

Preface to the Second Edition

Producing a new edition has given me the chance to discuss some of the many interesting results that have been proven since 2007. New sections have been added to later chapters of the book describing these more recent advances in our understanding of resonance distribution and spectral asymptotics for hyperbolic surfaces.

In the last few years, we have also developed new techniques for the numerical computation of resonances. A new final chapter has been added describing these methods. The numerical computations are used to explore various conjectures related to resonance distribution.

While I have tried to incorporate as many new results as possible, the additions have been limited by the existing scope of the book. For example, extensions of results that were already known for hyperbolic surfaces and more general manifolds in higher dimensions have not been included. I have tried to update the notes at the end of each chapter to mention these developments. (I apologize in advance for any errors or omissions in these notes.)

One of the most promising new developments not covered is an alternative approach to meromorphic continuation of the resolvent for asymptotically hyperbolic manifolds developed by Vasy [270, 271]. This new method is particularly well suited to semiclassical (high-frequency) analysis and has already inspired some important new results. A full expository treatment will appear in the forthcoming book of Dyatlov-Zworski [74, Ch. 5].

The new edition has also provided an opportunity to improve the organization in certain parts of the text. The most prominent example of this is a change in context for the central part of the book. For the first edition, I limited the main text to exact hyperbolic quotients exclusively, in order to keep the presentation as simple as possible. With the benefit of hindsight, it made sense to adopt the broader context of surfaces with hyperbolic ends for certain chapters, allowing the results in those sections to be stated in a stronger form.

I am extremely grateful to Catherine Crompton, Pascal Philipp, and Anke Pohl for providing lists of errata that needed to be fixed from the first edition. I would also like to thank Pierre Albin, Kiril Datchev, Semoy Dyatlov, Tanya Christiansen,

Frédéric Faure, Colin Guillarmou, Peter Hislop, Dmitry Jakobson, Frédéric Naud, Peter Perry, Tobias Weich, and Maciej Zworski for helping me to keep abreast of the new developments in this area of research. My thanks also go to Chris Tominich of Birkhäuser for encouraging the development of a second edition.

Atlanta, GA, USA
February 2016

David Borthwick

Preface to the First Edition

I first encountered the spectral theory of hyperbolic surfaces as an undergraduate physics student, through the intriguing expository article of Balazs-Voros [13] on relations between the Selberg theory of automorphic forms and quantum chaos. At the time, I was quite impressed at the range of topics represented, including quantum physics, discrete groups, differential geometry, number theory, complex analysis, and spectral theory. In my previous experience, these were completely separate realms, but here they were all mixed together in the same setting.

Twenty years later, these topics do not seem so far apart to me. However, I am no less amazed by the rich cross-fertilization of ideas in this subject area. The primary motivation for this book is the conviction that this sort of mathematics that bridges the divides between fields ought to be made accessible to as broad an audience as possible—to graduate students especially, for whom regular coursework often exaggerates the impression of boundaries between disciplines.

The spectral theory of compact and finite-area Riemann surfaces is a classical subject with a history going back to the pioneering work of Atle Selberg, who brought techniques from spectral theory and harmonic analysis into the study of automorphic forms. These cases have been thoroughly covered in various expository sources. In particular, Buser [51] develops the spectral theory for compact Riemann surfaces with a concrete approach based on hyperbolic geometry and cutting and pasting. Most treatments of the finite-area case, for example, Venkov [272], emphasize arithmetic surfaces and connections to number theory.

For infinite-area hyperbolic surfaces, a good understanding of the spectral theory has emerged only recently. The assumption of infinite area changes the character of the theory. The resolvent of the Laplacian takes on a predominant role, and the emphasis shifts from discrete eigenvalues to scattering theory and resonances. It has only been through dramatic advances in geometric scattering theory that the full development of the infinite-area theory has become possible.

My goal in this book is to present a relatively self-contained account of this recent development. Although many of the results could be stated in greater generality (e.g., higher dimensions), the book is restricted to the hyperbolic surface context for

the sake of accessibility. The notes at the end of each chapter include references to more general results.

The book assumes basic algebra and topology, at the level of a first graduate course. An undergraduate course on curves and surfaces should provide sufficient background in differential geometry. Because spectral theory is the primary topic, the analysis requirements are necessarily somewhat steeper. Beyond the basic real and complex analysis, a student would need basic functional analysis and some introduction to the analysis of linear partial differential equations. The appendix, while not a self-sufficient introduction to these topics, is meant to serve as a guide to readers who need more background information.

I would like to thank my collaborators, Chris Judge and Peter Perry, with whom I learned much of this material, and Edward Taylor, who introduced me to scattering theory on hyperbolic manifolds. Thanks also to Arthur Wightman, who supervised the undergraduate project where I first learned about the Selberg trace formula, and to Richard Melrose and Rafe Mazzeo, for encouragement when I first undertook to learn some scattering theory. I am very grateful to Colin Guillarmou and Peter Hislop for reading parts of the manuscript and offering corrections and suggestions.

Atlanta, GA, USA
February 2007

David Borthwick

Spectral Theory of Infinite-Area Hyperbolic Surfaces

Borthwick, D.

2016, XIII, 463 p. 64 illus., 37 illus. in color., Hardcover

ISBN: 978-3-319-33875-0

A product of Birkhäuser Basel