The History of Mathematics in the Progress of Mankind. Modifying the Narrative Around 1800

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Abstract

The narrative patterning of historiography changed profoundly around 1800. Instead of the accumulative, encyclopaedic format typical of the 18th century, a historical narrative hinged on the progress of mankind became viable (Condorcet, Fichte etc.). This also had an impact on the writing of the history of mathematics. An interesting testimony of this transition can be found in Alexander von Humboldt's project on the origin and development of the decimal positional numeral system. What originally started as an ethnographic and encyclopaedic project became a hypothetical history of ideas, inspired by the new philologies and the new mathematics.

Keywords

Narrative formats of historiography · History of mathematical notation · Condorcet · Fichte · Alexander von Humboldt

Near the end of the 18th century a new historiographic scheme appeared "that starts from the idea that mankind will for ever and ever progress in his cultivation and amelioration" and, as a consequence, "all analogies between past and present disappear" as a contemporary notes.[1] This new format broke with mid 18th century

[1]"Man arbeitet heut zu Tage an historischen Systemen, und unter andern an einem, welches von dem Gedanken ausgeht: daß das Menschengeschlecht *immer und immer* in seiner Kultur und Verbesserung vorwärts schreite, u.s.w. Dieses hat besonders der französische Bürger Condorcet zu behaupten und zu beweisen gesucht […] Es fallen folglich alle analogischen Schlüsse weg, welche man von den alten Begebenheiten auf das machen kann, was unter unseren Augen vorgeht; denn wir sind mehr kultivirt, als man sonst war, haben mehr Gewandtheit der Kräfte u.s.w." (Laukhard 1796, 315–316).

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modes of writing as can be typically found in Pierre Bayle's *Dictionaire historique et critique* (1697), Christian Wolff's oeuvre or J.H. Zedler's *Universal-Lexikon* (1731–1754). These books tried to exhaust all knowledge accessible, either framed in a systematic and accumulative format (Wolff), or through an accretion of quotes old and new, larded with comments and references to books and voyages (Bayle, Zedler). Around 1800 this abundance of historical and geographical materials made way for a historiography that first made a selection of facts and figures and then assembled them into an account that tells of the systematic progression of ideas.[2]

1 Origins of the 'Progress of Mankind'-Narrative

1.1 Paris 1800

The first person to introduce convincingly this new historiographic format seems to have been the Marquis de Condorcet (1743–1794). In the introduction to his posthumously published *Esquisse d'un tableau historique des progrès de l'esprit humain* (1795) he wrote:

> we have to show, by reasoning and with facts, that there is no limit in the perfection of the human faculties; […] that the progress of this perfection is now independent of every force that would try to stop it, and that it has no other limit than the duration that this world or nature gives us. Without any doubt, these progresses may follow a pace that is more or less fast, but it will never be retrograde.[3]

This idea of an infinite improvement resembles the classic religious eschatological scheme where mankind strives to perfection to ascend to God's kingdom, but in Condorcet's version it became secularised. Human reason and its main wings, science and technology, pushed mankind ever forwards to a better destination here on earth.

[2]It is interesting to note that a transition between genres of fiction occurred in parallel to this transition in historiography. Near the end of the 18th century the new literary genre 'Bildungsroman' appeared as a reaction to and transformation of the very popular mid-18th century genres of the picaresque novels, travel accounts and other forms of fiction that are essentially an accumulation of random events and experiences often held together by a loose narrative (Bakhtin 1986). The 'Bildungsroman' superimposes a teleological structure on the random sequence of events that the picaresque main character lives through. What at first sight appears a series of contingent occurrences, discloses itself at the end of the 'Bildungsroman' as a pre-arranged set of experiences that help to form (*bilden*) the hero's character, where the hero, as pars pro toto, stands for mankind.

[3]"il faut montrer, par le raisonnement et par les faits, qu'il n'a été marqué aucun terme au perfectionnement des facultés humaines; […]; que les progrès de cette perfectibilité désormais indépendante de toute puissance qui voudroit les arrêter, n'ont d'autre terme que la durée du globe où la nature nous a jetés. Sans doute, ces progrès pourront suivre une marche plus ou moins rapide, mais jamais elle ne sera rétrograde" (marquis de Condorcet 1795, 4).

Condorcet's *Tableau historique* was certainly the most influential and most elaborate version of the idea of eternal progress of mankind. The *Tableau historique* pursued and altered earlier 18th century ideas on history.[4] Its most direct ancestor was a *discours* by Turgot (1727–1781) on the 'successive progresses of the human mind'. In order to rationalise and defend the religious scheme of progressing towards perfection, Turgot pointed out that science and the arts could bring about just that perfection. The sciences and technological innovations led by mathematics guaranteed that the chaos and arbitrariness of mankind's opinions and sentiments could finally be appeased to contribute to his progress. Mathematics stood out as a stable haven against a background of chaos and disorder and served as the 'torch' that helps discern truth from error. Reason, or *in casu* logic or mathematics, as a torch enlightening mankind was a stock element of Enlightenment discourse.

