

Chapter 2

Control of the Functioning of Doubly-Fed Induction Generators

2.1 Outline

The chapter analyzes nonlinear control for Doubly-fed induction generators, through different approaches: (i) flatness-based control in cascading loops, (ii) flatness-based control through transformation to the canonical form, (iii) differential geometry-based control, (iv) nonlinear H-infinity control, (v) flatness-based adaptive control (vi) field orientation and backstepping control.

First, the chapter proposes a solution to the problem of sensorless control of doubly-fed induction generators, which is implemented in successive loops. Using the stator and rotor electrical equations the complete sixth order model of the doubly-fed induction generator is derived. It is proven that the doubly-fed induction generator model is differentially flat since all its state variables and control inputs can be written as functions of the flat output and of the associated derivatives. The differential flatness property shows that the design of a DFIG controller is possible using feed-forward control terms which are complemented by suitable error feedback terms. Next, sensorless control for the doubly-fed induction generator is implemented. The Extended Kalman Filter is used to estimate the non-measurable elements of the DFIG state vector such as the rotation speed and the magnetic flux through the processing of measurements of the rotor's angle and of the rotor currents. The efficiency of the proposed state estimation-based nonlinear control scheme is evaluated through simulation experiments.

Next, the chapter studies differential flatness properties and an input-output linearization procedure for doubly-fed induction generators (DFIGs). By defining flat outputs which are associated with the rotor's turn angle and the magnetic flux of the stator, an equivalent DFIG description in the Brunovsky (canonical) form is obtained. For the linearized canonical model of the generator a feedback controller is designed. Moreover, a comparison of the differential flatness theory-based control method against Lie algebra-based control is provided. At a second stage, a novel Kalman Filtering method (Derivative-free nonlinear Kalman Filtering) is introduced. The proposed Kalman Filter is redesigned as disturbance observer for estimating

additive input disturbances to the DFIG model. These estimated disturbance terms are finally used by a feedback controller that enables the generator's state variables to track desirable setpoints. The efficiency of the proposed state estimation-based control scheme is tested through simulation experiments.

Additionally, the chapter introduces a nonlinear H-infinity control method for feedback control of asynchronous electrical machines, with application example the control problem of doubly-fed induction generators (DFIGs). The control method consists of a repetitive solution of an H-infinity control problem for the DFIG, that makes use of a locally linearized model of the generator and takes place at each iteration of the control algorithm. The asynchronous generator's model is locally linearized round its current operating point through the computation of the associated Jacobian matrices. Using the linearized model of the generator an H-infinity feedback control law is computed. The known robustness features of H-infinity control enable to compensate for the errors of the approximate linearization, as well as to eliminate the effects of external perturbations. The efficiency of the proposed control scheme is shown analytically and is confirmed through simulation experiments.

Finally, the chapter proposes an adaptive control approach that is capable of compensating for model uncertainty and parametric changes of the DFIG, as well as for lack of measurements for the DFIG's state vector elements. First it is proven that the DFIG's model is a differentially flat one. This means that all its state variables and its control inputs can be written as differential functions of key state variables which are the so-called flat outputs. Moreover, this implies that the flat output and its derivatives are not connected to each other through a relation of the form of a differential equation. By exploiting differential flatness properties it is shown that the 6-th order DFIG model can be transformed into the linear canonical form. For the latter description, the new control inputs comprise unknown nonlinear functions which can be identified with the use of neurofuzzy approximators. The estimated dynamics of the generator is used by a feedback controller thus establishing an indirect adaptive control scheme. Moreover, to robustify the control loop a supplementary control term is computed using H-infinity control theory. Another problem that has to be dealt with comes from the inability to measure the complete state vector of the generator. Thus, a state-observer is implemented in the control loop. The stability of the considered observer-based adaptive control approach is proven using Lyapunov analysis. Moreover, the performance of the control scheme is evaluated through simulation experiments.

2.2 Flatness-Based Control of the DFIG in Successive Loops

2.2.1 Overview

As noted in Chap. 1, doubly-fed induction generators (DFIG) have been widely used in variable-speed fixed frequency hydro-power generation systems, wind-power

generation systems and turbine engine power generation systems. Doubly-fed induction generators have proven to be more efficient than squirrel-cage induction generator systems (SCIG) and the synchronous generator systems in terms of cost and losses of the associated power electronics converters. DFIG systems can operate either in grid-connected mode or in stand-alone mode [56, 64, 114, 140, 141, 148, 295, 345]. Moreover, several field-oriented control schemes have been proposed for both operation modes. Additionally, several results have been published on sensorless control of DFIG [8, 11, 62, 69, 143, 473, 486]. Taking into account that the installation and maintenance of sensors for measuring several parameters of the generator's state vector can be technically difficult or costly, the need for developing sensorless control schemes for DFIG becomes apparent. In this section a novel sensorless control scheme is developed using flatness-based control theory and a state estimation method that is based on Extended Kalman Filtering.

Using the electric equations of the stator and rotor a dynamic model for the doubly fed induction generator is derived. The doubly-fed induction generator is analogous to the induction motor. In an induction motor the stator voltage plays the role of an input variable, while the rotor voltage is a constant. In case of the doubly-fed induction machine it is quite similar but the other way round, with a dual analogy to hold between the stator and rotor parameters of the generator and the motor. This means that the rotor voltage now acts as an input, while the stator voltage is a constant parameter. The stator's and rotor's voltages, currents and magnetic flux are represented as vectors in a rotating orthogonal axis frame. The complete sixth order model of the DFIG captures efficiently transients at both the stator and the rotor side.

In this section, it is shown that the doubly-fed induction generator model is differentially flat. The theory of differential flatness has significant advantages over other approaches such as backstepping control [280, 137, 388, 369, 456]. In the DFIG case, the proposed flat output is the pair consisting of the rotor's angle θ and of the orientation of the magnetic flux ρ . It is shown that the doubly-fed induction generator model is differentially flat since all its state variables and control inputs can be written as functions of the flat output and of the associated derivatives. The differential flatness property shows that the design of a DFIG controller is possible using feed-forward control terms which are complemented by suitable error feedback terms. The design of the DFIG controller consists of two stages: (i) in the outer-loop the controller enables convergence of the stator's magnetic flux and of the rotor's angular velocity to the associated reference setpoint. The control input consists of the rotor currents expressed in the rotating reference frame (ii) in the inner-loop the controller enables convergence of the rotor's dq currents to the associated reference setpoints, where these setpoints are the control inputs used by the outer control loop. The control inputs for the inner loop are now the rotor voltages also expressed in the rotating reference frames.

Next, sensorless control for the doubly-fed induction generator is implemented. A main difficulty in the machine's control is that some of its state vector elements are not directly measured and their monitoring requires the installation of specific sensors that can be costly, or difficult to install and maintain. The Extended Kalman Filter is used to estimate the non-measurable elements of the DFIG state vector such

as the rotation speed and the magnetic flux through the processing of measurements of the rotor's angle and of the rotor's currents. The Kalman Filter is a special kind of observer that provides optimal filtering of process and measurement noises if the covariances of these noises are known. For linear systems subjected to Gaussian measurement or process noise the Kalman Filter is the optimal state estimator, since it results in minimization of the trace of the estimation error's covariance matrix [45, 209, 341]. For nonlinear systems, subjected to Gaussian noise one can use the generalization of the Kalman Filter as formulated in terms of the Extended Kalman Filter (EKF). The Extended Kalman Filter is based on a linearization of the systems' dynamics using a first order Taylor expansion [334, 358]. In this section it is shown that with the use of nonlinear Kalman Filtering it is possible to reduce the number of sensors involved in the control loops of the doubly-fed induction generator and to implement efficient state estimation-based control.

2.2.2 Field Orientation for Induction Machines

The classical method for induction machines control was introduced by Blascke (1971) and in the DFIG case is based on a transformation of the rotor's currents ($i_{r\alpha}$) and (i_{rb}) and of the magnetic fluxes of the stator ($\psi_{s\alpha}$ and ψ_{sb}) to the reference frame $d - q$ which rotates together with the rotor. Thus the controller's design uses the currents i_{rd} and i_{rq} and the fluxes ψ_{sd} and ψ_{sq} [353]. The angle of the vectors that describe the magnetic fluxes $\psi_{s\alpha}$ and ψ_{sb} is first defined, i.e.

$$\rho = \tan^{-1}\left(\frac{\psi_{sb}}{\psi_{s\alpha}}\right) \quad (2.1)$$

The angle between the inertial reference frame of the stator and the rotating reference frame of the rotor is taken to be equal to ρ . The transition from ($i_{r\alpha}$, i_{rb}) to (i_{rd} , i_{rq}) is given by

$$\begin{pmatrix} i_{rd} \\ i_{rq} \end{pmatrix} = \begin{pmatrix} \cos(\rho) & \sin(\rho) \\ -\sin(\rho) & \cos(\rho) \end{pmatrix} \begin{pmatrix} i_{r\alpha} \\ i_{rb} \end{pmatrix} \quad (2.2)$$

The transition from ($\psi_{s\alpha}$, ψ_{sb}) to (ψ_{sd} , ψ_{sq}) is given by

$$\begin{pmatrix} \psi_{sd} \\ \psi_{sq} \end{pmatrix} = \begin{pmatrix} \cos(\rho) & \sin(\rho) \\ -\sin(\rho) & \cos(\rho) \end{pmatrix} \begin{pmatrix} \psi_{s\alpha} \\ \psi_{sb} \end{pmatrix} \quad (2.3)$$

Moreover, it holds that $\cos(\rho) = \frac{\psi_{s\alpha}}{\|\psi\|}$, $\sin(\rho) = \frac{\psi_{sb}}{\|\psi\|}$, and $\|\psi\| = \sqrt{\psi_{s\alpha}^2 + \psi_{sb}^2}$. Using the above transformation ones obtains

$$\begin{aligned} i_{rd} &= \frac{\psi_{s\alpha} i_{r\alpha} + \psi_{sb} i_{rb}}{\|\psi\|} & \psi_{sd} &= \|\psi\| \\ i_{rq} &= \frac{\psi_{s\alpha} i_{rb} - \psi_{sb} i_{r\alpha}}{\|\psi\|} & \psi_{sq} &= 0 \end{aligned} \quad (2.4)$$

Therefore, in the rotating frame $d-q$ of the generator there will be only one non-zero component of the magnetic flux ψ_{sd} , while the component of the flux along the d axis equals 0. The new inputs of the system are considered to be v_{rd}, v_{rq} , which are connected to v_{ra}, v_{rb} according to the relation

$$\begin{pmatrix} v_{r\alpha} \\ v_{rb} \end{pmatrix} = \|\psi\| \cdot \begin{pmatrix} \psi_{sa} & \psi_{sb} \\ \psi_{sb} & \psi_{sa} \end{pmatrix}^{-1} \begin{pmatrix} v_{rd} \\ v_{rq} \end{pmatrix} \quad (2.5)$$

In the new coordinates, the induction generator model has been described in Eqs. (1.20)–(1.25). The state-space model of the induction generator has been defined in Eqs. (1.26) and (1.27). Using the state variables notation the DFIG model was written in the form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{K_m}{J} x_2 - \frac{T_m}{J} + \frac{\eta}{J} (x_6 x_3 - x_5 x_4) \\ \dot{x}_3 &= -\frac{1}{\tau_s} x_3 + \omega_{dq} x_4 + \frac{M}{\tau_s} x_5 + v_{sd} \\ \dot{x}_4 &= -\omega_{dq} x_3 - \frac{1}{\tau_s} x_4 + \frac{M}{\tau_s} x_6 + v_{sq} \\ \dot{x}_5 &= -\beta x_2 x_4 + \frac{\beta}{\tau_s} x_3 + (\omega_{dq} - x_2) x_6 - \gamma_2 x_5 + \frac{1}{\sigma L_r} v_{rd} - \beta v_{sd} \\ \dot{x}_6 &= -\frac{\beta}{\tau_s} x_4 + \beta x_2 x_3 - (\omega_{dq} - x_2) x_5 - \gamma_2 x_6 + \frac{1}{\sigma L_r} v_{rq} - \beta v_{sq} \end{aligned} \quad (2.6)$$

Next, the following nonlinear feedback control law is defined

$$\begin{pmatrix} v_{rd} \\ v_{rq} \end{pmatrix} = \sigma L_r \begin{pmatrix} \beta x_2 x_4 - \frac{\beta}{\tau_s} x_3 - (\omega_{dq} - x_2) x_6 + \beta v_{sd} + \beta v_1 \\ \frac{\beta}{\tau_s} x_4 - \beta x_2 x_3 + (\omega_{dq} - x_2) x_5 + \beta v_{sq} + \beta v_2 \end{pmatrix} \quad (2.7)$$

The terms in Eq. (2.7) have been selected so as to linearize the fifth and sixth row of the state space model of the induction generator in Eq. (2.6) and to produce first-order linear differential equations. The control signal in the inertial coordinates system $a-b$ is

$$\begin{pmatrix} v_{r\alpha} \\ v_{rb} \end{pmatrix} = \|\psi\| \sigma L_r \begin{pmatrix} \psi_{sa} & \psi_{sb} \\ -\psi_{sb} & \psi_{sa} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \beta x_2 x_4 - \frac{\beta}{\tau_s} x_3 - (\omega_{dq} - x_2) x_6 + \beta v_{sd} + \beta v_1 \\ \frac{\beta}{\tau_s} x_4 - \beta x_2 x_3 + (\omega_{dq} - x_2) x_5 + \beta v_{sq} + \beta v_2 \end{pmatrix} \quad (2.8)$$

Substituting Eq. (2.7) into Eq. (2.6) one obtains:

$$\dot{x}_1 = x_2 \quad (2.9)$$

$$\dot{x}_2 = -\frac{K_m}{J} x_2 - \frac{T_m}{J} + \frac{\eta}{J} (x_6 x_3 - x_5 x_4) \quad (2.10)$$

$$\dot{x}_3 = -\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{sd} \quad (2.11)$$

$$\dot{x}_4 = -\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{sq} \quad (2.12)$$

$$\dot{x}_5 = -\gamma_2x_5 + \beta v_1 \quad (2.13)$$

$$\dot{x}_6 = -\gamma_2x_6 + \beta v_2 \quad (2.14)$$

The system of Eqs. (2.9)–(2.14) comprises two linear subsystems, where the first one has as output the magnetic flux $x_3 = \psi_{sd}$ and the second has as output the rotation speed $x_2 = \omega$ [353].

Thus, from Eqs. (2.11) and (2.13) one obtains

$$\dot{x}_3 = -\frac{1}{\tau_s}x_3 + \frac{M}{\tau_s}x_5 + v_{sd} \quad (2.15)$$

$$\dot{x}_5 = -\gamma_2x_5 + \beta v_1 \quad (2.16)$$

while from Eqs. (2.10) and (2.14) one obtains

$$\dot{x}_2 = -\frac{K_m}{J}x_2 - \frac{T_m}{J} + \frac{\eta}{J}x_3x_6 \quad (2.17)$$

$$\dot{x}_6 = -\gamma_2x_6 + \beta v_2 \quad (2.18)$$

For $x_3 = \psi_{sd}$, it holds that if $\psi_{sd} \rightarrow \psi_{sd}^{\text{ref}}$, i.e. the transient phenomena for ψ_{sd} have been eliminated and therefore ψ_{sd} has converged to a steady state value, then the two subsystems described by Eqs. (2.15)–(2.16) and Eqs. (2.17)–(2.18) are decoupled.

The subsystem that is described by Eqs. (2.15) and (2.16) is linear with control input v_1 , and can be controlled using methods of linear control, such as optimal control, or PID control. For instance the following PI controller has been proposed for the control of the magnetic flux [353]

$$v_1(t) = -k_{d1}(\psi_{sd} - \psi_{sd}^{\text{ref}}) - k_{d2} \int_0^t (\psi_{sd}(\tau) - \psi_{sd}^{\text{ref}}(\tau)) d\tau \quad (2.19)$$

Thus, if Eq. (2.19) is applied to the subsystem that is described by Eqs. (2.15) and (2.16), one anticipates to succeed $\psi_{sd}(t) \rightarrow \psi_{sd}^{\text{ref}}(t)$.

Now, the subsystem that consists of Eqs. (2.17) and (2.18) is examined. The term $T = \frac{\eta}{J}x_6x_3$ denotes the torque developed in the rotor. After achieving $\psi_{sd} \rightarrow \psi_{sd}^{\text{ref}}$, one can also control the generator's speed ω , using linear feedback control algorithms. A first approach to the control of the speed ω is to use nested PI loops, i.e.

$$\begin{aligned} v_2 &= -K_{q1}(T - T_{\text{ref}}) - K_{q2} \int_0^t (T(\tau) - T_{\text{ref}}(\tau)) d\tau \\ T_{\text{ref}} &= -K_{q3}(\omega - \omega_{\text{ref}}) - K_{q4} \int_0^t (\omega(\tau) - \omega_{\text{ref}}(\tau)) d\tau \end{aligned} \quad (2.20)$$

From the above analysis it becomes clear that a remaining problem in the implementation of field-oriented control for DFIGs is how to measure efficiently $x_3 = \psi_{sd}(t)$. Therefore one comes against the need for applying a state observer or Kalman Filtering. Besides, the tuning of the multiple PID and PI controllers that constitute the field-oriented control scheme, as described in Eqs. (2.19) and (2.20), remains valid only round local operating points and thus the stability and robustness of the field-oriented control for DFIGs cannot be assured.

2.2.3 Differential Flatness of the Doubly-Fed Induction Generator

2.2.3.1 Properties of Differentially Flat Systems

Having pointed out the shortfalls of field oriented control for DFIGs, the need for developing nonlinear control schemes of proven stability for such electrical machines becomes apparent. The main principles of flatness-based control are as follows [137, 280, 388, 456]. Flatness-based control can be applied to finite dimensional system of the form

$$S_i(w, \dot{w}, \ddot{w}, \dots, w^{(i)}), \quad i = 1, 2, \dots, q \quad (2.21)$$

The term w denotes the system variable while $w^{(i)}, i = 1, 2, \dots, q$ are its derivatives (these can be for instance the elements of the system's state vector). The system of Eq. (2.21) is said to be differentially flat if there exists a set of m functions $y = (y_1, \dots, y_m)$ of the system variables and of their time-derivatives, i.e.

$$y_i = \phi(w, \dot{w}, \ddot{w}, \dots, w^{(\alpha_i)}), \quad i = 1, \dots, m \quad (2.22)$$

such that the following two conditions are satisfied [137–456]:

1. There does not exist any differential relation of the form

$$R(y, \dot{y}, \dots, y^{(\beta)}) = 0 \quad (2.23)$$

which implies that the derivatives of the flat output are not coupled in the sense of an ordinary differential equation (ODE) denoted by function R , or equivalently it can be said that the flat output is differentially independent.

2. All system variables, i.e. the components of w (elements of the system's state vectors) can be expressed using only the flat output y and its time derivatives

$$w_i = \psi_i(y, \dot{y}, \dots, y^{(\gamma_i)}), \quad i = 1, \dots, s \quad (2.24)$$

An equivalent definition of differentially flat systems is as follows:

Definition: The system $\dot{x} = f(x, u)$, $x \in R^n$, $u \in R^m$ is differentially flat if there exist relations

$$\begin{aligned} h : R^n \times (R^m)^{r+1} &\rightarrow R^m \\ \phi : (R^m)^r &\rightarrow R^n \\ \psi : (R^m)^{r+1} &\rightarrow R^m \end{aligned} \quad (2.25)$$

such that

$$\begin{aligned} y &= h(x, u, \dot{u}, \dots, u^{(r)}) \\ x &= \phi(y, \dot{y}, \dots, y^{(r-1)}, y^{(r-1)}), \text{ and} \\ u &= \psi(y, \dot{y}, \dots, y^{(r-1)}, y^{(r)}) \end{aligned} \quad (2.26)$$

This means that all system dynamics can be expressed as a function of the flat output and its derivatives, therefore the state vector and the control input can be written as $x(t) = \phi(y(t), \dot{y}(t), \dots, y^{(r)}(t))$ and $u(t) = \psi(y(t), \dot{y}(t), \dots, y^{(r+1)}(t))$.

2.2.3.2 Classes of Differentially Flat Systems

Main classes of nonlinear differentially flat systems are [281]:

1. Affine in-the-input systems: The dynamics of such systems is given by:

$$\dot{x} = f(x) + \sum_{i=1}^m g_i(x) u_i \quad (2.27)$$

From Eq. (2.27) one can notice that the above state equation can also describe MIMO dynamical systems. Without out loss of generality it is assumed that $G = [g_1, \dots, g_m]$ is of rank m . In case that the flat outputs of the aforementioned system are only functions of states x , then this class of dynamical systems is called 0-flat. It has been proven that a dynamical affine system with n states and $n - 1$ inputs is 0-flat if it is controllable.

2. Driftless systems: These are systems of the form

$$\dot{x} = \sum_{i=1}^m f_i(x) u_i \quad (2.28)$$

For driftless systems with two inputs, i.e.

$$\dot{x} = f_1(x) u_1 + f_2(x) u_2 \quad (2.29)$$

flatness holds, if and only if the rank of matrix $E_{k+1} := \{E_k, [E_k, E_k]\}$, $k \geq 0$ (with $E_0 := \{f_1, f_2\}$ and $[E_k, E_k] := \{[X, Y], X, Y \in E_k\}$ denoting a Lie bracket) is equal to $k + 2$ for $k = 0, \dots, n - 2$. It has been proven that a driftless system that is differentially flat, is also 0-flat (which means that the flat output is a function of only the elements of the state vector of the initial nonlinear system).

Moreover, for flat systems with n states and $n - 2$ control inputs, i.e.

$$\dot{x} = \sum_{i=1}^{n-2} u_i f_i(x) \quad x \in \mathbb{R}^n \quad (2.30)$$

the flatness property holds, if controllability also holds. Furthermore, the system is 0-flat if n is even.

2.2.3.3 Conditions for Applying Differential Flatness Theory

Control based on differential flatness theory can be applied to systems that admit static feedback linearization. The generic class of nonlinear systems $\dot{x} = f(x, u)$ is considered. Such a system can be transformed to the form of an affine in the input system by adding an integrator to each input [57]

$$\dot{x} = f(x) + \sum_{i=1}^m g_i(x) u_i \quad (2.31)$$

If the system of Eq. (2.31) can be linearized by a diffeomorphism $z = \phi(x)$ and a static state feedback $u = \alpha(x) + \beta(x)v$ into the following form

$$\begin{aligned} \dot{z}_{i,j} &= z_{i+1,j} \text{ for } 1 \leq j \leq m \text{ and } 1 \leq i \leq v_j - 1 \\ \dot{z}_{v_i,j} &= v_j \end{aligned} \quad (2.32)$$

with $\sum_{j=1}^m v_j = n$, then $y_j = z_{1,j}$ for $1 \leq j \leq m$ are the 0-flat outputs which can be written as functions of only the elements of the state vector x . To define conditions for transforming the system of Eq. (2.31) into the canonical form described in Eq. (2.32) the following theorem holds [57]

Theorem: For the nonlinear systems described by Eq. (2.31) the following variables are defined: (i) $G_0 = \text{span}[g_1, \dots, g_m]$, (ii) $G_1 = \text{span}[g_1, \dots, g_m, \text{ad}_f g_1, \dots, \text{ad}_f g_m], \dots$ (k) $G_k = \text{span}\{\text{ad}_f^j g_i \text{ for } 0 \leq j \leq k, 1 \leq i \leq m\}$. Then, the linearization problem for the system of Eq. (2.31) can be solved if and only if: (1). The dimension of G_i , $i = 1, \dots, k$ is constant for $x \in X \subseteq \mathbb{R}^n$ and for $1 \leq i \leq n - 1$, (2). The dimension of G_{n-1} is of order n , (3). The distribution G_k is involutive for each $1 \leq k \leq n - 2$.

