. This behavior will become important in subsequent chapters.

Fig. 2.4 Probability density of the ground state wavefunction in a 30 nm-wide GaAs/AlGaAs quantum well. The density in the QW can be tuned electrostatically by a backgate. At the same time, changing the backgate voltage affects the shape of the wavefunction



2.2 Classical Electron Transport

The electron transport in two dimensions has many facets and depends on different parameters, such as sample quality, temperature and magnetic field. This section deals with the classical motion of electrons in two dimensions. We distinguish the situation with and without external magnetic field. The quantum-mechanical description of electron transport in two dimensions will be addressed in the next section. We limit the discussion to purely electronic transport.

Transport Without Magnetic Field

To approach the classical transport behavior, we employ the Drude model [17, 18]. In this model electrons are considered as point-like, massive particles. They get accelerated by an electric field $\mathbf{E}$. After a mean time τ electrons are scattered, for example by an impurity or a phonon. The scattering rate is treated as homogeneous and isotropic. Interactions between electrons are not taken into account. Furthermore, all electrons are assumed to contribute equally to the conductivity. Based on these assumptions, the equation of motion can be written as

$$m^* \frac{d\mathbf{v}}{dt} = -\frac{m^* \mathbf{v}}{\tau} - e\mathbf{E}, \quad (2.3)$$

where $\mathbf{v}$ denotes the velocity of the electron and e the positive elementary charge. This leads in the stationary state $d\mathbf{v}/dt = 0$ to a constant drift velocity

$$\mathbf{v}_D = -\frac{e\tau}{m^*} \cdot \mathbf{E}. \quad (2.4)$$

The mobility $\mu = e\tau/m^*$ is therefore proportional to the scattering time τ . The quality of a 2DES is often assessed by its mobility. In recent years, mobilities as high as $3 \times 10^7 \text{ cm}^2/\text{Vs}$ have been realized [10]. This corresponds to a mean free path of more than $200 \mu\text{m}$ between consecutive scattering events. Using Eq. 2.4, the current density $\mathbf{j} = -n_e e \mathbf{v}_D$ can be expressed as

$$\mathbf{j} = \frac{n_e e^2 \tau}{m^*} \cdot \mathbf{E}. \quad (2.5)$$

Thus, the conductivity at zero magnetic field σ_0 can be written as $\sigma_0 = n_e e \mu = n_e e^2 \tau / m^*$. It can be measured by applying a constant voltage and detecting the current flow through the sample. However, for the experiments discussed in this thesis, we impose a constant current and measure the resulting voltage drop. In this configuration, the resistivity ϱ of the sample is measured. In the absence of a magnetic field, ϱ_0 is simply the inverse of the conductivity $\varrho_0 = 1/n_e e \mu = m^* / n_e e^2 \tau$. Both quantities, conductivity and resistivity, can be conveniently used to determine the mobility of a 2DES provided that the electron density n_e is known. n_e can be extracted from the classical Hall effect with the help of an external magnetic field as described below.

Transport Under the Influence of a Magnetic Field

When applying a magnetic field B to the sample, the Lorentz force $\mathbf{F} = -e\mathbf{v} \times \mathbf{B}$ acts on the electrons and deflects them from a straight movement. As a consequence, the current flow $\mathbf{j}$ is no longer in line with the electric field direction $\mathbf{E}$. Thus, conductivity and resistivity need to be expressed as second-order tensors. In order to take the contribution of the magnetic field into account, Eq. 2.3 needs to be modified as

$$m^* \frac{d\mathbf{v}}{dt} = -\frac{m^* \mathbf{v}}{\tau} - e(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \quad (2.6)$$

If the magnetic field is perpendicular to the 2DES ($\mathbf{B} = B\mathbf{e}_z$), the electric field generated by the current is

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \varrho_{xx} & \varrho_{xy} \\ \varrho_{yx} & \varrho_{yy} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \frac{m^*}{n_e e^2 \tau} & \frac{B}{n_e e} \\ -\frac{B}{n_e e} & \frac{m^*}{n_e e^2 \tau} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix} =: \begin{pmatrix} \varrho_L & \varrho_H \\ -\varrho_H & \varrho_L \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}. \quad (2.7)$$

The equation above provides two important pieces of information:

- The longitudinal resistivity ϱ_L , which measures the resistivity along the direction of current flow, is equal to the zero field resistivity ϱ_0 . Further, it is independent of the current direction ($\varrho_{xx} = \varrho_{yy} =: \varrho_L$) as expected for an isotropic system.
- An additional electric field builds up perpendicular to the current flow. This effect is called the Hall effect [19]. The electric field is proportional to the current with the proportionality constant

$$\varrho_H = \frac{B}{n_e e}. \quad (2.8)$$

Thus, the Hall effect provides a convenient method to determine either the magnetic field for a known charge density or the charge density when knowing the magnetic field strength.

The conductivity tensor is the inverse of the resistivity tensor and can be deduced from Eq. 2.7:

$$\sigma = \frac{1}{\varrho_L^2 + \varrho_H^2} \begin{pmatrix} \varrho_L & -\varrho_H \\ \varrho_H & \varrho_L \end{pmatrix} = \frac{1}{\left(\frac{m^*}{n_e e^2 \tau}\right)^2 + \left(\frac{B}{n_e e}\right)^2} \begin{pmatrix} \frac{m^*}{n_e e^2 \tau} & -\frac{B}{n_e e} \\ \frac{B}{n_e e} & \frac{m^*}{n_e e^2 \tau} \end{pmatrix} =: \begin{pmatrix} \sigma_L & -\sigma_H \\ \sigma_H & \sigma_L \end{pmatrix} \quad (2.9)$$

The equation above implicitly defines the following relations:

$$\sigma_L = \frac{\varrho_L}{\varrho_L^2 + \varrho_H^2} \quad (2.10)$$

$$\sigma_H = \frac{\varrho_H}{\varrho_L^2 + \varrho_H^2} \quad (2.11)$$

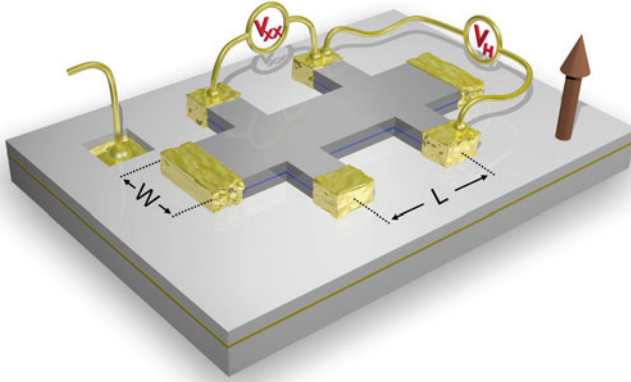


Fig. 2.5 Schematic of a Hall bar structure used for most of the transport experiments in this thesis. The current is imposed through the front contact covering the entire width of the Hall bar. The longitudinal resistance is measured on neighboring contacts along the perimeter of the Hall bar, the Hall resistance between contacts on opposite sides. Typical dimensions are $W = 0.4$ mm and $L = 1.25$ mm. In many cases a backgate is present, which is separately contacted. The magnetic field (brown arrow) is directed perpendicular to the 2DES

In order to relate the resistivity to the quantity measured in experiment, i.e. the resistance R , the sample geometry and current flow must be well defined. Often a Hall bar structure as shown in Fig. 2.5 is used. Here, W denotes the width of the Hall bar and L the distance between two neighboring contacts. For this configuration, the longitudinal and Hall resistivity can be calculated as $\varrho_{xx} = R_{xx} \cdot \frac{W}{L}$ and $\varrho_H = R_H$ provided that the current flow is homogeneous. The samples used in this thesis typically have the dimensions $W = 0.4$ mm and $L = 1.25$ mm.

2.3 The Integer Quantum Hall Effect

The Drude model introduced in the previous section predicts the Hall resistance to increase linearly with the magnetic field, while the longitudinal resistance is supposed to remain constant. This prediction is contrasted by the measurement of R_{xx} and R_H shown in Fig. 2.6.² In this measurement, a perpendicular magnetic field was applied to a high-quality sample at a temperature of roughly 20 mK. Already at a relatively low magnetic field of 100 mT, the Hall resistance deviates from its linear dependence, and R_{xx} starts to oscillate. These deviations from the predicted behavior get stronger when ramping up the magnetic field. Eventually, R_{xx} drops to zero in certain ranges of the magnetic field, interrupted by peaks of a finite resistance. In these regions

²Here, as in all other resistance measurements throughout this thesis, a low-frequency lock-in technique was used to improve the signal-to-noise ratio.

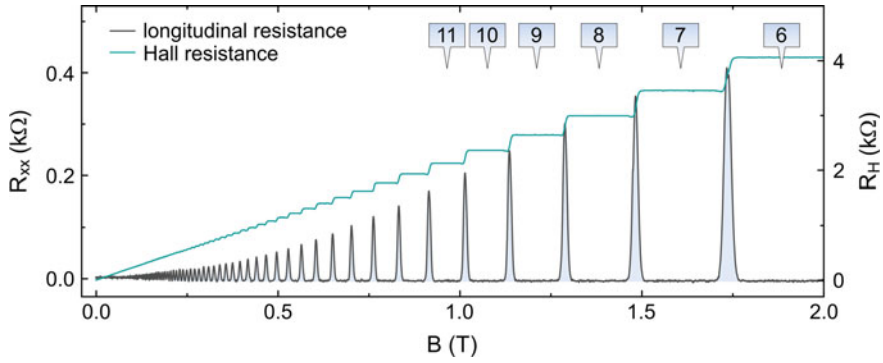


Fig. 2.6 Longitudinal and Hall resistance for magnetic fields up to 2 T. Characteristic of the integer quantum Hall effect is a vanishingly small longitudinal resistance accompanied by a plateau in the Hall resistance. The integer values of ν in expression 2.12 are indicated in blue boxes

of $R_{xx} \approx 0$, the Hall resistance exhibits a plateau. The resistance values of these plateaus are precisely quantized to

$$R_H = \frac{h}{e^2} \cdot \frac{1}{\nu}, \quad (2.12)$$

with ν being an integer number ($\nu = 1, 2, 3, \dots$). In view of these findings, this effect has been named the integer quantum Hall effect (IQHE). For its discovery, Klaus von Klitzing was awarded the Nobel Prize in 1985. The resistance values of the plateaus only depend on natural constants and are in particular independent of sample properties like charge density and spatial dimensions. For this reason, the IQHE is used as a resistance standard.

