

Chapter 2

Electron Optics

Chapter 1 represented an overview of various forms of microscopy, carried out using light, electrons, or mechanical probes. In each case, a microscope forms an enlarged image of the original object (the specimen) in order to show its internal or external structure. Before considering in more detail different aspects of electron microscopy, we will first examine some very general concepts involved in image formation. These ideas were derived during the development of visible-light optics but have a range of applications that is much wider.

2.1 Properties of an Ideal Image

Clearly, an optical image bears a close resemblance to the corresponding object, but what does this mean? What properties should the image have in relation to the object? The answer to this question was provided by the Scottish physicist James Clark Maxwell, who also developed the equations that underlie electrostatic and magnetic phenomena, including electromagnetic waves. In a journal article in 1858, he stated the requirements of a perfect imaging as follows:

1. For each point in the object, there is an *equivalent* point in the image.
2. The object and image are geometrically *similar*.
3. If the object is *planar* and perpendicular to the optic axis, so is the image.

Besides defining the desirable properties of an image, Maxwell's principles are useful for categorizing the **image defects** that occur when (in practice) the image is *not* ideal. To see this, we will discuss each rule in turn.

Rule 1 states that for each point in the object we can define a corresponding point in the image. In many forms of microscopy, the connection between these two points is made by some particle (e.g., electron or photon) that leaves the object and ends up at the corresponding image point. The particle is conveyed from object to

image through a focusing device (some form of lens) and its trajectory is referred to as a **ray** path. One particular ray path is called the **optic axis**; if no mirrors are involved, the optic axis is a straight line passing through the center of the lens or lenses.

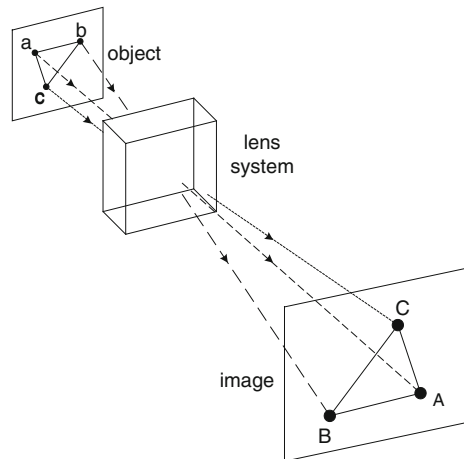
How closely Maxwell's first rule is obeyed depends on several properties of the lens. For example, if the focusing *strength* is incorrect, the image formed at the plane of observation will be **out-of-focus**; particles leaving a single object point arrive anywhere within a circular region, a **disk of confusion**. But even if the focusing power is appropriate, a real lens may produce a disk of confusion because of lens **aberrations**: particles having different energies, or which take different paths after leaving the object, arrive displaced from the "ideal" image point. The image then appears blurred, indicating a loss in spatial resolution.

Rule 2: If we consider object points that form a pattern, their equivalent points in the image should form a *similar* pattern, rather than being distributed at random. For example, any three object points occur at the corners of a triangle and the equivalent points in the image define a triangle which is similar in the geometric sense: it contains the same angles. The image triangle may have a different orientation; for example, it could be **inverted** (relative to the optic axis, as in Fig. 2.1) without violating Rule 2. Or the separations of the three image points may differ from those in the object by a **magnification factor** M , in which case the image is **magnified** (if $M > 1$) or **demagnified** (if $M < 1$).

Even when the light image formed by a glass lens appears similar to the object, a close inspection may reveal the presence of **distortion**. This effect is most easily observed if the object contains straight lines, which appear as curved lines in the distorted image.

The presence of distortion is equivalent to a variation of M with *position* in the object or image: **pincushion** distortion corresponds to M increasing with radial distance away from the optic axis (Fig. 2.2a), **barrel** distortion corresponds to

Fig. 2.1 A triangle imaged by an ideal lens, with magnification and inversion. Image points A, B, and C are *equivalent* to the object points a, b, and c, respectively



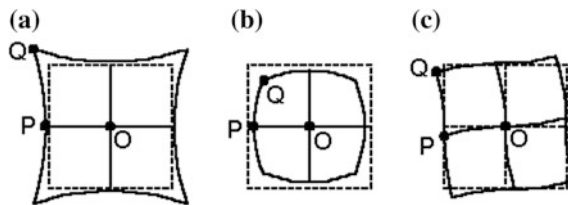


Fig. 2.2 **a** A square mesh (*dashed lines*) imaged with pincushion distortion (*solid curves*); magnification M is higher at point Q than at point P. **b** Image showing barrel distortion, with M at Q lower than at P. **c** Image of a square, showing spiral distortion; the counterclockwise rotation is higher at Q than at P

M decreasing away from the axis (Fig. 2.2b). As we will see, electron lenses may cause a *rotation* of the image and if this rotation changes with distance from the axis, the result is **spiral** distortion (Fig. 2.2c).

Rule 3: Images usually exist in two dimensions and occupy a *flat* plane. In fact, lenses are usually evaluated using a flat test chart and the sharpest image should ideally be produced on a flat screen. But if the focusing power of a lens depends on the distance of an object point from the optic axis, different regions of the image are brought to focus at different distances from the lens. The optical system then suffers from **curvature of field** and the sharpest image would be formed on a surface that is curved rather than planar. Cameras and telescopes have sometimes compensated for this defect by having a curved recording plane.

2.2 Imaging in Light Optics

Because electron optics involves many of the same concepts as light optics, we will first review some basic optical principles. Glass lenses are used to focus light, based on the property of **refraction**: deviation in direction of a light ray at a boundary where the **refractive index** changes. Refractive index is inversely related to the *speed* of light, which is $c = 3.00 \times 10^8$ m/s in vacuum (and almost the same in air) but c/n in a transparent material (such as glass) of refractive index n . If the angle of incidence (between the ray and a line drawn perpendicular to the interface) is θ_1 in material 1 (e.g., air, $n_1 \approx 1$), the corresponding angle θ_2 in material 2 (e.g. glass, $n_2 \approx 1.5$) differs by an amount given by Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (2.1)$$

Refraction can be demonstrated by means of a glass prism; see Fig. 2.3. The total deflection angle α , due to refraction at the air/glass *and* the glass/air interfaces, is independent of the thickness of the glass, but increases with increasing prism angle ϕ . For $\phi = 0$, corresponding to a flat sheet of glass, there is no overall angular deflection ($\alpha = 0$).

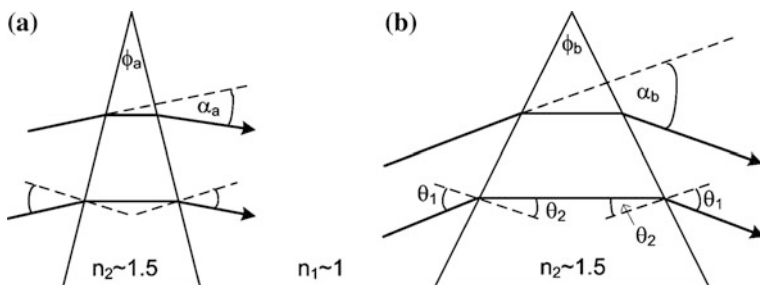


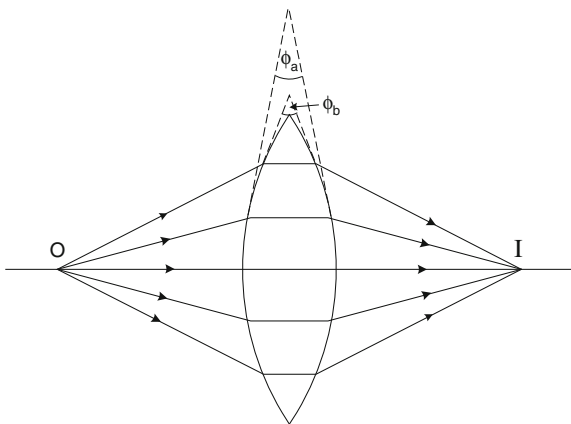
Fig. 2.3 Light refracted by a glass prism **a** of small angle ϕ_a and **b** of large angle ϕ_b . Note that the angle of deflection α is independent of the glass thickness but increases with increasing prism angle

A **convex lens** can be regarded as a prism whose angle increases with distance away from the optic axis. Therefore rays that arrive at the lens far from the optic axis are deflected more than those that arrive close to the axis (the so-called **paraxial** rays), even though the lens gets thinner towards its edge. To a first approximation, the deflection of a ray is proportional to its distance from the optic axis (at the lens), as needed to make rays starting from an on-axis object point *converge* towards a single (on-axis) point in the image (Fig. 2.4) and therefore satisfy the first of Maxwell's rules for image formation.