2.2.3.4 Differential Flatness for the DFIG Model

It will be shown that the doubly-fed induction generator model is differentially flat. First the stator's magnetic flux is written in a complex number form [280, 375]

$$\psi = \psi_{s_d} + j \psi_{s_q} = \alpha e^{j\rho} \quad (2.33)$$

where $\rho = \tan^{-1}(\frac{\psi_{sb}}{\psi_{sa}})$ denotes the orientation angle of the magnetic flux vector. According to Lorentz law the torque that is developed due to the magnetic flux is given by

$$\begin{aligned} T_e &= k_1 I_m (\psi_s^* \frac{d\psi_s}{dt}) \Rightarrow T_e = k_1 I_m (\alpha e^{-j\rho} \alpha j \dot{\rho} e^{j\rho}) \Rightarrow \\ T_e &= k_1 I_m (j \alpha^2 \dot{\rho}) \Rightarrow T_e = k_1 \alpha^2 \dot{\rho} \end{aligned} \quad (2.34)$$

The generator's equations of motion are

$$\begin{aligned} J \dot{\omega}_r &= T_m - b \dot{\omega}_r - T_e \Rightarrow \\ J \ddot{\theta} &= T_m - b \dot{\theta} - k_1 \alpha^2 \dot{\rho} \Rightarrow \alpha = \sqrt{\frac{(T_m - b \dot{\theta} - J \ddot{\theta})}{k_1 \dot{\rho}}} \end{aligned} \quad (2.35)$$

As flat output, the following pair of variables is considered $y = (\theta, \rho)$, where θ is the rotor's rotation angle and ρ is the orientation angle of the magnetic field. Thus, in the doubly-fed induction generator model the magnetic flux can be written as a function of the flat output y and of the associated derivatives. This is denoted as

$$\psi_s = f_g(y, \dot{y}, \ddot{y}) \quad (2.36)$$

Moreover, due to the field orientation it holds

$$\psi_{sd} = \text{Re}\{f_g(y, \dot{y}, \ddot{y})\}, \quad \psi_{sq} = \text{Im}\{f_g(y, \dot{y}, \ddot{y})\} = 0 \quad (2.37)$$

It is considered that for the stator feed voltages it holds that v_{sq} is constant (for a symmetrical grid), and v_{sd} is associated with the grid voltage and dependant on the rotor's turn angle [148]. Moreover due to field orientation it holds $\psi_{sq} = 0$, and from Eq. (1.3) one has

$$i_{sd} = \frac{1}{R_s} [v_{sd} + \frac{d\psi_{sd}}{dt} - \omega_{dq} \psi_{sq}] \Rightarrow i_{sd} = \frac{1}{R_s} f_{g1}(y, \dot{y}, \ddot{y}) \quad (2.38)$$

Moreover, from Eq. (1.4) one has

$$i_{sq} = \frac{1}{R_s} [v_{sq} - \frac{d\psi_{sq}}{dt} + \omega_{dq} \psi_{sd}] \Rightarrow i_{sq} = \frac{1}{R_s} f_{g2}(y, \dot{y}, \ddot{y}) \quad (2.39)$$

From Eq. (1.7) one obtains

$$i_{rd} = \frac{1}{M} [\psi_{sd} - L_s i_{sd}] \Rightarrow i_{rd} = f_{g3}(y, \dot{y}, \ddot{y}) \quad (2.40)$$

From Eq. (1.8) one obtains

$$i_{rq} = \frac{1}{M} [\psi_{sq} - L_s i_{sq}] \Rightarrow i_{rq} = f_{g4}(y, \dot{y}, \ddot{y}) \quad (2.41)$$

Additionally, from Eq. (1.9) it holds

$$\psi_{r_d} = L_s i_{r_d} + M i_{s_d} \Rightarrow \psi_{r_d} = f_{g5}(y, \dot{y}, \ddot{y}) \quad (2.42)$$

while from Eq. (1.10) one has

$$\psi_{r_q} = L_s i_{r_q} + M i_{s_q} \Rightarrow \psi_{r_q} = f_{g6}(y, \dot{y}, \ddot{y}) \quad (2.43)$$

Since variables $\psi_{r_d}, \psi_{r_q}, i_{r_d}$ and i_{r_q} have been expressed as functions of the flat output and of the associated derivatives the same can be done for the control inputs v_{r_d} and v_{r_q} .

From Eq. (1.5) and using Eqs. (2.44) and (2.42) one obtains

$$v_{r_d} = R_r i_{r_d} + \frac{d\psi_{r_d}}{dt} - \omega_{dq} \psi_{r_q} \Rightarrow v_{r_d} = f_{g7}(y, \dot{y}, \ddot{y}) \quad (2.44)$$

From Eq. (1.6) and using Eqs. (2.45) and (2.43) one has

$$v_{r_q} = R_r i_{r_q} + \frac{d\psi_{r_q}}{dt} - \omega_{dq} \psi_{r_d} \Rightarrow v_{r_q} = f_{g8}(y, \dot{y}, \ddot{y}) \quad (2.45)$$

Since all elements of the state vector $x = [\theta, \omega, \psi_{s_d}, \psi_{s_q}, i_{r_d}, i_{r_q}]^T$ and the control inputs v_{r_d} and v_{r_q} of the doubly-fed induction generator can be expressed as functions of the flat output $y = [\theta, \rho]$ and of the associated derivatives, it can be concluded that the DFIG model is differentially flat.

2.2.4 Control of the Doubly-Fed Induction Generator

The differential flatness property shows that the design of a DFIG controller is possible using feed-forward control terms which are complemented by suitable error feedback terms. The design of the controller of the doubly-fed induction generator consists of two stages: (i) in the outer-loop the controller enables convergence of the stator's magnetic flux to the associated reference setpoint. The control input consists the rotor currents on the d and q reference axes (ii) in the inner-loop the controller enables convergence of the rotor's dq currents to the associated reference setpoints, where these setpoints are the control inputs used by the outer control loop (see Fig. 2.1). The control inputs for the inner loop are now the rotor voltages v_{r_d} and v_{r_q} . For the outer control loop the stator flux dynamics is given by:

$$\frac{d\psi_{s_d}}{dt} = \omega_{dq} \psi_{s_q} - \frac{1}{\tau_s} \psi_{s_d} + \frac{M}{\tau_s} i_{r_d} + v_{s_d} \quad (2.46)$$

$$\frac{d\psi_{s_q}}{dt} = -\frac{1}{\tau_s} \psi_{s_q} - \omega_{dq} \psi_{s_d} + \frac{M}{\tau_s} i_{r_q} + v_{s_q} \quad (2.47)$$

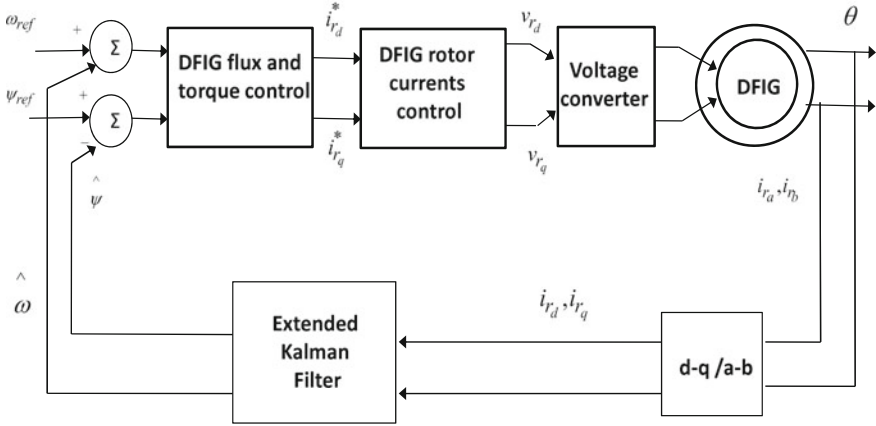


Fig. 2.1 DFIG control loop

Inner control loop of the rotor's currents:

For the inner control loop the rotor current dynamics is given by:

$$\frac{di_{rd}}{dt} = -\beta\omega_r\psi_{sq} + \frac{\beta}{\tau_s}\psi_{sd} + (\omega_{dq} - \omega_r)i_{rq} - \gamma_2i_{rd} - \beta v_{sd} + \frac{1}{\sigma L_r}v_{rd} \quad (2.48)$$

$$\frac{di_{rq}}{dt} = \frac{\beta}{\tau_s}\psi_{sq} - \beta\omega_r\psi_{sd} - \gamma_2i_{rq} - (\omega_{dq} - \omega_r)i_{rd} - \beta v_{sq} + \frac{1}{\sigma L_r}v_{rq} \quad (2.49)$$

The following control law is applied to the inner loop to make the rotor currents converge to the desirable reference setpoints

$$v_{rd} = \sigma L_r [\beta\omega_r\psi_{sq} - \frac{\beta}{\tau_s}\psi_{sd} - (\omega_{dq} - \omega_r)i_{rq} + \gamma_2i_{rd}^* + \beta v_{sd} + \frac{di_{rd}^*}{dt}] \quad (2.50)$$

$$v_{rq} = \sigma L_r [-\frac{\beta}{\tau_s}\psi_{sq} - \beta\omega_r\psi_{sd} + \gamma_2i_{rq}^* + (\omega_{dq} - \omega_r)i_{rd} + \beta v_{sq} + \frac{di_{rq}^*}{dt}] \quad (2.51)$$

The inner control loop is designed so as to succeed $i_{rq} \rightarrow i_{rq}^*$ and $i_{rd} \rightarrow i_{rd}^*$, where the setpoints i_{rq}^* and i_{rd}^* are the control inputs that make the stator's flux outer control loop converge to the flux reference setpoints ψ_{sd}^* and ψ_{sq}^* .

It is also noted that in DFIGs, the rotor-side converter is used to control the turbine output power and the voltage measured at the grid terminals. The power is controlled in order to follow a predefined power-speed characteristic, named tracking characteristic. The grid side converter is used to regulate the voltage of the dc bus capacitor. In addition, by using the grid side converter it is also possible to generate or absorb reactive power [62, 143].

Outer control loop of the stator's flux:

For the control of the magnetic flux, from Eqs. (2.46) and (2.47) the following matrix form is obtained:

$$\begin{pmatrix} \frac{d\psi_{sd}}{dt} \\ \frac{d\psi_{sq}}{dt} \end{pmatrix} = \begin{pmatrix} \omega_{dq} & -\frac{1}{\tau_s} \\ -\frac{1}{\tau_s} & -\omega_{dq} \end{pmatrix} \begin{pmatrix} \psi_{sd} \\ \psi_{sq} \end{pmatrix} + \begin{pmatrix} v_{sd} \\ v_{sq} \end{pmatrix} + \begin{pmatrix} \frac{M}{\tau_s} & 0 \\ 0 & \frac{M}{\tau_s} \end{pmatrix} \begin{pmatrix} i_{dr} \\ i_{qr} \end{pmatrix} \quad (2.52)$$

The magnetic flux reference setpoint is defined as $\psi_s^* = [\psi_{sd}^*, \psi_{sq}^*]^T$. Then the control law that assures that $\psi_s \rightarrow \psi_s^*$ is given by

$$i_{rd} = \frac{\tau_s}{M} [-v_{sd} + \frac{1}{\tau_s} \psi_{sd} - \omega_{dq} \psi_{sq} + \dot{\psi}_{sd}^* - r_1(\psi_{sd} - \psi_{sd}^*)] \quad (2.53)$$

$$i_{rq} = \frac{\tau_s}{M} [-v_{sq} + \frac{1}{\tau_s} \psi_{sq} + \omega_{dq} \psi_{sd} + \dot{\psi}_{sq}^* - r_2(\psi_{sq} - \psi_{sq}^*)] \quad (2.54)$$

To speed up convergence of the flux tracking error to zero it suffices to select appropriately coefficients r_1 and r_2 appearing in the right part of Eqs. (2.53) and (2.54).

Thus, in the outer control loop one defines the flux reference setpoint $\psi_s = [\psi_{sd}, \psi_{sq}]^T$ which can be attained by defining an appropriate control input i_{rd}, i_{rq} . The control input i_{rd}, i_{rq} becomes reference setpoint i_{rd}^*, i_{rq}^* for the inner control loop. The latter control loop assures convergence of i_{rd}, i_{rq} to i_{rd}^*, i_{rq}^* .

Finally, considering that in the dq reference frame $\psi_{sq} = 0$, the rotation speed of the rotor is controlled by an input associated to the magnetic flux

$$\psi_{sd} = \frac{J}{n_p \tau_{sq}} \left(\frac{T_m}{J} + \frac{K_m}{J} \omega_r \right) + \dot{\omega}_r^* - r_3(\omega_r - \omega_r^*) \quad (2.55)$$

The control signal ψ_{sd} computed through Eq. (2.55) becomes reference setpoint for the flux control loop shown in Fig. 2.1.

2.2.5 Flux and Rotation Speed Estimator

Using Eqs. (1.26) and (1.27) one can design a suitable state observer that enables estimation of non measurable state variables through the processing of measurements of a limited number of sensors. The rotor's angular velocity ω_r as well as state variables ψ_{sd} and ψ_{sq} appearing in the observer's innovation can be indirectly computed using Eqs. (1.7) and (1.8) and considering that the rotor's angle θ_r and the rotor's currents i_{rd} and i_{rq} can be measured.

The non-measurable elements of the DFIG state vector can be estimated from the measurements of a reduced number of sensors using the Extended Kalman Filter recursion and Eqs. (1.26) and (1.27). The DFIG model is subjected to discretization and the state model is now considered [352]:

$$\begin{aligned} x(k+1) &= \phi(x(k)) + L(k)u(k) + w(k) \\ z(k) &= \gamma(x(k)) + v(k) \end{aligned} \quad (2.56)$$

where $x \in R^{m \times 1}$ is the system's state vector, and $z \in R^{p \times 1}$ is the system's output, while $w(k)$ and $v(k)$ are uncorrelated, Gaussian zero-mean noise processes with covariance matrices $Q(k)$ and $R(k)$ respectively. The operators $\phi(x)$ and $\gamma(x)$ are $\phi(x) = [\phi_1(x), \phi_2(x), \dots, \phi_m(x)]^T$, and $\gamma(x) = [\gamma_1(x), \gamma_2(x), \dots, \gamma_p(x)]^T$, respectively. It is assumed that ϕ and γ are sufficiently smooth in x so that each one has a valid series Taylor expansion. Following a linearization procedure, ϕ is expanded into Taylor series about $\hat{x}$:

$$\phi(x(k)) = \phi(\hat{x}(k)) + J_\phi(\hat{x}(k))[x(k) - \hat{x}(k)] + \dots \quad (2.57)$$

where $J_\phi(x)$ is the $m \times m$ Jacobian of ϕ calculated at $\hat{x}(k)$:

$$J_\phi(x) = \frac{\partial \phi}{\partial x} \Big|_{x=\hat{x}(k)} = \begin{pmatrix} \frac{\partial \phi_1}{\partial x_1} & \frac{\partial \phi_1}{\partial x_2} & \dots & \frac{\partial \phi_1}{\partial x_m} \\ \frac{\partial \phi_2}{\partial x_1} & \frac{\partial \phi_2}{\partial x_2} & \dots & \frac{\partial \phi_2}{\partial x_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \phi_k}{\partial x_1} & \frac{\partial \phi_k}{\partial x_2} & \dots & \frac{\partial \phi_k}{\partial x_m} \end{pmatrix} \quad (2.58)$$

Likewise, γ is expanded about $\hat{x}^-(k)$

$$\gamma(x(k)) = \gamma(\hat{x}^-(k)) + J_\gamma[x(k) - \hat{x}^-(k)] + \dots \quad (2.59)$$

where $\hat{x}^-(k)$ is the estimation of the state vector $x(k)$ before measurement at the k -th instant to be received and $\hat{x}(k)$ is the updated estimation of the state vector after measurement at the k -th instant has been received. The $p \times m$ Jacobian $J_\gamma(x)$ is

$$J_\gamma(x) = \frac{\partial \gamma}{\partial x} \Big|_{x=\hat{x}^-(k)} = \begin{pmatrix} \frac{\partial \gamma_1}{\partial x_1} & \frac{\partial \gamma_1}{\partial x_2} & \dots & \frac{\partial \gamma_1}{\partial x_m} \\ \frac{\partial \gamma_2}{\partial x_1} & \frac{\partial \gamma_2}{\partial x_2} & \dots & \frac{\partial \gamma_2}{\partial x_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \gamma_p}{\partial x_1} & \frac{\partial \gamma_p}{\partial x_2} & \dots & \frac{\partial \gamma_p}{\partial x_m} \end{pmatrix} \quad (2.60)$$

The resulting expressions create first order approximations of ϕ and γ . Thus the linearized version of the plant is obtained:

$$\begin{aligned} x(k+1) &= \phi(\hat{x}(k)) + J_\phi(\hat{x}(k))[x(k) - \hat{x}(k)] + w(k) \\ z(k) &= \gamma(\hat{x}^-(k)) + J_\gamma(\hat{x}^-(k))[x(k) - \hat{x}^-(k)] + v(k) \end{aligned} \quad (2.61)$$

Now, the Extended Kalman Filter (EKF) recursion is as follows: First the time update is considered: by $\hat{x}(k)$ the estimation of the state vector at instant k is denoted. Given initial conditions $\hat{x}^-(0)$ and $P^-(0)$ the recursion proceeds as:

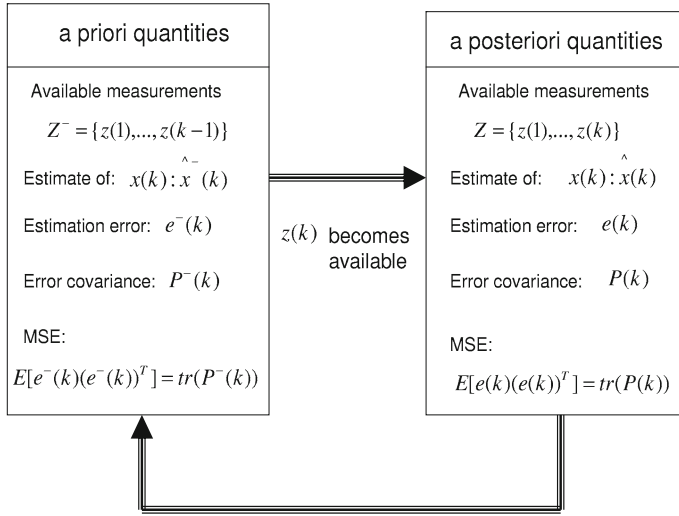


Fig. 2.2 Schematic diagram of the EKF loop

- *Measurement update.* Acquire $z(k)$ and compute:

$$\begin{aligned} K(k) &= P^-(k) J_\gamma^T(\hat{x}^-(k)) \cdot [J_\gamma(\hat{x}^-(k)) P^-(k) J_\gamma^T(\hat{x}^-(k)) + R(k)]^{-1} \\ \hat{x}(k) &= \hat{x}^-(k) + K(k) [z(k) - \gamma(\hat{x}^-(k))] \\ P(k) &= P^-(k) - K(k) J_\gamma(\hat{x}^-(k)) P^-(k) \end{aligned} \quad (2.62)$$

- *Time update.* Compute:

$$\begin{aligned} P^-(k+1) &= J_\phi(\hat{x}(k)) P(k) J_\phi^T(\hat{x}(k)) + Q(k) \\ \hat{x}^-(k+1) &= \phi(\hat{x}(k)) + L(k) u(k) \end{aligned} \quad (2.63)$$

The schematic diagram of the EKF loop is given in Fig. 2.2.

2.2.6 Implementation of the EKF for Sensorless Control of the DFIG

As analyzed, the state-space equation of the doubly-fed induction generator is given by

$$\dot{x} = f(x) + g_a v_{rd} + g_b v_{rq} \quad (2.64)$$

where the state vector has been defined as $x = [\theta, \omega, \psi_{sd}, \psi_{sq}, i_{rd}, i_{rq}]^T$ while functions $f(x)$, $g_a(x)$ and $g_b(x)$ have been defined as

$$f(x) = \begin{pmatrix} x_2 \\ -\frac{K_m}{J}x_2 - \frac{T_m}{J} + \frac{\eta}{J}(i_{s_q}x_3 - i_{s_d}x_4) \\ -\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d} \\ -\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q} \\ -\beta x_2x_4 + \frac{\beta}{\tau_s}x_3 + (\omega_{dq} - x_2)x_6 - \gamma_2x_3 \\ \frac{\beta}{\tau_s}x_4 + \beta x_2x_3 + (\omega_{dq} - x_2)x_5 - \gamma_2x_6 \end{pmatrix} \quad (2.65)$$

$$g_a(x) = \left(0 \ 0 \ 0 \ 0 \ \frac{1}{\sigma L_s} \ 0\right)^T \quad (2.66)$$

$$g_b(x) = \left(0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{\sigma L_s}\right) \quad (2.67)$$

The measured state variables are taken to be $x_1 = \theta$, $x_5 = i_{r_d}$ and $x_6 = i_{r_q}$. Then, the associated Jacobian matrices are:

$$J_\phi = (J_\phi^1 \ J_\phi^2 \ J_\phi^3 \ J_\phi^4 \ J_\phi^5 \ J_\phi^6)^T \quad (2.68)$$

where $J_\phi^1 = [0, 1, 0, 0, 0, 0]$, $J_\phi^2 = [0, \frac{K_m}{J}, \frac{\eta}{J}i_{s_q}, -\frac{\eta}{J}i_{s_d}, 0, 0]$, $J_\phi^3 = [0, 0, -\frac{1}{\tau_s}, \omega_{dq}, \frac{M}{\tau_s}, 0]$, $J_\phi^4 = [0, 0, -\omega_{dq}, -\frac{1}{\tau_s}, 0, \frac{M}{\tau_s}]$, $J_\phi^5 = [0, -\beta x_4 - x_6, -\gamma_2 + \frac{\beta}{\tau_s}, -\beta x_2, 0, \omega_{dq} - x_2]$ and $J_\phi^6 = [0, \beta x_3 - x_5, \beta x_2, \frac{\beta}{\tau_s}, \omega_{dq} - x_2, -\gamma_2]$.

$$J_\gamma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (2.69)$$

It is noted that: (i) the discrete-time model of Eq. (2.56) can be computed from Eqs. (2.64)–(2.67) after applying common discretization methods (e.g. first order Euler approximation of the continuous-time derivative), (ii) if a Jacobian matrix J_ϕ , associated to the drift term of the system's dynamics, is computed using the system's continuous-time description of Eq. (2.64), then in the EKF recursion of Eq. (2.62) and (2.63) it should be substituted by $I + T_s J_\phi$ where T_s is the sampling period and $I \in R^{n \times n}$ is the identity matrix.

