An Approach for Designing Order Size Dependent Lead Time Models for Use in Inventory and Supply Chain Management

Peter Nielsen and Zbigniew Michna

Abstract This paper addresses the issue of lead time behavior in supply chains. In supply chains without information sharing a supply chain member can only use the information they observe; orders/demand and their lead times. Using this information four different scenarios of lead time behavior are suggested and discussed. Based on this discussion an analytical approach is proposed that investigates the link between order quantities and lead times. This approach is then demonstrated on data from a company. In the particular case it is determined that there seems to be a link between order quantities and lead times, indicating that a complex lead time model may be necessary. It is also concluded that current state of supply chain management does not offer any methods to address this link between order quantities and lead times and that therefore further research is warranted.

Keywords Supply chain management • Lead times • Bullwhip effect • Stochastics

1 Introduction

The bullwhip effect is a term that covers the tendency for replenishment orders to increase in variability as one moves up-stream in a supply chain. The term is also often referred to as demand amplification from its technical definition as the variance of orders divided with the variance of the observed demands. In the current state of research typically five main causes of the bullwhip effect are considered

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(see e.g. Lee et al. [9]): demand forecasting, non-zero lead time, supply shortage, order batching and price fluctuation. Recently Michna et al. [10] has added lead time forecasting and variability of lead times as a sixth main cause of the bullwhip effect. Forecasting of lead times is necessary when a member of the supply chain places an order and the signal processing of lead times in a similar manner as signal processing of demands causes bullwhip effect. While demand forecasting is a well-known challenge in planning [15], lead time forecasting is a phenomenon that is of particular interest for the management of supply chains. A number of approaches to manage supply chains have been proposed ranging from simulation, to optimization and control theory (see e.g. Sitek [18]). This work focuses on lead times and their behavior. Despite it being well established that lead times are critical in terms of both supply chain management and bullwhip effect they have received surprisingly limited attention in literature. This work outlines a step in remedying this through proposing an approach for how a supply chain member using observations of their up-stream orders (lead times and order quantities) can develop a model of lead time behavior. Firstly we want to list the main problems arising in supply chains when lead times are not deterministic. Secondly we propose an approach for how a supply chain member using paired observations of their up-stream orders (lead times and order quantities) can develop a model of lead time behavior depending on orders. The aim is to use this model to improve inventory management and decision making. It is also the aim to establish whether complex models of lead time behavior should be studied further.

The remainder of the paper is structured as follows. First, the theoretical background is established and the relevant literature is reviewed. Second, an approach to investigate the link between lead times and order quantities is proposed. Third, a test [of the approach is conducted using data from a manufacturing company. Finally conclusions and further potential avenues of research are presented.

2 Theoretical Background and Literature Review

This research focuses on supply chains that do not use information sharing, but where each echelon acts solely based on the information it can observe. In the simplified case each member of a supply chain echelon can only observe (1) the demand received from the previous echelon and the orders it itself places in the next echelon (Q) (2) the lead time (LT) for the orders it places and the lead time it itself gives its customers. If we limit the scope of the research to using information that any echelon should be able to gather by observing its suppliers' behavior we arrive at four different cases of lead time models that are plausible.

In the first case LT's are deterministic. This case is trivial to establish and will not be subject to further study. However, it is worth noting that this particular scenario has received significant attention in literature. Chen et al. [3] conclude that the mean lead time and the information used to estimate demand are the single two most important factors in determining the bullwhip effect in a supply chain.

In the second case we assume that LT's are mutually independent identically distributed independent of demands and orders, i.e. lead times are exogenous. This case has received some attention in literature in the context of the bullwhip effect, see recent papers by Disney et al. [4], Do et al. [5], Duc et al. [6], Kim et al. [8], Michna et al. [10] and Nielsen et al. [13]. From Michna et al. [10] we know that the consequence of having i.i.d. lead times is a significant increase in the bullwhip effect and they indicate that lead time forecasting is another important cause of the bullwhip effect. This is seen in the equation below. Where the demand amplification (bullwhip measure BM) is expressed as the ratio $\frac{Var(q_t)}{Var(D_t)}$:

$$BM = \frac{Var(q_t)}{Var(D_t)} = \frac{2\sigma_L^2(m+n-1)}{m^2n^2} + \frac{2\sigma_L^2\mu_D^2}{m^2\sigma_D^2} + \frac{2\mu_L^2}{n^2} + \frac{2\mu_L}{n} + 1$$

where $\sigma_L^2, \sigma_D^2, \mu_L^2, \mu_D^2$ are the observed variance and means of the lead times and demand distributions and m and n are respectively the number of observations used to estimate the lead time and demand. It is a relatively trivial matter to develop an appropriate lead time model if in fact lead times are i.i.d. It is simply a matter of having sufficient observations available to estimate a distribution [13]. However, under the assumption of i.i.d. lead times there is a problem with the so-called crossovers which happens when replenishments are received in a different sequence than they were ordered see e.g. Bischak at el. [1, 2], Disney at el. [4] and Wang and Disney [19]. The crossover (see Fig. 1) effect is especially severe when a member of the supply chain forecasts lead times to place an appropriate order.

In the third case LT depends on Q i.e. the distribution of LT depends on the parameter m = Q

$$F = F(x,\ Q)$$

Consider the situation where a number of thresholds of order sizes exist i.e. several intervals $Q \in [Q_i;\ Q_j]$ exist for each of which there is a corresponding distribution of LT. In practice this seems like a potentially likely relationship between Q and LT. It also seems reasonable that LT unidirectionally depends on Q. From the perspective of inventory and supply chain management it is highly complicated if LT depends on Q in any form as the estimate of LT is used to determine an appropriate Q. The work presented by Nielsen et al. [12] indicates that such relationships may in certain cases be appropriate. However, to the best of the authors' knowledge no supply chain models exist that take this into account. In information sharing supply chains it could well be that Q or total demand depends

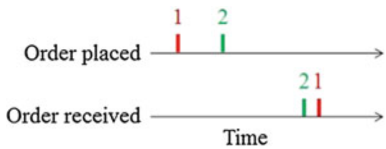

Fig. 1 Order crossover where order 1 is placed prior to order 2, but received after order 2

on LT. However, as only non-information sharing supply chains are considered this case is not investigated in this research.

In the final case the joint Q-LT distributions is not the product of the marginal distributions of Q and LT, that is we observe a stochastic dependence of lead times and orders which means that

$$H(x, y) = C(F(x), G(y))$$

where H is a joint distribution of lead times and orders, C is a copula function, F is a marginal distribution of lead time and G is a marginal distribution of orders see e.g. Joe [7] and Nelsen [11]. This case is by far the most complicated of the four, both to establish and to address. However, this scenario seems rather unlikely as it implies that Q is in fact a random variable (although jointly distributed with LT). From practice this is unlikely to be the case for any echelon of a supply chain baring the final echelon before the end-customer. It would also be potentially very complicated to manage inventory if such a relationship did in fact exist.

There is of course any number of other appropriate models for LT behavior. However, the models above have the benefit that they depend on information that a supply chain member can acquire through observation, i.e. observing actual lead times achieved for orders. Order quantities may be an important factor in determining lead times, but this is likely due to capacity constraints at up-stream echelons where the order quantity acts as a proxy estimator for capacity utilization. Pahl et al. [14] reviews a large body of literature for situations where lead times depend on capacity utilization. However, without information sharing, capacity utilization at the supplier is not known by the customer at the time of order placement. So the above proposed LT models seem easy to implement and appropriate to use in the general case.

3 Approach

The following approach to determine an appropriate lead time model relies on having a number of pair observations (Q_x, LT_x) of order quantities and their corresponding lead times. This information can be observed by any member of a supply chain by monitoring its orders to suppliers. It also assumes that it has been established that LT can be considered to be random variables, i.e. not constant.