Whereas Turgot allowed for error and decadence and saw science mainly as the light that guides through the chaos of history, Condorcet consciously only painted the progresses of the human mind. Instead of the 18th century tendency towards encyclopaedic accretion, Condorcet sifted through the huge amount of information available, both diachronically (through books) and synchronically (through the travel accounts that were so popular in the 18th century). A new historiographic master narrative also implied a new methodology of lining up historical facts and figures:

> it is necessary to choose [the facts of history] among those of several peoples, to bring them together, to combine them so as to derive from them a hypothetical history of only one people and to form a table of its progresses.[5]

Condorcet advocated a selection and combination of facts of possibly chronological and/or topological heterogenous origin to assemble the "hypothetical" history of mankind and its progress.

This led to an epistemological periodisation of history.[6] Condorcet's subdivision of history into 10 epochs reflected the succession of ideas and innovations in technology and industry that drive mankind's progress. From tribes to agricultural settlements, from the invention of alphabetic writing to the invention of printing, from there to the Renaissance of the sciences and finally to the French Revolution.[7]

[4]A more complete overview of the roots of Condorcet's *Tableau historique* can be found in the critical edition of (Marquis de Condorcet 1795, 32–36).

[5]"il est nécessaire de choisir [les faits de l'histoire] dans celles de différens peuples, de les rapprocher, de les combiner, pour en tirer l'histoire hypothétique d'un peuple unique, et former le tableau de ses progrès" (Marquis de Condorcet 1795, 13).

[6]The term 'epistemological periodisation' is borrowed from the Condorcet (1795).

[7]Condorcet's epochs are: (1) Les hommes sont réunis en peuplades; (2) Les peuples pasteurs. Passage de cet état à celui des peuples agriculteurs; (3) Progrès des peuples agriculteurs jusqu'à l'invention de écriture alphabétique; (4) Progrès de l'esprit humain dans la Grèce jusqu'au temps de la division des sciences vers le siècle d'Alexandre; (5) Progrès des sciences depuis leur division jusqu'à leur décadence; (6) Décadence des lumières, jusqu'à leur restauration vers le temps des croisades; (7) Depuis les premiers progrès des sciences vers leur restauration dans l'Occident jusqu'à l'invention de l'imprimerie; (8) Depuis l'invention de l'imprimerie jusqu'au temps où les

In each epoch, mathematics is duly mentioned. The predilection of Socrates and Plato for mathematics that saved them from sophistry, the combination with observation that made ancient Greek mathematics so successful, the invention of logarithms and the renaissance of mathematics (mentioned immediately after the horrors of the religious wars of the 16th–17th centuries), the birth of algebra and analysis that made the age of reason possible.[8] In the discussion of the last epoch, the future, mathematics is invoked once more:

> The progress of science assures the progress of the art of teaching, and this in its turn accelerates the progress of science. This reciprocal influence […] should be placed among the most active and powerful causes for the perfection of humankind. Today, a young man that leaves our schools knows more of mathematics than Newton had ever learned by profound studies or by genial discovery.[9]

Mathematics is also instrumental for the devices Condorcet proposes to accelerate progress in the future. These include a universal language, modeled after algebra, teaching of arithmetic in elementary schools or the application of mathematics for the organisation of a democratic state.[10]

Such an epistemological periodisation is also found in Bossut's contemporary history of mathematics, *Essai d'une histoire générale des mathématiques* (Bossut 1802). Charles Bossut (1730–1814) was a friend of Condorcet and they had both been involved in the redaction of the *Dictionnaire méthodique* in the 1780s. Instead of Montucla's (or Kästner's) 18th century histories of mathematics that are arranged according to the succession of the centuries, Bossut ordered his account after the succession of great ideas in mathematics: From the beginnings to the School of Alexandria, from the Arabs to the end of the 15th century (birth of symbolic algebra), from the 15th century to the invention of the calculus.[11] Instead of an absolute chronological timeframe, Condorcet and Bossut chose a relative timeframe

(Footnote 7 continued)
sciences et la philosophie secouèrent le joug de l'autorité; (9) Depuis Descartes jusqu'à la formation de la République Francoise; (10). Des progrès futurs de l'esprit humain.

[8]Pages 81–81; 102–107; 215–216; and 279–286 respectively.

[9]"Les progrès des sciences assurent les progrès de l'art d'instruire, qui eux-mêmes accélèrent ensuite ceux des sciences; et cette influence réciproque, dont l'action se renouvelle sans cesse, doit être placée au nombre des causes les plus actives, les plus puissantes du perfectionnement de l'espèce humaine. Aujourd'hui, un jeune homme, au sortir de nos écoles, sait en mathématiques, au-delà de ce que Newton avoit appris par de profondes études, ou découvert par son génie" (Marquis de Condorcet 1795, 372).