2.2.7 Estimation of the Wind-Generated Mechanical Torque Using EKF

The Extended Kalman Filter can be used not only for estimation of the non-measurable state vector elements of the DFIG (such as stator's magnetic flux and rotor's angular velocity) but also for estimation of the wind generated mechanical torque. One approach to succeed this is to redefine the parameters of the state vector of the induction generator as follows: $x_1 = \theta$, $x_2 = T_m$, $x_3 = \omega_r$, $x_4 = \dot{T}_m$, $x_5 = \psi_{s_d}$,

$x_6 = \psi_{sq}$, $x_7 = i_{rd}$ and $x_8 = i_{rq}$. Using the relation about the dependence of the mechanical torque on the design characteristics of the wind turbine as well as on the rotor's speed given in Eq. (1.2) and considering constant or piecewise constant wind speed v , the state equations of the DFIG can be written as:

$$f(x) = \begin{pmatrix} x_3 \\ x_4 \\ -\frac{K_m}{J}x_3 - \frac{x_2}{J} + \frac{\eta}{J}(i_{sq}x_5 - i_{sd}x_6) \\ \frac{1}{2}\rho\pi R^3 \frac{d^2}{dt^2}C_q(\lambda, \beta)v^2 \\ -\frac{1}{\tau_s}x_5 + \omega_{dq}x_6 + \frac{M}{\tau_s}x_7 + v_{sd} \\ -\omega_{dq}x_5 - \frac{1}{\tau_s}x_6 + \frac{M}{\tau_s}x_8 + v_{sq} \\ -\beta x_3x_6 + \frac{\beta}{\tau_s}x_5 + (\omega_{dq} - x_3)x_8 - \gamma_2x_5 \\ \frac{\beta}{\tau_s}x_6 + \beta x_3x_5 + (\omega_{dq} - x_3)x_7 - \gamma_2x_8 \end{pmatrix} \quad (2.70)$$

$$g_a(x) = \left(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{\sigma L_s} \ 0\right)^T \quad (2.71)$$

$$g_b(x) = \left(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{\sigma L_s}\right) \quad (2.72)$$

Introducing to the DFIG state vector the new state variables x_2 and x_4 and defining function

$$C^*(x_3) = \frac{1}{2}\rho\pi R^3 \ddot{C}_q\left(\frac{\omega R}{v}, \beta\right)v^2 \quad (2.73)$$

the Jacobian matrices J_ϕ and J_γ associated to the mechanical torque and used in the updated EKF recursion are written as:

$$J_\phi = (J_\phi^1 \ J_\phi^2 \ J_\phi^3 \ J_\phi^4 \ J_\phi^5 \ J_\phi^6 \ J_\phi^7 \ J_\phi^8)^T \quad (2.74)$$

where $J_\phi^1 = [0, 0, 1, 0, 0, 0, 0, 0]^T$, $J_\phi^2 = [0, 0, 0, 1, 0, 0, 0, 0]^T$

$$J_\phi^3 = [0, -\frac{1}{J}, -\frac{K_m}{J}, 0, \frac{n_p i_{sq}}{J}, -\frac{n_p i_{sd}}{J}, 0, 0], J_\phi^4 = [0, 0, \frac{\partial \ddot{C}^*(x_3)}{\partial x_3}, 0, 0, 0, 0, 0]$$

$$J_\phi^5 = [0, 0, 0, 0, -\frac{1}{\tau_s}, \omega_{dq}, \frac{M}{\tau_s}, 0], J_\phi^6 = [0, 0, 0, 0, -\omega_{dq}, -\frac{1}{\tau_s}, 0, \frac{M}{\tau_s}]$$

$$J_\phi^7 = [0, 0, -\beta x_6 - x_8, 0, \frac{\beta}{\tau_s} - \gamma_2, -\beta x_3, 0, \omega_{dq}], J_\phi^8 = [0, 0, \beta x_5 - x_7, 0, \beta x_3, \frac{\beta}{\tau_s}, \omega_{dq} - x_3, -\gamma_2].$$

$$J_\gamma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (2.75)$$

When the Jacobian matrix J_ϕ , associated to the drift term of the system's dynamics, is computed using the system's continuous-time description of Eqs. (1.26) and (1.27), then in the EKF recursion of Eqs. (2.62) and (2.63) it should be substituted by $I + T_s J_\phi$ where T_s is the sampling period and $I \in \mathbb{R}^{n \times n}$ is the identity matrix.

The estimation of the mechanical torque T_m using the EKF recursion can be more accurate than calculating T_m with the use of Eq. (1.2) since there may be inaccuracies about the values of parameters ρ , R , λ and ν as well in the modeling of the torque coefficient C_q . This type of modeling uncertainty is taken into account and compensated by the EKF, through the use of the process and measurement noise covariance matrices Q and R in the estimator's recursion described in Eqs. (2.62) and (2.63).

2.2.8 Simulation Tests

The efficiency of the proposed control scheme was tested in case of variable rotor speed setpoint assuming different wind profiles. The obtained results for the first wind torque profile are shown in Figs. 2.3, 2.4, and 2.5 while the results for the second wind torque profile are shown in Figs. 2.6, 2.7 and 2.8. The real state variable is denoted by the dashed blue line, the estimated state variable is denoted by the dashed green line, while the associated reference setpoint is denoted by the continuous red line. Although using a small number of sensors and measuring only specific elements of the DFIG state vector such as the rotor's angle and the stator's currents, the proposed control scheme succeeded accurate tracking of the reference setpoints and accurate estimation of the non-measurable parameters such as the rotation speed of the rotor, the magnetic flux of the stator and the wind-generated mechanical torque.

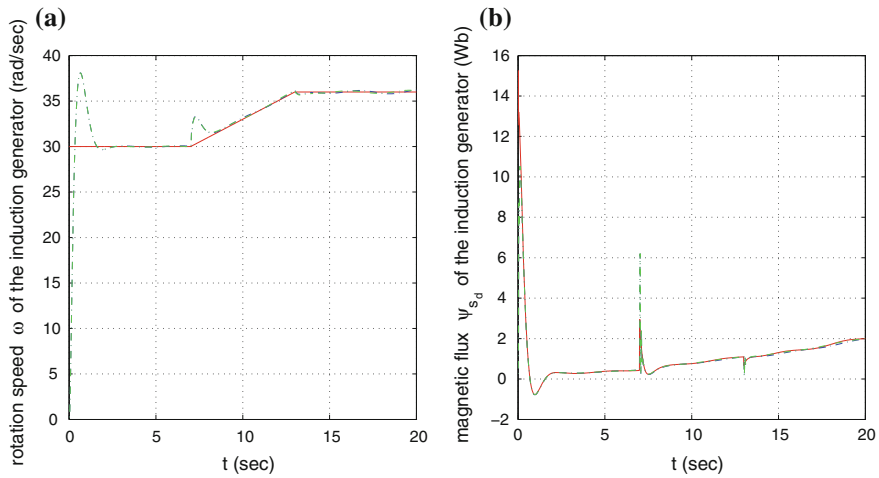


Fig. 2.3 Extended Kalman Filter-based control of the doubly-fed induction generator **a** estimation and setpoint tracking for rotor's angular velocity ω_{r_d} , **b** estimation and setpoint tracking for stator's magnetic flux ψ_{s_d}

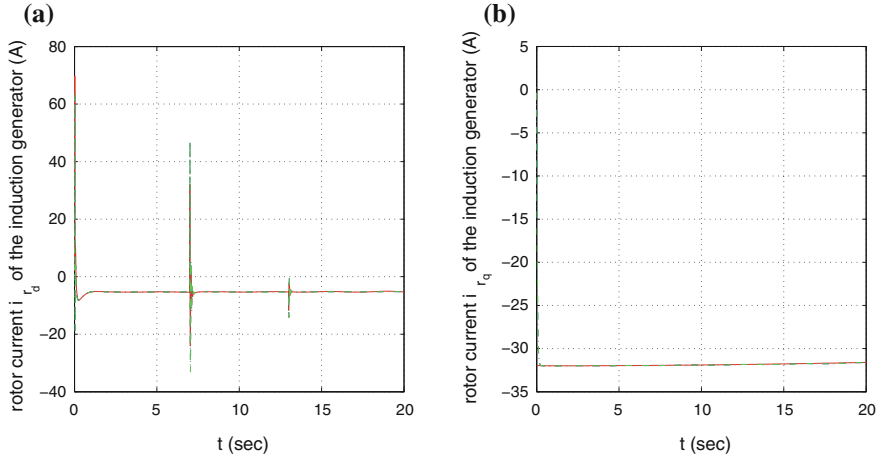


Fig. 2.4 Extended Kalman Filter-based control of the doubly-fed induction generator **a** rotor's control input current i_{rd} , **b** rotor's control input current i_{rq}

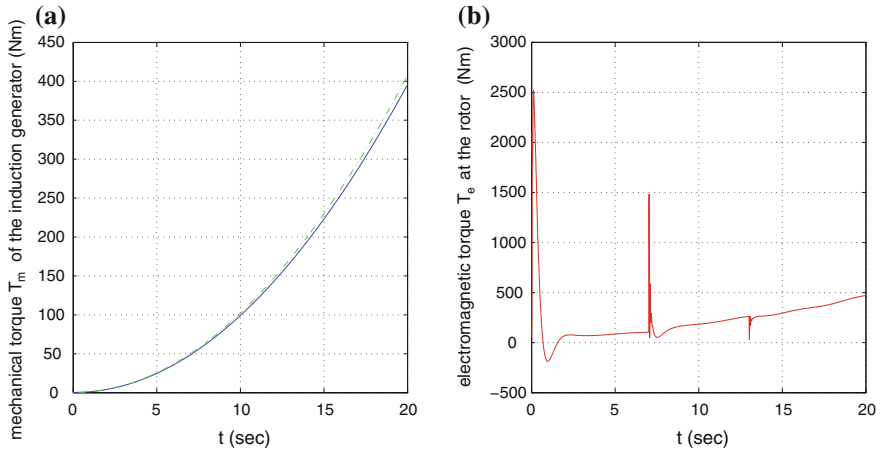


Fig. 2.5 **a** Mechanical torque T_m due to wind. **b** Electromagnetic torque T_e at the DFIG rotor

Taking into account that several variables of the induction generator's state vector (e.g. rotation speed and magnetic flux) are not directly measurable (due to sensors cost and limited reliability, sensors failures and difficulties in sensors installation) the significance of state estimation through Kalman Filtering becomes clear. It can be noticed that the Extended Kalman Filter is an efficient approach for the implementation of state estimation-based control of the sixth-order induction generator model. The Unscented Kalman Filter can be also used in place of the Extended Kalman Filter and in the latter case there will be no need to compute Jacobian matrices [202–204].

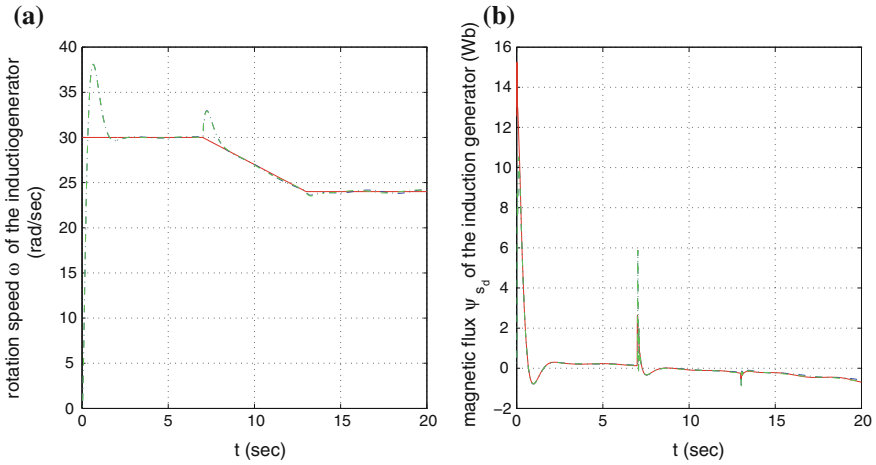


Fig. 2.6 Extended Kalman Filter-based control of the doubly-fed induction generator **a** estimation and setpoint tracking for rotor's angular velocity ω_{rd} , **b** estimation and setpoint tracking for stator's magnetic flux ψ_{sd}

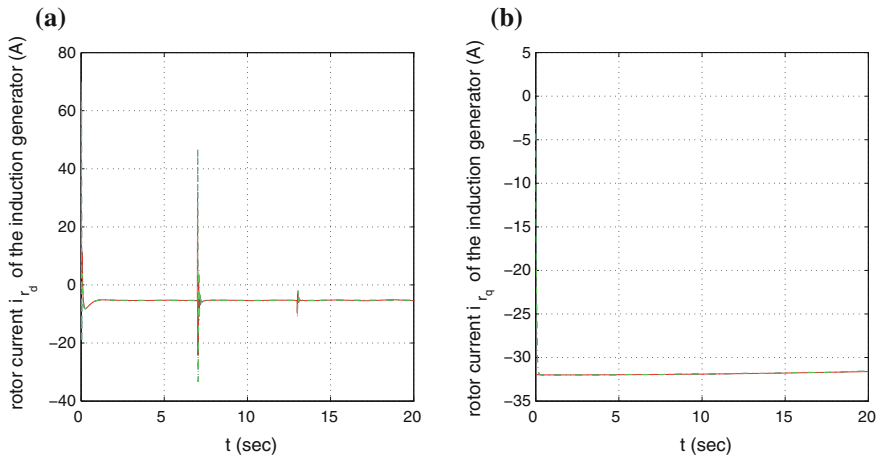


Fig. 2.7 Extended Kalman Filter-based control of the doubly-fed induction generator **a** rotor's control input current i_{rd} , **b** rotor's control input current i_{rq}

Finally, it is noted that to compensate for external disturbances and parameter variations in state estimation-based nonlinear control schemes the following approaches are possible (i) use of adaptive Kalman Filter, (ii) redesign of the Kalman filter in the form of a disturbance observer, (iii) redesign of the Kalman Filter in the form of a high-gain Extended Kalman Filter. Regarding (i), adaptive Kalman Filtering is implemented through the adaptation of the process and measurement noise covariance matrices in the Kalman Filter recursion so as to cope with variable noise levels

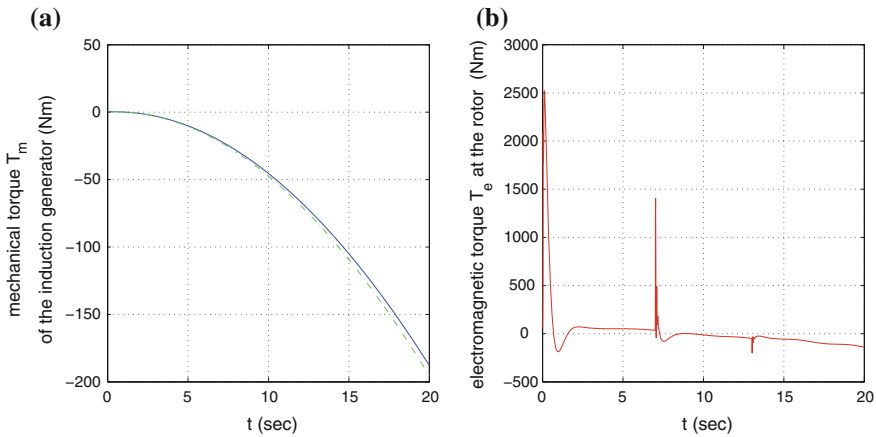


Fig. 2.8 **a** Mechanical torque T_m due to wind. **b** Electromagnetic torque T_e at the DFIG rotor

[14]. Regarding (ii), additive disturbances and parametric changes can be identified with the use of Kalman Filters that operate as disturbance observers (see work on Kalman Filtering and disturbance observers in [352]). Once the disturbance affecting the nonlinear system becomes known it can be compensated through the inclusion of an additional control term in the loop. Thus, it can be assured again that the estimation error $x - \hat{x}$ will be minimal and the performance of the control loop will be satisfactory. There are several results on disturbance observers (e.g. Extended State Observers, Unknown Input Observers or Perturbation Observers) and on their use within a Kalman Filter framework [224, 292]. Such results can be exploited so as to make Kalman Filtering for electric power generators as well as the associated state estimation-based control loop be more robust [352]. Regarding (iii) the high-gain Extended Kalman Filter can provide additional robustness to state estimation under external disturbances and parametric variations [47, 53].

2.3 Control of the DFIG Based on Global Linearization Approaches

2.3.1 Outline

In this section, flatness-based control for the doubly-fed induction generator is implemented through its transformation to the linear canonical form. As noted, differential flatness theory is currently a main direction in nonlinear dynamical systems and enables linearization and control for a wide class of systems, in a more efficient manner than Lie-algebra methods [83, 352, 388, 411, 473]. Previously, it was explained that, to find out if a dynamical system is differentially flat, the following should be

examined: (i) the existence of the so-called flat output, i.e. a new variable which is expressed as a function of the system's state variables. The flat output and its derivatives should not be coupled in the form of an ordinary differential equation, (ii) the components of the system (i.e. state variables and control input) should be expressed as functions of the flat output and its derivatives [57, 137, 241, 280, 412, 456]. Differential flatness theory enables transformation to a linearized form (canonical Brunovsky form) for which the design of the controller becomes easier. By showing that a system is differentially flat, one can easily design a reference trajectory as a function of the so-called flat output and can find a control law that assures tracking of this desirable trajectory [456, 137].

This chapter is concerned with proving differential flatness of the model of the doubly-fed induction generator and its description in the Brunovsky (canonical) form [388, 411]. By defining flat outputs which are associated with the rotor's angle and with the magnetic flux of the stator, an equivalent DFIG description in the Brunovsky (linear canonical) form is obtained. It is shown that for the linearized DFIG's model it is possible to design a feedback controller. At a second stage, a novel Kalman Filtering method, the Derivative-free nonlinear Kalman Filter, is proposed for estimating the state vector elements of the linearized system which are not directly measurable. With the redesign of the proposed Kalman Filter as a disturbance observer, it becomes possible to estimate also disturbance terms affecting the DFIG model and to use these terms in the feedback controller. By avoiding linearization approximations, the proposed filtering method, improves the accuracy of estimation, and results in smooth control signal variations and in minimization of the tracking error of the associated control loop [355, 368].

2.3.2 *Input-Output Linearization of the DFIG Using Lie Algebra Theory*

2.3.2.1 **Input-Output Linearization of the DFIG Model**

The following variables are defined

$$\begin{aligned} h_1(x) &= x_1 = \theta \\ h_2(x) &= x_3^2 + x_4^2 = \psi_{s_d}^2 + \psi_{s_q}^2 \end{aligned} \quad (2.76)$$

Next, based on h_1, h_2 the following transformed state variables are defined

$$z_1 = h_1(x) = \theta \quad (2.77)$$

$$\begin{aligned} z_2 &= L_f h_1(x) \Rightarrow \\ z_2 &= f_1 \Rightarrow z_2 = x_2 \Rightarrow z_2 = \omega \end{aligned} \quad (2.78)$$

Similarly, one has

$$\begin{aligned} z_3 &= L_f^2 h_1(x) = L_f z_2 \Rightarrow \\ z_3 &= f_2 \Rightarrow z_3 = -\frac{K_m}{J} x_2 - \frac{T_m}{J} + \frac{\eta}{J} (i_{r_q} x_3 - i_{r_d} x_4) \Rightarrow \\ z_3 &= -\frac{K_m}{J} x_2 - \frac{T_m}{J} + \frac{\eta}{J} (x_6 x_3 - x_5 x_4) \end{aligned} \quad (2.79)$$

For the transformed state variable z_4 one has

$$z_4 = h_2(x) = \psi_{sd}^2 + \psi_{sq}^2 = x_3^2 + x_4^2 \quad (2.80)$$

and

$$\begin{aligned} z_5 &= L_f h_2(x) \Rightarrow z_5 = 2x_3 f_3 + 2x_4 f_4 \Rightarrow \\ z_5 &= 2x_3 \left[-\frac{1}{\tau_s} x_3 + \omega_{dq} x_4 + \frac{M}{\tau_s} x_5 + v_{sd} \right] + \\ &+ 2x_4 \left[-\omega_{dq} x_3 - \frac{1}{\tau_s} x_4 + \frac{M}{\tau_s} x_6 + v_{sq} \right] \end{aligned} \quad (2.81)$$

After the change of the state variables it holds

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= L_f^3 h_1(x) + (L_{g_a} L_f^2 h_1(x)) u_1 + (L_{g_b} L_f^2 h_1(x)) u_2 \\ \dot{z}_4 &= z_5 \\ \dot{z}_5 &= L_f^2 h_2(x) + (L_{g_a} L_f h_2(x)) u_1 + (L_{g_b} L_f h_2(x)) u_2 \end{aligned} \quad (2.82)$$

The inputs of the above linearized and decoupled DFIG model are $u_1 = u_{r_d}$ and $u_2 = u_{r_q}$. The system of Eq. (2.82) can be written in the input-output linearized form

$$\begin{pmatrix} \dot{z}_1^{(3)} \\ \ddot{z}_4 \end{pmatrix} = f_a + \tilde{M} u \quad (2.83)$$

where

$$\begin{aligned} f_a(x) &= \begin{pmatrix} L_f^3 h_1(x) \\ L_f^2 h_2(x) \end{pmatrix} \\ \tilde{M} &= \begin{pmatrix} L_{g_a} L_f^2 h_1(x) & L_{g_b} L_f^2 h_1(x) \\ L_{g_a} L_f h_2(x) & L_{g_b} L_f h_2(x) \end{pmatrix} \end{aligned} \quad (2.84)$$

or equivalently one has the system's description in the MIMO canonical form

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (2.85)$$

where

$$\begin{aligned} v_1 &= L_f^3 h_1(x) + (L_{ga}(L_f^2 h_1(x))u_1 + (L_{gb}L_f^2 h_1(x))u_2 \\ v_2 &= L_f^2 h_2(x) + (L_{ga}L_f h_2(x))u_1 + (L_{gb}L_f h_2(x))u_2 \end{aligned} \quad (2.86)$$

Returning to the compact form of Eq. (2.84) one has

$$\begin{pmatrix} z_1^{(3)} \\ \ddot{z}_4 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (2.87)$$

and the control signal that assures convergence of the z_1 and z_4 to the reference setpoints z_1^d and z_4^d is given by

$$\begin{aligned} v_1 &= z_1^{d(3)} - k_1^{(1)}(\ddot{z}_1 - \ddot{z}_1^d) - k_2^{(1)}(\dot{z}_1 - \dot{z}_1^d) - k_3^{(1)}(z_1 - z_1^d) \\ v_2 &= \ddot{z}_4^d - k_1^{(2)}(\dot{z}_4 - \dot{z}_4^d) - k_2^{(2)}(z_4 - z_4^d) \end{aligned} \quad (2.88)$$

2.3.2.2 State Estimation-Based Control

For the implementation of the aforementioned control law, there is need to obtain measurements of all elements of the DFIG's state vector. The rotor's turn angle can be measured directly with the use of an encoder [33, 214]. Knowing the rotor's angle, and with the use of the decoupled induction machine's model of Eq. (2.85) it is possible to estimate the rotor's angular speed. Similarly, after obtaining measurements of the magnetic flux at the stator and with the use of the decoupled induction machine's model of Eq. (2.85) it is possible to estimate the derivatives of the magnetic flux. Due to the fact that the magnetic flux of the stator ψ_s cannot be measured directly, equations that provide indirect measurements of the flux (computed through measurements of the stator and rotor currents) will be used, that is

$$\begin{aligned} \psi_{sd} &= L_s i_{sd} + M i_{rd} \\ \psi_{sq} &= L_s i_{sq} + M i_{rq} \end{aligned} \quad (2.89)$$

It is noted that the currents are measured in the ab reference frame and their computation in the dq reference frame requires the application of the associated reference frame transformation.

Using the model of Eq. (2.85) the state estimator for the DFIG is given by

$$\dot{\hat{z}} = A\hat{z} + Bv + K(z^{meas} - C\hat{z}) \quad (2.90)$$

where the estimator's gain $K \in R^{5 \times 2}$

$$\begin{aligned} A &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} & B &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \\ C &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{aligned} \quad (2.91)$$

2.3.3 Differential Flatness for Nonlinear Dynamical Systems

2.3.3.1 Transformation of MIMO Nonlinear Systems into the Brunovsky Form

It is assumed now that after defining the flat outputs of the initial MIMO nonlinear model of the DFIG, and after expressing the system state variables and control inputs as functions of the flat output and of the associated derivatives, the system can be transformed in the Brunovsky canonical form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dots & \\ \dot{x}_{r_1-1} &= x_{r_1} \\ \dot{x}_{r_1} &= f_1(x) + \sum_{j=1}^p g_{1j}(x)u_j + d_1 & y_1 &= x_1 \\ \dot{x}_{r_1+1} &= x_{r_1+2} & \dots & \\ \dots & & y_p &= x_{n-r_p+1} \\ \dot{x}_{p-1} &= x_p \\ \dot{x}_p &= f_p(x) + \sum_{j=1}^p g_{pj}(x)u_j + d_p \end{aligned} \quad (2.92)$$

where $x = [x_1, \dots, x_n]^T$ is the state vector of the transformed system (according to the differential flatness formulation), $u = [u_1, \dots, u_p]^T$ is the set of control inputs, $y = [y_1, \dots, y_p]^T$ is the output vector, f_i are the drift functions and $g_{i,j}$, $i, j = 1, 2, \dots, p$ are smooth functions corresponding to the control input gains, while d_j is a variable associated to external disturbances. It holds that $r_1 + r_2 + \dots + r_p = n$. Having written the initial nonlinear system into the canonical (Brunovsky) form it holds

$$y_i^{(r_i)} = f_i(x) + \sum_{j=1}^p g_{ij}(x)u_j + d_j \quad (2.93)$$

Next the following vectors and matrices can be defined $f(x) = [f_1(x), \dots, f_n(x)]^T$, $g(x) = [g_1(x), \dots, g_n(x)]^T$ with $g_i(x) = [g_{1i}(x), \dots, g_{pi}(x)]^T$, and $A =$

$\text{diag}[A_1, \dots, A_p]$, $B = \text{diag}[B_1, \dots, B_p]$, $C = \text{diag}[C_1, \dots, C_p]$, $d = [d_1, \dots, d_p]^T$, where matrix A has the MIMO canonical form, i.e. with block-diagonal elements

$$\begin{aligned} A_i &= \begin{pmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{pmatrix}_{r_i \times r_i} \\ B_i^T &= (0 \ 0 \ \dots \ 0 \ 1)_{1 \times r_i} \\ C_i &= (1 \ 0 \ \dots \ 0 \ 0)_{1 \times r_i} \end{aligned} \quad (2.94)$$

Thus, Eq. (8.112) can be written in state-space form

$$\begin{aligned} \dot{x} &= Ax + Bv + B\tilde{d} \\ y &= Cx \end{aligned} \quad (2.95)$$

where the control input is written as $v = f(x) + g(x)u$.