The proposed approach contains three main steps as seen in Fig. 2. The first step is data cleaning, where any incorrect data entries are removed (negative values, non-integer values etc.). Following this each pair of observations (Q_x, LT_x) is removed if either the LT or the Q observation can be considered a statistical outlier. For simplicity in this research any observation of either Q or LT that is larger than the observed mean ($\bar{\mu}_Q$ and $\bar{\mu}_{LT}$) and four standard deviations is considered an outlier. Also for simplicity all outliers are removed in one step rather than iteratively.

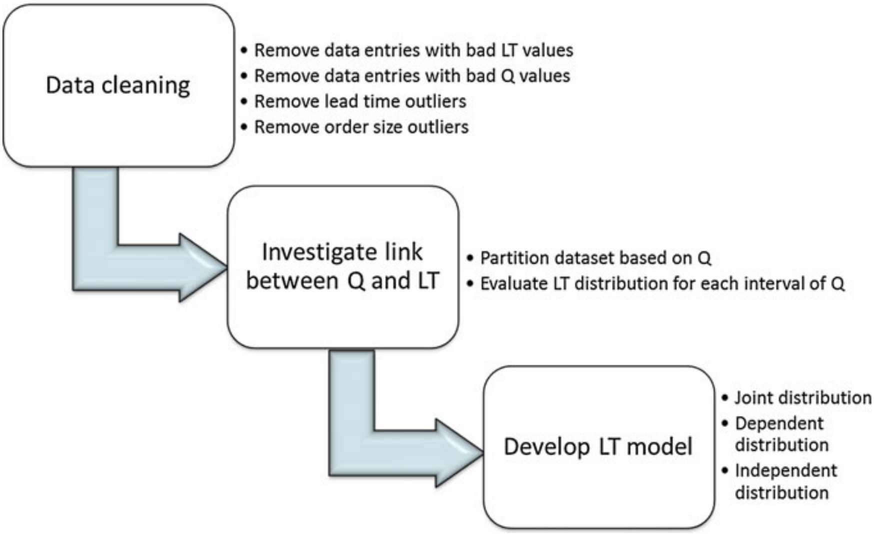

Fig. 2 A three step approach to determine an appropriate LT model

In the second step each pair of observations is portioned into buckets using Q to determine into which bucket the pair is placed. The buckets are derived such that the total range of Q is partitioned into a reasonable number of buckets covering an equal range. Subsequently the distribution of LT for each of these buckets is evaluated. The evaluation is a simple comparison of the main shape parameters of the LT distribution; mean, median, standard deviation and skewness. The mean and standard deviation of the LT distributions are used because we know from Duc et al. [6], Kim et al. [8] and Michna et al. [10] that they are critical for the size of the bullwhip effect incurred due to stochastic lead times. The median and skewness are included as they are critical for the time it takes for a supply chain to reach steady state. In this case a simple benchmark is used to determine if the different set of observations behave in conflicting manners. The benchmark is calculated in the same manner for all four shape parameters. Exemplified by the mean LT: $\bar{\mu}_{LT,set}/\bar{\mu}_{LT,all}$, where $\bar{\mu}_{LT,set}$ is the mean for the particular set of orders partitioned on Q and $\bar{\mu}_{LT,all}$ is the calculated mean LT for all observations. Values above 1 indicate that the particular subset has a higher mean than the whole data set. Values close to 1 indicate a similar mean on the particular parameter.

The third step involves evaluating the output from step two to determine an appropriate model of LT. Here the benchmarks are needed to determine whether there is any significant difference in LT behavior for the different ranges of Q, whether they cover a significant amount of observations or total demand to warrant modeling separately or a simple i.i.d. model for LT can be used instead.

4 Test Case

In the following data from four products from the same product family and a total of 29,897 observations of orders with corresponding order sizes (Q) and lead times (LT) are analyzed using R [16].

Figure 3 shows the cumulative data removed from the samples in each of the four steps and as can be seen, removing outliers, using a four standard deviations criteria, reduces the available data with between 2 and 3 %. This data removal is however necessary as there are several extreme observations in the tails of both the order size and lead time distributions that could potentially skew any subsequent analysis. In an application of the approach data cleaning must always be conducted in the given context.