Figure 2.4 shows only rays that originate from a *single point* in the object, which happens to lie on the optic axis. In practice, rays originate from all points in the two-dimensional object and travel at various angles relative to the optic axis. A diagram showing *all* of these rays would be highly confusing and in practice it is sufficient to represent just a few selected rays, as in Fig. 2.5.

One special ray travels *along* the optic axis. Since it passes through the center of the lens, where the prism angle is zero, this ray does not deviate from a straight line. Similarly, an oblique ray that leaves the object at a distance x_o from the optic axis

Fig. 2.4 A convex lens focusing rays from axial object point O to an axial image point I



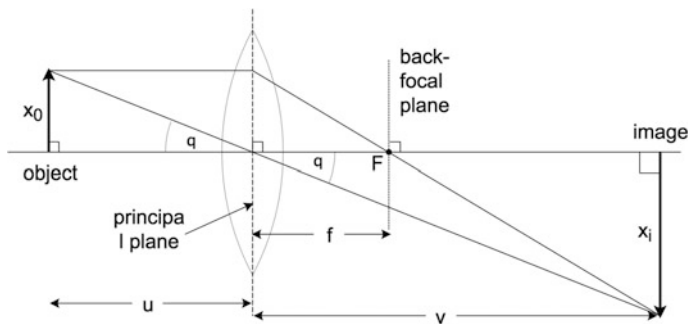


Fig. 2.5 A thin-lens ray diagram, in which bending of light rays is imagined to occur at the mid-plane of the lens (*dashed vertical line*). These special rays define the focal length f of the lens and the location of the back-focal plane (*dotted vertical line*)

and passes through the *center of the lens*, remains unchanged in direction. A third special ray leaves the object at distance x_o from the axis but travels *parallel to the optic axis*. The lens bends this ray towards the optic axis, so that it crosses the axis at point F, a distance f (the **focal length**) from the center of the lens. The plane that is perpendicular to the optic axis and passes through F is known as the **back-focal plane** of the lens.

In drawing these ray diagrams, we are using **geometric optics** to depict the image formation. Light is represented by rays rather than waves and so we ignore any diffraction effects, which would require **physical optics**.

It is convenient to assume that the bending of light rays takes place at a single plane (known as the **principal plane**) perpendicular to the optic axis (dashed line in Fig. 2.5). This assumption is reasonable if the lens is *physically thin*, so it is known as the **thin-lens approximation**. Within this approximation, the object distance u and the image distance v (both measured from the principal plane) are related to the focal length f by the **thin-lens equation**:

$$1/u + 1/v = 1/f \quad (2.2)$$

We can define image magnification as the ratio of the lengths x_i and x_o measured perpendicular to the optic axis. Since the two triangles defined in Fig. 2.5 are similar (both contain a right angle and the angle θ), the ratios of their horizontal and vertical dimensions must be equal. In other words,

$$v/u = x_i/x_o = M \quad (2.3)$$

From Fig. 2.5, we see that if a single lens forms a **real image** (one that could be viewed by inserting a screen at the appropriate plane), this image is *inverted*, equivalent to a 180° rotation about the optic axis. If a second lens is placed beyond this real image, the latter acts as an *object* for the *second* lens, which produces a *second* real image that is upright (non-inverted) relative to the original object.

The location of this second image is given by applying Eq. (2.2) with appropriate new values of u and v , while the additional magnification produced by the second lens is given by Eq. (2.3). The *total* magnification (between second image and original object) is then just the *product* of magnification factors of the two lenses.

If the second lens is placed *within* the image distance of the first, a real image may not occur. Instead, the first lens is said to form a **virtual image**, which acts as a virtual *object* for the second lens (having a *negative* object distance u). In this situation, the first lens produces *no* image inversion. A familiar example is a magnifying glass held within its focal length of the object; there is no inversion and the only real image is that produced on the retina of the eye, which the brain *interprets* as upright.

Most glass lenses have *spherical* surfaces (sections of a sphere) because these are the easiest to make by grinding and polishing. Such lenses suffer from **spherical aberration**, meaning that rays arriving at the lens at larger distances from the optic axis are focused to points that differ slightly from the focal point of the paraxial rays. Each image point then becomes a disk of confusion and the image produced on any given plane is blurred (reduced in resolution, as discussed in Chap. 1). *Aspherical* lenses have their surfaces tailored to the precise shape required for ideal focusing (for a given object distance) but are more expensive to produce.

Chromatic aberration arises when the light being focused has more than one wavelength present. A common example is white light, which contains a continuous range of wavelengths between its red and blue components. Because the refractive index of glass varies with wavelength (called **dispersion**, since it allows a glass prism to *separate* the component colors of the white light), the focal length f and the image distance v are slightly different for each wavelength present. Again, each image point is broadened into a disk of confusion and image sharpness is reduced.

2.3 Imaging with Electrons

Electron optics has much in common with light optics. We can imagine individual electrons leaving an object and being focused into an image, similar to visible-light photons. Because of this analogy, each electron trajectory is often referred to as a *ray* path.

To obtain the equivalent of a convex lens for electrons, the angular deviation of an electron ray must increase as its displacement from the optic axis increases. We cannot rely on refraction by a material such as glass, since electrons are strongly scattered and absorbed after entering a solid. However, we can take advantage of the fact that the electron has an electrostatic charge and is therefore deflected by an electric field. Alternatively, we can utilize the fact that the electrons in a beam are moving; the beam is equivalent to an electric current in a wire, and can be deflected by applying a magnetic field.

Electrostatic lenses

The simplest example of an electric field is the uniform field produced between two parallel conducting plates when a constant voltage is applied between them. An electron entering such a field would experience a constant force, regardless of its trajectory (ray path). This arrangement is suitable for *deflecting* an electron beam, but not for focusing.

The simplest electrostatic *lens* consists of a circular conducting electrode (disk or tube) connected to a negative potential and containing a circular hole (aperture) centered about the optic axis. An electron passing along the optic axis is repelled equally in all **azimuthal** directions (*around* the optic axis) and therefore suffers no deflection. But an off-axis electron is repelled more strongly by the negative charge that lies closest to it and is therefore deflected back towards the axis, as in Fig. 2.6. To a first approximation, the deflection angle is proportional to displacement from the optic axis and a point source of electrons is focused to a single image point.

A practical form of electrostatic lens (known as a **unipotential** or *einzel* lens, since electrons enter and leave it at the same potential) uses additional electrodes placed before and after, to limit the extent of the electric field produced by the central electrode, as illustrated in Fig. 2.6. Note that the electrodes, and therefore the electric fields that provide the focusing, have cylindrical or **axial symmetry**, which ensures that the focusing force depends only on *radial* distance of an electron from the axis and is independent of its *azimuthal* direction around the axis.

Electrostatic lenses were used in cathode-ray tubes and television picture tubes, to ensure that the electrons emitted from a heated filament were focused back into a small spot on the phosphor-coated inside face of the tube. Although some early electron microscopes used electrostatic optics, modern electron-beam instruments

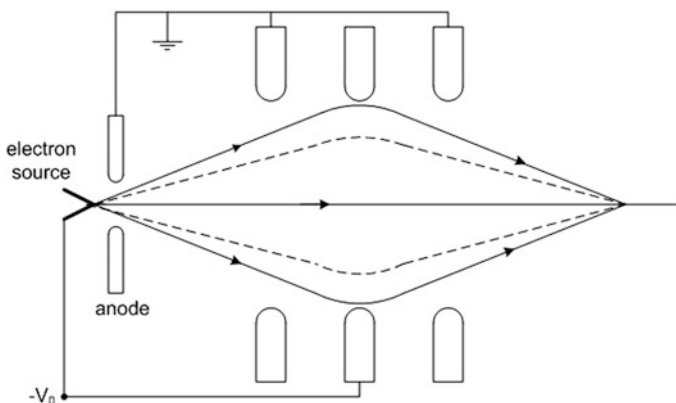


Fig. 2.6 Electrons emitted from an electron source, accelerated through a potential difference V_0 towards an anode, and then focused by a unipotential electrostatic lens. The electrodes, seen here in cross-section, are circular disks containing round holes (apertures) with a common axis, the optic axis of the lens

use electromagnetic lenses that do not require high-voltage insulation and have somewhat lower aberrations.