[10]See (Marquis de Condorcet 1795, 375–377), Condorcet, Condorcet's *Moyens d'apprendre à compter sûrement et avec facilité* (1800) and his *Tableau général de la science qui a pour objet l'application du calcul aux sciences politiques et morales* (1793) respectively. More generally on the role of mathematics in the plans of Turgot or Condorcet to organise and improve society in Brian (1994).

[11]The chapters in Bossut (1802) are: Etat des mathématiques depuis leur origine jusqu'à la destruction de l école d'Alexandrie; Etat des mathématiques depuis leur renouveau chez les Arabes jusque vers la fin du XVe siècle; Progrès des mathématiques depuis la fin du XVe siècle jusqu'à l'invention de l'Analise; Progrès des mathématiques depuis la découverte de l'Analise infinitésimale jusqu'à nos jours. Compare also with Novy (1996).

that follows and fits the internal logic of development of a field of knowledge. As a consequence, periods of decadence or lesser activity were mostly left out of the narrative. A further consequence was the disappearance of parallel narratives where, e.g., India and China would feature alongside of Europe, in favour of one master narrative that turns the rest into secondary or inferior plot developments.[12]

The new historiographic format would quickly become popular in many adaptations and versions. In France, people such as Lazare Carnot (1753–1823) or later Auguste Comte (1798–1857) with his positivism and corresponding philosophy of history would develop Condorcet's historiographic format further during the 19th century.

1.2 Berlin 1800

In the German-speaking states, Condorcet's *Tableau* was translated by E.L. Posselt (1763–1804) as early as 1796. The philosopher J.G. Fichte (1762–1814) provided one of the earliest and most influential adaptations of Condorcet's vision on history. In his *Grundzüge des gegenwärtigen Zeitalters* from 1804, Fichte wrote:

> The life of the human kind does not depend on blind arbitrariness, nor is it […] everywhere similar to itself […] but it goes along and it progresses always according to a fixed plan that *must* inevitably be realised. […] This plan is the following: The human kind has to form itself in this life with freedom into the pure image of reason. Their common life divises into five main epochs: a first, where reason rules as a blind instinct; a second where this instinct is transformed in an externally commanding authority; a third one where the rule of this authority, and therefore reason itself, is destroyed; a fourth one where reason and its laws are understood clearly and consciously; a fifth and final one where all relationships of the human kind will be ruled and ordered following those laws of reason.[13]

Fichte considered his own times as part of the third epoch in transition to the fourth one. Again, it is human reason that fronted mankind's progress and technology that played an important role. The invention of writing is capital for the second epoch, the invention of printing for the third, and general alphabetisation is, according to Fichte, one of the main instruments to pass over to the fourth epoch. Fichte also sharply criticised 18th century formats where the sciences were drawn hither and tither "by the blind tendency of the association between ideas" and where

[12] 18th century European historiography often saw Asia as an 'equal partner' or at the very least as a valid point of comparison, see Osterhammel (1998).

[13] "Das Leben der menschlichen Gattung hängt nicht ab vom blinden Ohngefähr, noch ist es […] sich selbst allenthalben gleich […] sondern es geht einher und rückt vorwärts immer nach einem festen Plane, der nothwendig erreicht werden *muss*. […] Dieser Plan ist der: dass die Gattung in diesem Leben mit Freiheit sich zum reinen Abdruck der Vernunft ausbilde. Ihr gesammtes Leben zertheilt sich in fünf Hauptepochen: diejenige, da die Vernunft als blinder Instinct herrscht; diejenige, da dieser Instinct in eine äusserlich gebietende Autorität verwandelt wird; diejenige, da die Herrschaft dieser Autorität, und mit ihr der Vernunft selber zerstört wird: diejenige, da die Vernunft und ihre Gesetze mit klarem Bewusstseyn begriffen werden: endlich diejenige, da durch fertige Kunst alle Verhältnisse der Gattung nach jenen Gesetzen der Vernunft gerichtet und geordnet werden" (Fichte 1846a, 17).

the master idea for representing facts and figures was “after the sequence of letters in the alphabet”, i.e. encyclopaedic or dictionary-like accretion.[14] Again, Antiquity’s mathematics gets quoted to prove that the sciences need not be such sundries.