2.3.1 Quantum Mechanical Treatment in the Single-Particle Picture

The time-independent Schrödinger equation of non-interacting electrons in a magnetic field can be written as

$$\hat{H} \cdot \psi(\mathbf{r}) = \left[\frac{1}{2m^*} (\hat{\mathbf{p}} - e\mathbf{A}(\hat{\mathbf{r}}))^2 + e\phi(\hat{\mathbf{r}}) \right] \psi(\mathbf{r}) = E \cdot \psi(\mathbf{r}). \quad (2.13)$$

The Hamilton operator $\hat{H}$ contains the momentum operator $\hat{\mathbf{p}}$, the electrostatic potential ϕ and the vector potential $\mathbf{A}$, which depends on the location operator $\hat{\mathbf{r}}$. ψ are the eigenfunctions of the Hamilton operator. To calculate the solutions of the Schrödinger equation in a homogeneous magnetic field, two different gauges for the vector poten-

tial are frequently used, both of which are outlined below. The corresponding calculations can be found in textbooks [1, 20].

The Landau Gauge

In the Landau gauge, the vector potential has only one component $\mathbf{A} = (0, -Bx, 0)$. This simplifies the algebra notably. In the case of free electrons, ϕ vanishes if no external electrical field is applied. For electrons confined to a quantum well, the electrostatic potential varies only along the growth direction (z -axis). In this case, the motion along the z -direction can be separated from the main Schrödinger equation, which leads to expression 2.1 in the first section. The normalized solution for the motion in the x - y -plane is shown in Eq. 2.14. Here, H_n represents the Hermite polynomial of order n . Figure 2.7 displays the wavefunctions $\psi_{n,m}$ for different parameters n and m . The eigenfunctions in Landau gauge comprise plane waves along the y -direction with wavevector k_m . For periodic boundary conditions, k_m must fulfill $k_m = \frac{2\pi}{L_y}m$ with m being an integer number ($m = 1, 2, 3, \dots$) and L_y representing the sample size in the y -direction. The plane waves are distributed equally along the x -axis and are centered at positions $x_m = l_B^2 k_m$.

$$\psi_{n,m}(x, y) = \frac{1}{\sqrt{2^n n! \sqrt{\pi} l_B}} H_n \left(\frac{x - l_B^2 k_m}{l_B} \right) e^{-\frac{(x - l_B^2 k_m)^2}{2l_B^2}} e^{ik_m y} \quad (2.14)$$

The solutions of the Schrödinger equation contain the magnetic length $l_B = \sqrt{\hbar/(eB)} \approx 25.7/\sqrt{B[T]} \text{ nm}$ —a characteristic length scale brought about by the magnetic field. The physical consequences of the eigenfunctions are discussed below. They must be independent of the chosen gauge. Properties of the wavefunctions that do depend on the gauge, like the orientation of the plane waves along the y -direction, have no physical meaning. In fact, one can also choose a gauge which is characterized by the circular symmetry of its eigenfunctions as described in the following.

Fig. 2.7 Electron wavefunctions in the Landau gauge for different parameters of m and n (based on [3]). Both energy and wavefunction amplitude are plotted along the z -axis

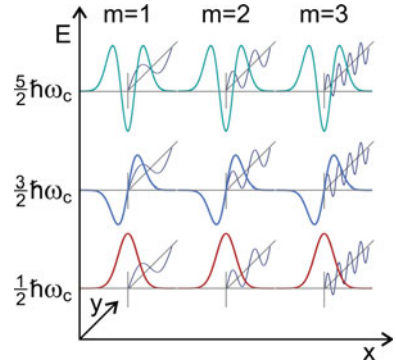
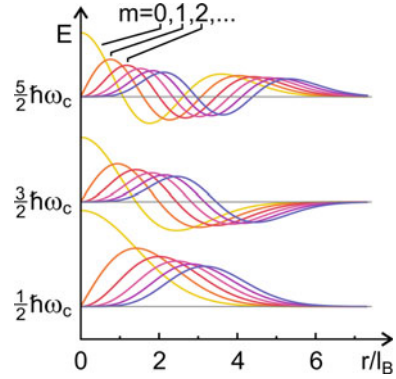


Fig. 2.8 Electron wavefunctions in the symmetric gauge for different parameters of m and n (based on [3]). Both energy and wavefunction amplitude are plotted along the ordinate



The Symmetric Gauge

The symmetric gauge is rotationally invariant about the z -axis and is defined as $\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}$. Consequently, the eigenfunctions in the symmetric gauge are best parameterized in polar coordinates r and θ :

$$\psi_{n_r, m}(r, \phi) = \frac{1}{l} \sqrt{\frac{n_r!}{(n_r + |m|)!}} \left(\frac{r}{l_B \sqrt{2}} \right)^{|m|} e^{-\frac{r^2}{4l_B^2}} L_{n_r}^{|m|} \left(\frac{r^2}{2l_B^2} \right) e^{im\theta}, \quad (2.15)$$

where $L_{n_r}^{|m|}$ are the associated Laguerre polynomials. The wavefunctions are shown graphically in Fig. 2.8. The radial quantum number n_r ($n_r \geq 0$) counts the zeros in the radial term of the wavefunction and relates to n (in the Landau gauge) via $n = n_r + (|m| - m)/2$. The angular momentum quantum number m is restricted to $m \geq -n$. For large values of m , the shape of the wavefunctions in the symmetric gauge is roughly equal to rings of width l_B and radius $r_{|m|} = \sqrt{2|m|}l_B$. As in the case of the Landau gauge, the wavefunctions are shaped along the vector potential $\mathbf{A}$. Because of the centrifugal potential, the wavefunctions get pushed away from the origin when increasing m . In general, the solutions in the symmetric gauge are more complicated to handle than those in the Landau gauge. Therefore, preferably the Landau gauge is used, except for cases where rotational symmetry is prevalent, e.g. in quantum dots.³

Gauge Independent Properties

The following consequences of the Schrödinger equation are independent of the gauge choice and describe important properties of the 2DES:

- **Formation of Landau levels**—The eigenvalues of the wavefunctions yield the energy spectrum of electrons in a magnetic field. They can be expressed as

³In some cases, which are not of relevance for this work, it is advantageous to use the von Neumann lattice gauge. Here, the wavefunctions describe a cyclotron motion of electrons centered around points of a von Neumann lattice [21].

$$E_n = \left(n + \frac{1}{2}\right) \cdot \hbar\omega_c, \quad (2.16)$$

using the classical cyclotron frequency $\omega_c = eB/m^*$. Thus, the energy spectrum has discrete values parameterized by the index n ($n = 0, 1, 2, \dots$). These energy levels are called Landau levels. They are separated by the cyclotron energy $E_c = \hbar\omega_c$. This result stands in contrast to classical calculations which predict a constant density of states (Eq. 2.2). The reduction of the electronic energy spectrum to discrete values brought about by the magnetic field has far-reaching consequences. In fact, the existence and robustness of the quantum Hall effect is deeply rooted in the formation of Landau levels.

- **Degeneracy of Landau levels**—The fact that the energy values in Eq. 2.16 only depend on the quantum number n and not on k (in Landau gauge) shows that the Landau levels are highly degenerate. The number of allowed states per unit area in a Landau level n_L depends on the sample dimensions. In the Landau gauge, we need to impose the requirement that the center coordinate x_m lies within the sample, i.e. $x_m = l_B^2 \frac{2\pi}{L_y} m < L_x$. This immediately leads to the expression

$$n_L = \frac{eB}{h} = \frac{B}{\Phi}. \quad (2.17)$$

Thus, a Landau level can accommodate as many electrons as magnetic flux quanta $\Phi = h/e$ penetrate the sample. In a sense, each electron state occupies the area of a single flux quantum. An important quantity in the context of the quantum Hall effect is the filling factor ν . It denotes how many Landau levels are filled with electrons. Using Eq. 2.17, the filling factor can be calculated as

$$\nu = \frac{n_e}{n_L} = \frac{n_e h}{eB}. \quad (2.18)$$

Strictly speaking, this expression is only valid on length scales much larger than the extent of the electron wavefunction. For small dimensions, the electron density n_e in Eq. 2.18 has to be replaced by the density of the wavefunctions' center coordinates. Of course, at macroscopic dimensions both definitions are equivalent.

The Spin Degree of Freedom

Up to now, the implications of the electron spin have been swept under the rug. If an electron spin is placed in a magnetic field, it contributes an additional energy E_z alongside the cyclotron energy E_c due to the Zeeman effect. The Zeeman energy depends linearly on the magnetic field:

$$E_z = g_e^* \mu_B B, \quad (2.19)$$

with $\mu_B = e\hbar/2m$ being the Bohr magneton and g_e^* the effective g-factor. For electrons in bulk GaAs, g_e^* has a value of -0.44 [22, 23]. However, in a 2DES, exchange

interaction effects might greatly modify g_e^* at low filling factors [23]. Equation 2.19 implicitly assumes that conduction band electrons in GaAs have total spin 1/2 (s-band, orbital spin 0). Hence, the Zeeman effect splits each Landau level into a spin-up and spin-down branch separated by E_z . The additional degree of freedom brought by the electron spin doubles the density of states in a Landau level (factor 2 in Eq. 2.17). Nevertheless, it is customary for the definition of the filling factor (Eq. 2.18) to count the spin branches as separate levels: $\nu = 1 \rightarrow$ only the spin-up branch ($n = 0, \uparrow$); $\nu = 2 \rightarrow$ the spin-down branch ($n = 0, \downarrow$) is filled as well.