Magnetic Lenses

To focus an electron beam, an electromagnetic lens generates a magnetic field by passing a direct current through a coil containing many turns of wire. As in the electrostatic case, a *uniform* field (applied perpendicular to the beam) would deflect but not focus a parallel beam. To achieve focusing, we require a field with rotational symmetry, similar to that of the einzel lens, and such a field is generated by the short coil illustrated in Fig. 2.7a. As an electron passes through this non-uniform magnetic field, the force acting on it varies in both magnitude and direction, so the force must be represented by a vector quantity $\mathbf{F}$. According to the electromagnetic theory,

$$\mathbf{F} = -e(\mathbf{v} \times \mathbf{B}) \quad (2.4)$$

In Eq. (2.4), $-e$ is the negative charge of the electron, $\mathbf{v}$ is its velocity vector, and vector $\mathbf{B}$ represents the magnetic field: its magnitude B (the **induction**, measured in Tesla) and its direction. The symbol $\times$ indicates a cross-product or **vector product** of $\mathbf{v}$ and $\mathbf{B}$, a mathematical operator that gives Eq. (2.4) the following two properties.

1. The direction of $\mathbf{F}$ is *perpendicular* to both $\mathbf{v}$ and $\mathbf{B}$. Consequently, $\mathbf{F}$ has *no* component in the direction of motion, implying that the electron speed v (the *magnitude* of the velocity $\mathbf{v}$) remains constant at all times. But since the

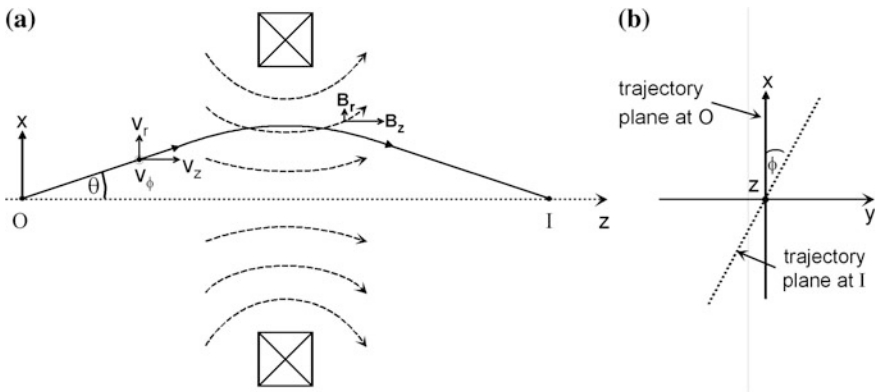


Fig. 2.7 **a** Magnetic flux lines (*dashed curves*) produced by a short coil, seen here in cross-section, together with the trajectory of an electron from an axial object point O to the equivalent image point I. **b** View along the optic axis (z -direction) showing the rotation ϕ of the plane of the electron trajectory, which is also the azimuthal rotation angle for an extended image produced at I

directions of $\mathbf{B}$ and $\mathbf{v}$ change as the electron moves through the field, the direction of $\mathbf{F}$ changes also.

2. The magnitude F of the force is given by:

$$F = e v B \sin(\varepsilon) \quad (2.5)$$

where ε is the instantaneous angle between $\mathbf{v}$ and $\mathbf{B}$ at the position of the electron. Because B changes continuously as an electron passes through the field, so does F . Note that for an electron traveling along the coil axis, $\mathbf{v}$ and $\mathbf{B}$ are both always in the axial direction, giving $\varepsilon = 0$ and $F = 0$ at every point, implying no deviation of the ray path from a straight line. Therefore the symmetry axis of the magnetic field is the optic axis.

For non-axial trajectories, the motion of the electron is more complicated. It can be analyzed in detail by using Eq. (2.4) in combination with Newton's second law ($\mathbf{F} = m \, d\mathbf{v}/dt$) and such analysis is simplified by considering $\mathbf{v}$ and $\mathbf{B}$ in terms of their vector components. Although we could take components parallel to three perpendicular axes (x , y , and z), it makes more sense to recognize that the magnetic field here possesses axial (cylindrical) symmetry. We therefore use cylindrical coordinates: z , r (=radial distance away from the z -axis) and ϕ (=azimuthal angle, representing the direction of the radial vector $\mathbf{r}$ relative to the plane of the initial trajectory).

In Fig. 2.7a, v_z , v_r , and v_ϕ are the axial, radial, and tangential components of electron velocity, while B_z and B_r are the axial and radial components of magnetic field. Equation (2.5) can then be rewritten to give the tangential, radial, and axial components of the magnetic force on an electron:

$$F_\phi = -e(v_z B_r) + e(B_z v_r) \quad (2.6a)$$

$$F_r = -e(v_\phi B_z) \quad (2.6b)$$

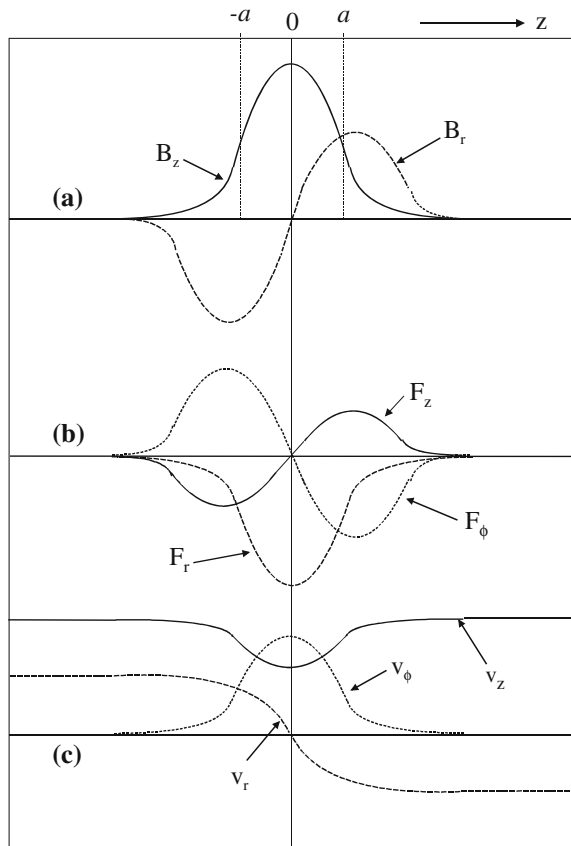
$$F_z = e(v_\phi B_r) \quad (2.6c)$$

We now trace the path of an electron that starts from an axial point O and enters the field at an angle θ relative to the symmetry (z) axis, as defined in Fig. 2.7. As the electron approaches the field, the main component is B_r and the predominant force comes from the term $(v_z B_r)$ in Eq. (2.6a). Since B_r is negative (field lines *approach* the z -axis), this contribution $(-e v_z B_r)$ to F_ϕ is positive, meaning that the tangential force F_ϕ is clockwise, viewed along the $+z$ direction. As the electron approaches the center of the field ($z = 0$) the magnitude of B_r decreases but the second term $e(B_z v_r)$ in Eq. (2.6a), which is also positive, increases. So as a result of both terms, the electron starts to spiral within the field, acquiring an increasing tangential velocity v_ϕ directed *out* of the plane of Fig. (2.7a). As a result of this tangential component, a new force F_r starts to act on the electron. According to Eq. (2.6b), this force is negative (towards the z -axis), so we have a focusing action: the non-uniform magnetic field acts like a convex lens.

Provided that the radial force F_r towards the axis is large enough, the radial motion of the electron will be *reversed* and the electron will approach the z -axis; v_r becomes negative and the *second* term in Eq. (2.6a) becomes negative. After the electron passes the $z = 0$ plane (the center of the lens), the field lines start to diverge so that B_r becomes positive and the *first* term in Eq. (2.6a) also becomes negative. As a result, F_ϕ becomes negative (reversed in direction) and the tangential velocity v_ϕ falls, as shown in Fig. 2.8c; by the time the electron leaves the field, its spiraling motion is reduced to zero. However, the electron is now traveling in a plane that has been *rotated* relative to its original (x - z) plane, as shown in Fig. 2.7b.

This rotation effect is *not* depicted in Fig. 2.7a or in other ray diagrams in this book, where for convenience we plot the radial distance r of the electron (from the axis) as a function of its axial distance z . This common convention allows the use of two-dimensional rather than three-dimensional diagrams; by deliberately ignoring the rotation effect, we can draw ray diagrams that resemble those of light optics. Even so, it is important to remember that when an electron passes through an axially symmetric magnetic lens, its motion has a rotational component.