Although Fichte’s scheme had much in common with Condorcet’s (the idea of progress, the role of reason and technology), it was a decidedly more abstract blueprint that also integrated a new dialectical mechanism that both endangered and empowered the idea of progress. The third epoch constituted that moment of dialectic articulation where reason as the exterior force from the second epoch is destroyed to make way for the interiorisation of reason in the fourth epoch. This rather abstract idea may be concretized by an example. In the follow-up to the *Grundzüge*, the *Reden an die deutsche Nation* (1808), Fichte wrote about the sciences that were fixated in a dead (Latin) or foreign (French) language and therefore were but a “riven collection of arbitrary and inexplicable signs of equally arbitrary ideas” that one could only memorise but not develop.[15] If German would be adopted as a scientific language, however, it would be a living language where each sign was alive and sensible as part of language, culture and life, both in the past and in the present.[16] Whereas reason used to be written and expressed in an external language, it should now be transmuted in the language of a nation’s inner life. Instead of following and even writing in foreign style and language, German science should develop its own German language and thereby overcome its enslavement to foreign manners. This is instance of the dialectic third epoch where reason is destroyed to be newly created again in a fourth epoch. The sciences old style should be replaced by sciences new style.

The dynamics of the words ‘Begriff’ (or ‘Gedanken’) and ‘Zeichen’ (or ‘Symbol’ or ‘Sinnbild’) is quite capital in Fichte’s discourse. ‘Begriff’ stands for the living idea, but the ‘Zeichen’ (sign) that ports and communicates the living idea between people has an ambivalent role. It may either obscure the idea and make it inexplicable as part of arbitrary combinations, or it may clarify the essence of the idea by showing its embedding in a real live discourse. In our example, it may be expressed clumsily in a foreign language or clearly revelated in your mother tongue. In the first case, it would cause reason to have a fallback to the second epoch, in the second, it would progress to a fourth epoch. It is the task of philosophy to guide the sciences in their communication of ideas.

[14]“In Absicht seiner Meinungen über diese Gegenstände wird es durch den blinden Hang der Ideenassociation bald dahin bald dorthin gezogen werden […] Ein Meisterfund für die Darstellung eines solchen Zeitalters wäre es, wenn es darauf geriethe, die Wissenschaften nach der Folge der Buchstaben im Alphabete vorzutragen” (Fichte 1846a).

[15]“zerrissenen Sammlung willkürlicher und durchaus nicht weiter zu erklärender Zeichen ebenso willkürlicher Begriffe” (Fichte 1846b, 325).

[16]“Dieser übersinnliche Theil ist in einer immerfort lebendig gebliebenen Sprache sinnbildlich, zusammenfassend bei jedem Schritte das Ganze des sinnlichen und geistigen, in der Sprache niedergelegten Lebens der Nation in vollendeter Einheit, um einen, ebenfalls nicht willkürlichen, sondern aus dem ganzen bisherigen Leben der Nation nothwendig hervorgehenden Begriff zu bezeichnen, aus welchem, und seiner Bezeichnung, ein scharfes Auge die ganze Bildungsgeschichte der Nation rückwärtsschreitend wieder müsste herstellen können” (Fichte 1846b, 325).

Fichte's historiographic scheme and its dialectic moment would have a lasting influence on the philosophy of history in early 19th century Prussia, especially at the newly founded university of Berlin. For philology we could quote August Boeckh (1785–1867) who in his study of Antiquity saw mythology as a symbolisation of exterior ideas that gets replaced by an epoch of art that symbolises the inner ideas.[17] More important to our topic is G.W.F. Hegel (1770–1831) who turned the dialectic moment into a general principle that drives history itself.

Already in the introduction to the *Phänomenologie des Geistes* (1807) but more explicitly in his *Enzyklopädie der Wissenschaften* (1817) Hegel wrote against a mathematisation of philosophy, or against a primacy of mathematics over philosophy.[18] Again, an 'external' language, mathematics, is regarded as an unsuitable medium for communicating philosophical ideas, because the arbitrariness with which mathematical symbols can be combined disrespects the precise determination of an idea. In his polemic, Hegel took up the same dichotomic pair *Gedanke-Symbol* (idea/concept—symbol/figure):

> It would further be a superfluous and ungrateful effort to use such a recalcitrant and inadequate medium as are spatial figures and numbers for the expression of ideas […]. The first simple figures and numbers can adequately and without misunderstandings be used as symbols because of their simplicity, but they remain a heterogenous and beggarly expression for the idea. The first attempts at pure thought have used them as a makeshift, the Pythagorean number system is its most famous example. But for many concepts, this means is completely unsatisfactory, because the external combination and the arbitrariness of the concatenation of these symbols is inadequate for the nature of a concept, also, it becomes totally ambiguous which of the many relationships that are possible between combinations of numbers and figures should be apprehended. Furthermore, the fluidity of a concept evaporates in such an external medium where every determination dissolves in indifference. This ambiguity can only be lifted by explanation. The essential expression of an idea is this explanation, and such a symbolisation thus only an unsubstantial exuberance.[19]

[17]"Man könnte vielleicht sagen, die Kunst als Symbolisirung der Ideen sei später zu betrachten als die Mythologie, weil jene das Aeussere, diese das zu Grunde liegende Innere darstelle" (Boeckh 1877, 62).