Figure 2.9 shows schematically the Landau level structure as a function of magnetic field. The energy spectrum has a fan-like structure since both Zeeman and cyclotron energy increase linearly with the magnetic field. The relative weight of the respective energies is understated in Fig. 2.9. For typical experimental conditions, the cyclotron energy is about 70 times larger than the Zeeman energy (for $g_e^* = -0.44$ and $m^* = 0.067 m$). This pronounced difference is the reason why integer quantum Hall states (IQHS) are generally more stable at even filling factors, i.e. they have a higher energy gap compared to odd filling factors (even in the presence of exchange enhancement at lower filling factors). Figure 2.9 also shows the evolution of the Fermi energy when increasing the magnetic field for a fixed electron density. E_F follows the linear increase of the Landau levels up to the point where the degeneracy in the lower levels is high enough to accommodate all electrons of the upper level. At this point, the Fermi energy jumps to the lower level, which creates the sawtooth-like

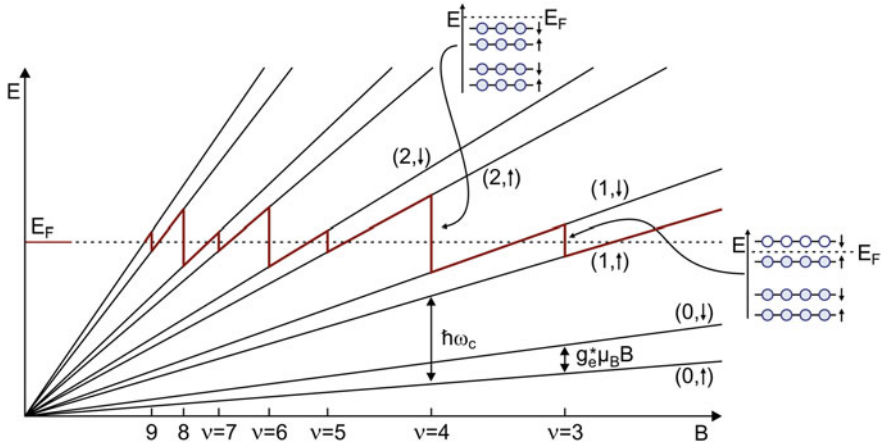


Fig. 2.9 Evolution of the discrete Landau level energies in a perpendicular magnetic field (based on [3]). The Landau levels are separated by the cyclotron energy. In addition, each Landau level is split by the Zeeman energy (splitting is exaggerated for clarity). Both energies depend linearly on the magnetic field. In addition, the Landau level degeneracy increases when raising B , and more electrons can be accommodated in the lower-lying Landau levels. Once a level is emptied completely, the Fermi energy (red line) drops abruptly

pattern shown in Fig. 2.9. Since the Fermi energy jumps between levels of opposite spin orientation, the total spin polarization exhibits an oscillatory behavior. At even filling factors, the electron system is unpolarized and becomes increasingly polarized towards (smaller) odd filling factors. Deviations from this general behavior arise from many-particle interactions between the electrons. We will return to this important point in later chapters.

2.3.2 Microscopic Picture

Many aspects of the QHE cannot be understood without looking at the electrostatic situation of the sample at a microscopic level. For instance, the width of the resistance plateaus in Fig. 2.6 depends decisively on the disorder potential experienced by the electrons, and the electrostatic potential at the edges strongly influences the current flow through the sample.

The Role of Disorder

The Landau levels introduced above as a solution of the Schrödinger equation have a sharp, peak-like density of states. In reality, the disorder present in the sample due to background impurities and remote ionized donors leads to an energetic broadening Γ of the Landau levels. The exact shape of the broadening is unclear and presumably depends on the details of the disorder potential. Some publications support a Gaussian shape [24–26], while others reported a Lorentzian one [27, 28]. Uncertainty governs also the question how the Landau level broadening evolves when changing the magnetic field. A square root dependence has been observed in references [25, 26], whereas references [24, 27] found no change of Γ with magnetic field. Figure 2.10a depicts the B -field dependence of the electron DOS in the case of a Gaussian broadening. A clear separation of the Landau levels requires $\hbar\omega_c > \Gamma$.

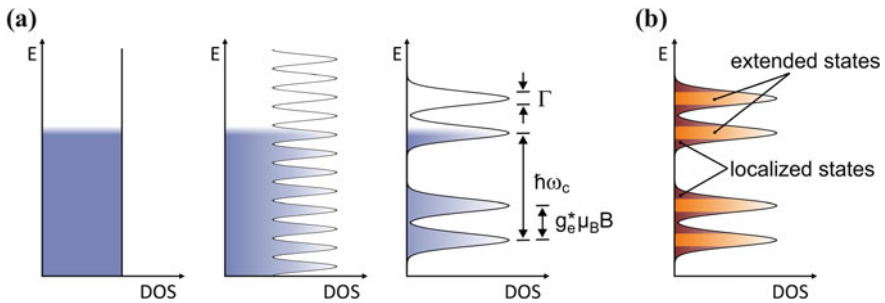


Fig. 2.10 **a** Evolution of the electron DOS with increasing magnetic field. Landau levels develop separated by the cyclotron energy. The Zeeman energy splits each Landau level into two branches of opposite spin orientation. **b** Due to the disorder in the sample, each Landau level branch is divided into extended states at its center and localized states at its periphery

The disorder potential not only leads to a broadening of the Landau levels but also divides the electron system into localized and extended states (Fig. 2.10b). Intuitively, electrons are localized either in the valleys or hills of the disorder landscape. The motion of the confined electrons can in a simplified picture be viewed as closed orbits of constant energy inside of the hills and valleys as shown in Fig. 2.11. Quantum mechanics dictates that these orbits enclose an integer number of flux quanta [2]. The localized states become populated predominantly for high and low fillings of a Landau level. For intermediate filling factors, the electrons are free to move across the sample. Yet, this simplified picture does not grasp the whole complexity of the problem. Ilani et al. found the number of localized states to be independent of the magnetic field by probing the 2DES with a single-electron transistor [29]. This is surprising in view of the magnetic field dependent Landau level degeneracy. As the magnetic field and with it the flux density increases, more and more localized states are expected to form in the valleys and hills of the disorder landscape [29]. Thus, in the single-particle picture, the density of localized states would depend on the magnetic field. Based on this discrepancy, the authors concluded that the localization physics in high-quality (low disorder) samples is dominated by Coulomb interaction. Further, each of the electron pockets has to be treated as a quantum dot (or anti-dot). Because of the Coulomb repulsion inside of the quantum dot (Coulomb blockade), each electron pocket can accommodate only a fixed number of electrons. As a consequence, the density of localized states is independent of the magnetic field.

Given the separation of the DOS into localized and extended states, it is possible to understand intuitively the vanishing of the longitudinal resistance over a wide magnetic field range in the IQHE (Fig. 2.6). Whenever the Fermi energy falls into a region of localized states, the conductivity σ_{xx} drops to zero, and according to Eq. 2.10, $\varrho_{xx} = 0$ follows. If extended states are present at the Fermi energy, R_{xx} has a finite value. The large number of localized states ensures $R_{xx} \approx 0$ over an extended filling factor range in contrast to the situation in Fig. 2.9, where zero longitudinal

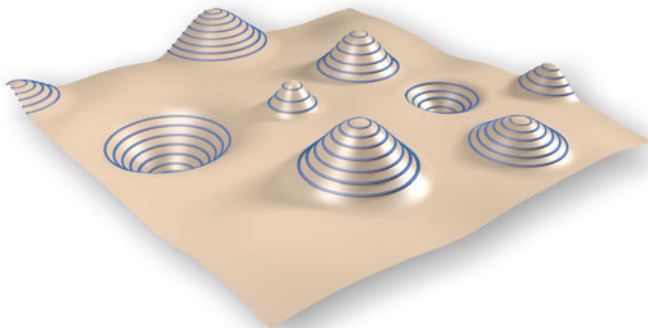


Fig. 2.11 Visualization of localized states in the single-particle picture. Shown is the spatial variation of the electrostatic potential in a disordered sample. Electrons get trapped in the valleys and hills of the disorder landscape. Their orbits are indicated in blue

resistance would only be observable at the point when E_F jumps from one Landau level to the other.

What we have not addressed so far is the question why the Hall resistance is quantized. To answer this question, we have to take a closer look at the edges of the sample.

The Landauer-Büttiker Picture

When going away from the interior of the sample towards the edges, the Landau levels have to bend upwards because the Fermi energy at the surface is pinned to a value within the GaAs bandgap (Fig. 2.12a). Consequently, the Landau levels coincide with the Fermi energy at the edges, and one-dimensional conductive channels are created, even if the interior part of the sample is insulating. For each completely filled Landau level branch, one transport channel is created on either side of the sample (Fig. 2.12b). The electron transport along the two sides occurs in opposite directions as determined by the potential gradient at the edges. The spatial separation of the counter-propagating current paths strongly suppresses dissipative backscattering of the charge carriers. As a consequence, a constant potential is maintained throughout each channel, and the voltage drop between contacts on the same side becomes vanishingly small. At the same time, each one-dimensional channel adds a value of e^2/h to the total conductance [30]. Based on this argument, it becomes obvious that the Hall resistance is precisely quantized to $R_H = h/(e^2\nu)$ simply because the Hall voltage is set by the potential difference between opposite sides of the sample. This approach of decomposing the transport in the quantum Hall regime into isolated current channels goes back to ideas by R. Landauer and M. Büttiker [30, 31].