Figure 4 shows a three dimensional frequency diagram with the frequency intensity as the z-axis. The data has for this purpose been cut into five intervals covering an equal range of Q and LT respectively. As can be seen, small order quantities (Q) and small lead times (LT) dominate the frequency plots for all four products. X^2-tests using the buckets depicted in the Fig. 4 shows that LT depends on Q on a better than 0.001 level in all four cases.

To evaluate the buckets' suitability in representing the actual data we investigate the number of observations in each bucket of Q and their contribution to total demand. The left hand side of Fig. 5 shows that for the particular data sets used in the analysis small orders (order size interval 1) very much dominate the data in the sense of observations as also seen on Fig. 4.

From the right hand side of Fig. 5 it should be noted that the total volume of demand is actually very dispersed between the order size intervals with no clear pattern between the four products. In the present analysis the observations have simply been partitioned into five groups of equal range for Q and LT. Another

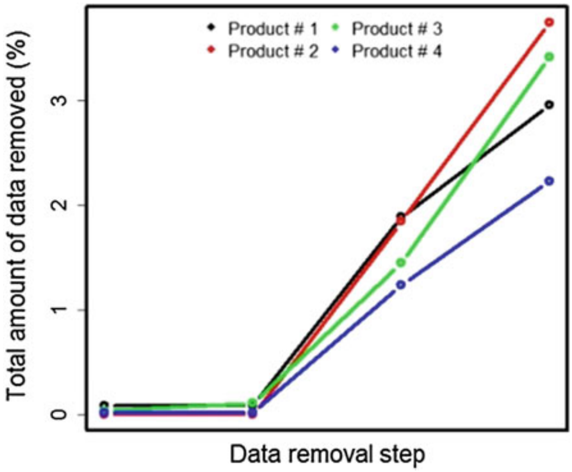

Fig. 3 Data removed in each of the four data cleaning steps

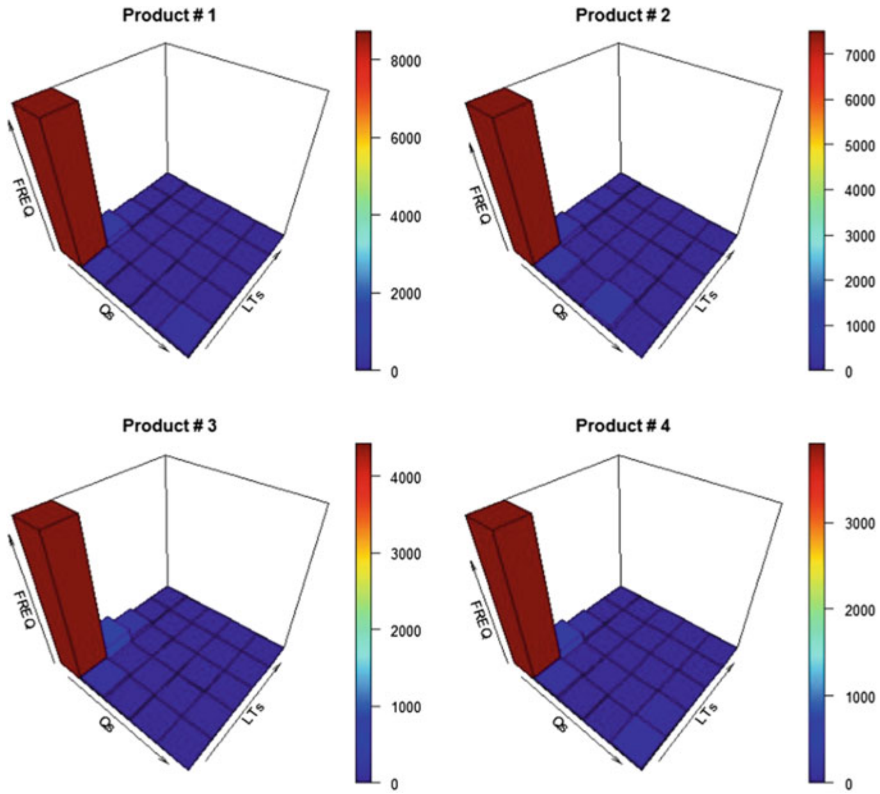

Fig. 4 3D frequency diagrams

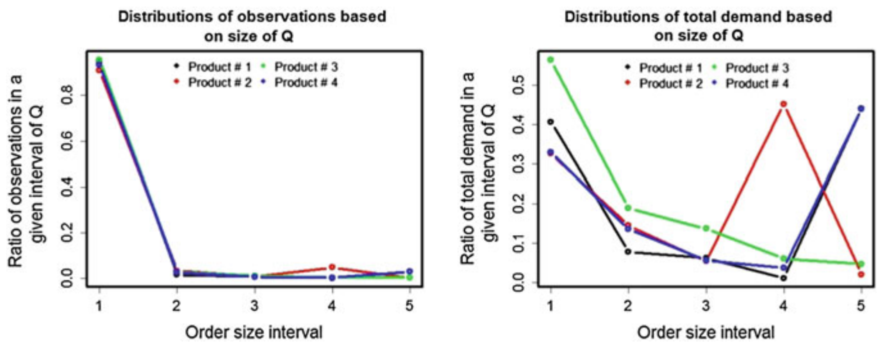

Fig. 5 Ratio of observations and the ratio of total demand in a given order interval for each of the four products