[18]The mathematisation of philosophy had been very popular during the 18th century, typical exponents of this trend were Christian Wolff (1679–1754) or J.H. Lambert (1728–1777), see, e.g., Arndt (1971).

[19]"Es würde ferner eine überflüssige und undankbare Mühe sein, für den Ausdruck der Gedanken ein solches widerspenstiges und inadäquates Medium, als Raumfiguren und Zahlen sind, gebrauchen zu wollen [und dieselben gewaltsam zu diesem Behufe zu behandeln]. Die einfachen ersten Figuren und Zahlen eignen sich ihrer Einfachheit wegen ohne Missverstandnisse zu Symbolen, die jedoch immer für den Gedanken ein heterogener und kümmerlicher Ausdruck sind, angewendet zu werden. Die ersten Versuche des reinen Denkens haben zu diesem Nothbehelfe gegriffen; das pythagoreische Zahlensystem ist das berühmte Beispiel davon. Aber bei manchen Begriffen werden diese Mittel völlig ungenügend, da deren äusserliche Zusammensetzung und die Zufälligkeit der Verknüpfung überhaupt der Natur des Begriffs unangemessen ist, und es völlig zweideutig macht, welche der vielen Beziehungen, die an zusammengesetzte[n] Zahlen und Figuren möglich sind, festgehalten werden sollen. Ohnehin verfliegt das Flüssige des Begriffs in solchem äusserlichen Medium, worin jede Bestimmung in das gleichgültige Aussereinander fällt. Jene Zweideutigkeit könnte allein durch die Erklärung gehoben werden. Der wesentliche Ausdruck des Gedankens ist alsdann diese Erklärung, und jenes Symbolisiren ein gehaltloser Ueberfluss" (Hegel 1845, § 259).

Mathematics is thus surely not suited for expressing philosophical ideas, but worse still, according to Hegel, mathematics, by its use of symbols and signs, is not even adequate to express the essence of time and space. Therefore Hegel advocated a new mathematics, a philosophical mathematics. Signs might still be useful for calculating, but for comprehension and explication, philosophy would be needed.

2 A Case of Epistemological History: Alexander Von Humboldt's Project on Numeration Systems

Adapting the history of mathematics to the new format of a progression of ideas implied, as was clear in Bossut's *Essai*, that the facts of mathematics' history should be rearranged according to the inner logic of mathematics as a discipline. They should line up to provide a self-consistent development of fundamental ideas that made up 19th century mathematics. The historical genesis of mathematics should ideally repeat or foreshadow the individual's learning curve in mathematics, the order of learning mathematics anno 1800. Such a narrative sequences key moments in the history of mathematics in order of ascending difficulty as does a school curriculum. It could start with Greek mathematics (Euclid's *Elements*, geometry and elementary arithmetic), than pursue with symbolic algebra that dates back to the early 17th century and go on with the calculus that was invented in the late 17th century. One key invention, however, resisted the format: the origin of the decimal place-value notation for numbers. Whereas in the order of things mathematical, it should come before the others, it historically only appeared in Europe during the 8th to 13th century.

As noted in the famous *Rapport historique sur les progrès des sciences mathématiques* (1810) even Napoleon himself had pointed out that the fact that the ancient Greek used another numeral system than the modern one was "une lacune très remarquable dans l'histoire des mathématiques" (Delambre 1810, 37). The notation system for numbers did fit neither chronologically nor geographically into a nice derivation from Greek Antiquity to mathematics anno 1800. This spurred two new kinds of investigation of our numerals' origin. A first strand focussed upon the (Greek) practice of doing arithmetic, here we may quote J.B. Delambre's (1749–1822) "De l'arithmétique des Grecs" (1807) or Reimer's (1772–1832) additions in his translation of Bossut's *Essai* (1804). A second strand was nurtured by the influx of new information on various (oriental) languages and the emergence of modern philology.

In a project that was never completed, Alexander von Humboldt (1769–1859) tried to combine both approaches to get at the intellectual origins of the decimal place-value system. Both by his being in between France and Prussia and his being in between generations of ethnologists and philologists, his project documents changing methodologies in the beginning of the 19th century.[20] Alexander von Humboldt most literally concretised Condorcet's methodology to "choose, join, combine all facts of history and of different people to make a hypothetical history" (as Condorcet wrote) in his studies on our modern numeral system.