The Interacting-Electron Picture

When taking a closer look, the Landauer-Büttiker picture of current flow has some obvious shortcomings. For instance, the sudden step-like drop of the electron density

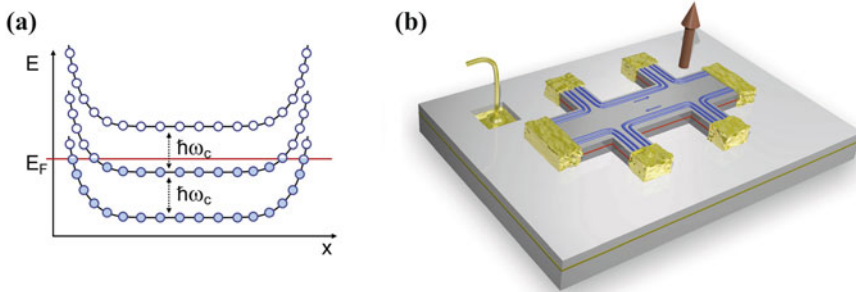


Fig. 2.12 The Landauer-Büttiker edge channel picture. **a** Spatial evolution of the Landau level energy across the sample. At the edges, the Landau levels are lifted by the confinement potential. As a consequence, partially filled one-dimensional channels are created which run along the sample edges. **b** Visualization of the edge channels in a Hall bar structure

at the edges of the sample is energetically unfavorable (Fig. 2.13a left). In a more realistic scenario, the Coulomb repulsion between electrons would lead to a smoothly varying density decrease towards the edges. This is confirmed by the self-consistent calculations of the electrostatic and chemical potential performed by Chklovskii et al. [32, 33]. Their findings are shown in Fig. 2.13a (right). In contrast to the single-particle picture of Landauer and Büttiker, the self-consistent calculations separate the sample into regions of opposing character—compressible and incompressible stripes. In the *compressible* stripes, the topmost filled Landau level is only partially occupied, and its energy is fixed to the Fermi energy. In the *incompressible* stripes, all occupied Landau levels are completely filled, and the electrons cannot screen the lateral confinement potential. Hence, the absolute value of the chemical potential decreases here towards the edge. All incompressible stripes have an integer value of the filling factor. Scattering of the electrons inside of these stripes is therefore strongly suppressed by the energy gap to the next, unoccupied level. As a result, the current can flow dissipationless inside of the incompressible stripes, and any externally injected current will preferentially distribute to flow in the incompressible stripes. More precisely, the current will favor the innermost incompressible stripe on either side of the sample because it is the most stable one and can sustain the highest currents [33–35]. If these incompressible stripes become too narrow (on the order of l_B), scattering into the central compressible regions is possible, and dissipation occurs. Hence, a quantized Hall plateau is only observable if at least one incompressible stripe exists in the sample which is wide enough to sustain a dissipationless current.

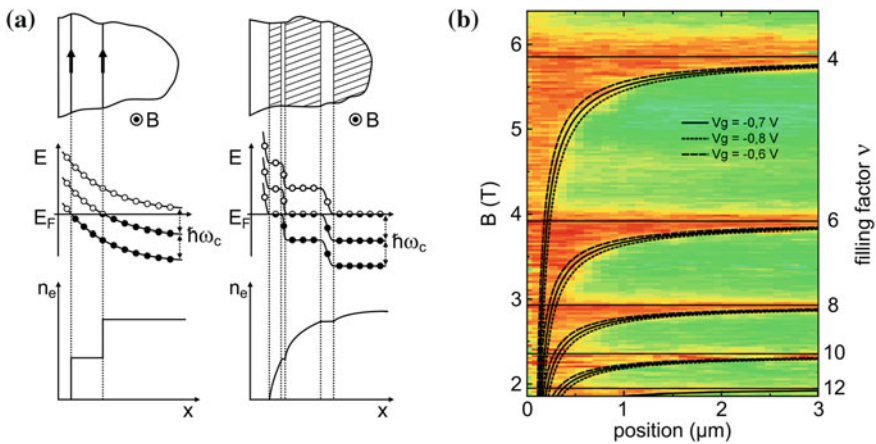


Fig. 2.13 The interacting-electron picture. **a** The Coulomb interaction favors a smooth density distribution at the edges of the sample (*right*) in contrast to the Landauer-Büttiker picture (*left*). This leads to the formation of compressible (hatched regions) and incompressible stripes (picture modified from [32]). **b** Scanning probe measurement of the Hall potential distribution from the edge towards the center of the sample (picture taken from [40]). Black lines indicate the theoretical position of the innermost incompressible stripe (see [40] for details)

The behavior described above has been confirmed experimentally by measuring the Hall potential distribution across the sample using different scanning probe techniques with either a metal tip [36, 37] or a single-electron transistor [38, 39] as a sensor. The spatial variation of the Hall potential provides information about the current distribution because the Hall potential drops in a region where the current flows. An example of a measured Hall potential distribution is depicted in Fig. 2.13b. It shows that the current distribution moves to the interior part of the sample as the magnetic field approaches an integer value of the filling factor. This observation is consistent with the theoretical movement of the innermost incompressible stripe (black lines). Details on these experiments are given in reference [40].

Yet, the question remains how the quantization of the Hall resistance comes about in this model. The very fact that in the quantum Hall effect the current flows in the innermost incompressible stripe is the reason why the Hall resistance is precisely quantized to $R_H = h/(e^2\nu)$. This explanation is straightforward when bringing to mind that the innermost incompressible stripe always has an integer filling factor and its value is the largest integer filling factor in the sample. If the current flow is limited to this filling factor region, the Hall voltage drop will be as well. Combining Eqs. 2.8 and 2.18 then directly leads to $R_H = h/(e^2\nu)$. The onset of the quantized resistance plateau therefore occurs at the point when the innermost incompressible stripe is wide enough to maintain its incompressible character. By increasing the magnetic field, this stripe will move to the center of the sample. Eventually, the plateau will disappear once the central stripe gets disrupted by disorder and vanishes [35].

2.4 The Fractional Quantum Hall Effect

In the previous section, we have introduced that a magnetic field perpendicular to the 2DES can cause a sequence of plateaus in the Hall resistance centered around integer values of the filling factor. The sample studied in Fig. 2.6 nicely follows this behavior up to a magnetic field of 3 T. If we increase the magnetic field beyond this point, deviations from the simple oscillatory behavior occur as shown in Fig. 2.14. Additional quantum Hall states occur in between the IQHS at fractional values of the filling factor. Clearly visible are for example the plateaus at $\nu = 5/3$, $4/3$ and $4/5$ accompanied by a vanishing longitudinal resistance. These states belong to the fractional quantum Hall effect (FQHE). The FQHE was first observed for $\nu = 1/3$ by Tsui et al. in 1982 [41] shortly after the discovery of the IQHE in 1980 [42]. In subsequent years, increasingly more fractional quantum Hall states (FQHS) were found, owing to improvements in sample quality and low-temperature measurements. At the present day, more than 80 different FQHS are known to exist [43]. The discovery and explanation of the FQHE has been honored with awarding the Nobel Prize to Robert Laughlin, Horst Störmer and Daniel Tsui in 1998.

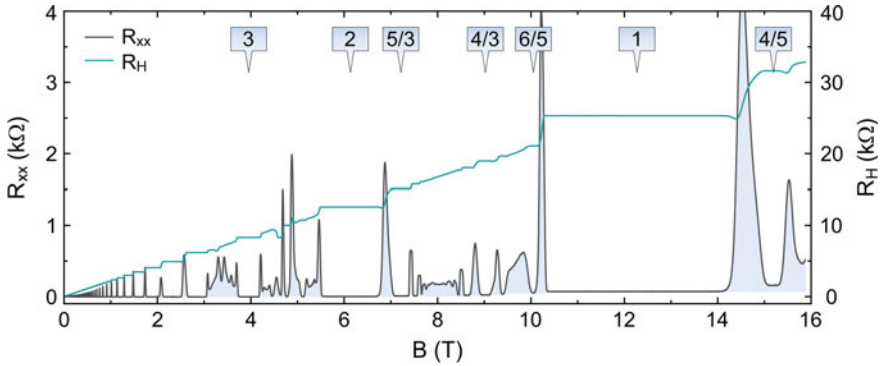


Fig. 2.14 Longitudinal and Hall resistance measured at base temperature (~ 20 mK) over a broad magnetic field range. Multiple fractional quantum Hall states are visible in between the IQHS. Filling factors are indicated in *blue* boxes

The existence of quantum Hall states at fractional filling factors came as a surprise because it cannot be explained by the single-particle theory introduced in the previous section. At a partial filling of Landau levels, the electron system is supposed to be compressible, and no excitation gap for the formation of quantum Hall states should exist. The FQHE must therefore originate from electron-electron interactions. In partially filled Landau levels, electrons can rearrange and thereby reduce their Coulomb energy while maintaining a constant kinetic energy. This is not possible for the IQHE because of the completely filled Landau levels. For this reason, the Coulomb interaction does not enter the IQHE in the first place. Yet, when taking a closer look, the Coulomb interaction can also significantly affect the IQHE as pointed out in the previous section. Describing these many-particle interactions theoretically is in general an exceptionally challenging endeavor. Nevertheless, several ingenious theoretical concepts have been put forward [2, 44–49]. The two most important ones are introduced hereinafter.