approach could have been to split the data into groups based on equal volume of sales or equal number of observations. However, these approaches would potentially be misleading with regards to the link between Q and LT.

Figure 6 shows the comparison across the range of Q for all four products and the four shape parameters. The four products appear to be behaving similarly so an overall evaluation will be given. With regards to the mean of the LT distributions the first interval of Q (i.e. the smallest order quantities in the data sets) (top left graph in Fig. 6) have uniformly the lowest expected mean where the remaining four intervals of Q have an expected mean LT of between 1.5 and 2.5 higher than the mean of the whole data set. The conclusion is the same for the median LT (top right graph in Fig. 6). The lowest range of Q has a much lower median LT than the four other intervals of Q regardless of product. It is interesting to note that the difference here is upwards to a factor 8 and thus much higher than the difference in mean, where the largest differences are a factor 2.5 larger. For the standard deviation of the LT distribution the picture is more complicated (bottom left graph in Fig. 6). There seems to be some indication that the standard deviation of the LT is lower for

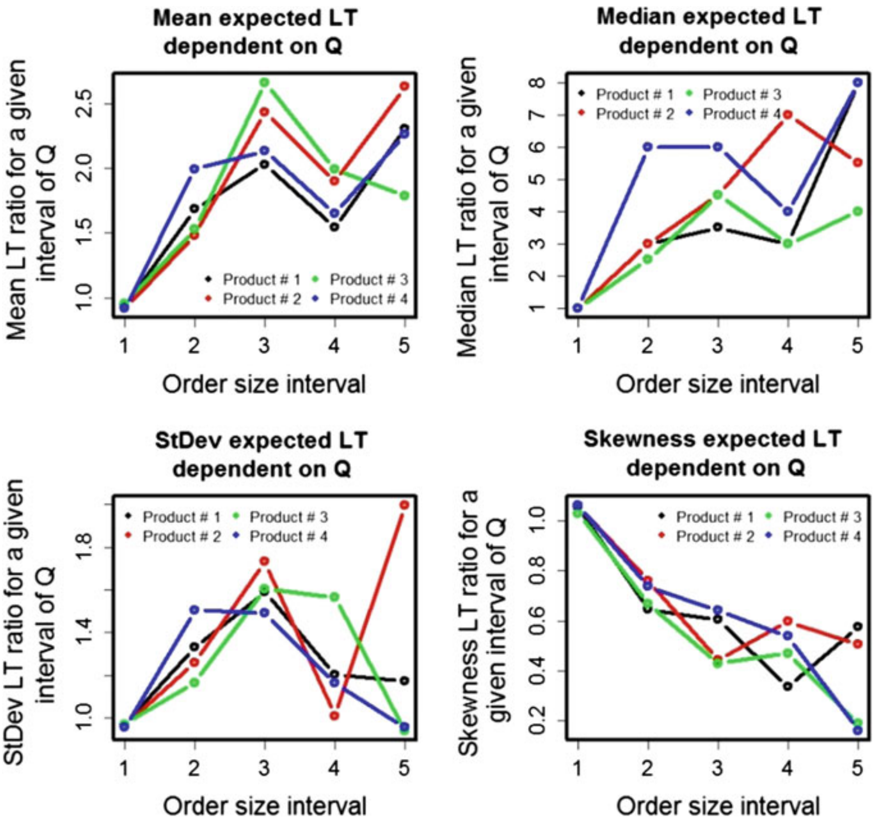

Fig. 6 Comparison for the four shape parameters for the four investigated products