2.1 Hypothetical History

The very beginning of A. v. Humboldt's interest in numeral systems seems to have been of ethnographic origin. In his *Monumens des peuples indigènes de l'Amérique* (1816, 1824) Humboldt described a numeral system developed by the Muyscas people. They used words for numbers that have the same roots as words that indicate the phases of the moon. According to Humboldt, "this would be one of the most remarkable facts in the philosophical history of language", because in all other languages the roots for numerals are completely independent of roots for words that express objects from the physical world (von Humboldt 1824, 241–242). Indeed, in 18th century linguistics, it was thought that all words stood in one-to-one correspondence with real world sensations, except for function words such as pronouns, articles, numerals etc. It was, however, thought that all function words did descend from object words but that their original meaning and expression had faded over time. For most of the function words, etymological reconstructions were found or proposed, but the numerals proved to be the hardest case.

Between 1808 and 1827, when A. von Humboldt lived in Paris, his initial ethnographic interest slowly grew into a rather panoramic project to both describe the variety of numerals used throughout the world and derive the origins of our positional decimal system. It remained a project but he described the main lines of his project in two lectures: One at the Académie royale des sciences in Paris, 20th September 1819 (von Humboldt 1819) and one at the Akademie der Wissenschaften zu Berlin, 2nd March 1829 (von Humboldt 1829). These texts lay bare a hub of communication between France, Germany and England on matters mathematical and philological.

The 'natural milieu' in Paris for Humboldt's study on numerals was the circle of researchers that would found the Société Asiatique in 1822 (Silvestre de Sacy, Abel-Rémusat and others). This was complemented by his own network in the German states and by close communication with his brother Wilhelm in Berlin who was also interested in the project because of his interest in Sanscrit and the Kawi-language. These networks are representative of a more general trend, that of the renewal of the studies of language at the beginning of the 19th century. Silvestre de Sacy (1758–1838) had been appointed as professor of Persian at the Collège de France in 1806. In 1814 August Boeckh had founded the *Philologisches Seminar* at the Berlin university that would renew classical philology. In 1816 Franz Bopp

[20]An interdisciplinary research group has recently been mounted at SPHERE (Paris) by M. Bullynck, A. Keller, I. Smadja and others to study the genesis and influence of A. von Humboldt's project.

(1791–1867) had published his *Über das Conjugationssystem der Sanskritsprache* that would found comparative linguistics. Also in 1816 Wilhelm von Humboldt (1767–1835) had secured Julius Klaproth (1783–1835) a pension from the Prussian state to pursue his Asiatic studies in Paris where Jean-Pierre Abel-Rémusat (1788–1832) held the newly founded chair of Chinese at the Collège de France.

Relying on those networks, A. von Humboldt managed to gather a variety of material on Arabic, Chinese, Indian, Ancient Egyptian, medieval Latin etc. use of numerals. He used these materials to derive an 'epistemological' reconstruction of the genesis of the decimal place-value system. The aim was to show how the Hindu-arabic notation could have been developed:

> I cannot historically develop that the origin of the Indian place-value system with 9 ciphers is really the one that I have indicated, but I believe I found a way in which the discovery could gradually have been made.[21]

To obtain this genesis of our numeral system, Humboldt proposed to abstract from the actual appearance of numeral signs and focus on the structural properties:

> In the studies of numerical signs, one has been occupied so far more with the characteristic physiognomy of the signs and their individual forms than with the idea of the methods […] I have made it myself a rule in this article to use no other signs than our usual arithmetic and algebraic ones. In this way, the attention is focused more on the essence of things, on the idea of the method.[22]

Individual symbols stood in the way of the general ideas that developed through history, therefore they had to be transformed to see the inner structure of methods—by transcribing everything into modern algebraic and numerical signs. By destroying the original symbols with modern signs, the idea itself should become visible.

In Humboldt's hypothetical history of the Hindu-arabic numerals, as explained in the 1819 and 1829 lectures, there were basically three phases. A first epoch was that of juxtaposition, repetition of simple signs (group signs) such as dashes: III. The group signs might evolve and be replaced and/or supplemented by a fixed sequence of arbitrary signs such as the sequence of the letters of the alphabet or another series of conventional signs (such as our 1,2,3, …). A second epoch advanced to the possible simplification of these group signs using an exponent to indicate repetition of the (group) sign, I^3. The 'diacritical' sign might be under (index) or over (exponent) the group sign or it might appear left of the group sign, as a coefficient, $3I$. In cases, the group sign might appear under or over the

[21]"Ich kann nicht historisch entwickeln, dass der Ursprung des indischen Stellenwerthes von 9 Ziffern wirklich der sei, welchen ich angegeben, aber ich glaube einen Weg gefunden zu haben, auf welchen allmählich die Entdeckung gemacht werden konnte" (von Humboldt 1829, 207).