Fig. 2.8 Qualitative behavior of the radial (r), axial (z), and azimuthal (ϕ) components of **a** magnetic field, **b** force on an electron, and **c** the resulting electron velocity, as a function of the z -coordinate of an electron going through the electron lens shown in Fig. 2.7a



Since the overall speed v of an electron in a magnetic field remains constant, the appearance of tangential and radial components of velocity implies that v_z must decrease, as depicted in Fig. 2.8c. This is in accord with Eq. (2.6c), which predicts the existence of a third force F_z that is negative for $z < 0$ (because $B_r < 0$) and therefore acts in the $-z$ direction. After the electron passes the center of the lens, z , B_r and F_z all become positive and v_z increases back to its original value. The fact that the total speed v remains constant contrasts with the case of the einzel electrostatic lens, where an electron slows down as it passes through the retarding field.

We have seen that the radial component of magnetic induction B_r plays an essential part in electron focusing. If a long coil (solenoid) were used to generate the field, B_r would be present only in the **fringing field** at either end. (The uniform field inside the solenoid can focus electrons radiating from a point source but not a broad beam of electrons traveling parallel to its axis). So rather than using an *extended* magnetic field, we should make the field as short as possible by partially enclosing the current-carrying coil by ferromagnetic material such as soft iron, as shown in Fig. 2.9a. Due to its high permeability, the iron conducts most of the magnetic flux lines. However, the magnetic circuit contains a **gap** filled with nonmagnetic material, so that magnetic flux is forced to appear within the internal **bore** of the lens. The magnetic field experienced by an electron can be increased and further localized by the use of ferromagnetic (soft iron) **polepieces** having small internal diameter, as illustrated in Fig. 2.9b. These polepieces are machined to high precision to ensure that the magnetic field has the high degree of axial (rotational) symmetry needed for good focusing.

A cross-section through a typical magnetic lens is shown in Fig. 2.10, where the optic axis is now vertical. A typical electron-beam instrument contains several lenses and stacking them vertically (in a lens **column**) provides a mechanically robust structure in which the weight of each lens acts parallel to the optic axis. There is then no tendency for the column to gradually bend under its own weight, leading to lens **misalignment**. The strong magnetic field (up to about 2 T) in each lens gap is generated by a relatively large coil that contains many turns of wire and may carry a few amps of direct current. To remove heat generated in the coil, due to its ohmic resistance, water flows into and out of each lens. Water cooling ensures

Fig. 2.9 **a** Use of ferromagnetic soft iron to concentrate magnetic induction within a small volume. **b** Use of ferromagnetic polepieces to further concentrate the field

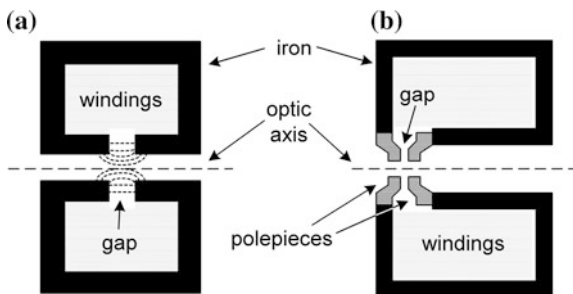
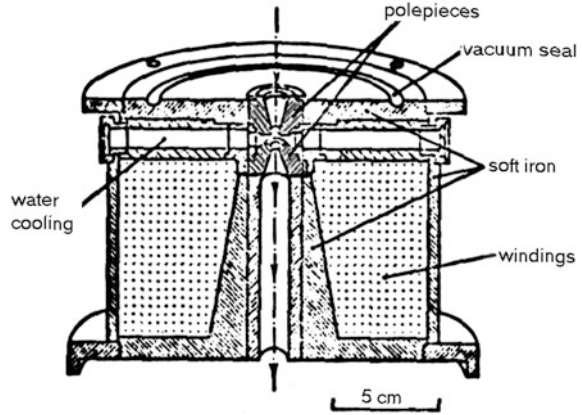


Fig. 2.10 Cross-section through a magnetic lens whose optic axis (*dash-dot line*) is vertical



that the temperature of the lens column reaches a stable value, so that thermal expansion (which could result in column misalignment) is minimized. Temperature changes are further reduced by controlling the temperature of the cooling water, using a refrigeration system to circulate water through the lenses (in a closed cycle) and remove the heat generated.

Rubber **o-rings** (of circular cross-section) provide an airtight seal between the interior of the lens column (under vacuum) and the exterior, which is at atmospheric pressure. The absence of air in the vicinity of the electron beam is essential to prevent collisions and scattering of the electrons by gas molecules. Some internal components (such as apertures) must be located close to the optic axis but adjusted in position by external controls. Sliding o-ring seals or thin-metal bellows are used to allow this motion, while preserving the internal vacuum.

2.4 Focusing Properties of a Thin Magnetic Lens

The use of ferromagnetic polepieces results in a focusing field that extends only a few mm along the optic axis, so that to a first approximation the lens can be considered thin. Deflection of the electron trajectory then occurs close to a single principal plane, allowing thin-lens formulas such as Eqs. (2.2) and (2.3) to be used to describe the optical properties of the lens.

The thin-lens approximation also simplifies analysis of the motion of a particle of charge e and mass m , leading to a compact expression for the **focusing power** (reciprocal of focal length) of a magnetic lens:

$$1/f = [e^2/(8mE_0)] \int B_z^2 dz \quad (2.7)$$

Since we are considering electrons, $e = 1.6 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg; E_0 represents the kinetic energy of the electrons passing through the lens, expressed in Joule and given by $E_0 = (e)(V_0)$ where V_0 is the potential difference used to accelerate the electrons from rest. The integral ($\int B_z^2 dz$) can be interpreted as the total area under a graph of B_z^2 plotted against distance z along the optic axis, B_z being the z -component of magnetic field (in Tesla). Because the field is non-uniform, B_z is a function of z and depends also on the lens current and polepiece geometry.

There are two simple cases in which the integral in Eq. (2.7) can be solved analytically. One of these corresponds to the assumption that B_z has a constant value B_0 in a region $-a < z < a$ but falls abruptly to zero outside this region. The total area is then that of a rectangle and the integral becomes $2aB_0^2$. This rectangular distribution is unphysical but would approximate the field produced by a long solenoid of length $2a$.

For a typical electron lens, a more realistic assumption is that B increases smoothly towards its maximum value B_0 (at the center of the lens) according to a symmetric *bell-shaped* curve described by the **Lorentzian** function:

$$B_z = B_0 / (1 + z^2/a^2) \quad (2.8)$$

As seen by substituting $z = a$ in Eq. (2.8), a is the half-width at half maximum (HWHM) of a plot of B_z versus z , meaning the distance (away from the central maximum) at which B_z falls to *half* of its maximum value; see Fig. 2.8a. Alternatively, $2a$ is the *full* width at half maximum (FWHM) of the field: the length (along the optic axis) over which the field exceeds $B_0/2$. If Eq. (2.8) is a good approximation, the integral in Eq. (2.7) becomes $(\pi/2)aB_0^2$ and the focusing power of the lens is:

$$1/f = (\pi/16) [e^2/(mE_0)] aB_0^2 \quad (2.9)$$

As an example, we can take $B_0 = 0.3$ T and $a = 3$ mm. If the electrons entering the lens have been accelerated from rest by applying a voltage $V_0 = 100$ kV, we have $E_0 = eV_0 = 1.6 \times 10^{-14}$ J. Equation (2.9) then gives the focusing power $1/f = 93 \text{ m}^{-1}$ and focal length $f = 11$ mm. Since f turns out to be less than twice the full width of the field ($2a = 6$ mm) we might question the accuracy of the thin-lens approximation in this case. More exact theory (Reimer and Kohl 2008) shows that the thin-lens formula underestimates f by about 14 % for these parameters. For larger B_0 and a , Eqs. (2.7) and (2.9) become unrealistic (see Fig. 2.13 later). In other words, strong lenses have to be treated as *thick* lenses, for which (as in light optics) the mathematical description is more complicated.

In addition, our thin-lens formula for $1/f$ is based on *non-relativistic* mechanics, in which the mass of the electron is assumed to be equal to its rest mass. The relativistic increase in mass, predicted by Einstein's Special Relativity, can be incorporated by replacing E_0 by $E_0 (1 + V_0/1022 \text{ kV})$ in Eq. (2.9). This modification increases f by about 1 % for each 10 kV of accelerating voltage, that is by 10 % for $V_0 = 100$ kV, 20 % for $V_0 = 200$ kV, and so on.

Although only approximate, Eq. (2.9) allows us to see how the focusing power of a magnetic lens depends on the strength and spatial extent of the magnetic field, and on the kinetic energy, charge, and mass of the particles being imaged. Since kinetic energy E_0 appears in the denominator of Eq. (2.7), the focusing power decreases as the accelerating voltage is increased. As might be expected, electrons are deflected less in the magnetic field as their momentum increases.