2.4.1 Laughlin's Wavefunction

In an attempt to describe the formation of quantum Hall states at fractional filling factors $\nu = 1/q$ in the lowest Landau level, Laughlin proposed a succinct trial wavefunction

$$\psi_{1/q} = \prod_{i < j} (z_i - z_j)^q \exp\left(-\frac{1}{4l_B^2} \sum_i |z_i|^2\right), \quad (2.20)$$

where z_i denotes the position of the i th electron in complex numbers [44, 50]. The properties of this wavefunction are described below:

- Based on the assumption that all electrons in the lowest Landau levels are spin polarized (symmetric spin wavefunction), the orbital wavefunction in Eq. 2.20 must be anti-symmetric. This forces q to be an odd integer number. Assuming a spin-polarized electron system is in fact only justified for filling factors $1/q$. At other FQHS the electron system can be partially polarized or even unpolarized. We will come back to this important issue at a later point.
- The electron system has a homogeneous density distribution [2].
- The second part of Eq. 2.20 consists of a product of Gaussian wavefunctions reminiscent of the single-particle solutions discussed in the previous section.
- The product term at the beginning contains the electron-electron interactions by imposing a q -fold zero point whenever two electrons are at the same position. This ensures that all electrons are well separated and thereby reduces the Coulomb energy. In a more advanced interpretation, the zeros in the wavefunction can be understood as vortices, i.e. as points of vanishing probability density in the center and a phase change of 2π when circulating around the vortex. Intuitively, one can imagine each electron to carry q vortices with it. The correlation of electrons and vortices ensures that no two electrons can occur in the same place. One vortex would be enough to fulfill Pauli's exclusion principle. However, the additional $q - 1$ vortices further reduce the electrostatic Coulomb energy considerably [51]. This is the basic idea behind the electron-electron correlations in a 2DES exposed to a strong magnetic field. Besides the q vortices at each electron's location, no further vortices are present in the 2DES. Hence, the number of vortices equals the number of magnetic flux quanta piercing through the sample simply because of the respective filling factor $1/q$. In fact, one can think of each vortex in the 2DES as embodiment of a single flux quantum [52]. This close relation between flux quanta and vortices as well as their affinity to electrons forms the basis of the composite fermion theory detailed below.

Above, we have introduced that a 2DES at $\nu = 1/q$ can minimize its Coulomb energy by attaching q vortices to each electron. When moving away from this commensurate electron-vortex ratio, e.g. by increasing the magnetic field and adding another vortex, a substantial amount of Coulomb energy has to be paid [46, 51]. This determines the energy gap of the FQHE and thereby identifies the Coulomb energy as the prevalent energy scale for FQHE physics. At the same time, introducing additional vortices in the 2DES excites quasiparticles that come along with very surprising properties. Because of the strong tie between vortices and electrons in Laughlin's theory, introducing one additional flux quantum can be understood equivalently as adding a quasiparticle with a fractional charge $e^* = e/q$ [44]. This startling property of Laughlin's quasiparticles has far-reaching consequences as these particles obey neither Fermi-Dirac nor Bose-Einstein statistics [46]. Instead, they follow a more general anyonic statistics, which will be addressed in Sect. 2.6.

Yet, the question remains whether Laughlin's wavefunction is indeed correctly describing FQHE physics and, if so, to what extent. The existence of fractionally

charged excitations has been confirmed experimentally by tunneling [53] and shot noise measurements [54–56] as well as more recently in experiments using single-electron transistors [57]. The verification of these counterintuitive entities provides strong support for Laughlin’s theory. Apart from that, it is a beautiful example of fractionalization in solid state physics, according to which the properties of quasiparticles cannot be understood simply as a linear combination of its elementary constituents. This general phenomenon has been encountered in different manifestations in recent years, for example as magnetic monopoles in spin ice materials [58–60] or as the deconfinement of an electron into a holon, spinon and orbiton [61–63]. Further support for Laughlin’s theory comes from exact diagonalization studies [64] and Monte Carlo simulations [65], where it was shown that Laughlin’s wavefunction yields the correct ground state for short range interactions.

Laughlin’s concept has been successfully generalized to FQHS at filling factors different from $1/q$. Quantum Hall states at filling factors $\nu = 1 - 1/q$ follow directly from particle-hole symmetry [51, 66]. Wavefunctions for other fractional filling factors were constructed by Halperin and Haldane using a hierarchical approach based on Laughlin’s quasiparticles [45, 46, 51]. However, these wavefunctions are more complex and partly lead to discrepancies between experiment and theory, such as the relative strength of FQHS [2]. An alternative and at the same time very elegant approach to construct higher-order FQHS is the composite fermion model.

2.4.2 The Composite Fermion Theory

In 1989 Jainendra Jain enriched the understanding of the FQHE by proposing a new concept [67]. Again, it is based on the coupling of electrons and vortices, but this time an electron dressed with an even number of magnetic flux quanta is considered as a new entity—the composite fermion (CF). With this transformation in mind, the FQHE can be elegantly rephrased as the IQHE of composite fermions. In the following, we illustrate this statement at an intuitive level using Fig. 2.15. For a deeper understanding, we refer to reference [2]. Figure 2.15 shows the longitudinal resistance for the FQHS around $\nu = 1/2$ in the lower panel. Plotted above are the IQHS appearing at smaller magnetic fields. Filling factor $\nu = 1/2$ corresponds to the situation where twice as many magnetic flux quanta penetrate the sample as electrons are present. After attaching two flux quanta to each electron, the resulting composite particles essentially move around in a vanishing effective magnetic field because all flux quanta were already taken into account in the construction of the composite fermions. Hence, a compressible Fermi sea of composite fermions is formed. When increasing the magnetic field and going away from $\nu = 1/2$, the effective magnetic field $B^* = B - B(\nu = 1/2)$ increases likewise. As a consequence, quantum Hall states start to develop for integer values of the effective filling factor $\nu^* = n_e h / (e B^*)$. Figure 2.15 highlights the essence and beauty of the CF model: The IQHE of composite fermions in their effective magnetic field coincides with the FQHE of electrons in the total magnetic field. In this sense, the FQHE at

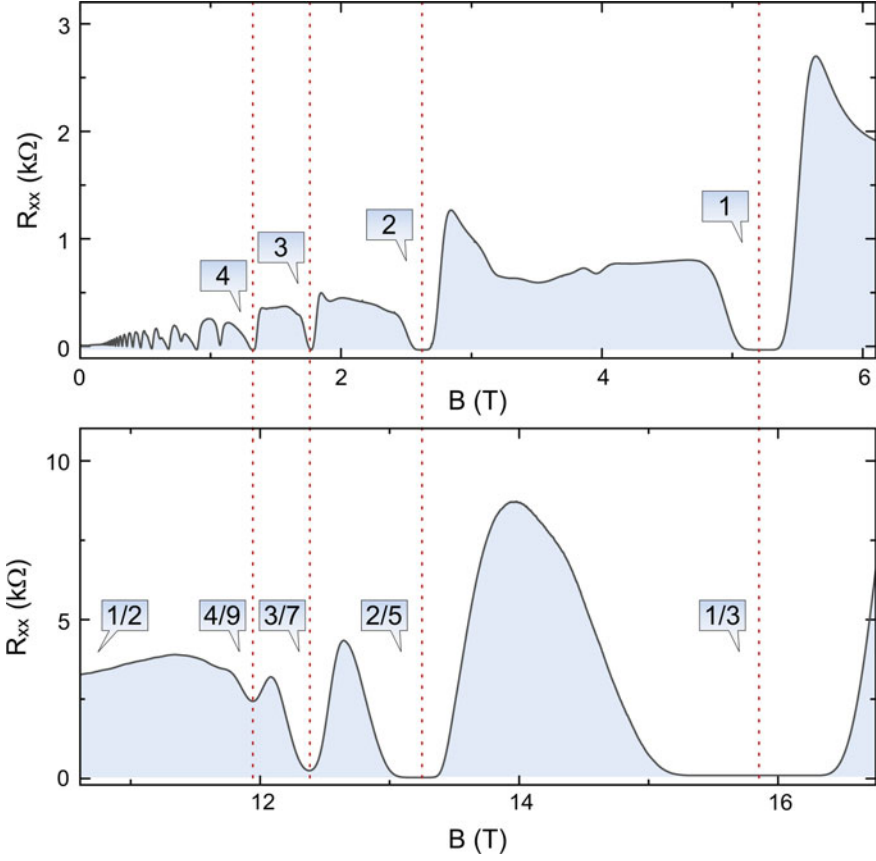


Fig. 2.15 Visualization of the CF picture (plotted according to [78]). Shown is the longitudinal resistance up to $\nu = 1$ (*upper panel*) and between $\nu = 1/2$ and $\nu = 1/3$ (*lower panel*). The positions of the FQHS with respect to $\nu = 1/2$ coincide with the IQHS in the total magnetic field. The *upper panel* was measured at roughly 400 mK in order to locate more easily the position of each IQHS

$\nu = 1/3$ ($2/5$, $3/7$, ...) can be understood as the IQHE of composite fermions with $\nu^* = 1$ (2, 3, ...). Thus, the CF theory transforms the strongly interacting 2DES into a system of weakly interacting composite fermions whose energy spectrum develops discrete Landau levels if B^* is strong enough. These CF Landau levels are sometimes referred to as Λ levels. Drawing further on the analogy to the IQHE, the excitation gap of composite fermions can be interpreted as cyclotron energy $E_c = \hbar e B^* / m_{CF}^*$, with m_{CF}^* being the effective CF mass. m_{CF}^* must be proportional to $\sqrt{B}$ since the energy gap arises solely from Coulomb interactions.

In general, the CF theory accounts for FQHS at filling factors

$$\nu = \frac{\nu^*}{p \cdot \nu^* \pm 1}. \quad (2.21)$$

In this expression, p is an even integer number and denotes the number of flux quanta attached to each electron. Equation 2.21 covers only filling factors in the lowest Landau level branch. However, it can be extended to the second Landau level by ignoring the completely filled lower-lying levels. This accounts for the FQHE at $\nu = 7/3$ and $\nu = 10/3$, for instance. In addition, particle-hole symmetry within each Landau level might be observable, leading to states such as $\nu = 4/5$ and $\nu = 5/7$.

The composite fermion theory has been proven very helpful in explaining various experimental results [68–70]. The validity of the CF concept is further supported by the determination of the CF Fermi wavevector [71, 72] and the demonstration of a quasi-classical motion of the composite fermions in their effective magnetic field B^* [73–75]. Cyclotron resonances of composite fermions have been measured directly by microwave absorption [76]. Interestingly, also the FQHE of composite fermions has been observed, which highlights the existence of residual interactions between composite fermions [77].