[22]"Man hat sich bisher, in den Untersuchungen über die numerischen Zeichen […] ernster mit der characteristischen Physiognomik der Zeichen und ihrer individuellen Gestaltung, als mit dem Geist der Methoden beschäftigt […] Ich habe es mir in dieser Abhandlung zum Gesetz gemacht, keine anderen Zeichen, als die gewöhnlichsten arithmetischen und algebraischen zu gebrauchen. Die Aufmerksamkeit wird auf diese Weise mehr auf das Wesentliche, auf den Geist der Methode, gerichtet" (von Humboldt 1829, 204–214).

diacritical sign, 3^I reversing the positions of the group sign and the repetition sign. This last reversal was a critical transition point in Humboldt's reconstruction since it provided the link between juxtaposing numeral system (as the Chinese system) to a positional system such as the Indian number system. A final epoch dropped the (group) signs and kept only the exponents. Adding a 'zero' closed the deal.

2.2 Philologies

Three clusters of material were important 'cornerstones' for building Humboldt's argument (cf. Table 1). First, the discovery of the gobar numerals in the North of Africa by Silvestre de Sacy for the transition from I^3 to 3^I. Second, the book *The philosophy of arithmetic* (1817) by John Leslie (1766–1832), who basically compared the words and signs used to count in many cultures and introduced a distinction between palpable and figurative arithmetic. Palpable arithmetic meant doing arithmetic using moveable objects (from hands to calculating instruments), figurative arithmetics is characterised by the use of symbols to denote numbers and to operate on them. And finally, Henry Thomas Colebrooke's (1765–1837) work on Indian mathematics (that Alexander got via mediation of comparative philologist Franz Bopp) and his brother Wilhelm's studies on Sanscrit provided ample new information on numeral systems on the Indian subcontinent. These materials were key to Humboldt's history of transmission and transformation of numeral signs, providing the cultural substratum (either linguistic or material) that could accomodate for the transformations of numerals throughout their history (say, explain how it was transmitted and how it was transformed here rather than there).

The first question was of course: Where did the first impetus towards our numeral system come from before it got its present form in India? This was the most speculative aspect of Humboldt's paper and drew extensively on Leslie's differentiation between palpable and figurative arithmetic. Humboldt defended the idea that the Chinese suanpan (our abacus) might be at the origin of our number system.

> The usage of the suanpan made the people familiar with the idea of many ranks of groups; they showed an empty place (sifroun) where an intermediary group was lacking. The Chinese artifice that placed the unities as multiplicators over the group signs probably completed the discovery. It transplanted so to say the germ of the indian method from the domain of palpable arithmetic into the domain of figurative or graphical arithmetic.[23]

In a letter to Franz Bopp, A. v. Humboldt called this 'proof' of the Chinese origin of the Hindu-arabic numerals the main result of his 1819 paper.

[23]"L'usage du suanpan accoutumait les peuples à l'idée de plusieurs rangs de groupes; ils montraient un place vide (un sifroun) là où manquait un groupe intermédiaire. L'artifice chinois de placer des unités comme multiplicateurs au-dessus des signes de groupes acheva probablement la découverte. Il transplanta, pour ainsi dire, le germe de la méthode indienne du domaine de l'arithmétique palpable dans le domaine de l'arithmétique figurative ou graphique." (von Humboldt 1819, 100).

Table 1 Overview of some of A.v. Humboldt's sources with indication of the year(s) of the most important publication(s) and/or interaction(s)

Source	Year	Topic
A. v. Humboldt	1810	Muyscas-system
S. de Sacy	1810	Arabic gobar-notation
W. v. Humboldt	1810–1835	South-Indian and Malay numerals
A. Boeckh	1811–…	Greece and Rome
J. Leslie	1817	Palpable vs. figurative arithmetic
Th. Young	1814–1823	Egyptian numerals
J. Klaproth	1815–1835	Asiatic languages
H. Th. Colebrooke	1817	Indian numerals
Franz Bopp	1819–20	Indo-Germanic numerals
J.-F. Champollion	1822–1832	Egyptian numerals
Karl Otfried Müller	1820–1828	Rome and Etruscan
J.-P. Abel-Rémusat	1825	Chinese

A second development in the evolution and transmission of decimal positional number systems that Humboldt wanted to explain is: Why did it emerge in Sanscrit, in an Indian context? Humboldt had learned from Colebrooke that the Indians do and did not use only one system of notation, they used a variety of systems in parallel, both in writing and in speech.[24] They used a set of words for each numeral to be juxtaposed according to the metre, they used the same words but with position (without zero) also to fit in a metre; they used syllables for ciphers, indian ciphers, both from left to right as well as right to left, etc. Looking at the (non-indo-european) languages spoken on the Indian subcontinent, that his brother Wilhelm von Humboldt was studying, the variety became even more overwhelming. According to Humboldt, this was fertile ground once the Chinese way of notating numbers arrived in the Indian subcontinent. The rich variety of notations that were all linear and sequential proved the perfect substratum in which the new notation could settle, find its expression and slowly fixate the numeral system that would be ours. One thing changed, the direction of writing: "the order that had been established in a perpendicular writing must have been conservated in horizontal writing".[25]

The final transmission to be accounted for was the one between the Arabic world and the Indian world. Here, the gobar or dust writing provided the missing link. Silvestre de Sacy had signalled the gobar numerals in his *Grammaire arabe à l'usage des élèves de l'école spéciale des langues orientales vivantes* (1810) in a footnote. Borrowing the idea of diacritical marks often used in semitic languages to indicate vowels in an otherwise consonantic writing system, this notation used *n*

[24]For a modern overview, see Singh (1997).