smaller Q. However, the evidence is not conclusive. The final parameter is the skewness (bottom right graph in Fig. 6). Here the results are the same for all four products. The skewness of the distribution is lower for larger ranges of Q and although they are significantly less all LT distributions are skewed in the same direction regardless of which interval of Q is considered.

The overall conclusion is that there seems to be ample evidence that in the particular case lead times can be considered to depend to some extend on the order quantities. From the number of observations available in each order size interval (regardless of product) one could consider to merge the four order size intervals that cover the larger orders into one interval. In all four cases they behave similar and doing this would mean that 59.4, 67.4, 43.6 and 67.0 % of the total volume would be in one order size interval (i.e. large orders). This would allow for simplifying the LT model to two functions:

$$F_1(x, Q)\, for\, Q \in [Q_i; Q_j]$$
$$F_2(x, Q)\, for\, Q \in [Q_j; Q_h]$$

where $Q_i; Q_j$ limits the range of the first order interval for a given product, and Q_h is the largest order size included in the final range and F_1 and F_2 are density functions for each of the respective intervals of Q. In the data preparation values of Q larger than Q_h have been removed, so in practice some approximation of expectations for LT for $Q > Q_h$ must be included. For illustrative purposes the corresponding density functions have been numerically approximated [17] for product 1 for all intervals of Q and for the last four intervals of Q combined. The results are illustrated in Fig. 7.

While the presented example is just one case, there is no reason to believe that this case is unique and it underlines the need to conduct research into supply chains with order quantity dependent lead time distributions. No doubt in practice this type

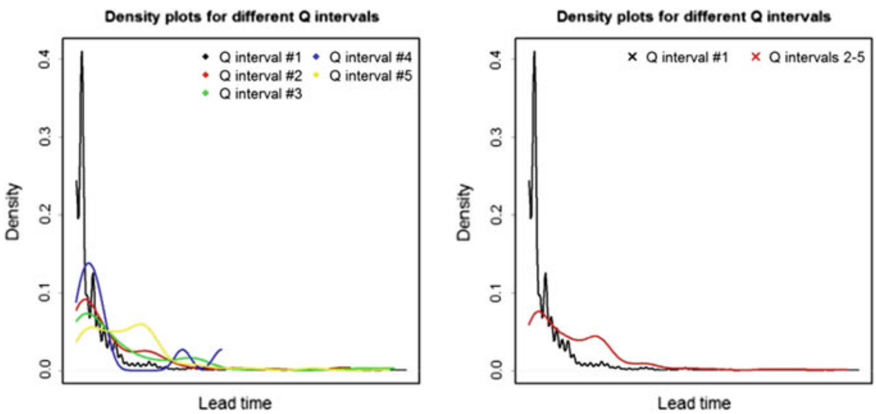

Fig. 7 Density plots of lead times for product 1 for different intervals of Q

of supply chain without information sharing is more difficult to manage than one where lead times are independent of order quantities or even more ideally one where lead times can be considered to be deterministic.

5 Conclusions and Future Research

From literature it is established that lead times and their behavior are a major source of bullwhip effect in supply chains. It is also established that there are limited studies of how actual lead times behave. To remedy this, a simple approach for analyzing lead times in supply chains with no information sharing is proposed. The approach utilizes data that any company should have readily available to establish if there is a link between order quantities and lead times. The approach is tested on data from a company and it is concluded that the lead times for four products appear to depend on the order quantity. Future research will focus on analyzing the impact of quantity dependent lead times on supply chains.

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