2.5 Density-Modulated Phases in the Quantum Hall Regime

When looking at Fig. 2.14 in the previous section, it becomes apparent that the FQHS only appear at high magnetic fields in the first and second Landau level ($n = 0, 1$). At lower magnetic fields, i.e. when higher Landau levels are occupied, density-modulated phases are energetically favored at partial fillings [79]. Their existence is not directly visible in Fig. 2.14 but will be demonstrated experimentally later on. For these density-modulated phases, the homogeneous electron system in the topmost Landau level breaks up into alternating regions of high and low electron density. Two types of density-modulated phases are believed to exist [80]: the bubble phases characterized by a twofold periodicity and the stripe phases, which consists in the simplest case of parallel stripes with a strictly one-dimensional (1D) periodicity (Fig. 2.16). In both cases, the system alternates between regions where the topmost Landau level branch is completely filled and regions where it is completely empty. The reason for the formation of density-modulated phases is rooted in the competition between the repulsive direct Coulomb interaction on the one hand and the attractive exchange interaction on the other hand [80, 81]. Whenever the 2DES deviates from a homogeneous distribution, this comes at the cost of direct Coulomb energy. Despite the abrupt change of the filling factor in Fig. 2.16, the impact on the Coulomb interaction is weakened by the spatial extent of the electron wavefunction in the plane of the 2DES as it smoothens the electron density distribution. At dimensions on the order of the wavefunction width, it is important to bear in mind that the filling factor is given by the density of the wavefunctions' center coordinates and not by the electron density (see Sect. 2.3.1). If the modulation period is similar to the width of the wavefunction ($\sim l_B$), the density modulation and therefore also the increase in Coulomb energy will be small in comparison to the step-like change of the filling factor. The

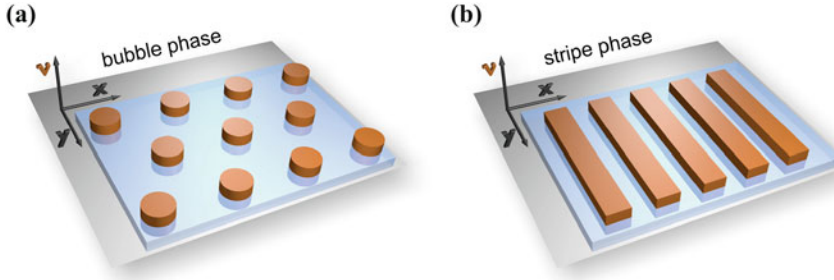


Fig. 2.16 Spatial modulation of the filling factor in the topmost Landau level for the bubble and stripe phase. **a** In the case of the bubble phase, the homogeneous electron system contracts to islands which arrange in a triangular lattice. **b** The stripe phase may in the simplest case be understood as a periodic arrangement of parallel stripes with a strictly one-dimensional periodicity

gain in exchange energy, however, is rather determined by the modulation of the filling factor [82]. In total, the 2DES can minimize its energy by modulating the electron density at certain fillings of the topmost occupied spin branch [79, 82, 83]. The average filling factor determines the phase symmetry, i.e. whether the bubble or the stripe phase is favored energetically. In general, the energy gain of density-modulated phases is rather small. Hence, to observe such phases, low temperatures (~ 100 mK) as well as high mobilities ($\sim 10^7$ cm²/Vs) are required.

2.5.1 The Bubble Phase

The bubble phase is present in Fig. 2.14 though not yet visible. Its presence becomes evident as one slightly raises the temperature to about 35 mK. The result is shown in Fig. 2.17. At higher temperatures, the physics appears to be much richer than the low-temperature data suggests. The blue curve reveals that the IQHS breaks down rather quickly when increasing (and lowering) the magnetic field. However, the IQHE reappears at even higher (lower) magnetic fields, where R_{xx} drops to zero and R_H returns to the value of the nearby IQHE plateau. These states are therefore called reentrant integer quantum Hall states (RIQHS). In the case of $n \geq 2$, the RIQHS occur at elevated temperatures around the partial filling factors $\tilde{\nu} = 1/4$ and $\tilde{\nu} = 3/4$, where $\tilde{\nu}$ denotes the filling factor of the highest occupied Landau level branch [84–86]. If the electron temperature is low enough, a seamless transition to the IQHE is observed in Fig. 2.17. The reappearance of the IQHE can be understood as a manifestation of the bubble phase: Since the lattice of electron clusters is pinned by disorder, it cannot participate in transport. Therefore, the system behaves in a standard charge transport experiment as if all Landau levels are filled, and the IQHE occurs. The same arguments hold for the RIQHS at $\tilde{\nu} = 3/4$ with the exception that there clusters of holes are embedded in the electron system of the next higher integer filling factor.

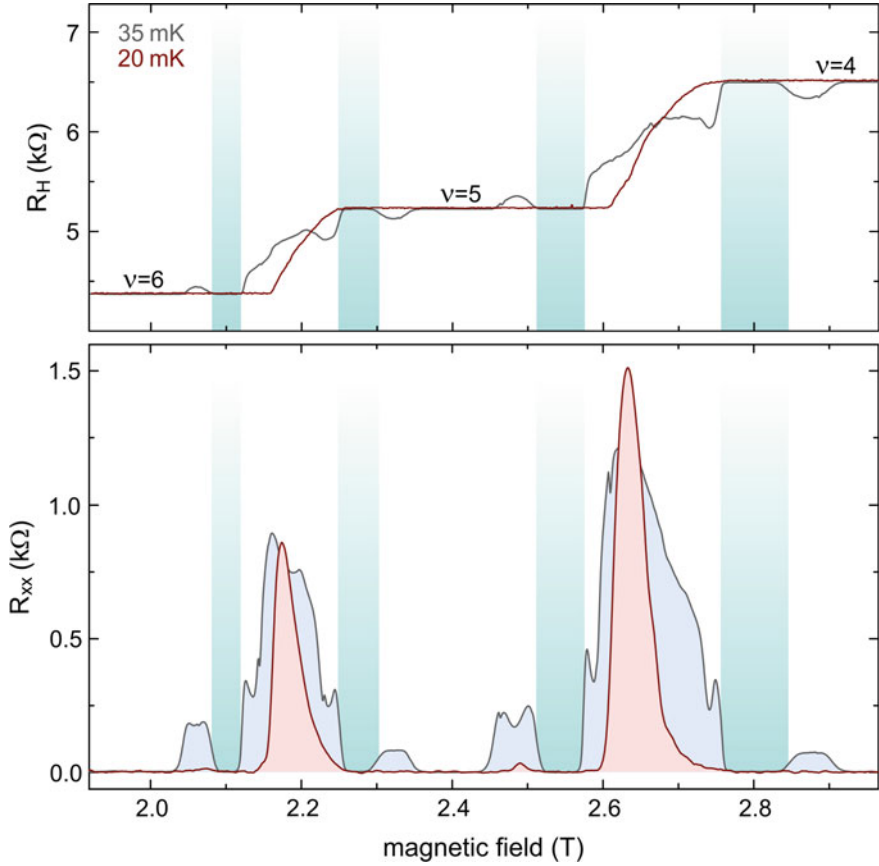


Fig. 2.17 Longitudinal and Hall resistance at base temperature (~ 20 mK, in red) and at slightly higher temperatures (~ 35 mK, in blue). At elevated temperatures a reappearance of the IQHE is visible (marked in green)

Extensive theoretical studies of the bubble phase and, in general, the density-modulated phases have been carried out. They predict that the electron bubbles arrange in a triangular lattice with a period of roughly $3 \cdot R_c$, where $R_c = \sqrt{2\pi n_e \hbar / (eB)}$ denotes the cyclotron radius [79, 82]. The number of electrons varies according to the partial filling $\tilde{\nu}$ with the restriction that the maximum number of electrons per bubble is limited to n in the n th Landau level (for $n > 1$) [79, 87].⁴ The optimal number of electrons per bubble can be well approximated by $3\tilde{\nu}n$ [80, 87]. Thus, when increasing $\tilde{\nu}$ from $\tilde{\nu} = 0$, at first, only a single electron occupies a bubble site.

⁴Côté et al. consider a maximum number of $n+1$ electrons per bubble. However, as the $n+1$ bubble phase appears around $\tilde{\nu} = 1/2$, it is unstable against the formation of a stripe phase [81].

This is the famous Wigner crystal [88]. It can be considered a special case of the bubble phase. For higher $\tilde{\nu}$, the system undergoes a succession of phases separated by first order phase transitions with an increasing number of electrons per bubble [87].

From an experimental point of view, little is known about the microscopic structure of the bubble phases. The interpretation as bubbles pinned by disorder is corroborated by the observation of a strongly non-linear I - V characteristic, which points to the sudden depinning of the bubble crystal [86]. Apart from that, resonances in the microwave absorption have been interpreted as pinning modes of the bubble crystal in the surrounding disorder potential [89, 90]. Using this technique, the coexistence of the Wigner crystal and the multi-electron bubble phase has been shown over a broad filling factor range, which indicates a first order transition between the respective phases [91].