[25]"l'ordre établi dans l'écriture perpendiculaire a dû être conservé dans l'écriture horizontale" (von Humboldt 1819, 101).

points above a cipher to indicate that the cipher should be multiplied with ten to the n-th power. According to Alexander von Humboldt's interpretation the gobar notation, used by the Mauretian customs, might be a variant of our numeral system with zero, though still displaying some older stage of its development. It was evidence of that transitional point where multiples of a number are expressed by marks placed above or under the signs and thus for the slow process of transmission between the Indian subcontinent and the Arabic world.

3 Beginnings of Modern History of Mathematics

From the year 1819 onwards the initial speculative enthusiasm present in A. v. Humboldt's first version of his 'Zahlzeichen'-project gradually dimmed. In the 1829 incarnation for Crelle's Journal and once again in its last appearance, as an endnote in Part 2 of the *Kosmos* (1847), the focus shifted. These shifts correspond to disciplinary evolutions during those years.

The 1820s were years of *rétablissement* in the German states after the wars with Napoleon. Not only the new universities of Berlin and Bonn were founded, but new professorships and new disciplines altogether were created. For instance, classical philology and Sanscrit studies began to flourish in the German states and this had an impact on the history of mathematics too. Alexander's brother Wilhelm had been working in the 1820s on his opus magnum *Ueber die Kawi-Sprache auf der Insel Java* (4 volumes, written 1830–1833, publ. 1836–1838) collecting many new sources on Sanscrit. Franz Bopp was working on his *Vergleichende Grammatik* (6 volumes, 1833-1852) of which a capital part was the comparison of the numerals in different languages. At Berlin university Boeckh himself had edited the texts of Philolaos (1819) and had published on ancient metrology (1817, 1838) and chronology (1855–6; 1862).[26] A number of his students, such as J.E. Nizze (1788–1872) and his colleague C.F.J. Hasenbalg would embark on historical critical editions of Greek mathematicians (Theodosius (1826), Hero (1826)). Others such as K.O. Müller (1797–1840) would study the Minean, Doric and Etruscan cultures (1820–1828). This influx of material from the Indo-European realm refocused A. v. Humboldt's project. It now featured lots of information on Sanscrit and other languages from the Indian subcontinent[27] and as a consequence the idea of palpable arithmetic and of Chinese influences disappeared from the main argument, though it was still mentioned. The main result of the 1829 paper was rather a summary of philologic findings and in this spirit Humboldt closed the paper with the wish that the philologists might find and exploit more material in a near future.

[26] On Humboldt and Boeckh, see also Knobloch (2011).

[27] The explanation is only briefly mentioned in 1819 (p. 101), but given more space in 1829 (pp. 212–213, 215–216, 219, 226–228).

In 1847 Humboldt's project had shrunk to a mere two-page endnote in his opus magnum *Kosmos*. By that time, a new generation of researchers, both competent in the new philologies and in mathematics, had begun to study historical sources of mathematics. Friedrich August Rosen (1805–1837), a student of Franz Bopp, went to work in London to edit Sanscrit sources and translated Al-Khwarizmi's *Algebra* in 1831. Franz Woepcke (1826–1864), another student of Berlin university, got a stipend through A. v. Humboldt to go work in Paris where he published and translated a variety of Arabic works on mathematics (1851–1863). G.H.F. Nesselmann (1811–1881), a student of C.G.J. Jacobi (1804–1851, himself once a student of Hegel and Boeckh in Berlin) wrote a *Versuch einer kritischen Geschichte der Algebra* (1842) that analysed the original texts and posited three phases in the history of algebra.[28] This had made Humboldt's own project nearly redundant. As (pure) mathematics had emancipated itself in the beginning of the 19th century to become a blossoming discipline in the 1850s, so it began to become self-conscious too. Using the new philologies that had matured in parallel, mathematics started to write its own histories.

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[28]The first one is a rhetorical stage where everyday words are used for stating and solving a problem, the second stage is a syncopated stage of abbreviating words, and a final stage would be the symbolic stage that manipulates its elements with the utmost economy. This three-phase scheme has a close connection with Humboldt 1829-article, compare pp. 62–64 with p. 302 in Nesselmann (1842). More on these authors that mainly worked on Arabic sources can be found in Sonja Brentjes'contribution to this volume.

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