Since B_0 is proportional to the current supplied to the lens windings, changing this current allows the focusing power of the lens to be varied. This ability to vary the focal length means that an electron image can be focused by adjusting the lens current, without mechanical motion. But it also implies that the lens current must be highly stabilized (typically to within a few parts per million) to prevent *unwanted* changes in focusing power, which would cause the image to drift out-of-focus. In *light optics*, change in f can only be achieved mechanically: by changing the curvature of the lens surfaces (in the case of the eye) or by changing the spacing between elements of a compound lens, as in the zoom lens of a camera.

When discussing the action of a magnetic field, we saw that an electron passing through a lens executes a spiral motion. As a result, the plane containing the exit ray is rotated through an angle ϕ relative to the plane containing the incoming electron. By again making a thin-lens approximation ($a \ll f$) and assuming a Lorentzian field distribution, the equations of motion can be solved to give:

$$\phi = \left[e / (8mE_0)^{1/2} \right] \int B_z dz = \left[e / (8mE_0)^{1/2} \right] \pi a B_0 \quad (2.10)$$

Using $B_0 = 0.3$ T, $a = 3$ mm, and $V_0 = 100$ kV as before: $\phi = 1.33$ rad = 76° , so the rotation is significant. Note that this rotation is *in addition* to the inversion about the z -axis that occurs when a real image is formed (Fig. 2.5). In other words, the rotation of a *real* electron image, relative to the object, is actually $\pi \pm \phi$ radians.

Note that the image rotation would reverse (e.g., go from clockwise to counter-clockwise) if the current through the lens windings were reversed, as B_z would be reversed in sign. On the other hand, reversing the current has no effect on the focusing power, since the integral in Eq. (2.7) involves B_z^2 , which is always positive. In fact, all of the terms in Eq. (2.7) are positive, implying that we cannot use an axially symmetric magnetic field to produce the electron-optical equivalent of a diverging (concave) lens having negative focal length.

Equations (2.7)–(2.10) apply equally well to the focusing of other charged particles, such as protons and ions, provided e and m are replaced by the appropriate charge and mass. Equation (2.7) shows that, for the same lens current and kinetic energy (same accelerating potential), the focusing power of a magnetic lens is much less for these particles, whose mass is thousands of times larger than that of the electron. For this reason, **ion optics** commonly involves *electrostatic* lenses.

2.5 Comparison of Magnetic and Electrostatic Lenses

Because the electrostatic force on a charged particle is parallel (or in the case of an electron, antiparallel) to the field and since axially symmetric fields have no tangential component, electrostatic lenses offer the convenience of no image rotation. Their low weight and near-zero power consumption have made them attractive for space-based equipment, such as planetary probes.

Electrostatic lenses are also used in focused-ion-beam (FIB) machines, which are commonly used for the fabrication of nm-scale devices and for the preparation of TEM specimens; see Sect. 4.10. Ions such as Ga⁺ are accelerated by voltages of typically 5–30 kV and can remove atoms of the specimen by sputtering. Because of the large mass of these ions compared to that of an electron, magnetic lenses would be inefficient. Electrostatic lenses can generate a probe with a diameter of a few nm, to provide scanning-mode images of the specimen using secondary electrons or secondary ions.

If the voltage V_0 that accelerates electrons is also used as the voltage applied to an electrostatic lens (as in Fig. 2.6), the lens focusing becomes insensitive to small voltage changes. If V_0 increases, the electrons travel faster but require a higher lens voltage to focus them, so the effect of the voltage change cancels to first order. Electrostatic lenses were used in some early electron microscopes because their high-voltage supplies were prone to slow change (drift) and fluctuating components (ripple). Nowadays, high-voltage supplies can be stabilized to one part in a million, although this requires careful and expensive manufacture.

The fact that a voltage comparable to the accelerating voltage V_0 must be applied to an electrostatic lens means that insulation and safety problems become severe for $V_0 > 50$ kV. Because higher accelerating voltages permit better image resolution, magnetic lenses came to be preferred for electron microscopy. Magnetic lenses also provide somewhat lower aberrations, for the same focal length, thereby improving the image resolution.

As we will see later in this chapter, lens aberrations are also minimized by making the focal length of a lens small, implying an *immersion* objective lens with the specimen present *within* the lens field. This situation gives rise to problems in the case of ferromagnetic specimens, which distort the local field of a magnetic lens and prevent good imaging (unless the objective excitation is turned off, which impairs the image resolution). However, introducing *any* specimen into the field of an electrostatic lens would change its field distribution, making an electrostatic immersion lens impractical.

With the advent of spherical-aberration correction (Sect. 2.7), an immersion lens of small focal length becomes less essential to good resolution, so electrostatic lenses begin to look more attractive, at least at lower voltages where insulation is less problematic. Chromatic aberration becomes limiting at low electron energies but can also be corrected; see Sect. 2.7.

Table 2.1 Comparison of electrostatic and electromagnetic lens designs

Advantages of electrostatic lenses	Advantages of magnetic lenses
No image rotation	Lower lens aberrations
Lightweight, consume no power	No high-voltage insulation needed
Highly stable voltage unnecessary	Can function as an immersion lens

Especially if used with ions, which can contaminate an optical column with sputtered atoms, electrostatic lenses may require periodic cleaning to avoid electrostatic discharge across their insulator surfaces. The differences between electrostatic and magnetic optics are summarized in Table 2.1.

2.6 Defects of Electron Lenses

In the case of a microscope, the most important focusing defects are lens *aberrations*, since they reduce the spatial resolution of the image, even when it has been optimally focused. We will discuss two kinds of **axial aberration** that lead to image blurring even for object points that lie *on the optic axis*.

Spherical Aberration

The focusing of light by a glass lens with spherical surfaces is imperfect: a **marginal** ray that is refracted at the edge of the lens is focused more strongly than a **paraxial** ray that travels close to the optic axis. A similar effect occurs with electron lenses, magnetic or electrostatic.

Spherical aberration can be defined by means of a diagram that shows electrons arriving at a thin lens after traveling parallel to the optic axis but not necessarily along it; see Fig. 2.11. Paraxial rays (represented by dashed lines in Fig. 2.11) are brought to a focus F , a distance f from the center of the lens, at the **Gaussian** image plane. Electrons arriving at a distance x from the axis are focused to a different point F_1 , located a shorter distance f_1 from the center of the lens.

We might expect the axial shift in focus ($\Delta f = f - f_1$) to depend on the initial x -coordinate of the electron and on the degree of imperfection of the lens focusing. Without knowing the details of this imperfection, we can represent the x -dependence in terms of a power series:

$$\Delta f = c_2 x^2 + c_4 x^4 + \text{higher even powers of } x \quad (2.11)$$

with c_2 and c_4 as unknown coefficients. Note that odd powers of x have been omitted: provided the magnetic field that focuses the electrons is rotationally symmetric, the deflection angle α should be identical for electrons that arrive with coordinates $+x$ and $-x$, as in Fig. 2.11. This would not be the case if terms involving x or x^3 were present in Eq. (2.11).

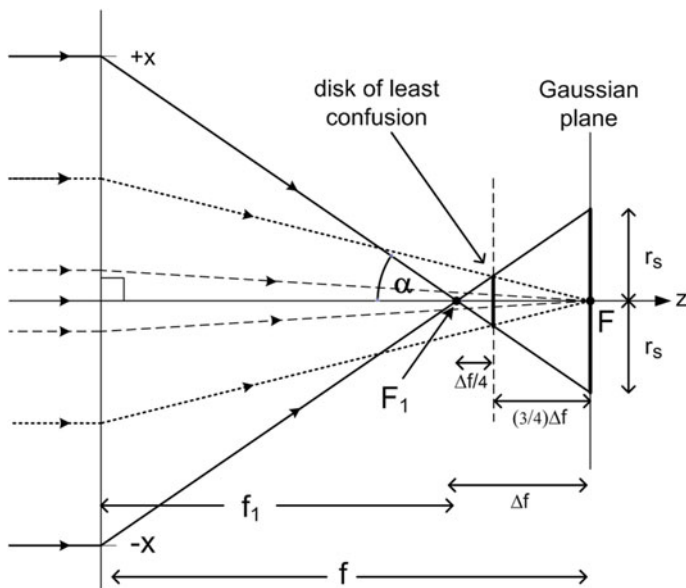


Fig. 2.11 Definition of the disk of confusion (at the Gaussian image plane) arising from spherical aberration, in terms of the focusing of parallel rays by a thin lens

From the geometry of the large right-angled triangle in Fig. 2.11,

$$x = f_1 \tan \alpha \approx f \tan \alpha \approx f \alpha \quad (2.12)$$

Here we have assumed that $x \ll f$, taking the angle α to be small, and that $\Delta f \ll f$, supposing spherical aberration to be a *small* effect for electrons that deviate by no more than a few degrees from the optic axis. These approximations are reasonable for typical electron-accelerating voltages.