2.5.2 The Stripe Phase

To unveil the existence of the stripe phase, the longitudinal resistance along two orthogonal crystal directions must be compared. This is done in Fig. 2.18. The measurement was performed on a square-shaped structure in order to obtain a symmetric setup. Figure 2.18 demonstrates that the longitudinal resistance depends strongly on the crystal direction probed in experiment. This observation is most evident at half

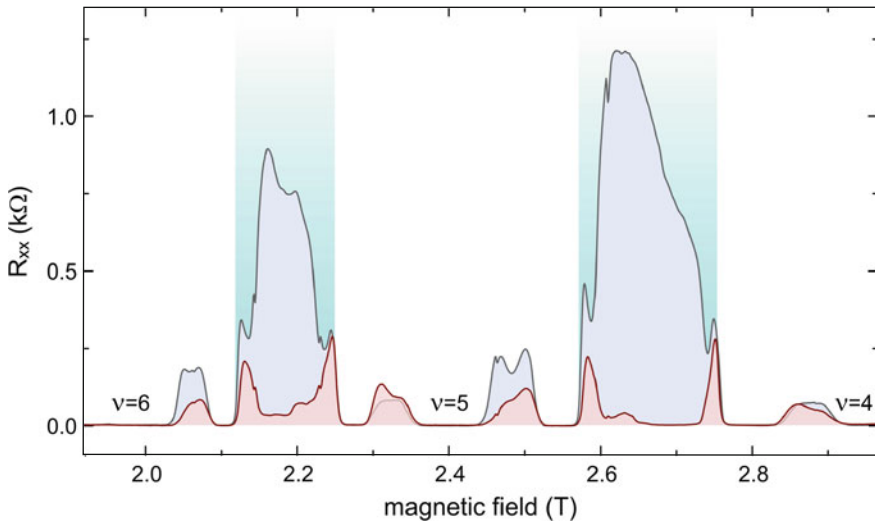


Fig. 2.18 Longitudinal resistance along two orthogonal current directions (sample temperature ~ 35 mK). A strong transport anisotropy is visible at each half-filled Landau level branch (marked in green), which indicates the formation of a stripe phase

filling of each Landau level branch ($\tilde{\nu} = 1/2$). Since both crystal directions, $[110]$ and $[1\bar{1}0]$, are identical in an ideal gallium arsenide crystal, the strong transport anisotropy indicates the existence of an anisotropic, stripe-like electron phase. If the current flows along the stripes (“easy” axis), the resulting voltage drop will be smaller than in the orthogonal direction (“hard” axis). This anisotropy is amplified by an effect called current channeling: If the current is sent perpendicular to the stripes, it gets spread out stronger towards the sample edges causing a higher voltage drop as compared to the orthogonal current direction [92, 93].

The physical mechanism leading to a preferred stripe orientation along specific crystal axes remains an open question up to date. Cooper et al. excluded substrate morphology, steps induced by miscuts as well as an anisotropic effective mass as possible explanations [94]. Instead, piezoelectric effects [95] and a combination of Rashba and Dresselhaus spin-orbit interactions have been proposed [96]. Intriguing is the fact that the stripe orientation depends on the electron density. Below a density of $2.9 \times 10^{11} \text{ cm}^{-2}$ the easy axis is directed along the $[110]$ direction, whereas for higher densities the $[1\bar{1}0]$ direction is preferred [97]. In addition, the stripes can be oriented by applying a magnetic field component in the plane of the 2DES [97–99]. Yet, the exact behavior of the stripes in an in-plane magnetic field seems to depend on the details of the sample [97].

Different theoretical models have been proposed for the microscopic structure of the stripe phase [80]. The charge density wave picture has been mentioned before (Fig. 2.16). In this model, the local filling factor is assumed to be modulated in parallel stripes with a strictly one-dimensional periodicity. Fradkin and Kivelson studied the effects of quantum and thermal fluctuations on the stripe formation and predicted in analogy to liquid crystal behavior the existence of a different class of stripe phases—the electron liquid crystal phases [100]. They are categorized according to their symmetry as smectic, nematic and stripe crystal phase. These phases will be discussed in detail in Chap. 5. Which of the three electron liquid crystal phases is lowest in energy depends on the strength of the shape fluctuations and the average filling factor [100]. As a consequence, different types of stripe phases might be present within the anisotropic region in Fig. 2.18, and transitions between the respective phases may occur as a function of the magnetic field.

Similar to the bubble phase, experimental results providing a detailed microscopic understanding of the stripe phase are scarce. The temperature dependence of the longitudinal resistance points towards the existence of a nematic phase [101, 102]. Recently, the collective modes of the stripe phase at $\nu = 9/2$ were probed in a sophisticated experiment involving surface acoustic waves, microwaves and photoluminescence [103]. In the same experiment, the period of the stripe pattern was determined to $3.6 \cdot R_c$. In addition, pinning mode resonances have been observed also for the stripe phase as reported earlier for the bubble phase [104, 105]. Interestingly, in the case of the stripe phase, a resonance only occurs if the electric field of the microwave signal is oriented along the hard axis.

2.6 The Second Landau Level

In the previous sections, it has been shown that the physics in partially filled Landau levels is governed by two classes of competing, interaction-driven phases. At high magnetic fields, when the first Landau level is partially occupied, FQHS are prevalent. In contrast, starting from the third Landau level towards higher filling factors, density-modulated phases, i.e. the bubble and stripe phases, take over. Their existence is heralded either by a reappearance of the IQHE or a strong transport anisotropy. Their dominance over the FQHS in higher Landau levels is rooted in the different exchange and direct Coulomb interactions, which originate from the altered shape and extent of the wavefunction. The competition between density-modulated phases and FQHE is most prominent in the second Landau level. Here, density-modulated phases as well as FQHS coexist, and minute changes of the magnetic field or electron density can lead to transitions between them. This is shown in Fig. 2.19. It displays the longitudinal and Hall resistance between filling factors $\nu = 2$ and $\nu = 4$, measured on a high-quality sample. A large number of different FQHS as well as reentrant states are visible, which emphasizes the superior quality of the sample. The variety of competing phases in the second Landau level is a beautiful example of the enormous richness and complexity of quantum Hall physics. When taking a closer look at Fig. 2.19, it becomes apparent that the second Landau level is exceptional in many regards. Not only the coexistence of density-modulated phases and FQHS is conspicuous but also the appearance of four reentrant states in each branch of the second Landau level leaps to the eye. Moreover, the observation of a fractional quantum Hall state at half filling of each spin branch is striking ($\nu = 5/2$ and $\nu = 7/2$). Both points are addressed in more detail hereinafter.

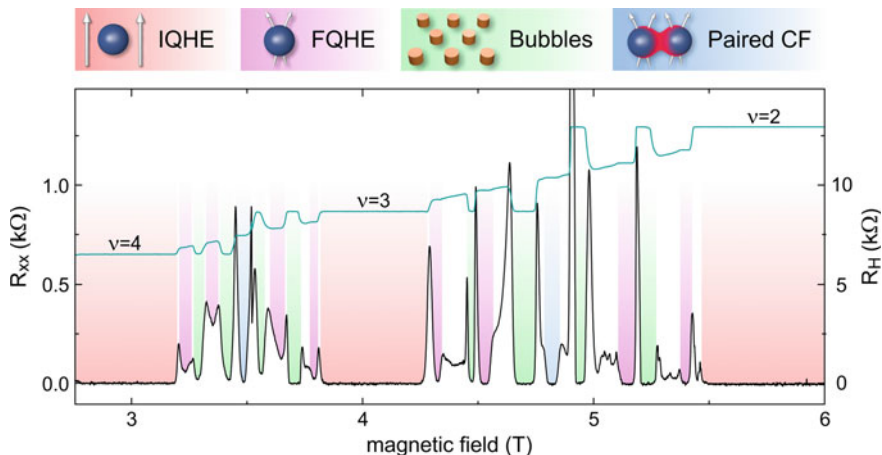


Fig. 2.19 Longitudinal and Hall resistance in the second Landau level measured on a high-quality sample. Multiple FQHS (marked in *pink*) and reentrant states (marked in *green*) are visible. Remarkable is the appearance of a quantum Hall state at half filling of each spin-split Landau level ($\nu = 5/2$ and $7/2$, marked in *blue*)

2.6.1 *Reentrant States in the Second Landau Level*

As mentioned earlier, the second Landau level differs from higher levels in the number of reentrant states. Figure 2.19 shows four reentrant states per spin branch, separated by different FQHS. They appear at partial filling $\tilde{\nu} = 0.29, 0.43, 0.57$ and 0.70 in agreement with previous publications [106–108]. That raises the question whether the reentrant states in the second Landau level are of equal nature as the bubble phases in higher levels. Initially, theories did not consider the second Landau level for bubble phase formation at all [82, 83]. This deficiency was remedied more recently, when the bubble phase consisting of one-electron and two-electron bubbles was shown to be lowest in energy for certain filling factors in the second Landau level [81, 109]. Besides the IQHE transport behavior, other similarities to the bubble phases in higher Landau levels have been observed, such as sharp and hysteretic features in the I – V characteristics [106] as well as pinning modes in microwave absorption [110]. In addition, it has been demonstrated that the onset temperatures of the RIQHS in the second Landau level scale with the Coulomb energy, which highlights the importance of electron-electron interactions for the formation of these states [108]. However, also discrepancies to the bubble phases in higher Landau levels are evident. The onset temperatures normalized by the Coulomb energy differ strongly between the second and third Landau level [111]. This observation is in conflict with existing theories and emphasizes that the bubble phases in the second Landau level might be of a more complex nature. Further work is needed to clarify the microscopic structure of the bubble phases in the second Landau level and beyond.

It is important to mention that also the close relative of the bubble phase, the stripe phase, can be observed in the second Landau level. In Fig. 2.19 its position at half filling is taken in by the $\nu = 5/2$ and $7/2$ FQHS. However, tilting the sample with respect to the external magnetic field causes the stripe phase to prevail energetically over the FQHE [98, 99, 112]. The microscopic nature of this tilt-induced stripe phase is the focus of Chap. 5.