2.3.4 Input-Output Linearization of the DFIG Using Differential Flatness Theory

2.3.4.1 Differential Flatness Properties of the DFIG

The flat outputs of the system are defined as

$$\begin{aligned} y_1 &= \theta \text{ or } y = x_1 \\ y_2 &= \psi_{s_d}^2 + \psi_{s_q}^2 \text{ or } y_2 = x_3^2 + x_4^2 \end{aligned} \quad (2.96)$$

It holds that

$$\begin{aligned} \dot{y}_1 &= \omega \text{ or } \dot{y}_1 = x_2 \Rightarrow \\ \ddot{y}_1 &= \dot{\omega} = -\frac{K_m}{J}x_2 - \frac{T_m}{J} + \frac{\eta}{J}(x_6x_3 - x_5x_4) \Rightarrow \\ \ddot{y}_1 &= \dot{\omega} = -\frac{K_m}{J}\dot{y}_1 - \frac{T_m}{J} + \frac{\eta}{J}(x_6x_3 - x_5x_4) \end{aligned} \quad (2.97)$$

Deriving the last row of Eq. (2.97) with respect to time one obtains

$$\begin{aligned} y_1^{(3)} &= -\frac{K_m}{J}\ddot{y}_1 + \frac{\eta}{J}(\dot{x}_6x_3 + x_6\dot{x}_3 - \dot{x}_5x_4 - x_5\dot{x}_4) \Rightarrow \\ y_1^{(3)} &= -\frac{K_m}{J}\ddot{y}_1 + \frac{\eta}{J}x_3\left\{\left[\frac{\beta}{\tau_s}x_4 + \beta x_2x_3 + (\omega_{dq} - x_2)x_5 - \right. \right. \\ &\quad \left. \left. - \gamma_2x_6 - \beta v_{s_q}\right] + \frac{1}{\sigma L_r}u_1\right\} + \frac{\eta}{J}x_6\left[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}\right] \\ &\quad - \frac{\eta}{J}x_4\left\{[-\beta x_2x_4 + \frac{\beta}{\tau_s}x_3 + (\omega_{dq} - x_2)x_6 - \gamma_2x_5 - \right. \\ &\quad \left. - \beta v_{s_d}] + \frac{1}{\sigma L_r}u_2\right\} - \frac{\eta}{J}x_5\left[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}\right] \end{aligned} \quad (2.98)$$

Moreover, about the second flat output it holds

$$\begin{aligned} \dot{y}_2 &= 2x_3\dot{x}_3 + 2x_4\dot{x}_4 \Rightarrow \\ \dot{y}_2 &= 2x_3[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}] + \\ &+ 2x_4[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}] \Rightarrow \end{aligned} \quad (2.99)$$

Consequently, it holds

$$\begin{aligned} \ddot{y}_2 &= 2\dot{x}_3[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}] + \\ &+ 2x_3[-\frac{1}{\tau_s}\dot{x}_3 + \omega_{dq}\dot{x}_4 + \frac{M}{\tau_s}\dot{x}_5] + \\ &2\dot{x}_4[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}] + \\ &+ 2x_4[-\omega_{dq}\dot{x}_3 - \frac{1}{\tau_s}\dot{x}_4 + \frac{M}{\tau_s}\dot{x}_6] \end{aligned} \quad (2.100)$$

or equivalently

$$\begin{aligned} \ddot{y}_2 &= 2[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}]^2 + \\ &- \frac{2}{\tau_s}x_3[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}] \\ &- 2\omega_{dq}x_3[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}] \\ &+ \frac{2M}{\tau_s}x_3\{[-\beta x_2x_4 + \frac{\beta}{\tau_s}x_3 + (\omega_{dq} - x_2)x_6 - \\ &- \gamma_2x_5 - \beta v_{s_d}] + \frac{1}{\sigma L_r}u_1\} \\ &+ 2[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}]^2 \\ &- 2\omega_{dq}x_4[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}] \\ &- \frac{2}{\tau_s}x_4[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}] + \\ &+ 2x_4\frac{M}{\tau_s}\{[\frac{\beta}{\tau_s}x_4 + \beta x_2x_3 + (\omega_{dq} - x_2)x_5 - \\ &- \gamma_2x_6 - \beta v_{s_q}] + \frac{1}{\sigma L_r}u_2\} \end{aligned} \quad (2.101)$$

It holds that $x_1 = y_1$, $x_2 = \dot{y}_1$. From the second row of Eq. (2.96) and considering that the field orientation condition requires $x_4 = \psi_{s_q} = 0$ one obtains that $x_3 = \sqrt{y_2}$. Moreover, from Eq. (2.97) it holds

$$\begin{aligned} \ddot{y}_1 &= -\frac{K_m}{J}\dot{y}_1 - \frac{T_m}{J} + \frac{\eta}{J}\sqrt{y_2}x_6 \Rightarrow \\ x_6 &= \frac{\ddot{y}_1 + \frac{K_m}{J}\dot{y}_1 + \frac{T_m}{J}}{\frac{\eta}{J}\sqrt{y_2}}, \quad y_2 \neq 0 \end{aligned} \quad (2.102)$$

From Eq. (2.99) one obtains

$$\begin{aligned} \dot{y}_2 &= -\frac{2}{\tau_s}x_3^2 + \frac{2M}{\tau_s}x_3x_5 + 2v_{s_d}x_3 \Rightarrow \\ \dot{y}_2 + (\frac{2}{\tau_s}x_3 - 2v_{s_d})x_3 &= \frac{2M}{\tau_s}x_3x_5 \Rightarrow \\ x_5 &= \frac{\dot{y}_2 + (\frac{2}{\tau_s}\sqrt{y_2} - 2v_{s_d})\sqrt{y_2}}{\frac{2M}{\tau_s}\sqrt{y_2}}, \quad y_2 \neq 0 \end{aligned} \quad (2.103)$$

Therefore, x_5 is also a function of the flat output and of its derivatives. Additionally, by solving the system of Eqs. (2.98) and (2.101) with respect to the control inputs u_1 and u_2 one obtains that the control inputs are functions of the flat output and its derivatives. Therefore, the model of the DFIG is a differentially flat one.

Next, to design the flatness-based controller for the DFIG the following transformation of the state variables is introduced: $z_1 = y_1$, $z_2 = \dot{y}_1$, $z_3 = \ddot{y}_1$, $z_4 = y_2$, $z_5 = \dot{y}_2$ for which holds

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= L_f^3 h_1(x) + (L_{g_a} L_f^2 h_1(x))u_1 + (L_{g_b} L_f^2 h_1(x))u_2 \\ \dot{z}_4 &= z_5 \\ \dot{z}_5 &= L_f^2 h_2(x) + (L_{g_a} L_f h_2(x))u_1 + (L_{g_b} L_f h_2(x))u_2 \end{aligned} \quad (2.104)$$

Therefore, one obtains the decoupled and linearized representation of the system

$$\begin{pmatrix} \ddot{z}_1^{(3)} \\ \ddot{z}_4 \end{pmatrix} = \begin{pmatrix} L_f^3 h_1(x) \\ L_f^2 h_2(x) \end{pmatrix} + \begin{pmatrix} L_{g_a} L_f^2 h_1(x) & L_{g_b} L_f^2 h_1(x) \\ L_{g_a} L_f h_2(x) & L_{g_b} L_f h_2(x) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (2.105)$$

or equivalently

$$\begin{pmatrix} \ddot{z}_1^{(3)} \\ \ddot{z}_4 \end{pmatrix} = f_a + \tilde{M}u \quad (2.106)$$

where

$$f_a = \begin{pmatrix} L_f^3 h_1(x) \\ L_f^2 h_2(x) \end{pmatrix} \quad \tilde{M} = \begin{pmatrix} L_{g_a} L_f^2 h_1(x) & L_{g_b} L_f^2 h_1(x) \\ L_{g_a} L_f h_2(x) & L_{g_b} L_f h_2(x) \end{pmatrix} \quad (2.107)$$

By defining the control inputs $v_1 = L_f^3 h_1(x) + (L_{g_a} L_f^2 h_1(x))u_1 + (L_{g_b} L_f^2 h_1(x))u_2$ and $v_2 = L_f^2 h_2(x) + (L_{g_a} L_f h_2(x))u_1 + (L_{g_b} L_f h_2(x))u_2$ one can also have the description in the MIMO canonical form

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (2.108)$$

The control input for the linearized and decoupled model of the DFIG is chosen as follows

$$\begin{aligned} v_1 &= z_1^{d(3)} - k_1^{(1)}(\ddot{z}_1 - \ddot{z}_1^d) - k_2^{(1)}(\dot{z}_1 - \dot{z}_1^d) - k_3^{(1)}(z_1 - z_1^d) \\ v_2 &= \ddot{z}_4^d - k_1^{(2)}(\dot{z}_4 - \dot{z}_4^d) - k_2^{(2)}(z_4 - z_4^d) \end{aligned} \quad (2.109)$$

and finally the control input that is applied to the system is

$$u = \tilde{M}^{-1}(-f_a + v) \quad (2.110)$$

The proposed control scheme can work with the use of measurements from a small number of sensors. That is, there is need to obtain measurements of only $y_1 = \theta$ which is the turn angle of the generator's rotor, and of the magnetic flux $y_2 = \psi_s^2 = \psi_{s_d}^2 + \psi_{s_q}^2$, or due to the orientation of the magnetic field $y_2 = \psi_s^2 = \psi_{s_d}^2$. The stator flux (ψ_s) cannot be measured directly from a sensor (e.g. the use of Hall sensor in an electric machine with a rotating part would not be efficient), however Eq. (2.89) that relates stator flux and stator and rotor currents can be used to calculate ψ_s . Thus one has:

$$\begin{aligned}\psi_{s_d} &= L_s i_{s_d} + M i_{r_d} \\ \psi_{s_q} &= 0\end{aligned}\quad (2.111)$$

which means that by measuring stator and rotor currents one can obtain an indirect measurement of the stator's magnetic flux ψ_{s_d} . Next, one can compute the dynamics of the magnetic flux, jointly with the dynamics of the rotor's motion through the use of the Kalman Filter. This estimation method is based on the application of the Kalman Filter recursion to the linearized equivalent of the generator's model which is given by Eq. (2.108), and is known as Derivative-free nonlinear Kalman Filter [375]. Actually, Eq. (2.108) can be written in the state-space form

$$\begin{aligned}\dot{z} &= Az + Bv \\ z^{meas} &= Cz\end{aligned}\quad (2.112)$$

where

$$\begin{aligned}A &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} & B &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \\ C &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}\end{aligned}\quad (2.113)$$

The estimator's dynamics is:

$$\dot{\hat{z}} = A \cdot \hat{z} + B \cdot v + K(z^{meas} - C\hat{z}) \quad (2.114)$$

where $K \in R^{5 \times 2}$ is the state estimator's gain. Defining as $\tilde{A}_d$, $\tilde{B}_d$, and $\tilde{C}_d$, the discrete-time equivalents of matrices A , B and C respectively, the associated Kalman Filter-based estimator is given by [34, 209, 341, 480]

measurement update:

$$\begin{aligned}K(k) &= P^-(k)C_d^T[C_d \cdot P^-(k)C_d^T + R]^{-1} \\ \hat{z}(k) &= \hat{z}^-(k) + K(k)[z^{meas}(k) - C_d\hat{z}^-(k)] \\ P(k) &= P^-(k) - K(k)C_dP^-(k)\end{aligned}\quad (2.115)$$

time update:

$$\begin{aligned} P^-(k+1) &= A_d(k)P(k)A_d^T(k) + Q(k) \\ \hat{z}^-(k+1) &= A_d(k)\hat{z}(k) + B_d(k)v(k) \end{aligned} \quad (2.116)$$

2.3.5 Kalman Filter-Based Disturbance Observer for the DFIG Model

2.3.5.1 Application of a Disturbance Observer to the DFIG Model

Next, it will be considered that additive input disturbances (e.g. due to load variations) affect the DFIG model. The simultaneous estimation of the non-measurable elements of the DFIG state vector as well as the estimation of additive disturbance terms affecting the generator is possible with the use of a disturbance estimator [99, 169, 224, 292].

It is assumed that the third and fifth row of the state-space equations of the Doubly-Fed Induction Generator of Eq. (2.104) include a disturbance term

$$\begin{aligned} \dot{z}_3 &= L_f^3 h_1(x) + L_{g_a}(L_f^2 h_1(x))u_1 + L_{g_b}(L_f^2 h_2(x))u_2 + \tilde{d}_1 \\ \dot{z}_5 &= L_f^2 h_2(x) + L_{g_a}(L_f h_2(x))u_1 + L_{g_b}(L_f h_2(x))u_2 + \tilde{d}_2 \end{aligned} \quad (2.117)$$

Without loss of generality, the dynamics of the disturbance terms is described by their second order derivatives and the associated initial conditions, i.e. $\ddot{\tilde{d}}_1 = \tilde{f}_a(x)$ and $\ddot{\tilde{d}}_2 = \tilde{f}_b(x)$. Next, an extended state-space model of the system is defined that comprises as additional state variables the disturbance terms $z_6 = \tilde{d}_1$, $z_7 = \dot{\tilde{d}}_1$, while $z_8 = \tilde{d}_2$, and $z_9 = \dot{\tilde{d}}_2$. Thus, the extended state-space model is written as $\dot{z}_1 = z_2$, $\dot{z}_2 = z_3$, $\dot{z}_3 = v_1 + z_6$, $\dot{z}_4 = z_5$, and $\dot{z}_5 = v_2 + z_8$, $\dot{z}_6 = z_7$, $\dot{z}_7 = \tilde{f}_a$, $\dot{z}_8 = z_9$ and $\dot{z}_9 = \tilde{f}_b$, or in matrix form one has

$$\begin{aligned} \dot{\tilde{z}} &= \tilde{A}\tilde{z} + \tilde{B}\tilde{v} \\ \tilde{z}^{meas} &= \tilde{C}\tilde{z} \end{aligned} \quad (2.118)$$

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \\ \dot{z}_6 \\ \dot{z}_7 \\ \dot{z}_8 \\ \dot{z}_9 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \\ z_9 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \tilde{f}_a \\ \tilde{f}_b \end{pmatrix} \quad (2.119)$$

$$\begin{pmatrix} z_1^{meas} \\ z_4^{meas} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \\ z_9 \end{pmatrix} \quad (2.120)$$

The associated state estimator is

$$\hat{\dot{z}} = \tilde{A}_o \hat{z} + \tilde{B}_o \tilde{v}_1 + K_o (\tilde{z}^{meas} - \tilde{C} \hat{z}) \quad (2.121)$$

where

$$\tilde{A}_o = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \tilde{B}_o = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.122)$$

$$\tilde{C}_o = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

while the estimator's gain $K_o \in R^{9 \times 2}$ is obtained from the standard Kalman Filter recursion [34, 209, 341, 480].

Defining as $\tilde{A}_d$, $\tilde{B}_d$, and $\tilde{C}_d$, the discrete-time equivalents of matrices $\tilde{A}_o$, $\tilde{B}_o$ and $\tilde{C}_o$ respectively, a Derivative-free nonlinear Kalman Filter can be designed for the aforementioned representation of the system dynamics [352, 355]. The associated Kalman Filter-based disturbance estimator is given by

measurement update:

$$\begin{aligned} K(k) &= P^-(k) \tilde{C}_d^T [\tilde{C}_d P^-(k) \tilde{C}_d^T + R]^{-1} \\ \hat{\tilde{z}}(k) &= \hat{\tilde{z}}^-(k) + K(k) [\tilde{C}_d \tilde{z}(k) - \tilde{C}_d \hat{\tilde{z}}^-(k)] \\ P(k) &= P^-(k) - K(k) \tilde{C}_d P^-(k) \end{aligned} \quad (2.123)$$

time update:

$$\begin{aligned} P^-(k+1) &= \tilde{A}_d(k)P(k)\tilde{A}_d^T(k) + Q(k) \\ \hat{\tilde{z}}^-(k+1) &= \tilde{A}_d(k)\hat{\tilde{z}}(k) + \tilde{B}_d(k)\tilde{v}(k) \end{aligned} \quad (2.124)$$

2.3.6 Simulation Tests

The structure of the flatness-based control scheme is depicted in Fig. 2.9. The control scheme comprises (i) the flatness-based control part which computes the control signal for the system's equivalent model that is transformed to the linear canonical form, (ii) a Kalman Filter-based disturbances estimator which provides estimates for the elements of the state vector of the DFIG, such as rotor's speed, magnetic flux at the stator as well as disturbances affecting the generator's model.

Indicative numerical values for the parameters of the considered doubly-fed induction generator model are given in Table 2.1.

Simulation tests were carried out for two different setpoints of the turn speed of the generator's rotor. The values of the generator's state vector elements are actually measured in SI units, however in the simulation results they are expressed in the per unit (p.u.) system. The results obtained for the first setpoint are depicted in Fig. 2.10

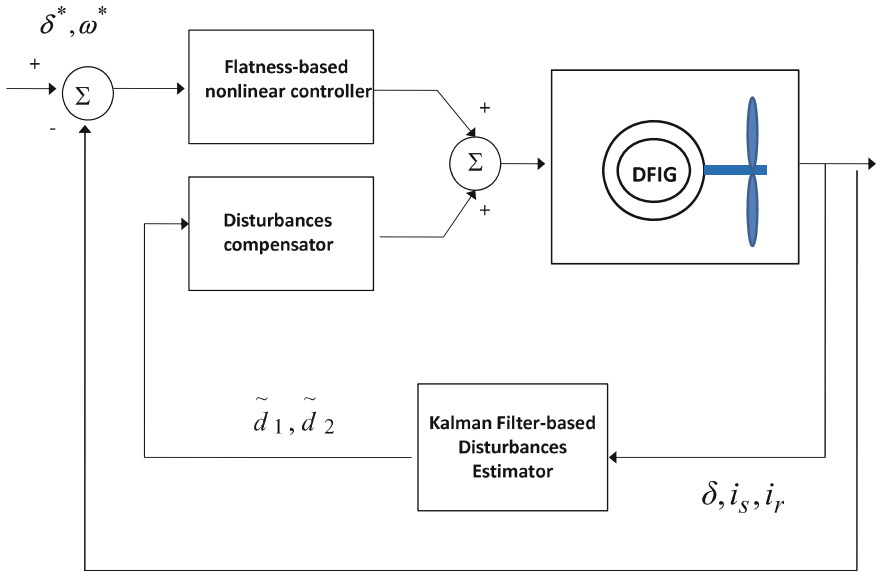
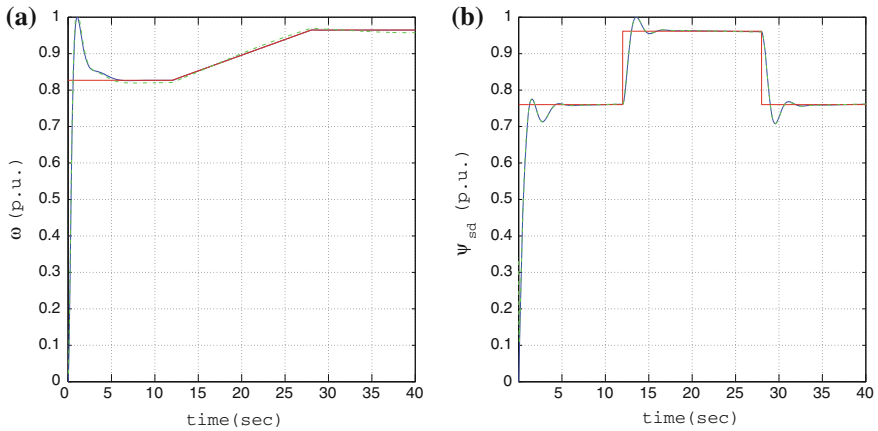


Fig. 2.9 Control loop of the DFIG comprising a flatness-based control element and an estimator for disturbances compensation

Table 2.1 Ratings of the modeled DFIG

Rated power	15.5 kW
Number of pole pairs	4
Stator resistance	0.58 Ω
Stator inductance	13 mH
Rotor resistance	1.30 Ω
Rotor inductance	3 mH
Mutual inductance	10 mH
Rotor's inertia	20.0 kg m ²

**Fig. 2.10** DFIG setpoint 1: **a** Control of state variable $x_2 = \omega$. **b** Control of state variable $x_3 = \psi_{sd}$

and Fig. 2.11. Similarly, the results obtained for the second setpoint are depicted in Figs. 2.12 and 2.13. It can be observed that the proposed control scheme assures that the rotor's turn speed follows a specific setpoint, while tracking of reference setpoints is succeeded for the components of the magnetic flux and for the rotor's currents. Several reference setpoints have been defined for the DFIG state variables, i.e. rotor's angular speed ω , rotor currents i_{rd} , i_{rq} and the magnetic flux ψ_{sd} and as it can be observed from the associated diagrams, the proposed control scheme resulted in fast and accurate convergence to these setpoints. The disturbance observer that was based on the Derivative-free nonlinear Kalman Filter was capable of estimating the unknown and time-varying input disturbances affecting the DFIG model.

The selection of the magnetic flux setpoints appearing in Figs. 2.10 and 2.12 did not aim at being restricted only to the case that the DFIG is connected to a grid, which is characterized by constant voltage amplitude and frequency. The purpose of the simulation experiments was to show the capability of the proposed nonlinear control scheme to succeed convergence to time-varying magnetic flux setpoints (e.g. piecewise constant ones). Of course, the analyzed control method for the DFIG

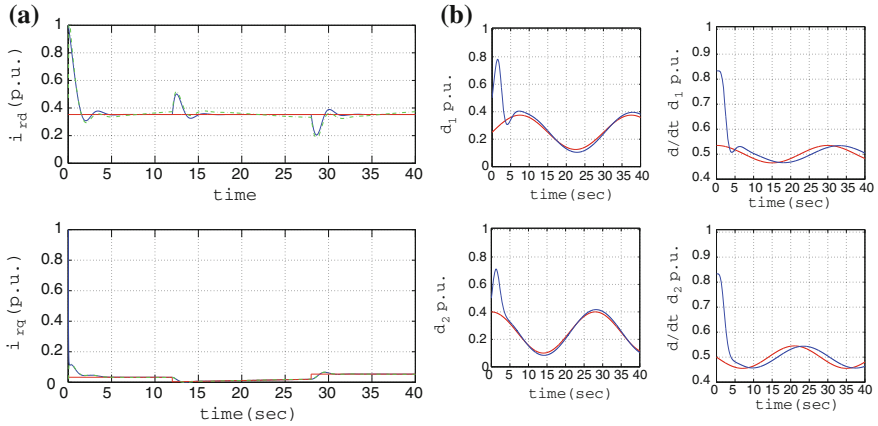


Fig. 2.11 DFIG setpoint 1: **a** Control of state variable $x_5 = i_{rd}$ and of state variable $x_6 = i_{rq}$. **b** Estimation of disturbance inputs d_i , $i = 1, 2$ and of their derivatives

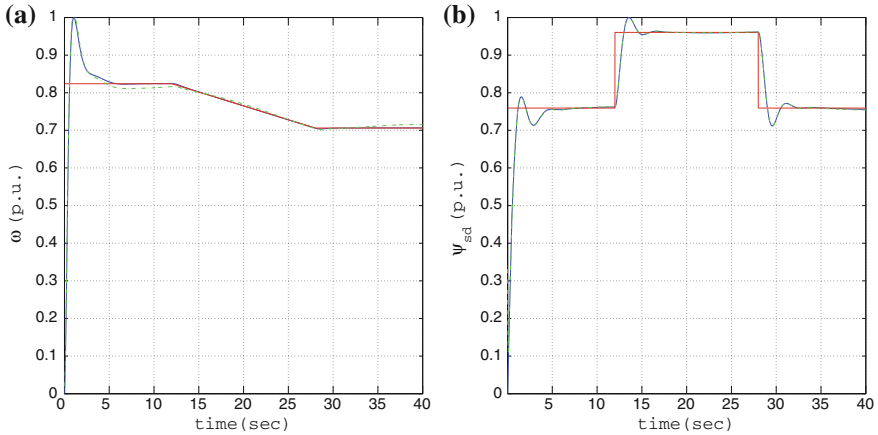


Fig. 2.12 DFIG setpoint 2: **a** Control of state variable $x_2 = \omega$. **b** Control of state variable $x_3 = \psi_{sd}$

enables also convergence of the stator's flux to constant setpoints, but this is a subcase of what has already been presented.

