Preface

The concept of entropy is a powerful tool not only in physics and chemistry but also in mathematics. Entropies allow for the development of new techniques to analyze partial differential equations (PDEs). This book is a collection and summary of some entropy methods for diffusive PDEs devised by many researchers in recent decades. These methods enable the understanding of the qualitative behavior of solutions to diffusive equations (and Markov diffusion processes). Applications are the large-time asymptotics of solutions, the derivation of convex Sobolev inequalities, the existence and uniqueness of weak solutions, and the analysis of discrete and geometric structures of the PDEs.

The purpose of this book is to give an introduction to selected entropy methods which can be found in the literature. In order to highlight the ideas, the results are not stated in the widest generality and most of the arguments are only formal (in the sense that the functional setting is not specified or sufficient regularity is supposed). My hope is that in this way the text will be accessible for advanced master's and PhD students and may be useful in special courses and seminars.

The book consists of five chapters. Chapter 1 gives a summary of concepts of entropy in physics and mathematics and introduces some ideas and definitions. Entropy methods for Fokker–Planck equations are presented in Chap. 2. This is a huge topic, investigated by many mathematicians, and I focus on some aspects only. In particular, the approach of Bakry and Emery is reviewed, both from the original stochastic viewpoint and from the PDE viewpoint formulated by Arnold, Markowich, Toscani, and Unterreiter. Furthermore, extensions to nonlinear equations are given, based on the works by Otto, Carrillo and Toscani, and Del Pino and Dolbeault. Many aspects are left out, such as connections to hypocoercivity, displacement convexity, Ricci curvature, Wasserstein gradient flows, and large deviation principles. More complete and rigorous expositions can be found, for instance, in the monographs of Villani and Bakry, Gentil, and Ledoux.

Chapter 3 is concerned with systematic integration by parts, which was elaborated by Matthes, Bukal, me, and others. Systematic integration by parts is a tool to perform some computations needed in entropy methods in an efficient, computer-aided way, in particular for highly nonlinear higher order equations.

Cross-diffusion systems are analyzed in Chap. 4. Entropy methods are here related to principles of thermodynamics and help to prove the global existence of solutions as well as, in some cases, their boundedness. These techniques were developed by Chen and me and extended by Burger, Di Francesco, Pietschmann, Schlake, and others. The proofs of the existence theorems are rather technical since approximation schemes as well as compactness and weak convergence arguments are needed. Nonstandard auxiliary results are recalled in the appendix.

Chapter 5 deals with discrete entropy methods, motivated by the aim of preserving the entropy structure of diffusive equations on the numerical level. Since this field is under development, I only sketch some approaches taken from the literature: the Bakry–Emery approach for Markov chains (recently investigated by Caputo, Dai Pra, and Posta; Chow, Huang, and Zhou; Mielke; Fathi and Maas; and others), the connection to finite-volume approximations of Fokker–Planck equations, and entropy-dissipating time-discrete schemes.

First versions of this text were written for summer schools in Vienna (Austria) in 2007, Sendai (Japan) in 2008, Bielefeld (Germany) in 2012, Kacov (Czech Republic), L'Aquila (Italy), and Krakow (Poland) in 2015. Some material in this book is taken from various sources, notably the lecture notes of Matthes and Evans, both on entropy methods, and papers by Arnold, Bothe, Burger, Gajewski, Mielke, Villani, and others. Precise references are given in the corresponding sections. In view of the limited space, the references are far from exhaustive. I do not claim completeness and apologize for possible omissions.

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