2.6.2 *The $\nu = 5/2$ and $7/2$ States*

The observation of FQHS at half fillings comes as a surprise in view of the CF theory introduced in Sect. 2.4.2. There, the partial filling factor $\tilde{\nu} = 1/2$ was described as a compressible Fermi sea of composite fermions. It was concluded that the CF theory only accounts for FQHS at odd-denominator filling factors. Since its discovery in 1987 [113], the $5/2$ state and its particle-hole conjugate, the $7/2$ state, attracted a lot of attention.⁵ This is partly due to the fact that these even-denominator FQHS do not comply with the standard CF picture but also because of some interesting theoretical

⁵In the following, we will focus mainly on the $5/2$ state because of its higher stability and therefore greater experimental relevance. Since the $7/2$ state is considered the particle-hole conjugate of the $5/2$ state, all statements should apply equally to the $7/2$ state.

proposals which came up to explain the occurrence of the $5/2$ state. In particular the theory of G. Moore and N. Read (together with Greiter et al. [114]) raised a lot of excitement as it ascribes the origin of the $5/2$ state to a pairing of two CFs in analogy to the p-wave pairing in a superconductor [115]. By the nature of the p-wave pairing, the resulting state would be spin polarized, and the quasiparticles would have a fractional charge of $e/4$. Even more intriguing is the prediction that the excitations of the Moore-Read state constitute a novel type of quasiparticle called non-abelian anyon. Their exceptional properties are described below. A recent review on the $5/2$ state and its potentially non-abelian nature is given in reference [116].

Non-abelian Anyons and Their Relevance for Topological Quantum Computation

Quasiparticles in the FQHE not only carry fractional charges as mentioned in Sect. 2.4 but also exhibit another fascinating property—they are anyons. Anyons can be considered a generalization of bosons and fermions. They can be distinguished by their exchange statistics. The interchange of anyons results in a general phase factor $e^{i\theta}$ to the wavefunction and is not limited to -1 and 1 as in the case of fermions and bosons, respectively [117]. This generalized exchange statistics can only occur in two-dimensional systems. To illustrate this statement, we consider the twofold exchange of two particles with each other. This process is equivalent to describing a closed loop of one particle around the other. When moving a particle around another in a three-dimensional space, the trace can always be deformed continuously down to a point (Fig. 2.20). Therefore, a closed loop of particles is topologically equivalent to no movement at all and multiplies the wavefunction with a phase factor of 1 . In this case, the only possible solutions for a single interchange are the factors -1 and 1 , corresponding to the exchange of fermions and bosons, respectively. In two dimensions, however, a closed loop can no longer be contracted to a point without cutting through the other particle, i.e. the system does not necessarily come back to the same state [118]. Hence, also the phase acquired during particle exchange can have any value and is not bound to 0 and π as in the case of fermions and bosons. For the FQHE at $\nu = 1/3$, for instance, a counterclockwise exchange of quasiparticles leads to a phase change of $\pi/3$ [119].

Non-abelian anyons are special in the sense that the particle exchange is non-commutative. Suppose we have three particles and want to mutually change their positions (Fig. 2.21). The non-abelian nature brings the system into a different state

Fig. 2.20 Particle exchange in two and three dimensions (based on [3])

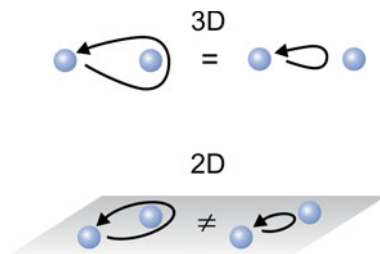
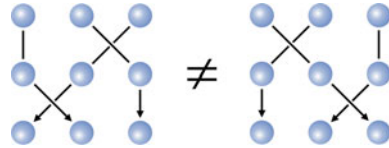


Fig. 2.21 The exchange of non-abelian particles is not commutative (based on [3])



if first particles on position 1 and 2 interchange and then on positions 2 and 3 in comparison to the particle exchange in the opposite order. Put differently, the operators for particle exchange do not commute. A necessary requirement for non-abelian states of matter is the existence of a highly degenerate ground state which is protected by an energy gap. When particles encircle one another or, in other words, particles are made to “braid”, the system transforms from one ground state to another [120]. The transformation is determined solely by the topology of the braiding and is in particular independent of the precise trace taken by the particles. This robustness to local perturbations in combination with the non-abelian braiding statistics provides the basis for topological quantum computation [118–121]. In this particular type of quantum computer, the quantum operations are performed by braiding of non-abelian particles. A topological quantum computer is superior in the sense that it is rather immune to local perturbations of the environment [118]. Standard schemes of quantum computation suffer from the fact that the quantum bits couple to the environment, which causes decoherence and loss of quantum information. In a topological quantum computer local perturbations would affect the path of the particle, but as long as it maintains a loop around the other particle, i.e. as long as the topology is preserved, the quantum information is preserved as well. Instead, the lifetime of quantum information is mainly determined by the sample temperature with respect to the energy gap which protects the degenerate ground state [118, 119].

Besides the mentioned odd-denominator FQHS, other phases of matter are predicted to host non-abelian particles, such as cold atoms [122], p-wave superconductors [118] as well as hybrid systems made out of superconductors in combination with topological insulators [123] or semiconductors [124, 125]. Among these, the most prominently studied candidate for fault-tolerant quantum computation is probably the $5/2$ state. In the following, we briefly summarize what is known about the potentially non-abelian nature of the $5/2$ state.

Is the $5/2$ State Indeed Home to Non-abelian Anyons?

This question would be answered most convincingly by braiding of $\nu = 5/2$ quasiparticles and thereby testing the underlying exchange statistics. Several proposals for interference experiments at the $5/2$ state have been made [126–130], and also first steps towards their realization were undertaken [131, 132]. Even weak signatures of non-abelian anyons were already reported in interferometer structures at $\nu = 5/2$ [133, 134]. The results are, however, difficult to interpret and discussed rather controversially. Further experiments will be necessary to directly prove the non-abelian nature of the $5/2$ quasiparticles. Unfortunately, efforts in this direction are hampered by the high fragility of the $5/2$ state. The fabrication of gates on top

of the sample, a basic requirement to control the interferometer structure, can easily destroy the $\nu = 5/2$ state. A different way to probe the potentially non-abelian properties might be thermopower [135] or entropy measurements [136]. More precisely, it has been predicted that the thermopower at $\nu = 5/2$ is temperature independent and changes linearly when going away from the exact filling factor if non-abelian anyons are present. Recently, first experiments in this direction have been performed [137]. The results were tentatively claimed to be consistent with the predicted behavior. Even though being still in its infancy, this technique might well prove fruitful in the exploration of the $\nu = 5/2$ state.

In view of the hurdles which need to be taken to directly measure the non-abelian properties, research in the past has focused stronger on the other predictions of the Moore-Read theory. In particular, the verification of the $e/4$ fractional charge and the full spin polarization would provide strong support for the wavefunction proposed by Moore and Read (sometimes called Pfaffian wavefunction).

The question for the charge of the $\nu = 5/2$ quasiparticles has been addressed in a number of experiments by different means: tunneling through a quantum point contact [132], noise measurements [138], Aharonov-Bohm interference [133] and experiments using a single-electron transistor [139]. The obtained results are consistently in agreement with a fractional charge of $e/4$. However, establishing the charge $e/4$ does not allow to discriminate between the Pfaffian state and other proposed wavefunctions such as the anti-Pfaffian and the so-called 331 state. The anti-Pfaffian state is similar to the Pfaffian and is also expected to give rise to non-abelian excitations but in addition is particle-hole symmetric, a property lacked by the Pfaffian state [140, 141]. The 331 state was originally proposed by Halperin as a description of a two-component FQHS [51]. It is unpolarized and obeys abelian statistics [142].

Information on the spin polarization of the $\nu = 5/2$ state would shed further light on its true nature. In view of the verified $e/4$ fractional charge, a polarized state would indirectly provide strong support for the non-abelian character of the $\nu = 5/2$ quasiparticles. First experiments on the spin polarization examined the behavior of the $\nu = 5/2$ state when tilting the sample with respect to the magnetic field [143]. This selectively enhances the Zeeman energy relative to the Coulomb energy since the latter only depends on the perpendicular magnetic field component. The disappearance of the $\nu = 5/2$ state under tilt was therefore taken as an indication for an unpolarized state. In later years, the vanishing of the $\nu = 5/2$ state was attributed rather to the emergence of an anisotropic stripe phase under the influence of the in-plane magnetic field [98, 99, 112, 144, 145]. Pan et al. approached the spin polarization of the $\nu = 5/2$ state in a different way [146]. They determined the energy gap at $\nu = 5/2$ over a wide density range and compared its evolution with the behavior of the FQHS at $\nu = 8/5$, a state known to be unpolarized at low electron densities. From the observation that the $\nu = 8/5$ state undergoes a spin transition whereas the $\nu = 5/2$ state remains mostly unaffected, the authors concluded a spin-polarized state at $\nu = 5/2$. In contrast, a more recent study of the $\nu = 5/2$ energy gap at various perpendicular and in-plane magnetic fields came to the conclusion that their data is more consistent with a spin-unpolarized state [147]. Besides transport and tilted-field experiments, also other techniques have been used to assess the spin polarization at $\nu = 5/2$. Rhone et al. probed the $\nu = 5/2$ state by

inelastic light scattering and ruled out a fully polarized state from the absence of a long-wavelength spin wave mode [148]. However, shortly after, such a spin mode was found by Wurstbauer et al. [149]. Apart from that, Stern et al. claimed the observation of an unpolarized $5/2$ state using photoluminescence spectroscopy [150]. Yet, the difficulty behind optical experiments is that the excitation of electron-hole pairs might disturb the electron system and may impair the proper assignment of the filling factor. A different access to the spin polarization provides nuclear magnetic resonance spectroscopy. It uses a characteristic shift of the nuclear resonance frequency to detect the presence of a spin-polarized electron system. This technique has been employed in two similar studies, both of which found a fully spin-polarized state [151, 152].

After all, contradictory conclusions from different experiments keep the issue of the spin polarization at $\nu = 5/2$ open [116]. Since the discrepancies between the different experiments might arise from the fragility of the $5/2$ state, it is required to repeat the measurement of the spin polarization on samples with a higher quality showing a well-developed $5/2$ state. This task will be addressed in Chap. 4.

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