The improvement in the performance of the control loop that is due to the use of a disturbance observer based on the Derivative-free nonlinear Kalman Filter is explained as follows: (i) compensation of the disturbance terms which are generated by parametric uncertainty or unknown external inputs (ii) more accurate estimation of the disturbance terms because the filtering procedure is based on an exact linearization of the system's dynamics and does not introduce numerical errors (as for example in the case of the Extended Kalman Filter). This is shown in Fig. 2.14.

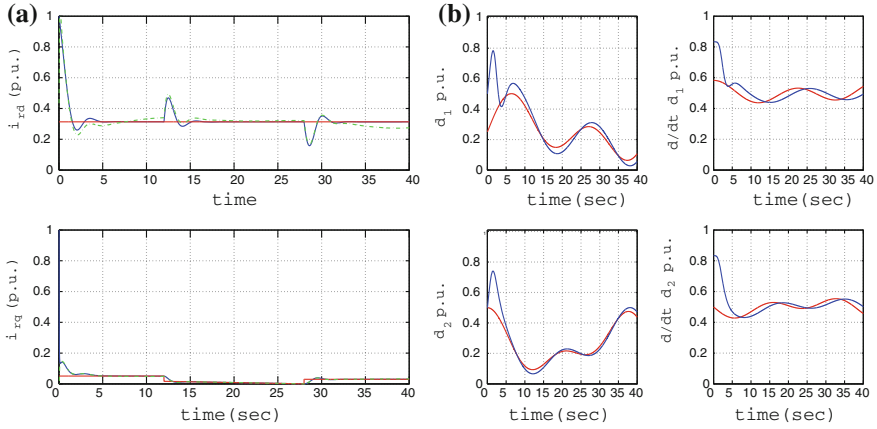


Fig. 2.13 DFIG setpoint 2: **a** Control of state variable $x_5 = i_{rd}$ and of state variable $x_6 = i_{rq}$. **b** Estimation of disturbance inputs d_i , $i = 1, 2$ and of their derivatives

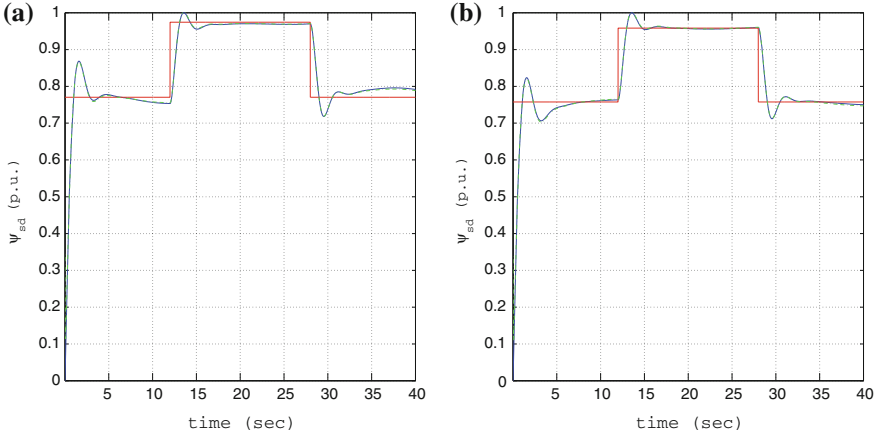


Fig. 2.14 Convergence of the stator's magnetic flux ψ_{sd} to the reference setpoint **a** without using the disturbance observer, **b** when using the disturbance observer

2.3.7 Input-Output Linearization of the DFIG Model with Use of Lie Algebra

Linearization of the DFIG model can be also performed with the use of Lie algebra. The following variables have been defined

$$\begin{aligned} h_1(x) &= x_1 = \theta \\ h_2(x) &= x_3^2 + x_4^2 = \psi_{sd}^2 + \psi_{sq}^2 \end{aligned} \quad (2.125)$$

Next, based on h_1, h_2 the following transformed state variables are defined

$$z_1 = h_1(x) = \theta \quad (2.126)$$

$$\begin{aligned} z_2 &= L_f h_1(x) \Rightarrow \\ z_2 &= f_1 \Rightarrow z_2 = x_2 \Rightarrow z_2 = \omega \end{aligned} \quad (2.127)$$

Similarly, one has

$$\begin{aligned} z_3 &= L_f^2 h_1(x) = L_f z_2 \Rightarrow \\ z_3 &= f_2 \Rightarrow z_3 = -\frac{K_m}{J} x_2 - \frac{T_m}{J} + \frac{\eta}{J} (i_{r_q} x_3 - i_{r_d} x_4) \Rightarrow \\ z_3 &= -\frac{K_m}{J} x_2 - \frac{T_m}{J} + \frac{\eta}{J} (x_6 x_3 - x_5 x_4) \end{aligned} \quad (2.128)$$

Moreover, one has

$$\dot{z}_3 = L_f^3 h_1(x) + (L_{g_a} L_f^2 h_1(x)) u_1 + (L_{g_b} L_f^2 h_1(x)) u_2 \quad (2.129)$$

It holds that

$$L_f^3 h_1(x) = L_f z_3 \quad (2.130)$$

$$\begin{aligned} L_f^3 h_1(x) &= -\frac{K_m}{J} \left[-\frac{K_m}{J} x_2 - \frac{T_m}{J} + \frac{\eta}{J} (x_6 x_3 - x_5 x_4) \right] \\ &\quad + \frac{\eta}{J} x_6 \left[-\frac{1}{\tau_s} x_3 + \omega_{dq} x_4 + \frac{M}{\tau_s} x_5 + v_{s_d} \right] \\ &\quad - \frac{\eta}{J} x_5 \left[-\omega_{dq} x_3 - \frac{1}{\tau_s} x_4 + \frac{M}{\tau_s} x_6 + v_{s_q} \right] \\ &\quad - \frac{\eta}{J} x_4 \left[-\beta x_2 x_4 + \frac{\beta}{\tau_s} x_3 + (\omega_{dq} - x_2) x_6 - \gamma_2 x_5 - \beta v_{s_d} \right] \\ &\quad + \frac{\eta}{J} x_3 \left[\frac{\beta}{\tau_s} x_4 + \beta x_2 x_3 + (\omega_{dq} - x_2) x_5 - \gamma_2 x_6 - \beta v_{s_q} \right] \end{aligned} \quad (2.131)$$

Equivalently one has

$$\begin{aligned} L_{g_a}(L_f^2 h_1(x)) &= L_{g_a} z_3 \Rightarrow \\ L_{g_a}(L_f^2 h_1(x)) &= -\frac{\eta}{J} \frac{1}{\sigma L_r} x_4 \end{aligned} \quad (2.132)$$

and similarly

$$\begin{aligned} L_{g_b}(L_f^2 h_1(x)) &= L_{g_b} z_3 \Rightarrow \\ L_{g_b}(L_f^2 h_1(x)) &= \frac{\eta}{J} \frac{1}{\sigma L_r} x_3 \end{aligned} \quad (2.133)$$

For the transformed state variable z_4 one has

$$z_4 = h_2(x) = \psi_{s_d}^2 + \psi_{s_q}^2 = x_3^2 + x_4^2 \quad (2.134)$$

and

$$\begin{aligned} z_5 &= L_f h_2(x) \Rightarrow z_5 = 2x_3 f_3 + 2x_4 f_4 \Rightarrow \\ z_5 &= 2x_3 \left[-\frac{1}{\tau_s} x_3 + \omega_{dq} x_4 + \frac{M}{\tau_s} x_5 + v_{s_d} \right] + \\ &\quad + 2x_4 \left[-\omega_{dq} x_3 - \frac{1}{\tau_s} x_4 + \frac{M}{\tau_s} x_6 + v_{s_q} \right] \end{aligned} \quad (2.135)$$

and equivalently one has

$$\dot{z}_5 = L_f^2 h_2(x) + L_{g_a}(L_f h_2(x))u_1 + L_{g_b}(L_f h_2(x))u_2 \quad (2.136)$$

It holds that

$$\begin{aligned} L_f^2 h_2(x) = & \left(-\frac{4}{\tau_s}x_3 - \frac{2M}{\tau_s}x_5 + 2v_{s_d}\right)\left[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}\right] + \\ & \left(-\frac{4}{\tau_s}x_4 + \frac{2M}{\tau_s}x_6 + 2v_{s_q}\right)\left[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}\right] + \\ & \left(\frac{2M}{\tau_s}x_3\right)\left[-\beta x_2x_4 + \frac{\beta}{\tau_s}x_3 + (\omega_{dq} - x_2)x_6 - \gamma_2x_5 - \beta v_{s_d}\right] + \\ & \left(\frac{2M}{\tau_s}x_3\right)\left[\frac{\beta}{\tau_s}x_4 + \beta x_2x_3 + (\omega_{dq} - x_2)x_5 - \gamma_2x_6 - \beta v_{s_q}\right] \end{aligned} \quad (2.137)$$

Moreover, it holds that

$$L_{g_a}(L_f h_2(x)) = \frac{2M}{\tau_s}x_3g_{a5} \Rightarrow L_{g_a}(L_f h_2(x)) = \frac{2M}{\tau_s} \frac{1}{\sigma L_s}x_3 \quad (2.138)$$

and in a similar manner

$$\begin{aligned} L_{g_b}(L_f h_2(x)) &= \frac{2M}{\tau_s}x_4g_{a6} \Rightarrow \\ L_{g_b}(L_f h_2(x)) &= \frac{2M}{\tau_s} \frac{1}{\sigma L_s}x_4 \end{aligned} \quad (2.139)$$

Next, it is confirmed that after change of the state variables it holds

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= L_f^3 h_1(x) + L_{g_a}(L_f^2 h_1(x))u_1 + L_{g_b}(L_f^2 h_1(x))u_2 \\ \dot{z}_4 &= z_5 \\ \dot{z}_5 &= L_f^2 h_2(x) + L_{g_a}(L_f h_2(x))u_1 + L_{g_b}(L_f h_2(x))u_2 \end{aligned} \quad (2.140)$$

It holds that $z_1 = \theta$, $\dot{z}_1 = \omega = z_2$, $\dot{z}_2 = \dot{\omega} = f_2(x) + g_{a2}u_1 + g_{b2}u_2 \Rightarrow \dot{z}_2 = f_2(x) + 0u_1 + 0u_2$ which finally gives $\dot{z}_2 = f_2(x)$. Moreover, it has been proven that $z_3 = f_2$ therefore it holds $\dot{z}_2 = z_3$. Additionally, it holds that

$$\dot{z}_3 = \frac{\partial z_3}{\partial x_1}\dot{x}_1 + \frac{\partial z_3}{\partial x_2}\dot{x}_2 + \frac{\partial z_3}{\partial x_3}\dot{x}_3 + \frac{\partial z_3}{\partial x_4}\dot{x}_4 + \frac{\partial z_3}{\partial x_5}\dot{x}_5 + \frac{\partial z_3}{\partial x_6}\dot{x}_6 \quad (2.141)$$

which in turn gives

$$\begin{aligned} \dot{z}_3 &= \frac{\partial z_3}{\partial x_1}f_1 + \frac{\partial z_3}{\partial x_2}f_2 + \frac{\partial z_3}{\partial x_3}f_3 + \frac{\partial z_3}{\partial x_4}f_4 + \\ &+ \frac{\partial z_3}{\partial x_5}(f_5 + \frac{1}{\sigma L_r}u_1) + \frac{\partial z_3}{\partial x_6}(f_6 + \frac{1}{\sigma L_r}u_2) \end{aligned} \quad (2.142)$$

that is also written as

$$\dot{z}_3 = L_f^3 h_1(x) + L_{g_a}(L_f^2 h_1(x))u_1 + L_{g_b}(L_f^2 h_1(x))u_2 \quad (2.143)$$

Similarly, one has

$$\begin{aligned} z_4 &= x_3^2 + x_4^2 \Rightarrow \\ \dot{z}_4 &= 2x_3\dot{x}_3 + 2x_4\dot{x}_4 \Rightarrow \dot{z}_4 = 2x_3f_3 + 2x_4f_4 \\ \dot{z}_4 &= 2x_3[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{sd}] + \\ &+ 2x_4[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{sq}] \Rightarrow \dot{z}_4 = z_5 \end{aligned} \quad (2.144)$$

Additionally, it holds

$$\dot{z}_5 = \frac{\partial z_5}{\partial x_1}\dot{x}_1 + \frac{\partial z_5}{\partial x_2}\dot{x}_2 + \frac{\partial z_5}{\partial x_3}\dot{x}_3 + \frac{\partial z_5}{\partial x_4}\dot{x}_4 + \frac{\partial z_5}{\partial x_5}\dot{x}_5 + \frac{\partial z_5}{\partial x_6}\dot{x}_6 \quad (2.145)$$

which in turn gives

$$\begin{aligned} \dot{z}_5 &= \frac{\partial z_5}{\partial x_1}f_1 + \frac{\partial z_5}{\partial x_2}f_2 + \frac{\partial z_5}{\partial x_3}f_3 + \frac{\partial z_5}{\partial x_4}f_4 + \\ &+ \frac{\partial z_5}{\partial x_5}(f_5 + \frac{1}{\sigma L_r}u_1) + \frac{\partial z_5}{\partial x_6}(f_6 + \frac{1}{\sigma L_r}u_2) \end{aligned} \quad (2.146)$$

which subsequently gives

$$\dot{z}_5 = L_f^2 h_2(x) + L_{g_a}(L_f h_2(x))u_1 + L_{g_b}(L_f h_2(x))u_2 \quad (2.147)$$

which is the anticipated relation about $\dot{z}_5$. Consequently, Eq. (2.82) is confirmed to hold. The system of Eq. (2.82) can be written in the input-output linearized form

$$\begin{pmatrix} \dot{z}_1^{(3)} \\ \dot{z}_4 \end{pmatrix} = f_a + \tilde{M}u \quad (2.148)$$

where

$$\begin{aligned} f_a(x) &= \begin{pmatrix} L_f^3 h_1(x) \\ L_f^2 h_2(x) \end{pmatrix} \\ \tilde{M} &= \begin{pmatrix} L_{g_a} L_f^2 h_1(x) & L_{g_b} L_f^2 h_2(x) \\ L_{g_a} L_f h_1(x) & L_{g_b} L_f h_2(x) \end{pmatrix} \end{aligned} \quad (2.149)$$

or equivalently one has the system's description in the MIMO canonical form

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (2.150)$$

where

$$\begin{aligned} v_1 &= L_f^3 h_1(x) + L_{g_a}(L_f^2 h_1(x))u_1 + L_{g_b}(L_f^2 h_2(x))u_2 \\ v_2 &= L_f^2 h_2(x) + L_{g_a}(L_f h_1(x))u_1 + L_{g_b}(L_f h_2(x))u_2 \end{aligned} \quad (2.151)$$

2.4 Nonlinear H-Infinity Control of DFIGs

2.4.1 Outline

Control methods for DFIGs based on local linearization approaches have been developed [182, 208, 517]. In particular about nonlinear H-infinity control, the method has been primarily applied to other electromechanical systems but exhibits also the potential to be applied to DFIGs too [373, 443]. In all these methods the objective is to succeed the reliable operation of the DFIG under variable speed so as to optimize power generation and to modify the amount of the produced power in accordance to the grid's needs [161].

In this section a new control method for DFIGs is developed, based on nonlinear H-infinity control theory. The application of an approximate linearization scheme for the dynamic model of the DFIG is proposed, based on Taylor series expansion round the generator's present operating point. To perform this linearization the computation of Jacobian matrices is needed while the induced linearization error terms are treated as disturbances. For the linearized equivalent of the power generator's model an H_∞ feedback control scheme is developed. The formulation of the H_∞ control problem is based on the minimization of a quadratic cost function that comprises both the disturbance and the control input effects. The disturbance tries to maximize the cost function while the control signal tries to minimize it, within a mini-max differential game. The efficiency of the proposed nonlinear H_∞ control scheme has been tested through simulation experiments, which have shown a satisfactory performance.

Comparing to nonlinear feedback control approaches which are based on exact feedback linearization of the DFIG (as the ones based on differential flatness theory and analyzed in the previous sections and in Ref. [352, 366, 375]) the proposed H_∞ control scheme is assessed as follows: (i) it uses an approximate linearization approach of the system's dynamic model which does not follow the elaborated transformations (diffeomorphisms) of the exact linearization methods, (ii) it introduces additional disturbance error which is due to the approximate linearization of the system dynamics coming from the application of Taylor series expansion, (iii) it requires the computation of Jacobian matrices, which in the case of the sixth-order DFIG model can be also a tedious procedure, (iv) unlike exact feedback linearization, the H_∞ control term has to compensate not only for modelling uncertainties and external disturbances but needs also to annihilate the effects of the cumulative linearization error, (v) the H_∞ control approach follows an optimal control method for the computation of the control signal, however unlike exact feedback linearization

control it requires the solution of Riccati equations which for the sixth-order DFIG model can be again a burdensome procedure.

2.4.2 Approximate Linearization of the Doubly-Fed Induction Generator's Dynamic Model

The nonlinear state space equation of the doubly-fed induction generator is given by

$$\dot{x} = f(x) + g_a v_{r_d} + g_b v_{r_q} \quad (2.152)$$

where the state vector has been defined as $x = [\theta, \omega, \psi_{s_d}, \psi_{s_q}, i_{r_d}, i_{r_q}]^T$ while functions $f(x)$, $g_a(x)$ and $g_b(x)$ have been defined as

$$f(x) = \begin{pmatrix} x_2 \\ -\frac{K_m}{J}x_2 - \frac{T_m}{J} + \frac{\eta}{J}(i_{r_q}x_3 - i_{r_d}x_4) \\ -\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d} \\ -\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q} \\ -\beta x_2x_4 + \frac{\beta}{\tau_s}x_3 + (\omega_{dq} - x_2)x_6 - \gamma_2x_3 - \beta v_{s_d} \\ \frac{\beta}{\tau_s}x_4 + \beta x_2x_3 + (\omega_{dq} - x_2)x_5 - \gamma_2x_6 - \beta v_{s_q} \end{pmatrix} \quad (2.153)$$

$$g_a(x) = \left(0 \ 0 \ 0 \ 0 \ \frac{1}{\sigma L_s} \ 0\right)^T \quad (2.154)$$

$$g_b(x) = \left(0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{\sigma L_s}\right)^T \quad (2.155)$$

The measured state variables are taken to be $x_1 = \theta$, $x_5 = i_{r_d}$ and $x_6 = i_{r_q}$. Then, the associated Jacobian matrices are:

$$A = J_f = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{K_m}{J} & \frac{\eta}{J}x_6 & -\frac{\eta}{J}x_5 & -\frac{\eta}{J}x_4 & \frac{\eta}{J}x_3 \\ 0 & 0 & -\frac{1}{\tau_s} & \omega_{dq} & \frac{M}{\tau_s} & 0 \\ 0 & 0 & -\omega_{dq} & -\frac{1}{\tau_s} & 0 & \frac{M}{\tau_s} \\ 0 & -\beta x_4 - x_6 & -\gamma_2 + \frac{\beta}{\tau_s} & -\beta x_2 & 0 & \omega_{dq} - x_2 \\ 0 & \beta x_3 - x_5 & \beta x_2 & \frac{\beta}{\tau_s} & \omega_{dq} - x_2 & -\gamma_2 \end{pmatrix} \quad (2.156)$$

Moreover, linearization of the the generator's dynamics with respect to the control input variables u_1, u_2 gives

$$B = [J_{g_a} J_{g_b}] = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \end{pmatrix} \quad (2.157)$$

After Taylor series expansion linearization round its current operating point, the DFIG's dynamic model is written as

$$\dot{x} = Ax + Bu + d_1 \quad (2.158)$$

Parameter d_1 stands for the linearization error in the generator's dynamic model appearing in Eq. (2.158). The reference setpoints for the doubly-fed induction generator are denoted by $\mathbf{x}_d = [x_1^d, \dots, x_6^d]$. Tracking of this trajectory is succeeded after applying the control input u^* . At every time instant the control input u^* is assumed to differ from the control input u appearing in Eq. (2.158) by an amount equal to Δu , that is $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2 \quad (2.159)$$

The dynamics of the controlled system described in Eq. (2.158) can be also written as

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \quad (2.160)$$

and by denoting $d_3 = -Bu^* + d_1$ as an aggregate disturbance term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \quad (2.161)$$

By subtracting Eq. (2.159) from Eq. (2.161) one has

$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2 \quad (2.162)$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\tilde{d} = d_3 - d_2$, the tracking error dynamics becomes

$$\dot{e} = Ae + Bu + \tilde{d} \quad (2.163)$$

The above linearized form of the generator's model can be efficiently controlled after applying an H-infinity feedback control scheme.

2.4.3 The Nonlinear H -Infinity Control

2.4.3.1 Mini-Max Control and Disturbance Rejection

The initial nonlinear model of the induction generator is in the form

$$\dot{x} = f(x, u) \quad x \in R^n, \quad u \in R^m \quad (2.164)$$

Linearization of the system (doubly-fed induction generator) is performed at each iteration of the control algorithm round its present operating point $(x^*, u^*) = (x(t), u(t - T_s))$. The linearized equivalent of the system is described by

$$\dot{x} = Ax + Bu + L \quad x \in R^n, \quad u \in R^m, \quad d \in R^q \quad (2.165)$$

where matrices A and B are obtained from the computation of the Jacobians

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \Big|_{(x^*, u^*)} \quad B = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \dots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \dots & \frac{\partial f_2}{\partial u_m} \\ \dots & \dots & \dots & \dots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \dots & \frac{\partial f_n}{\partial u_m} \end{pmatrix} \Big|_{(x^*, u^*)} \quad (2.166)$$

and vector d denotes disturbance terms due to linearization errors. The problem of disturbance rejection for the linearized model that is described by

$$\begin{aligned} \dot{x} &= Ax + Bu + Ld \\ y &= Cx \end{aligned} \quad (2.167)$$

where $x \in R^n$, $u \in R^m$, $d \in R^q$ and $y \in R^p$, cannot be handled efficiently if the classical LQR control scheme is applied. This is because of the existence of the perturbation term d . The disturbance term d apart from modeling (parametric) uncertainty and external perturbation terms can also represent noise terms of any distribution.

In the H_∞ control approach, a feedback control scheme is designed for trajectory tracking by the system's state vector and simultaneous disturbance rejection, considering that the disturbance affects the system in the worst possible manner. The disturbances' effect are incorporated in the following quadratic cost function:

$$J(t) = \frac{1}{2} \int_0^T [y^T(t)y(t) + ru^T(t)u(t) - \rho^2 d^T(t)d(t)] dt, \quad r, \rho > 0 \quad (2.168)$$

The significance of the negative sign in the cost function's term that is associated with the perturbation variable $d(t)$ is that the disturbance tries to maximize the cost function $J(t)$ while the control signal $u(t)$ tries to minimize it. The physical meaning of the relation given above is that the control signal and the disturbances compete to each other within a mini-max differential game. This problem of mini-max optimization can be written as

$$\min_u \max_d J(u, d) \quad (2.169)$$

The objective of the optimization procedure is to compute a control signal $u(t)$ which can compensate for the worst possible disturbance, the latter being externally imposed to the system. However, the solution to the mini-max optimization problem is directly related to the value of parameter ρ . This means that there is an upper bound in the disturbances magnitude that can be annihilated by the control signal.