When non-paraxial electrons arrive at the Gaussian image plane, they are displaced radially from the optic axis by an amount r_s given by:

$$r_s = \Delta f \tan \alpha \approx \Delta f \alpha \quad (2.13)$$

where we once again assume that α is small. Since small α implies small x , we can to a first approximation neglect powers higher than x^2 in Eq. (2.11) and combine this equation with Eqs. (2.12) and (2.13) to give:

$$r_s \approx [c_2 (f\alpha)^2] \alpha = c_2 f^2 \alpha^3 = C_s \alpha^3 \quad (2.14)$$

where we have combined c_2 and f into a single constant C_s , the **coefficient of spherical aberration** of the lens. Because α (in radian) is dimensionless, C_s has the dimensions of length.

Figure 2.11 illustrates a limited number of off-axis electron trajectories. More typically, we have a broad entrance beam of circular cross-section, containing

electrons arriving at the lens with a continuous range of radial displacement within the x - z plane (that of the diagram), within the y - z plane (perpendicular to the diagram), and within all intermediate planes that contain the optic axis. Due to the rotational symmetry, all these electrons arrive at the Gaussian image plane *within* the disk of confusion (radius r_s). The angle α now represents the *maximum* angle of the focused electrons, which might be determined by the internal diameter of the lens bore or by a circular aperture placed in the optical system.

Figure 2.11 is directly relevant to a scanning electron microscope (SEM), where the objective lens focuses a near-parallel beam into an electron probe of very small diameter at the specimen. Because the spatial resolution of a secondary-electron image cannot be better than the probe diameter, spherical aberration might be expected to limit the spatial resolution to a value of the order of $2r_s$. In fact, this conclusion is too pessimistic: if the specimen is advanced towards the lens, the illuminated disk gets smaller and at a certain location (represented by the dashed vertical line in Fig. 2.11), its diameter has a minimum value ($=r_s/2$) corresponding to the **disk of least confusion**. Advancing the specimen any closer to the lens would make the disk larger, due to contributions from medium-angle rays, shown dotted in Fig. 2.11.

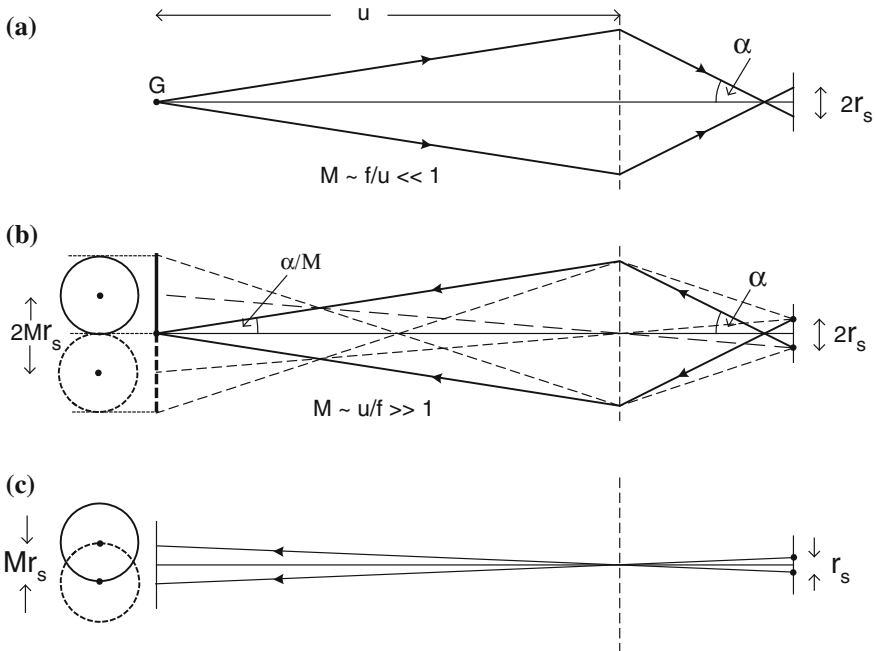


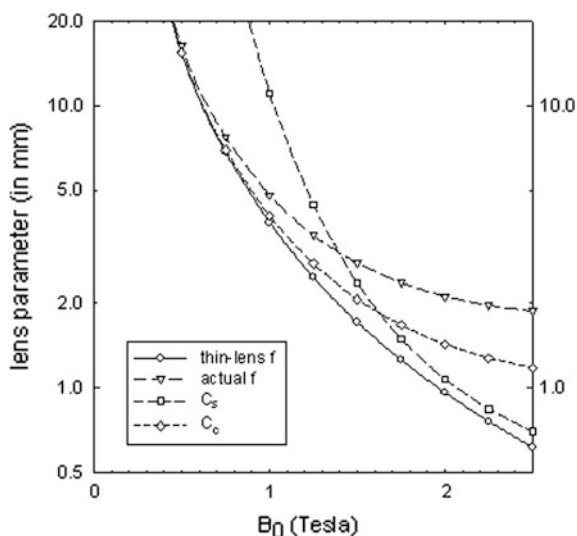
Fig. 2.12 **a** Ray diagram as in Fig. 2.11 but with an object distance u large but finite, a situation that applies to demagnification of an electron source G by an SEM objective lens. **b** Equivalent diagram with the solid rays reversed, showing two image disks of confusion arising from object points whose separation is $2r_s$. **c** Same diagram but with object-point separation reduced to r_s such that the two points are barely resolved in the image, according to the Rayleigh criterion

In the case of a transmission electron microscope (TEM), a relatively broad beam of electrons arrives at the specimen and an objective lens simultaneously images each object point. Figure 2.11 can be adapted to this case by first imagining the lens to be weakened slightly, so that F is the Gaussian image of an object point G located at a large but finite distance u from the lens, as in Fig. 2.12a. Although r_s is still given approximately by Eq. (2.14), this diagram depicts large *demagnification* ($M \ll 1$). To make it relevant to a TEM objective ($M \gg 1$), we can simply reverse the ray paths, a procedure that is permissible in both light and electron optics, resulting in Fig. 2.12b. By adding extra dashed rays as shown, Fig. 2.12b illustrates how electrons emitted from two points a distance $2r_s$ apart are focused into two magnified disks of confusion in the image (on the left) of radius Mr_s and separation $2Mr_s$. Although these disks touch at their periphery, the image would still be recognizable as representing two separate point-like objects in the specimen. If the separation between the object points is now reduced to r_s (as in Fig. 2.12c), the disks overlap enough to satisfy the Rayleigh criterion for resolution (Sect. 1.1). We can therefore take r_s as the spherical-aberration limit to the **point resolution** of a TEM objective lens. In microscopy we always specify blurring in the *object plane* because it is the object resolution that we care about.

Spherical aberration occurs in TEM lenses *after* the objective but is much less significant. This situation arises from the fact that each lens *reduces* the maximum angle of electrons (relative to the optic axis) by a factor equal to its magnification (as illustrated in Fig. 2.12b) and the spherical-aberration blurring depends on the *third power* of this angle, according to Eq. (2.14).

So far, we have said nothing about the *value* of the spherical-aberration coefficient C_s . On the assumption of a Lorentzian (bell-shaped) field, C_s can be calculated from a somewhat-complicated formula (Glaser 1952). Figure 2.13 shows the calculated C_s and focal length f as a function of the maximum field B_0 , for

Fig. 2.13 Focal length and the coefficients of spherical and chromatic aberration for a magnetic lens containing a Lorentzian field with peak field B_0 and half-width $a = 1.8$ mm, focusing 200 keV electrons. Thin-lens values were calculated from Eq. (2.7), other values taken from Glaser (1952)



200 kV accelerating voltage and a field half-width of $a = 1.8$ mm. The thin-lens formula, Eq. (2.9), is seen to be quite good at predicting the focal length of a weak lens (low B_0) but gives too low a value for a strong lens. The aberration coefficient C_s exceeds the actual focal length for a weak lens but is less than the focal length for a strong lens (and closer to f given by the thin-lens formula). For a typical TEM objective lens, we might take $f = 2$ mm and $C_s \approx 0.5$ mm; for a point resolution $r_s < 1$ nm, the maximum angle of the electrons (relative to the optic axis) must satisfy: $C_s \alpha^3 < r_s$, giving $\alpha \approx 10^{-2}$ rad = 10 mrad. This low value justifies our use of small-angle approximations in the preceding analysis.