2.4.3.2 H-Infinity Feedback Control

For the linearized system given by Eq. (2.167) the cost function of Eq. (2.168) is defined, where the coefficient r determines the penalization of the control input and the weight coefficient ρ determines the reward of the disturbances' effects. It is assumed that:

It is assumed that (i) The energy that is transferred from the disturbances signal $d(t)$ is bounded, that is $\int_0^\infty d^T(t)d(t)dt < \infty$, (ii) the matrices $[A/B]$ and $[A/L]$ are stabilizable, (iii) the matrix $[A/C]$ is detectable. Then, the optimal feedback control law is given by

$$u(t) = -Kx(t) \quad (2.170)$$

with

$$K = \frac{1}{r} B^T P \quad (2.171)$$

where P is a positive semi-definite symmetric matrix which is obtained from the solution of the Riccati equation

$$A^T P + PA + Q - P(\frac{1}{r} BB^T - \frac{1}{2\rho^2} LL^T)P = 0 \quad (2.172)$$

where Q is also a positive definite symmetric matrix. The worst case disturbance is given by

$$d(t) = \frac{1}{\rho^2} L^T P x(t) \quad (2.173)$$

2.4.3.3 The Role of Riccati Equation Coefficients in H_∞ Control Robustness

The parameter ρ in Eq. (2.168), is an indication of the closed-loop system robustness. If the values of $\rho > 0$ are excessively decreased with respect to r , then the solution of the Riccati equation is no longer a positive definite matrix. Consequently there is a lower bound ρ_{min} of ρ for which the H_∞ control problem has a solution. The acceptable values of ρ lie in the interval $[\rho_{min}, \infty)$. If ρ_{min} is found and used in the design of the H_∞ controller, then the closed-loop system will have increased

robustness. Unlike this, if a value $\rho > \rho_{min}$ is used, then an admissible stabilizing H_∞ controller will be derived but it will be a suboptimal one. The Hamiltonian matrix

$$H = \begin{pmatrix} A & -(\frac{1}{\rho}BB^T - \frac{1}{\rho^2}LL^T) \\ -Q & -A^T \end{pmatrix} \quad (2.174)$$

provides a criterion for the existence of a solution of the Riccati equation Eq. (2.172). A necessary condition for the solution of the algebraic Riccati equation to be a positive semi-definite symmetric matrix is that H has no imaginary eigenvalues [352].

2.4.4 Lyapunov Stability Analysis

Through Lyapunov stability analysis it will be shown that the proposed nonlinear control scheme assures H_∞ tracking performance, and that in case of bounded disturbance terms asymptotic convergence to the reference setpoints is succeeded. The tracking error dynamics for the doubly-fed induction generator is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d} \quad (2.175)$$

where in the generator's application example $L = I \in \mathbb{R}^2$ with I being the identity matrix. The following Lyapunov equation is considered

$$V = \frac{1}{2}e^T P e \quad (2.176)$$

where $e = x - x_d$ is the tracking error. By differentiating with respect to time one obtains

$$\begin{aligned} \dot{V} &= \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} \Rightarrow \\ \dot{V} &= \frac{1}{2}[Ae + Bu + L\tilde{d}]^T P + \frac{1}{2}e^T P [Ae + Bu + L\tilde{d}] \Rightarrow \end{aligned} \quad (2.177)$$

$$\begin{aligned} \dot{V} &= \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T] P e + \\ &+ \frac{1}{2}e^T P [Ae + Bu + L\tilde{d}] \Rightarrow \end{aligned} \quad (2.178)$$

$$\begin{aligned} \dot{V} &= \frac{1}{2}e^T A^T P e + \frac{1}{2}u^T B^T P e + \frac{1}{2}\tilde{d}^T L^T P e + \\ &\frac{1}{2}e^T P A e + \frac{1}{2}e^T P B u + \frac{1}{2}e^T P L \tilde{d} \end{aligned} \quad (2.179)$$

The previous equation is rewritten as

$$\begin{aligned} \dot{V} &= \frac{1}{2}e^T (A^T P + P A) e + (\frac{1}{2}u^T B^T P e + \frac{1}{2}e^T P B u) + \\ &+ (\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2}e^T P L \tilde{d}) \end{aligned} \quad (2.180)$$

Assumption: For given positive definite matrix Q and coefficients r and ρ there exists a positive definite matrix P , which is the solution of the following matrix equation

$$A^T P + P A = -Q + P \left(\frac{2}{r} B B^T - \frac{1}{\rho^2} L L^T \right) P \quad (2.181)$$

Moreover, the following feedback control law is applied to the system

$$u = -\frac{1}{r} B^T P e \quad (2.182)$$

By substituting Eq. (2.181) and Eq. (2.182) one obtains

$$\begin{aligned} \dot{V} = & \frac{1}{2} e^T [-Q + P \left(\frac{1}{r} B B^T - \frac{1}{2\rho^2} L L^T \right) P] e + \\ & + e^T P B \left(-\frac{1}{r} B^T P e \right) + e^T P L \tilde{d} \Rightarrow \end{aligned} \quad (2.183)$$

$$\begin{aligned} \dot{V} = & -\frac{1}{2} e^T Q e + \left(\frac{1}{r} P B B^T P e - \frac{1}{2\rho^2} e^T P L L^T P e \right. \\ & \left. - \frac{1}{r} e^T P B B^T P e \right) + e^T P L \tilde{d} \end{aligned} \quad (2.184)$$

which after intermediate operations gives

$$\dot{V} = \frac{1}{2} e^T Q e - \frac{1}{2\rho^2} e^T P L L^T P e + e^T P L \tilde{d} \quad (2.185)$$

or, equivalently

$$\begin{aligned} \dot{V} = & \frac{1}{2} e^T Q e - \frac{1}{2\rho^2} e^T P L L^T P e + \\ & + \frac{1}{2} e^T P L \tilde{d} + \frac{1}{2} \tilde{d}^T L^T P e \end{aligned} \quad (2.186)$$

Lemma: The following inequality holds

$$\frac{1}{2} e^T L \tilde{d} + \frac{1}{2} \tilde{d}^T L^T P e - \frac{1}{2\rho^2} e^T P L L^T P e \leq \frac{1}{2} \rho^2 \tilde{d}^T \tilde{d} \quad (2.187)$$

Proof: The binomial $(\rho a - \frac{1}{\rho} b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\begin{aligned} \rho^2 a^2 + \frac{1}{\rho^2} b^2 - 2ab & \geq 0 \Rightarrow \frac{1}{2} \rho^2 a^2 + \frac{1}{2\rho^2} b^2 - ab \geq 0 \Rightarrow \\ ab - \frac{1}{2\rho^2} b^2 & \leq \frac{1}{2} \rho^2 a^2 \Rightarrow \frac{1}{2} ab + \frac{1}{2} ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2} \rho^2 a^2 \end{aligned} \quad (2.188)$$

The following substitutions are carried out: $a = \tilde{d}$ and $b = e^T P L$ and the previous relation becomes

$$\frac{1}{2} \tilde{d}^T L^T P e + \frac{1}{2} e^T P L \tilde{d} - \frac{1}{2\rho^2} e^T P L L^T P e \leq \frac{1}{2} \rho^2 \tilde{d}^T \tilde{d} \quad (2.189)$$

Equation (2.189) is substituted in Eq. (2.186) and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (2.190)$$

Equation (2.190) shows that the H_∞ tracking performance criterion is satisfied. The integration of $\dot{V}$ from 0 to T gives

$$\begin{aligned} \int_0^T \dot{V}(t) dt &\leq -\frac{1}{2} \int_0^T \|e\|_Q^2 dt + \frac{1}{2} \rho^2 \int_0^T \|\tilde{d}\|^2 dt \Rightarrow \\ 2V(T) + \int_0^T \|e\|_Q^2 dt &\leq 2V(0) + \rho^2 \int_0^T \|\tilde{d}\|^2 dt \end{aligned} \quad (2.191)$$

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^\infty \|\tilde{d}\|^2 dt \leq M_d \quad (2.192)$$

then one gets

$$\int_0^\infty \|e\|_Q^2 dt \leq 2V(0) + \rho^2 M_d \quad (2.193)$$

Thus, the integral $\int_0^\infty \|e\|_Q^2 dt$ is bounded. Moreover, $V(T)$ is bounded and from the definition of the Lyapunov function V in Eq. (2.176) it becomes clear that $e(t)$ will be also bounded since $e(t) \in \Omega_e = \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}$. According to the above and with the use of Barbalat's Lemma one obtains $\lim_{t \rightarrow \infty} e(t) = 0$.

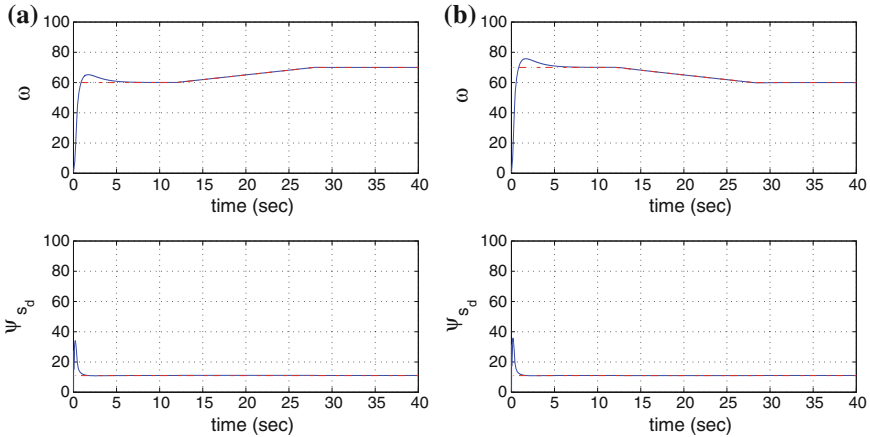


Fig. 2.15 Nonlinear H_∞ control of the doubly-fed induction generator. **a** Convergence of the rotor's angular speed and stator's magnetic flux to setpoint 1 **b** Convergence of the rotor's angular and stator's magnetic flux to setpoint 2

2.4.5 Simulation Tests

The performance of the proposed nonlinear H_∞ control scheme for DFIGs is tested in tracking of various setpoints. As it can be observed in Fig. 2.15 the feedback control scheme of the DFIG enabled accurate convergence to the reference setpoints. Yet simple, the considered H_∞ control law succeeded precise tracking of the reference signals. In comparison to feedback control methods for asynchronous generators which are based on exact linearization, the nonlinear H_∞ control requires the solution of an algebraic Riccati equation at each iteration of the control algorithm. The known robustness features of H_∞ control are the ones that permit to compensate for the approximation errors which were induced to the linearized model of the generator.

2.5 Flatness-Based Adaptive Fuzzy Control of DFIGs

2.5.1 Overview

This section presents results on model-free control of Doubly-fed Induction Generators (DFIGs) with the use of differential flatness theory and adaptive control theory. As previously explained, the problem of control of the DFIGs is a nontrivial one because the dynamic model of such electric machines is a multi-variable and nonlinear one. Moreover, under real operating conditions it exhibits parametric variations and is subjected to external perturbations. As a result empirical model-free control approaches of the PID-type lack stability and robustness, remain functional only round local operating points and in general are proven to be inefficient. DFIG control in stand-alone functioning mode has been analyzed in [56, 62, 140, 148, 279, 319, 357, 378, 379].

In particular control schemes exhibiting adaptivity features have been presented in [1, 463]. Distributed control of DFIGs and their functioning when interacting with the power grid has been studied in [473, 482]. The performance of specific DFIG control schemes under external perturbations has been presented in [8, 481, 489, 490]. Moreover, the problem of sensorless control of DFIGs has been studied in [141, 320, 486].

To cope with the DFIG control problem, in this section the differential flatness properties of the DFIG model are exploited once more. It is proven that the rotor's turn speed and the stator's magnetic flux of this electric machine are flat outputs which means that all other state variables and the control inputs of the DFIG can be expressed as differential functions of these outputs [57, 137, 241, 352, 388, 411, 412, 456]. Moreover, it is proven that the flat outputs and their derivatives are differentially independent. This comes to complete the proof about the differential flatness of the DFIG model [280, 375].

The differential flatness of the DFIG implies also that the transformation of its dynamic model to the linear canonical (Brunovsky) form is possible. After such a transformation the implementation of a stabilizing feedback controller for the generator

is enabled. This solution to the control problem would be complete if no model uncertainty and parametric variations of the DFIG existed. Actually, it is assumed that only the order of the DFIG's state-space model is known while its parameters and the specific form of its state-space equations are completely unknown. In the linearized equivalent model of the generator this uncertainty takes the form of unknown functions which appear as part of the system's transformed control inputs. This unknown part of the system's dynamics is identified with the use of neurofuzzy approximators. The estimates of the DFIG's dynamics are used by a state feedback controller thus establishing an indirect adaptive control scheme. The learning procedure for these estimators is defined by the requirement to assure negative definiteness for the system's Lyapunov function [352, 366, 375]. Another issue that arises is that the state vector that is fed into the controller is partially measurable, and the missing state vector elements have to be estimated with the use of a state observer. By including such an observer in the control loop an output feedback-based control scheme is established. The stability of the control method is proven through Lyapunov stability analysis.

2.5.2 Flatness-Based Adaptive Neurofuzzy Control

2.5.2.1 Transformation of MIMO Nonlinear Systems into the Brunovsky Form

Following the previous procedure, that is after defining the flat outputs of the initial MIMO nonlinear model of the DFIG, that was described in Sect. 2.3 and after expressing the system's state variables and control inputs as functions of the flat output and of the associated derivatives, the DFIG is transformed into the Brunovsky canonical form of Eq. (2.108), or equivalently into the form [352, 366, 375, 378]:

$$\begin{aligned}
 \dot{x}_1 &= x_2 \\
 \dot{x}_2 &= x_3 \\
 &\dots \\
 \dot{x}_{r_1-1} &= x_{r_1} \\
 \dot{x}_{r_1} &= f_1(x) + \sum_{j=1}^p g_{1j}(x)u_j + d_1 & y_1 &= x_1 \\
 & & y_2 &= x_{r_1-1} \\
 & & \dots & \\
 \dot{x}_{r_1+1} &= x_{r_1+2} & y_p &= x_{n-r_p+1} \\
 \dot{x}_{r_1+2} &= x_{r_1+3} \\
 &\dots \\
 \dot{x}_{p-1} &= x_p \\
 \dot{x}_p &= f_p(x) + \sum_{j=1}^p g_{pj}(x)u_j + d_p
 \end{aligned} \tag{2.194}$$

where $x = [x_1, \dots, x_n]^T$ is the state vector of the transformed system (according to the differential flatness formulation), $u = [u_1, \dots, u_p]^T$ is the set of control inputs, $y = [y_1, \dots, y_p]^T$ is the output vector, f_i are the drift functions and $g_{i,j}$, $i, j = 1, 2, \dots, p$ are smooth functions corresponding to the control input gains, while d_j

is a variable associated to external disturbances. It holds that $r_1 + r_2 + \dots + r_p = n$. Having written the initial nonlinear system into the canonical (Brunovsky) form it holds

$$y_i^{(r_i)} = f_i(x) + \sum_{j=1}^p g_{ij}(x)u_j + d_j \quad (2.195)$$

Equivalently, in vector form, one has the following description for the system dynamics

$$y^{(r)} = f(x) + g(x)u + d \quad (2.196)$$

where the following vectors and matrices can be defined

$$\begin{aligned} y^{(r)} &= [y_1^{(r_1)}, \dots, y_p^{(r_p)}] \\ f(x) &= [f_1(x), \dots, f_p(x)]^T \\ g(x) &= [g_1(x), \dots, g_p(x)] \\ \text{with } g_i(x) &= [g_{1i}(x), \dots, g_{pi}(x)]^T \\ A &= \text{diag}[A_1, \dots, A_p], \quad B = \text{diag}[B_1, \dots, B_p] \\ C^T &= \text{diag}[C_1, \dots, C_p], \quad d = [d_1, \dots, d_p]^T \end{aligned} \quad (2.197)$$

where matrix A has the MIMO canonical form, i.e. with elements

$$A_i = \begin{pmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{pmatrix}_{r_i \times r_i} \quad B_i = \begin{pmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 1 \end{pmatrix}_{r_i \times 1} \quad C_i^T = \begin{pmatrix} 1 \\ 0 \\ \dots \\ 0 \\ 0 \end{pmatrix}_{r_i \times 1} \quad (2.198)$$

Thus, Eq. (2.195) can be written in state-space form

$$\begin{aligned} \dot{x} &= Ax + B[f(x) + g(x)u + \tilde{d}] \\ y &= C^T x \end{aligned} \quad (2.199)$$

which can be also written in the equivalent form:

$$\begin{aligned} \dot{x} &= Ax + Bv + B\tilde{d} \\ y &= C^T x \end{aligned} \quad (2.200)$$

where $v = f(x) + g(x)u$. The reference setpoints for the system's outputs $y_1, \dots, y_p$ are denoted as $y_{1m}, \dots, y_{pm}$, thus for the associated tracking errors it holds

$$\begin{aligned} e_1 &= y_1 - y_{1m} \\ e_2 &= y_2 - y_{2m} \\ &\dots \\ e_p &= y_p - y_{pm} \end{aligned} \quad (2.201)$$

The error vector of the outputs of the transformed MIMO system is denoted as

$$\begin{aligned} E_1 &= [e_1, \dots, e_p]^T \\ y_m &= [y_{1m}, \dots, y_{pm}]^T \\ &\dots \\ y_m^{(r)} &= [y_{1m}^{(r)}, \dots, y_{pm}^{(r)}]^T \end{aligned} \quad (2.202)$$

where $y_{im}^{(r)}$ denotes the r -th order derivative of the i -th reference output of the MIMO dynamical system. Thus, one can also define the following vectors: (i) a vector containing the state variables of the system and the associated derivatives, (ii) a vector containing the reference outputs of the system and the associated derivatives

$$x = [x_1, \dots, x_1^{r_1-1}, \dots, x_p, \dots, x_p^{r_p-1}]^T \quad (2.203)$$

$$Y_m = [y_{1m}, \dots, y_{1m}^{r_1-1}, \dots, y_{pm}, \dots, y_{pm}^{r_p-1}]^T \quad (2.204)$$

while in a similar manner one can define a vector containing the tracking error of the system's outputs and the associated derivatives

$$e = Y_m - x = [e_1, \dots, e_1^{r_1-1}, \dots, e_p, \dots, e_p^{r_p-1}]^T \quad (2.205)$$

It is assumed that matrix $g(x)$ is a nonsingular one, i.e. $g^{-1}(x)$ exists and is bounded for all $x \in U_x$, where $U_x \subset R^n$ is a compact set. In any case, the problem of singularities in matrix $g(x)$ can be handled by appropriately modifying the state feedback-based control input.

The objective of the adaptive neurofuzzy controller, denoted as $u = u(x, e|\theta)$ is: (i) all the signals involved in the controller's design are bounded and it holds that $\lim_{t \rightarrow \infty} e = 0$, (ii) the H_∞ tracking performance criterion is succeeded for a prescribed attenuation level.

In the presence of non-gaussian disturbances w_d , successful tracking of the reference signal is denoted by the H_∞ criterion [375, 379]:

$$\int_0^T e^T Q e dt \leq \rho^2 \int_0^T w_d^T w_d dt \quad (2.206)$$

where ρ is the attenuation level and corresponds to the maximum singular value of the transfer function $G(s)$ of the linearized model associated to Eqs. (2.199) and (2.200).

2.5.2.2 Control Law

The control signal of the MIMO nonlinear DFIG model which has been transformed into the Brunovsky form as described by Eq. (2.199) contains the unknown nonlinear functions $f(x)$ and $g(x)$. In case that the complete state vector x is measurable these unknown functions can be approximated by

$$\begin{aligned}\hat{f}(x|\theta_f) &= \Phi_f(x)\theta_f \\ \hat{g}(x|\theta_g) &= \Phi_g(x)\theta_g\end{aligned}\quad (2.207)$$

where

$$\Phi_f(x) = (\xi_f^1(x), \xi_f^2(x), \dots, \xi_f^n(x))^T \quad (2.208)$$

with $\xi_f^i(x)$, $i = 1, \dots, n$ being the vector of kernel functions (e.g. normalized fuzzy Gaussian membership functions), where

$$\xi_f^i(x) = (\phi_f^{i,1}(x), \phi_f^{i,2}(x), \dots, \phi_f^{i,N}(x)) \quad (2.209)$$

thus giving

$$\Phi_f(x) = \begin{pmatrix} \phi_f^{1,1}(x) & \phi_f^{1,2}(x) & \dots & \phi_f^{1,N}(x) \\ \phi_f^{2,1}(x) & \phi_f^{2,2}(x) & \dots & \phi_f^{2,N}(x) \\ \dots & \dots & \dots & \dots \\ \phi_f^{n,1}(x) & \phi_f^{n,2}(x) & \dots & \phi_f^{n,N}(x) \end{pmatrix} \quad (2.210)$$

while the weights vector is defined as

$$\theta_f^T = (\theta_f^1, \theta_f^2, \dots, \theta_f^N) \quad (2.211)$$

$j = 1, \dots, N$ is the number of basis functions that is used to approximate the components of function f which are denoted as $i = 1, \dots, n$. Thus, one obtains the relation of Eq. (2.207), i.e. $\hat{f}(x|\theta_f) = \Phi_f(x)\theta_f$.

In a similar manner, for the approximation of function $g(x)$ one has

$$\Phi_g(x) = (\xi_g^1(x), \xi_g^2(x), \dots, \xi_g^N(x))^T \quad (2.212)$$

with $\xi_g^i(x)$, $i = 1, \dots, N$ being the vector of kernel functions (e.g. normalized fuzzy Gaussian membership functions), where

$$\xi_g^i(x) = (\phi_g^{i,1}(x), \phi_g^{i,2}(x), \dots, \phi_g^{i,N}(x)) \quad (2.213)$$

thus giving

$$\Phi_g(x) = \begin{pmatrix} \phi_g^{1,1}(x) & \phi_g^{1,2}(x) & \cdots & \phi_g^{1,N}(x) \\ \phi_g^{2,1}(x) & \phi_g^{2,2}(x) & \cdots & \phi_g^{2,N}(x) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_g^{n,1}(x) & \phi_g^{n,2}(x) & \cdots & \phi_g^{n,N}(x) \end{pmatrix} \quad (2.214)$$

while the weights vector is defined as

$$\theta_g = (\theta_g^1, \theta_g^2, \dots, \theta_g^p) \quad (2.215)$$

where the components of matrix θ_g are defined as

$$\theta_g^j = (\theta_{g_1}^j, \theta_{g_2}^j, \dots, \theta_{g_N}^j)^T \quad (2.216)$$

$j = 1, \dots, p$ is the number of basis functions that is used to approximate the components of function g which are denoted as $i = 1, \dots, n$. Thus one obtains about matrix $\theta_g \in R^{N \times p}$

$$\theta_g = \begin{pmatrix} \theta_{g_1}^1 & \theta_{g_1}^2 & \cdots & \theta_{g_1}^p \\ \theta_{g_2}^1 & \theta_{g_2}^2 & \cdots & \theta_{g_2}^p \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{g_N}^1 & \theta_{g_N}^2 & \cdots & \theta_{g_N}^p \end{pmatrix} \quad (2.217)$$

It holds that

$$g = \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix} = \begin{pmatrix} g_1^1 & g_1^2 & \cdots & g_1^p \\ g_2^1 & g_2^2 & \cdots & g_2^p \\ \vdots & \vdots & \ddots & \vdots \\ g_n^1 & g_n^2 & \cdots & g_n^p \end{pmatrix} \quad (2.218)$$

Using the above, one finally has the relation of Eq. (2.207), i.e. $\hat{g}(x|\theta_g) = \Phi_g(x)\theta_g$. If the state variables of the system are available for measurement then a state-feedback control law can be formulated as

$$u = \hat{g}^{-1}(x|\theta_g)[- \hat{f}(x|\theta_f) + y_m^{(r)} - K^T e + u_c] \quad (2.219)$$

where $\hat{f}(x|\theta_f)$ and $\hat{g}(x|\theta_g)$ are neurofuzzy models to approximate $f(x)$ and $g(x)$, respectively. u_c is a supplementary control term, e.g. an H_∞ control term that is used to compensate for the effects of modelling inaccuracies and external disturbances. Moreover, K^T is the feedback gain matrix which assures that the characteristic polynomial of matrix $A - BK^T$ will be a Hurwitz one.

2.5.3 Estimation of the State Vector

The control of the system described by Eq. (2.196) becomes more complicated when the state vector x of the DFIG is not directly measurable and has to be reconstructed through a state observer. The following definitions are used

- error of the state vector $e = x - x_m$
- error of the estimated state vector $\hat{e} = \hat{x} - x_m$
- observation error $\tilde{e} = e - \hat{e} = (x - x_m) - (\hat{x} - x_m)$

When an observer is used to reconstruct the state vector, the control law of Eq. (2.219) is written as

$$u = \hat{g}^{-1}(\hat{x}|\theta_g)[- \hat{f}(\hat{x}|\theta_f) + y_m^{(r)} - K^T \hat{e} + u_c] \quad (2.220)$$

Applying Eq. (2.220) to the nonlinear system described by Eq. (2.196), results into

$$\begin{aligned} y^{(r)} &= f(x) + g(x)\hat{g}^{-1}(\hat{x})[- \hat{f}(\hat{x}) + y_m^{(r)} - K^T \hat{e} + u_c] + d \Rightarrow y^{(r)} = f(x) + [g(x) - \\ &\hat{g}(\hat{x}) + \hat{g}(\hat{x})]\hat{g}^{-1}(\hat{x})[- \hat{f}(\hat{x}) + y_m^{(r)} - K^T \hat{e} + u_c] + d \Rightarrow y^{(r)} = [f(x) - \hat{f}(\hat{x})] + \\ &[g(x) - \hat{g}(\hat{x})]u + y_m^{(r)} - K^T \hat{e} + u_c + d. \end{aligned}$$

It holds $e = x - x_m \Rightarrow y^{(r)} = e^{(r)} + y_m^{(r)}$. Substituting $y^{(r)}$ in the above equation gives

$$\begin{aligned} e^{(r)} + y_m^{(r)} &= y_m^{(r)} - K^T \hat{e} + u_c + [f(x) - \hat{f}(\hat{x})] + \\ &+ [g(x) - \hat{g}(\hat{x})]u + d \end{aligned} \quad (2.221)$$

and equivalently

$$\begin{aligned} \dot{e} &= Ae - BK^T \hat{e} + Bu_c + B\{[f(x) - \hat{f}(\hat{x})] + \\ &+ [g(x) - \hat{g}(\hat{x})]u + \tilde{d}\} \end{aligned} \quad (2.222)$$

$$e_1 = C^T e \quad (2.223)$$

where $e = [e^1, e^2, \dots, e^p]^T$ with $e^i = [e_i, \dot{e}_i, \ddot{e}_i, \dots, e_i^{r_i-1}]^T$, $i = 1, 2, \dots, p$ and also $\hat{e} = [\hat{e}^1, \hat{e}^2, \dots, \hat{e}^p]^T$ with $\hat{e}^i = [\hat{e}_i, \dot{\hat{e}}_i, \ddot{\hat{e}}_i, \dots, \hat{e}_i^{r_i-1}]^T$, $i = 1, 2, \dots, p$. Matrices A , B and C have been defined in Eq. (8.115).

A state observer is designed according to Eqs. (2.222) and (2.223) and is given by [375]:

$$\dot{\hat{e}} = A\hat{e} - BK^T \hat{e} + K_o[e_1 - C^T \hat{e}] \quad (2.224)$$

$$\hat{e}_1 = C^T \hat{e} \quad (2.225)$$

The feedback gain matrix is denoted as $K \in R^{n \times p}$. The observation gain matrix is denoted as $K_o \in R^{p \times n}$ and its elements are selected so as to assure the asymptotic elimination of the observation error.

2.5.4 Application of Flatness-Based Adaptive Neurofuzzy Control to the DFIG

2.5.4.1 Tracking Error Dynamics Under Feedback Control

If the state vector in the linearized description of the DFIG of Eq. (2.104) is constrained into $x = [x_1, x_2, x_3, x_4]^T = [\omega, \dot{\omega}, \psi_{sd}, \dot{\psi}_{sd}]^T$, it holds that

$$\begin{aligned}\ddot{x}_1 &= f_1(x) + g_1(x)u \\ \ddot{x}_3 &= f_2(x) + g_2(x)u\end{aligned}\tag{2.226}$$

or equivalently

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(x) + g_1(x)u \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(x) + g_2(x)u\end{aligned}\tag{2.227}$$

Next, taking into account also the effects of additive disturbances the dynamic model becomes

$$\begin{aligned}\ddot{x}_1 &= f_1(x, t) + g_1(x, t)u + d_1 \\ \ddot{x}_3 &= f_2(x, t) + g_2(x, t)u + d_2\end{aligned}\tag{2.228}$$

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x, t) \\ f_2(x, t) \end{pmatrix} + \begin{pmatrix} g_1(x, t) \\ g_2(x, t) \end{pmatrix} u + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}\tag{2.229}$$

The following control input is defined

$$u = \begin{pmatrix} \hat{g}_1(x, t) \\ \hat{g}_2(x, t) \end{pmatrix}^{-1} \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x, t) \\ \hat{f}_2(x, t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c1} \\ u_{c2} \end{pmatrix} \right\}\tag{2.230}$$

where $[u_{c1} \ u_{c2}]^T$ is a robust control term that is used for the compensation of the model's uncertainties as well as of the external disturbances and the feedback control gain is $K_i^T = [k_1^i, k_2^i, \dots, k_{n-1}^i, k_n^i]$. Substituting Eq. (2.230) into Eq. (2.229) the closed-loop tracking error dynamics is obtained

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x, t) \\ f_2(x, t) \end{pmatrix} + \begin{pmatrix} g_1(x, t) \\ g_2(x, t) \end{pmatrix} \begin{pmatrix} \hat{g}_1(x, t) \\ \hat{g}_2(x, t) \end{pmatrix}^{-1} \cdot \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x, t) \\ \hat{f}_2(x, t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c1} \\ u_{c2} \end{pmatrix} \right\} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \quad (2.231)$$

Equation (2.231) can now be written as

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x, t) \\ f_2(x, t) \end{pmatrix} + \left\{ \begin{pmatrix} g_1(x, t) - \hat{g}_1(x, t) \\ g_2(x, t) - \hat{g}_2(x, t) \end{pmatrix} + \begin{pmatrix} \hat{g}_1(x, t) \\ \hat{g}_2(x, t) \end{pmatrix} \right\} \begin{pmatrix} \hat{g}_1(x, t) \\ \hat{g}_2(x, t) \end{pmatrix}^{-1} \cdot \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x, t) \\ \hat{f}_2(x, t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c1} \\ u_{c2} \end{pmatrix} \right\} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \quad (2.232)$$

and using Eq. (2.230) this results into

$$\begin{pmatrix} \ddot{e}_1 \\ \ddot{e}_3 \end{pmatrix} = \begin{pmatrix} f_1(x, t) - \hat{f}_1(x, t) \\ f_2(x, t) - \hat{f}_2(x, t) \end{pmatrix} + \begin{pmatrix} g_1(x, t) - \hat{g}_1(x, t) \\ g_2(x, t) - \hat{g}_2(x, t) \end{pmatrix} u - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c1} \\ u_{c2} \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \quad (2.233)$$

The following description for the approximation error is defined

$$w = \begin{pmatrix} f_1(x, t) - \hat{f}_1(x, t) \\ f_2(x, t) - \hat{f}_2(x, t) \end{pmatrix} + \begin{pmatrix} g_1(x, t) - \hat{g}_1(x, t) \\ g_2(x, t) - \hat{g}_2(x, t) \end{pmatrix} u \quad (2.234)$$

Moreover, the following matrices are defined

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.235)$$

$$K^T = \begin{pmatrix} K_1^1 & K_2^1 & K_3^1 & K_4^1 \\ K_1^2 & K_2^2 & K_3^2 & K_4^2 \end{pmatrix}$$

Using matrices A , B , K^T , Eq. (2.233) is written in the following form

$$\begin{aligned} \dot{e} &= (A - BK^T)e + Bu_c + B \left\{ \begin{pmatrix} f_1(x, t) - \hat{f}_1(x, t) \\ f_2(x, t) - \hat{f}_2(x, t) \end{pmatrix} + \right. \\ &\quad \left. + \begin{pmatrix} g_1(x, t) - \hat{g}_1(x, t) \\ g_2(x, t) - \hat{g}_2(x, t) \end{pmatrix} u + \tilde{d} \right\} \end{aligned} \quad (2.236)$$

When the estimated state vector $\hat{x}$ is used in the feedback control loop, equivalently to Eq. (2.222) one has

$$\begin{aligned} \dot{e} = & Ae - BK^T \hat{e} + Bu_c + B \left\{ \begin{pmatrix} f_1(x, t) - \hat{f}_1(\hat{x}, t) \\ f_2(x, t) - \hat{f}_2(\hat{x}, t) \end{pmatrix} + \right. \\ & \left. + \begin{pmatrix} g_1(x, t) - \hat{g}_1(\hat{x}, t) \\ g_2(x, t) - \hat{g}_2(\hat{x}, t) \end{pmatrix} u + \tilde{d} \right\} \end{aligned} \quad (2.237)$$

and considering that the approximation error w is now denoted as

$$w = \begin{pmatrix} f_1(x, t) - \hat{f}_1(\hat{x}, t) \\ f_2(x, t) - \hat{f}_2(\hat{x}, t) \end{pmatrix} + \begin{pmatrix} g_1(x, t) - \hat{g}_1(\hat{x}, t) \\ g_2(x, t) - \hat{g}_2(\hat{x}, t) \end{pmatrix} u \quad (2.238)$$

Equation (2.237) can be also written as

$$\dot{e} = Ae - BK^T \hat{e} + Bu_c + Bw + B\tilde{d} \quad (2.239)$$

The associated state observer will be described again by Eqs. (2.224) and (2.225).

2.5.4.2 Dynamics of the Observation Error

The observation error is defined as $\tilde{e} = e - \hat{e} = x - \hat{x}$. Subtracting Eq. (2.224) from Eq. (2.222) as well as Eq. (2.225) from Eq. (2.223) one gets

$$\begin{aligned} \dot{e} - \dot{\hat{e}} = & A(e - \hat{e}) + Bu_c + B \{ [f(x, t) - \hat{f}(\hat{x}, t)] + \\ & + [g(x, t) - \hat{g}(\hat{x}, t)]u + \tilde{d} \} - K_o C^T (e - \hat{e}) \\ e_1 - \hat{e}_1 = & C^T (e - \hat{e}) \end{aligned}$$

or equivalently

$$\begin{aligned} \dot{\tilde{e}} = & A\tilde{e} + Bu_c + B \{ [f(x, t) - \hat{f}(\hat{x}, t)] + \\ & + [g(x, t) - \hat{g}(\hat{x}, t)]u + \tilde{d} \} - K_o C^T \tilde{e} \\ \tilde{e}_1 = & C^T \tilde{e} \end{aligned}$$

which can be written as

$$\begin{aligned} \dot{\tilde{e}} = & (A - K_o C^T) \tilde{e} + Bu_c + B \{ [f(x, t) - \hat{f}(\hat{x}, t)] + \\ & + [g(x, t) - \hat{g}(\hat{x}, t)]u + \tilde{d} \} \end{aligned} \quad (2.240)$$

$$\tilde{e}_1 = C^T \tilde{e} \quad (2.241)$$

or equivalently, it can be written as

$$\dot{\tilde{e}} = (A - K_o C^T) \tilde{e} + B u_c + B w + \tilde{d} \quad (2.242)$$

$$\tilde{e}_1 = C^T \tilde{e} \quad (2.243)$$

2.5.4.3 Approximation of Functions $f(x, t)$ and $g(x, t)$

Next, the following neurofuzzy approximators of the unknown system dynamics are defined

$$\hat{f}(\hat{x}) = \begin{pmatrix} \hat{f}_1(\hat{x}|\theta_f) & \hat{x} \in R^{4 \times 1} & \hat{f}_1(\hat{x}|\theta_f) \in R^{1 \times 1} \\ \hat{f}_2(\hat{x}|\theta_f) & \hat{x} \in R^{4 \times 1} & \hat{f}_2(\hat{x}|\theta_f) \in R^{1 \times 1} \end{pmatrix} \quad (2.244)$$

with kernel functions

$$\phi_f^{i,j}(\hat{x}) = \frac{\prod_{j=1}^n \mu_{A_j}^i(\hat{x}_j)}{\sum_{i=1}^N \prod_{j=1}^n \mu_{A_j}^i(\hat{x}_j)} \quad (2.245)$$

where $l = 1, 2$, $\hat{x}$ is the estimate of the state vector and $\mu_{A_j}^i(\hat{x})$ is the i -th membership function associated with the l -th hidden layer node (fuzzy rule), as shown in Fig. 2.16. Similarly, the following approximators of the unknown system dynamics are defined

$$\hat{g}(\hat{x}) = \begin{pmatrix} \hat{g}_1(\hat{x}|\theta_g) & \hat{x} \in R^{4 \times 1} & \hat{g}_1(\hat{x}|\theta_g) \in R^{1 \times 2} \\ \hat{g}_2(\hat{x}|\theta_g) & \hat{x} \in R^{4 \times 1} & \hat{g}_2(\hat{x}|\theta_g) \in R^{1 \times 2} \end{pmatrix} \quad (2.246)$$

The values of the weights that result in optimal approximation are

$$\begin{aligned} \theta_f^* &= \arg \min_{\theta_f \in M_{\theta_f}} [\sup_{\hat{x} \in U_{\hat{x}}} (f(x) - \hat{f}(\hat{x}|\theta_f))] \\ \theta_g^* &= \arg \min_{\theta_g \in M_{\theta_g}} [\sup_{\hat{x} \in U_{\hat{x}}} (g(x) - \hat{g}(\hat{x}|\theta_g))] \end{aligned} \quad (2.247)$$

where the variation ranges for the weights are defined as

$$\begin{aligned} M_{\theta_f} &= \{\theta_f \in R^h : \|\theta_f\| \leq m_{\theta_f}\} \\ M_{\theta_g} &= \{\theta_g \in R^h : \|\theta_g\| \leq m_{\theta_g}\} \end{aligned} \quad (2.248)$$

The value of the approximation error defined in Eq. (2.234) that corresponds to the optimal values of the weights vectors θ_f^* and θ_g^* one has

$$w = \left(f(x, t) - \hat{f}(\hat{x}|\theta_f^*) \right) + \left(g(x, t) - \hat{g}(\hat{x}|\theta_g^*) \right) u \quad (2.249)$$

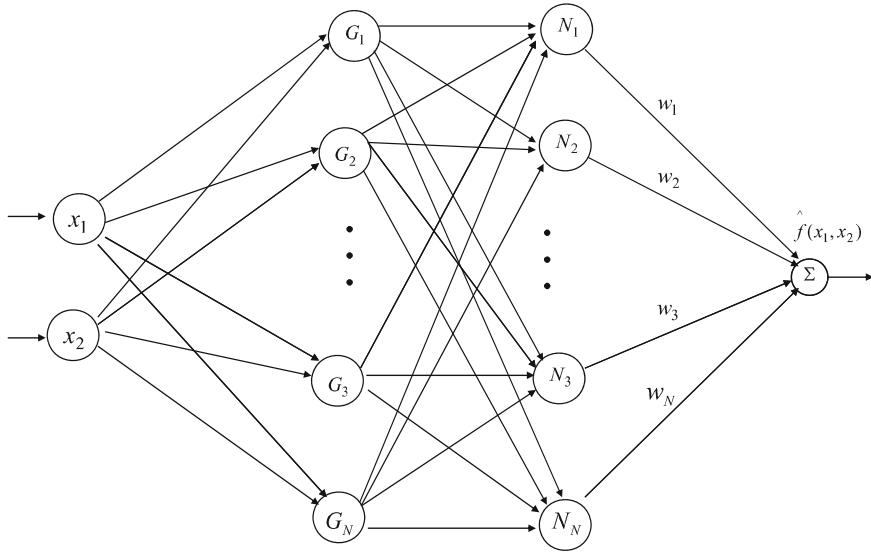


Fig. 2.16 Neurofuzzy approximator used for estimating the unknown dynamics of the DFIG: G_i Gaussian basis function, N_i : normalization unit

which is next written as

$$w = \left(f(x, t) - \hat{f}(\hat{x}|\theta_f) + \hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*) \right) + \left(g(x, t) - \hat{g}(\hat{x}|\theta_g) + \hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*) \right) u \quad (2.250)$$

which can be also written in the following form

$$w = (w_a + w_b) \quad (2.251)$$

where

$$w_a = \{ [f(x, t) - \hat{f}(\hat{x}|\theta_f)] + [g(x, t) - \hat{g}(\hat{x}|\theta_g)] \} u \quad (2.252)$$

$$w_b = \{ [\hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)] + [\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*)] \} u \quad (2.253)$$

Moreover, the following weights' error vectors are defined

$$\begin{aligned} \tilde{\theta}_f &= \theta_f - \theta_f^* \\ \tilde{\theta}_g &= \theta_g - \theta_g^* \end{aligned} \quad (2.254)$$

It is noted that both neural networks and neurofuzzy networks are function approximators using non-orthogonal kernel functions. This is to point out the

difference to other approximators using orthogonal basis functions as for instance Fourier series expansions. The distinctive features for neurofuzzy networks are (i) they use as kernel functions in their hidden layer nodes some typical membership functions met in fuzzy logic (for instance Gaussian functions) (ii) the outputs of the hidden layer nodes are normalized so as to sum up to 1 (this is a property of most fuzzy membership functions). On the other hand in neural networks (i) it is possible to have several types of membership functions (e.g. wavelet functions, Gauss-Hermite polynomials, Bezier-Bernstein polynomials, B-splines), (ii) the outputs of the hidden layer nodes are not necessarily normalized. The learning (weights update) in both neural networks and neurofuzzy networks is performed with gradient algorithms of first or higher order (Fig. 2.16).

2.5.5 Lyapunov Stability Analysis

2.5.5.1 Design of the Lyapunov Function

The adaptation law of the neurofuzzy approximators weights θ_f and θ_g as well as the equation of the supervisory control term u_c are derived from the requirement for negative definiteness of the Lyapunov function

$$V = \frac{1}{2}\hat{e}^T P_1 \hat{e} + \frac{1}{2}\tilde{e}^T P_2 \tilde{e} + \frac{1}{2\gamma_1}\tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\gamma_2}tr[\tilde{\theta}_g^T \tilde{\theta}_g] \quad (2.255)$$

The selection of the Lyapunov function is based on the following principle of indirect adaptive control $\hat{e} : \lim_{t \rightarrow \infty} \hat{x}(t) = x_d(t)$ and $\tilde{e} : \lim_{t \rightarrow \infty} \hat{x}(t) = x(t)$. This yields $\lim_{t \rightarrow \infty} x(t) = x_d(t)$. Substituting Eqs. (2.224), (2.225), (2.240) and (2.241) into Eq. (2.255) and differentiating results into

$$\begin{aligned} \dot{V} = & \frac{1}{2}\dot{\hat{e}}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \dot{\hat{e}} + \frac{1}{2}\dot{\tilde{e}}^T P_2 \tilde{e} + \frac{1}{2}\tilde{e}^T P_2 \dot{\tilde{e}} + \\ & + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g] \Rightarrow \end{aligned} \quad (2.256)$$

$$\begin{aligned} \dot{V} = & \frac{1}{2}\{(A - BK^T)\hat{e} + K_o C^T \tilde{e}\}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \{(A - BK^T)\hat{e} + K_o C^T \tilde{e}\} + \frac{1}{2}\{(A - \\ & K_o C^T)\tilde{e} + Bu_c + B\tilde{d} + Bw\}^T P_2 \tilde{e} + \frac{1}{2}\tilde{e}^T P_2 \{(A - K_o C^T)\tilde{e} + Bu_c + B\tilde{d} + Bw\} + \\ & + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g] \Rightarrow. \end{aligned}$$

or equivalently,

$$\begin{aligned} \dot{V} = & \frac{1}{2}\{\hat{e}^T (A - BK^T)^T + \tilde{e}^T C K_o^T\} P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \{(A - BK^T)\hat{e} + K_o C^T \tilde{e}\} + \frac{1}{2}\{\tilde{e}^T (A - \\ & K_o C^T)^T + u_c^T B^T + w^T B^T + \tilde{d}^T B^T\} P_2 \tilde{e} + \frac{1}{2}\tilde{e}^T P_2 \{(A - K_o C^T)\tilde{e} + Bu_c + Bw + \\ & B\tilde{d}\} + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g] \Rightarrow \end{aligned}$$

$$\begin{aligned}
\dot{V} = & \frac{1}{2}\dot{\tilde{e}}^T (A - BK^T)^T P_1 \hat{e} + \frac{1}{2}\tilde{e}^T CK_o^T P_1 \hat{e} + \\
& \frac{1}{2}\dot{\tilde{e}}^T P_1 (A - BK^T) \hat{e} + \frac{1}{2}\dot{\tilde{e}}^T P_1 K_o C^T \tilde{e} + \frac{1}{2}\tilde{e}^T (A - K_o C^T)^T P_2 \tilde{e} \\
& + \frac{1}{2}(u_c^T + w^T + \tilde{d}^T) B^T P_2 \tilde{e} + \frac{1}{2}\tilde{e}^T P_2 (A - K_o C^T) \tilde{e} + \\
& + \frac{1}{2}\tilde{e}^T P_2 B(u_c + w + \tilde{d}) + \frac{1}{\gamma_1} \dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]
\end{aligned} \tag{2.257}$$

Assumption 1: For given positive definite matrices Q_1 and Q_2 there exist positive definite matrices P_1 and P_2 , which are the solution of the following Riccati equations [340, 352, 366, 375]

$$(A - BK^T)^T P_1 + P_1 (A - BK^T) + Q_1 = 0 \tag{2.258}$$

$$\begin{aligned}
& (A - K_o C^T)^T P_2 + P_2 (A - K_o C^T) - \\
& - P_2 B \left(\frac{2}{r} - \frac{1}{\rho^2} \right) B^T P_2 + Q_2 = 0
\end{aligned} \tag{2.259}$$

The conditions given in Eq. (2.258) to (2.259) are related to the requirement that the systems described by Eqs. (2.224), (2.225), (2.240) and (2.241) have a decreasing Lyapunov function. Substituting Eq. (2.258) to (2.259) into $\dot{V}$ yields

$$\begin{aligned}
\dot{V} = & \frac{1}{2}\dot{\tilde{e}}^T \{ (A - BK^T)^T P_1 + P_1 (A - BK^T) \} \hat{e} + \tilde{e}^T CK_o^T P_1 \hat{e} + \frac{1}{2}\tilde{e}^T \{ (A - K_o C^T)^T P_2 + \\
& P_2 (A - K_o C^T) \} \tilde{e} + \tilde{e}^T P_2 B(u_c + w + \tilde{d}) + \frac{1}{\gamma_1} \dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g] \\
\text{that is} \\
\dot{V} = & -\frac{1}{2}\dot{\tilde{e}}^T Q_1 \hat{e} + \tilde{e}^T CK_o^T P_1 \hat{e} - \frac{1}{2}\tilde{e}^T \{ Q_2 - P_2 B \left(\frac{2}{r} - \frac{1}{\rho^2} \right) B^T P_2 \} \tilde{e} + \tilde{e}^T P_2 B(u_c + \\
& w + \tilde{d}) + \frac{1}{\gamma_1} \dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]
\end{aligned}$$

The supervisory control u_c is decomposed in two terms, u_a and u_b .