The basic physical properties of a magnetic field dictate that the spherical aberration of a rotationally symmetric electron lens *cannot* be eliminated by careful design of the lens polepieces. However, spherical aberration can be *minimized* by using a strong objective lens in the TEM. The shortest possible focal length is determined by the maximum field ($B_0 \approx 2.6$ T) obtainable, which is limited by magnetic saturation of the lens polepieces. In practice, f and C_s can be made small enough to achieve atomic resolution in suitably thin specimens. Further improvement requires the use of an aberration corrector to reduce C_s , as discussed in Sect. 2.7.

Chromatic Aberration

In light optics, chromatic aberration occurs when there is a spread in the wavelength of the light passing through a lens, combined with a variation of refractive index with wavelength (dispersion). In the case of an electron, the de Broglie wavelength depends on the particle momentum, and therefore on its kinetic energy, while Eq. (2.7) shows that the focusing power of a magnetic lens varies inversely with the kinetic energy E_0 . So if electrons are present with different kinetic energies, they will be focused at different distances from a lens; at an image plane, there will be a *chromatic* disk of confusion rather than a point focus. The spread in kinetic energy can arise from several causes.

- (1) Variations in the energy of the electrons emitted from the source. For example, electrons emitted by a thermionic source have a **thermal spread** of the order of kT , where T is the temperature of the emitting surface, due to the statistics of the electron-emission process.
- (2) Fluctuations in the potential V_0 applied to accelerate the electrons. Although high-voltage supplies are stabilized as well as possible, there is always some drift (slow variation) and ripple (alternating component) in the accelerating voltage, and therefore in the kinetic energy eV_0 .
- (3) Energy loss due to **inelastic scattering** in the specimen, a process in which energy is transferred from an electron to the specimen. This scattering is a statistical process: not all electrons lose the same amount of energy, resulting in an energy spread within the transmitted beam. Because the TEM *imaging* lenses focus electrons *after* they have passed through the specimen, inelastic scattering will cause chromatic aberration in the magnified image.

We can estimate the radius of the *chromatic* disk of confusion by the use of Eq. (2.7) and thin-lens geometric optics, setting aside spherical aberration and other

lens defects. Consider an axial point source P of electrons (distance u from the lens) that is focused to a point Q in the image plane (distance v from the lens) for electrons of energy E_0 as shown in Fig. 2.14. Since $1/f$ increases as the electron energy decreases, electrons whose kinetic energy is $E_0 - \Delta E_0$ will have a smaller image distance $v - \Delta v$ and arrive at the image plane a radial distance r_i from the optic axis. If the angle β of the arriving electrons is small,

$$r_i = \Delta v \tan \beta \approx \beta \Delta v \quad (2.15)$$

As in the case of spherical aberration, we need to know the x -displacement of a second point object P' whose disk of confusion partially overlaps the first, as shown in Fig. 2.14. Following again the Rayleigh criterion, we take the required displacement in the image plane to be equal to the disk radius r_i , corresponding to a displacement *in the object plane* of $r_c = r_i/M$. The image magnification M is given by:

$$M = v/u = \tan \alpha / \tan \beta \approx \alpha / \beta \quad (2.16)$$

From Eqs. (2.15) and (2.16), we have:

$$r_c \approx \beta \Delta v / M \approx \alpha \Delta v / M^2 \quad (2.17)$$

Assuming a thin lens, $1/u + 1/v = 1/f$ and taking derivatives of this equation (for a fixed object distance u) gives: $0 + (-2) v^{-2} \Delta v = (-2) f^{-2} \Delta f$, leading to:

$$\Delta v = (v^2/f^2) \Delta f \quad (2.18)$$

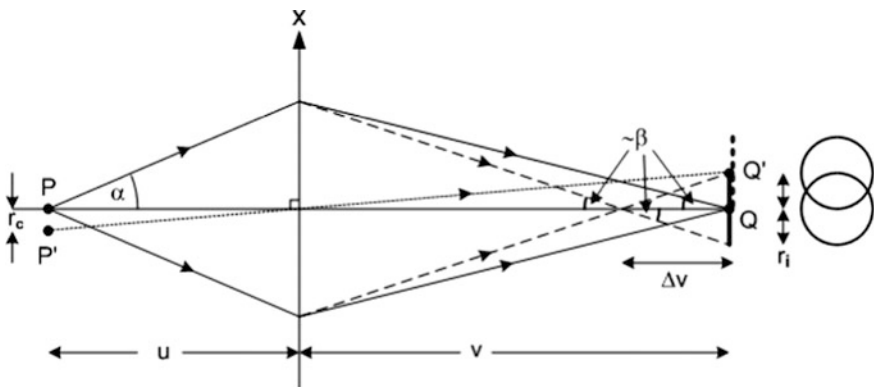


Fig. 2.14 Ray diagram illustrating the change in focus and the disk of confusion resulting from chromatic aberration. With two object points, the image disks overlap; the Rayleigh criterion (about 15 % reduction in intensity between the current-density maxima) is satisfied when the separation PP' in the object plane is given by Eq. (2.20)

For $M \gg 1$, the thin-lens equation, $1/u + 1/(Mu) = 1/f$, implies that $u \approx f$ and $v \approx Mf$, so Eq. (2.18) becomes $\Delta v \approx M^2 \Delta f$ and Eq. (2.17) gives:

$$r_c \approx \alpha \Delta f \quad (2.19)$$

From Eq. (2.7), the focal length of the lens can be written as $f = A E_0$ where A is independent of electron energy, so taking derivatives gives: $\Delta f = A \Delta E_0 = (f/E_0) \Delta E_0$. The loss of spatial resolution due to chromatic aberration is therefore:

$$r_c \approx \alpha f (\Delta E_0 / E_0) \quad (2.20)$$

More generally, a **coefficient of chromatic aberration** C_c is defined by the equation:

$$r_c \approx \alpha C_c (\Delta E_0 / E_0) \quad (2.21)$$

Our analysis has shown that $C_c = f$ in the thin-lens approximation. A more exact (thick-lens) treatment gives C_c slightly smaller than f for a weak lens and closer to $f/2$ for a strong lens; see Fig. 2.13. As in the case of spherical aberration, chromatic aberration cannot be eliminated through lens design but is minimized by making the lens as strong as possible (large focusing power, small f), by using an angle-limiting aperture (restricting α), and by using a high electron-accelerating voltage (large E_0) as Eq. (2.21) indicates. Chromatic effects can also be minimized by reducing ΔE_0 through the use of an electron-beam monochromator, as discussed in Sect. 2.7.

Axial Astigmatism

So far, we have assumed complete rotational symmetry of the magnetic field that focuses the electrons. In practice, lens polepieces cannot be machined with perfect accuracy and the polepiece material may be slightly inhomogeneous, resulting in local variations in relative permeability. In either case, the departure from rotational symmetry will cause the magnetic field at a given radius r from the z -axis to depend on the *plane of incidence* of an incoming electron (i.e., on its **azimuthal** angle ϕ , viewed along the z -axis). According to Eq. (2.9), this difference in magnetic field will give rise to a difference in focusing power; the lens is said to suffer from **axial astigmatism**.

Figure 2.15a shows electrons leaving an *on-axis* object point P at equal angles to the z -axis but traveling in the x - z and y - z planes. They cross the optic axis at different points, F_x and F_y that are displaced along the z -axis. In Fig. 2.15a, the x -axis corresponds to the *lowest* focusing power and the perpendicular y -direction corresponds to the *highest* focusing power.

In practice, electrons leave P with *all* azimuthal angles and at all angles (up to α) relative to the z -axis. At the plane containing F_y , the electrons lie within a **caustic figure** that approximates to an ellipse whose long axis lies parallel to the x -direction. At F_x they lie within an ellipse whose long axis points in the y -direction. At some intermediate plane F, the electrons define a circular disk of confusion of radius R , rather than a single point. If that plane is used as the image (magnification M), astigmatism will limit the point resolution to a value R/M .