- The control term u_a is given by

$$u_a = -\frac{1}{r} \tilde{e}^T P_2 B + \Delta u_a \tag{2.260}$$

where assuming that the measurable elements of vector $\tilde{e}$ are $\{\tilde{e}_1, \tilde{e}_3, \dots, \tilde{e}_k\}$, the term Δu_a is such that

$$-\frac{1}{r} \tilde{e}^T P_2 B + \Delta u_a = -\frac{1}{r} \begin{pmatrix} p_{11}\tilde{e}_1 + p_{13}\tilde{e}_3 + \dots + p_{1k}\tilde{e}_k \\ p_{13}\tilde{e}_1 + p_{33}\tilde{e}_3 + \dots + p_{3k}\tilde{e}_k \\ \dots \dots \dots \\ p_{1k}\tilde{e}_1 + p_{3k}\tilde{e}_3 + \dots + p_{kk}\tilde{e}_k \end{pmatrix} \tag{2.261}$$

- The control term u_b is given by

$$u_b = -[(P_2 B)^T (P_2 B)]^{-1} (P_2 B)^T C K_o^T P_1 \hat{e} \tag{2.262}$$

- u_a is an H_∞ control used for the compensation of the approximation error w and the additive disturbance $\tilde{d}$. Its first component $-\frac{1}{r} \tilde{e}^T P_2 B$ has been chosen so

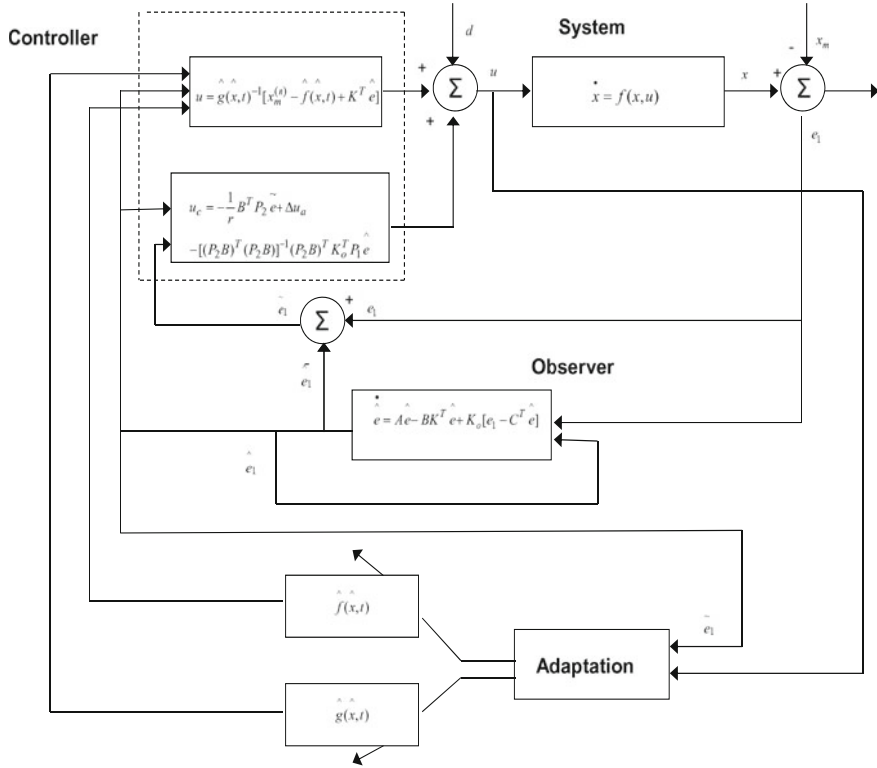


Fig. 2.17 The proposed output feedback-based adaptive fuzzy control scheme for the DFIG

as to compensate for the term $\frac{1}{r} \tilde{e}^T P_2 B B^T P_2 \tilde{e}$, which appears in the previously computed function about $\dot{V}$. By including also the second component Δu_a , one has that u_a is computed based on the feedback of only the measurable variables $\{\tilde{e}_1, \tilde{e}_3, \dots, \tilde{e}_k\}$, out of the complete vector $\tilde{e} = [\tilde{e}_1, \tilde{e}_2, \dots, \tilde{e}_n]$. Equation (8.182) is finally rewritten as $u_a = -\frac{1}{r} \tilde{e}^T P_2 B + \Delta u_a$.

- u_b is a control used for the compensation of the observation error (the control term u_b has been chosen so as to satisfy the condition $\tilde{e}^T P_2 B u_b = -\tilde{e}^T C K_o^T P_1 \hat{e}$).

The control scheme is depicted in Fig. 2.17

Substituting Eqs. (2.260) and (2.262) in $\dot{V}$ and assuming that Eqs. (2.258) and (2.259) hold, one gets

$$\begin{aligned}
 \dot{V} = & -\frac{1}{2} \hat{e}^T Q_1 \hat{e} + \tilde{e}^T C K_o^T P_1 \hat{e} - \frac{1}{2} \tilde{e}^T Q_2 \tilde{e} + \\
 & + \frac{1}{r} \tilde{e}^T P_2 B B^T P_2 \tilde{e} - \frac{1}{2\rho^2} \tilde{e}^T P_2 B \tilde{B}^T P_2 \tilde{e} + \\
 & + \tilde{e}^T P_2 B u_a + \tilde{e}^T P_2 B u_b + \tilde{e}^T P_2 B (w + \tilde{d}) + \\
 & + \frac{1}{\gamma_1} \dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} \text{tr}[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]
 \end{aligned} \tag{2.263}$$

or equivalently,

$$\begin{aligned} \dot{V} = & -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + \\ & + \tilde{e}^T P_2 B(w + \tilde{d} + \Delta u_a) + \frac{1}{\gamma_1}\tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2}tr[\tilde{\theta}_g^T \tilde{\theta}_g] \end{aligned} \quad (2.264)$$

It holds that $\dot{\tilde{\theta}}_f = \dot{\theta}_f - \dot{\theta}_f^* = \dot{\theta}_f$ and $\dot{\tilde{\theta}}_g = \dot{\theta}_g - \dot{\theta}_g^* = \dot{\theta}_g$. The following weight adaptation laws are considered:

$$\begin{aligned} \dot{\theta}_f &= -\gamma_1 \Phi(\hat{x})^T B^T P_2 \tilde{e} \\ \dot{\theta}_g &= -\gamma_2 \Phi(\hat{x})^T B^T P_2 \tilde{e} u^T \end{aligned} \quad (2.265)$$

where assuming N fuzzy rules and associated kernel functions the matrices dimensions are $\theta_f \in R^{N \times 1}$, $\theta_g \in R^{N \times 2}$, $\Phi(x) \in R^{2 \times N}$, $B \in R^{4 \times 2}$, $P \in R^{4 \times 4}$ and $\tilde{e} \in R^{4 \times 1}$.

The update of θ_f is a gradient type algorithm [341, 344]. The update of θ_g is also a gradient type algorithm, where u_c implicitly tunes the adaptation gain γ_2 . Substituting Eq. (2.265) in $\dot{V}$ gives

$$\begin{aligned} \dot{V} = & -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + B^T P_2 \tilde{e}(w + d + \Delta u_a) \\ & + \frac{1}{\gamma_1}(-\gamma_1)\tilde{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \frac{1}{\gamma_2}(-\gamma_2)tr[u\tilde{e}^T P_2 B \Phi(\hat{x})(\theta_g - \theta_g^*)] \end{aligned}$$

or equivalently

$$\begin{aligned} \dot{V} = & -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + B^T P_2 \tilde{e}(w + \tilde{d} + \Delta u_a) \\ & + \frac{1}{\gamma_1}(\gamma_1)\tilde{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \frac{1}{\gamma_2}(\gamma_2)tr[u\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))] \end{aligned}$$

Taking into account that $u \in R^{2 \times 1}$ and $\tilde{e}^T P B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)) \in R^{1 \times 2}$ it holds

$$\begin{aligned} \dot{V} = & -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + B^T P_2 \tilde{e}(w + \tilde{d} + \Delta u_a) + \\ & + \frac{1}{\gamma_1}(-\gamma_1)\tilde{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \frac{1}{\gamma_2}(-\gamma_2)tr[\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u] \end{aligned}$$

Since $\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u \in R^{1 \times 1}$ it holds

$$\begin{aligned} tr(\tilde{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u) = \\ = \tilde{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u \end{aligned} \quad (2.266)$$

Therefore, one finally obtains

$$\begin{aligned} \dot{V} = & -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + B^T P_2 \tilde{e}(w + \tilde{d} + \Delta u_a) + \\ & + \frac{1}{\gamma_1}(-\gamma_1)\tilde{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \frac{1}{\gamma_2}(-\gamma_2)\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u \end{aligned}$$

Next, the following approximation error is defined

$$w_\alpha = [\hat{f}(\hat{x}|\theta_f^*) - \hat{f}(\hat{x}|\theta_f)] + [\hat{g}(\hat{x}|\theta_g^*) - \hat{g}(\hat{x}|\theta_g)]u \quad (2.267)$$

Thus, one obtains

$$\begin{aligned} \dot{V} = & -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + \\ & + B^T P_2 \tilde{e}(w + \tilde{d}) + \tilde{e}^T P_2 B w_\alpha \end{aligned} \quad (2.268)$$

Denoting the aggregate approximation error and disturbances vector as

$$w_1 = w + \tilde{d} + w_\alpha + \Delta u_a \quad (2.269)$$

the derivative of the Lyapunov function becomes

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + \tilde{e}^T P_2 B w_1 \quad (2.270)$$

which in turn is written as

$$\begin{aligned} \dot{V} = & -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + \\ & + \frac{1}{2}\tilde{e}^T P B w_1 + \frac{1}{2}w_1^T B^T P_2 \tilde{e} \end{aligned} \quad (2.271)$$

Lemma: The following inequality holds

$$\frac{1}{2}\tilde{e}^T P_2 B w_1 + \frac{1}{2}w_1^T B^T P_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} \leq \frac{1}{2}\rho^2 w_1^T w_1 \quad (2.272)$$

Proof: The binomial $(\rho a - \frac{1}{\rho}b)^2 \geq 0$ is considered. Expanding the left part of the above inequality one gets

$$\begin{aligned} \rho^2 a^2 + \frac{1}{\rho^2} b^2 - 2ab & \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2} b^2 - ab \geq 0 \Rightarrow \\ ab - \frac{1}{2\rho^2} b^2 & \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2}\rho^2 a^2 \end{aligned} \quad (2.273)$$

The following substitutions are carried out: $a = w_1$ and $b = \tilde{e}^T P_2 B$ and the previous relation becomes

$$\frac{1}{2}w_1^T B^T P_2 \tilde{e} + \frac{1}{2}\tilde{e}^T P_2 B w_1 - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} \leq \frac{1}{2}\rho^2 w_1^T w_1 \quad (2.274)$$

The above relation is used in $\dot{V}$, and the right part of the associated inequality is enforced

$$\dot{V} \leq -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} + \frac{1}{2}\rho^2 w_1^T w_1 \quad (2.275)$$

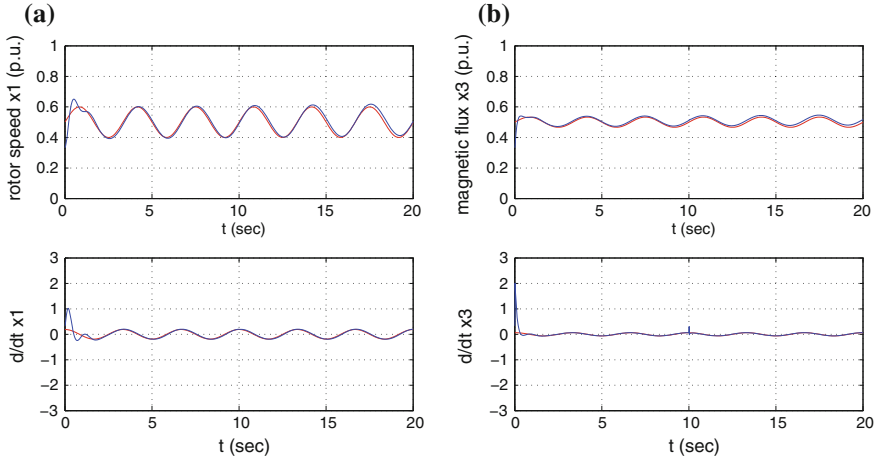


Fig. 2.18 Tracking of setpoint 1 (red lines) from state variables of the DFIG (blue lines): **a** rotor's speed x_1 and its derivative x_2 , **b** magnetic flux of the stator x_3 and its derivative x_4

Thus, Eq. (2.275) can be written as

$$\dot{V} \leq -\frac{1}{2}E^T Q E + \frac{1}{2}\rho^2 w_1^T w_1 \quad (2.276)$$

where

$$E = \begin{pmatrix} \hat{e} \\ \tilde{e} \end{pmatrix}, \quad Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} = \text{diag}[Q_1, Q_2] \quad (2.277)$$

Hence, the H_∞ performance criterion is derived. For ρ sufficiently small Eq. (2.275) will be true and the H_∞ tracking criterion will be satisfied. In that case, the integration of $\dot{V}$ from 0 to T gives

$$\begin{aligned} \int_0^T \dot{V}(t) dt &\leq -\frac{1}{2} \int_0^T \|E\|^2 dt + \frac{1}{2} \rho^2 \int_0^T \|w_1\|^2 dt \Rightarrow \\ 2V(T) - 2V(0) &\leq -\int_0^T \|E\|_Q^2 dt + \rho^2 \int_0^T \|w_1\|^2 dt \Rightarrow \\ 2V(T) + \int_0^T \|E\|_Q^2 dt &\leq 2V(0) + \rho^2 \int_0^T \|w_1\|^2 dt \end{aligned} \quad (2.278)$$

It is assumed that there exists a positive constant $M_w > 0$ such that $\int_0^\infty \|w_1\|^2 dt \leq M_w$. Therefore for the integral $\int_0^T \|E\|_Q^2 dt$ one gets

$$\int_0^\infty \|E\|_Q^2 dt \leq 2V(0) + \rho^2 M_w \quad (2.279)$$

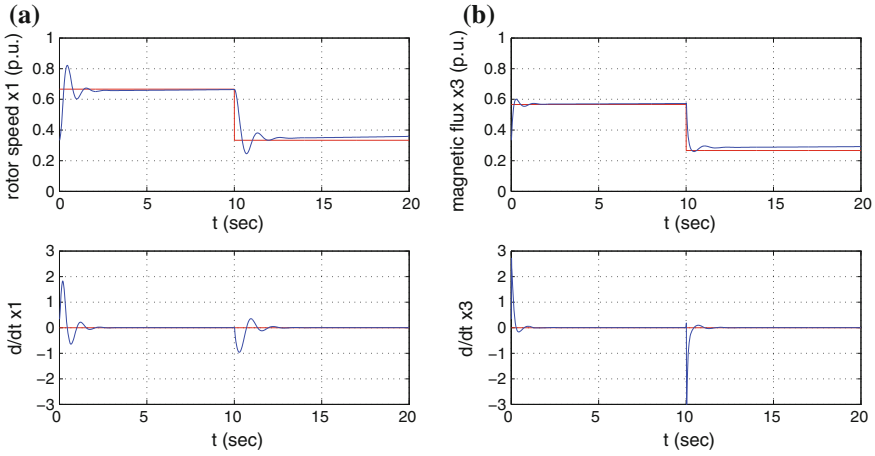


Fig. 2.19 Tracking of setpoint 2 (red lines) from state variables of the DFIG (blue lines): **a** rotor's speed x_1 and its derivative x_2 , **b** magnetic flux of the stator x_3 and its derivative x_4

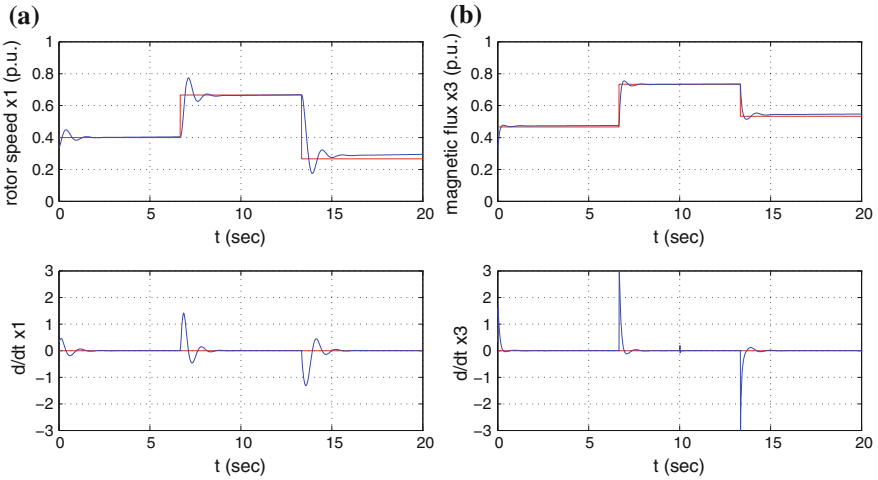


Fig. 2.20 Tracking of setpoint 3 (red lines) from state variables of the DFIG (blue lines): **a** rotor's speed x_1 and its derivative x_2 , **b** magnetic flux of the stator x_3 and its derivative x_4

Thus, the integral $\int_0^\infty \|E\|_Q^2 dt$ is bounded and according to Barbalat's Lemma

$$\begin{aligned} \lim_{t \rightarrow \infty} E(t) &= 0 \Rightarrow \\ \lim_{t \rightarrow \infty} \hat{e}(t) &= 0 \\ \lim_{t \rightarrow \infty} \tilde{e}(t) &= 0 \end{aligned} \quad (2.280)$$

Therefore $\lim_{t \rightarrow \infty} e(t) = 0$.

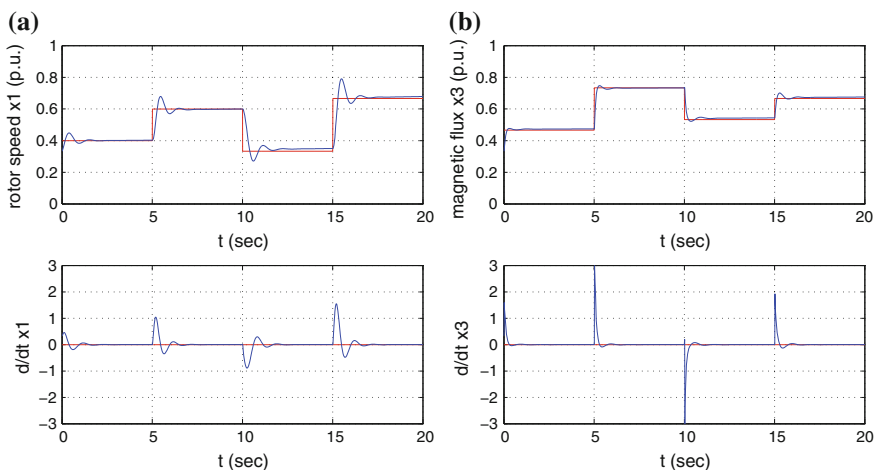


Fig. 2.21 Tracking of setpoint 4 (red lines) from state variables of the DFIG (blue lines): **a** rotor's speed x_1 and its derivative x_2 , **b** magnetic flux of the stator x_3 and its derivative x_4

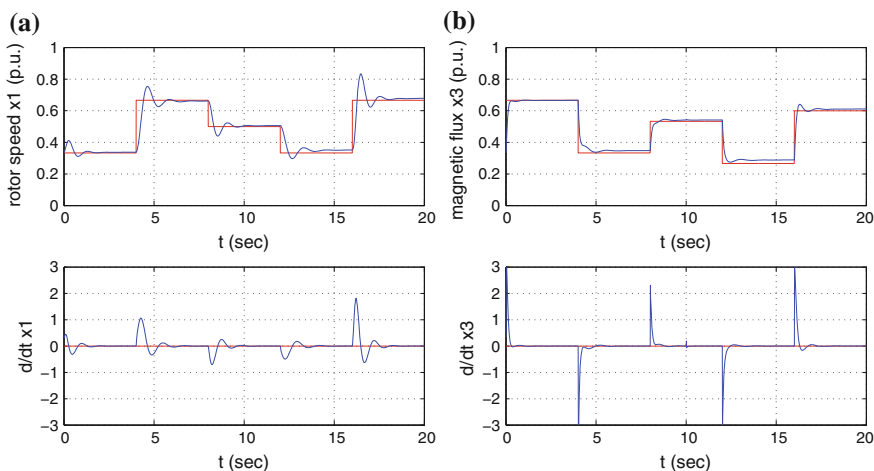


Fig. 2.22 Tracking of setpoint 5 (red lines) from state variables of the DFIG (blue lines): **a** rotor's speed x_1 and its derivative x_2 , **b** magnetic flux of the stator x_3 and its derivative x_4

2.5.6 Simulation Tests

The performance of the proposed adaptive neurofuzzy control scheme for the Doubly-fed induction generator, using only output feedback, has been evaluated through simulation experiments. Indicative numerical values for the parameters of the considered doubly-fed induction generator model are given in Table 2.1, appearing in Sect. 2.3.6. The turn speed of the rotor ω was considered to be

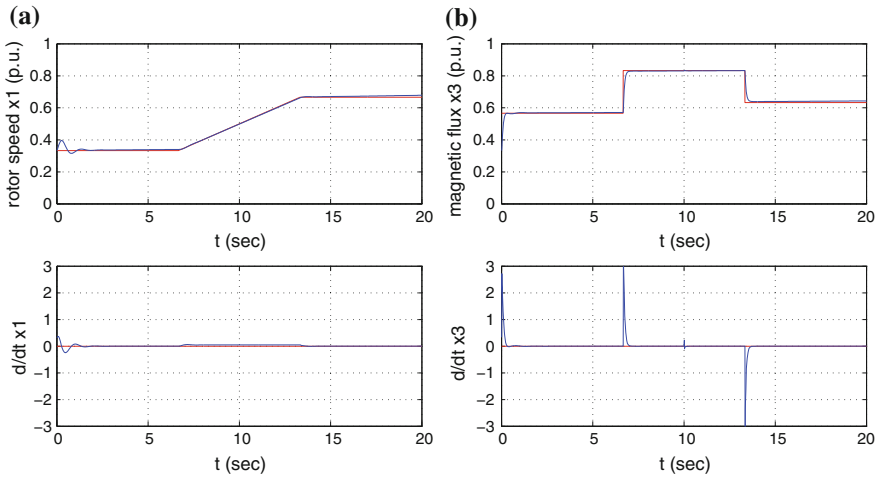


Fig. 2.23 Tracking of setpoint 6 (red lines) from state variables of the DFIG (blue lines): **a** rotor's speed x_1 and its derivative x_2 , **b** magnetic flux of the stator x_3 and its derivative x_4

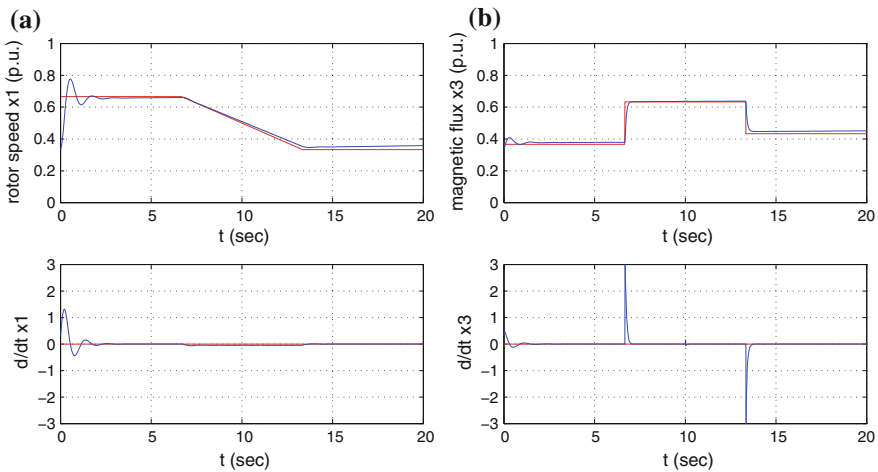


Fig. 2.24 Tracking of setpoint 7 (red lines) from state variables of the DFIG (blue lines): **a** rotor's speed x_1 and its derivative x_2 , **b** magnetic flux of the stator x_3 and its derivative x_4

measurable. Moreover, the magnetic flux of the stator ψ_{s_d} was considered to be computed from measurements of stator and rotor currents, following the previously explained procedure. The obtained experimental results are presented in Figs. 2.18, 2.19, 2.20, 2.21, 2.22, 2.23 and 2.24. It can be noticed that the proposed control scheme succeeded in all cases fast and accurate tracking of the reference setpoints.

The treated control problem, that is (i) simultaneous estimation of the unknown dynamics of the generator and estimation of the machine's state vector, and

(ii) use of this information by an adaptive control scheme, is of elevated difficulty. The provided Lyapunov analysis has come to confirm the stability properties of the control loop. The presented adaptive control method of the DFIG is a model-free one. This means that no prior knowledge about the dynamic model of the generator is needed. Moreover, no knowledge is needed about the mechanical torque exerted on the generator's rotor (e.g. due to wind, steam or water fall). Through the identification procedure that was analyzed in Sect. 2.5.4 it becomes possible to estimate in real-time the unknown dynamical model of the generator and subsequently to use this information in the control loop. Besides, this adaptive control method does not need to measure the complete state vector of the DFIG and is based only on output feedback. The analyzed adaptive neurofuzzy control approach exhibits robustness to external perturbation thus assuring the reliable functioning of the generator under harsh conditions.

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