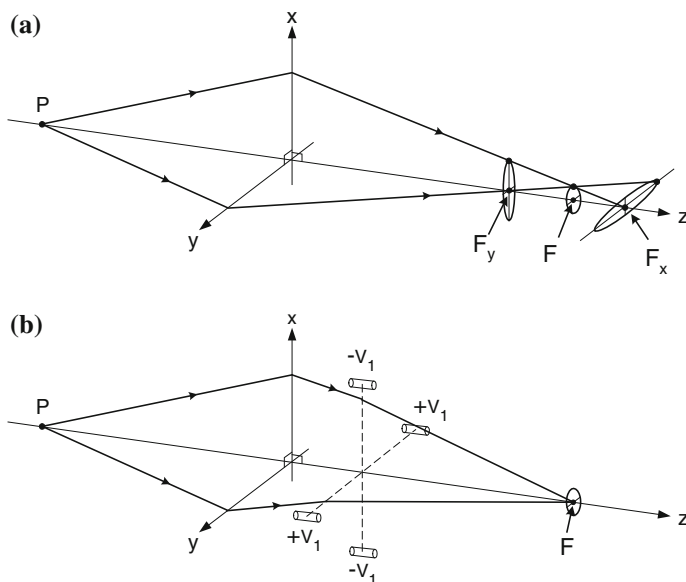


Fig. 2.15 **a** Rays leaving an axial image point, focused by a lens (with axial astigmatism) into ellipses centered on F_x and F_y or into a circle of radius R at some intermediate plane. **b** Use of an electrostatic stigmator to correct for the axial astigmatism of an electron lens

Axial astigmatism also occurs in the human eye, when there is a lack of rotational symmetry. It can be *corrected* by wearing lenses whose focal length varies with azimuthal direction by an amount just sufficient to compensate for the azimuthal variation in focusing power of the eyeball. In *electron* optics, the device that corrects for astigmatism is called a **stigmator** and it takes the form of a weak **quadrupole lens**.

An *electrostatic* quadrupole consists of four electrodes, in the form of short conducting rods aligned parallel to the z -axis and located at equal distances along the $+x$, $-x$, $+y$, and $-y$ directions; see Fig. 2.15b. A power supply generating a voltage $-V_1$ is connected to the two rods that lie in the x - z plane, so electrons traveling in that plane are repelled *towards* the axis, resulting in a *positive* focusing power (convex-lens effect). A potential $+V_1$ is applied to the other pair, which therefore *attracts* electrons traveling in the y - z plane and provides a *negative* focusing power in that plane. By adding the stigmator to an astigmatic lens and choosing V_1 appropriately, the focal lengths (of the entire system) in the x - z and y - z planes can be made equal; the two foci F_x and F_y are brought together to a single point and the axial astigmatism is eliminated, as in Fig. 2.15b.

In practice, we cannot predict which azimuthal direction will correspond to the smallest or largest focusing power of an astigmatic lens. Therefore the *direction* of the stigmator correction must be adjustable, as well as its strength. One way of achieving this is to mechanically rotate the quadrupole around the z -axis. A more

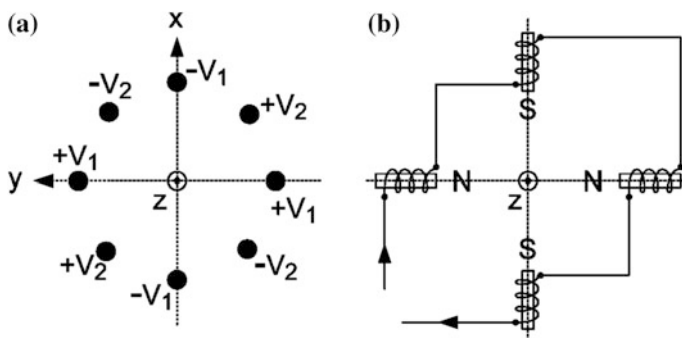


Fig. 2.16 **a** Electrostatic stigmator, viewed along the optic axis. **b** Magnetic quadrupole, which is the basis of an electromagnetic stigmator

convenient arrangement is to add four more electrodes, connected to a second power supply that generates potentials of $+V_2$ and $-V_2$, as in Fig. 2.16a. Varying the magnitude and polarity of the two voltage supplies is equivalent to varying the strength and orientation of a single quadrupole, without the need for mechanical rotation.

A more common form of stigmator consists of a *magnetic* quadrupole: four short solenoid coils with their axes pointing towards the optic axis. The coils that generate the magnetic field are connected in series and carry a common current I_1 . They are wired so that similar (north or south) magnetic poles face each other, as in Fig. 2.16b. The magnetic force on an electron is perpendicular to the magnetic field, but in other respects, a magnetic stigmator acts similarly to an electrostatic one. Astigmatism correction could be achieved by adjusting the current I_1 and the azimuthal orientation of the quadrupole, but in practice a second set of four coils is inserted at 45° to the first and carries a current I_2 that can be varied independently. Independent adjustment of I_1 and I_2 enables the astigmatism to be corrected without mechanical rotation.

The stigmators found in electron-beam columns are *weak* quadrupoles, designed to correct for *small* deviations in focusing power of a much stronger lens. *Strong* quadrupoles are used in synchrotrons and nuclear-particle accelerators, to focus high-energy electrons or other charged particles. Their focusing power is positive in one plane and negative (diverging, equivalent to a concave lens) in the perpendicular plane. However, a series combination of *two* quadrupole lenses can result in an overall convergence in both planes, without image rotation and with less power dissipation than required by an axially symmetric lens. This last consideration favors the use of quadrupoles for focusing ions or in situations where the electron optics must be compact.

In light optics, the surfaces of a glass lens can be machined with sufficient accuracy that *axial* astigmatism is negligible. But for rays coming from an *off-axis* object point, the lens appears elliptical rather than round, so *off-axis* astigmatism is unavoidable.

In electron optics, this kind of astigmatism is less significant because the electrons are confined to small angles relative to the optic axis (to avoid excessive spherical and chromatic aberration). Another off-axis aberration, called *coma*, is of importance if a TEM is to achieve its highest possible resolution, and can be minimized by a suitable lens-alignment procedure.

Distortion and Curvature of Field

In an undistorted image, the distance R of an image point from the optic axis is given by $R = M r$, where r is the distance of the corresponding object point from the axis and the image magnification M is a constant. The presence of distortion changes this ideal relation to:

$$R = M r + C_d r^3 \quad (2.22)$$

where C_d is a constant. If $C_d > 0$, each image point is displaced outwards, particularly those further from the optic axis, and the entire image suffers from pincushion distortion (Fig. 2.2c). If $C_d < 0$, each image point is displaced inward relative to the ideal image and barrel distortion is present (Fig. 2.2b).

As might be expected from the third-power dependence in Eq. (2.22), distortion is related to spherical aberration. In fact, a rotationally symmetric electron lens (for which $C_s > 0$) will give $C_d > 0$ and pincushion distortion. Barrel distortion is produced in a two-lens system in which the second lens magnifies a *virtual* image produced by the first lens. In a multi-lens system, it is therefore possible to combine the two types of distortion to achieve a distortion-free image.

In the case of magnetic lenses, a third type of distortion arises from the fact that the image rotation ϕ may depend on the distance r of the object point from the optic axis. This spiral distortion was illustrated in Fig. 2.2c. Again, compensation is possible in a multi-lens system.

For most purposes, distortion is a less serious lens defect than aberration, since it does not result in a loss of image detail. In fact, it may not be noticeable unless the microscope specimen contains straight-line features. In some TEMs, distortion is observable when the final (projector) lens is operated at a reduced current (giving larger C_d) to achieve low overall magnification.

Curvature of field is not a serious problem in the TEM or SEM, since the angular deviation of electrons from the optic axis is small. This results in a large *depth of focus* (the image remains acceptably sharp as the viewing plane is moved along the optic axis), as discussed in Chap. 3.

2.7 Aberration Correctors and Monochromators

As early as 1936, Otto Scherzer proved that any electron lens will suffer from spherical and chromatic aberration if it has rotational symmetry, produces a real image, uses static (time-independent) focusing fields, and involves no on-axis

electrostatic charge. Since then, numerous scientists have aimed to construct aberration-free systems by violating one or other of these constraints. Reflecting electrons from an electrode connected to a potential higher than the electron-source potential represents a special case of time dependence (the electron reverses its velocity vector) and such electrostatic mirrors have been used to correct the aberrations of low-energy and photoelectron microscopes (Rempfer et al. 1997). But for the TEM and SEM, success has depended on the use of multipole lenses that involve non-axial electric or magnetic fields.

Multipole Optics

The stigmator used to correct for axial astigmatism (Figs. 2.15 and 2.16) is a weak **quadrupole** lens, consisting of *four* elements that generate a symmetric pattern of electric or magnetic fields, perpendicular to the optic axis. *Strong* quadrupoles (used in pairs) are used in the spectrometer optics (Sect. 6.9) and for focusing high-energy electrons or heavier particles in particle accelerators. Electrostatic or magnetic **dipoles** contain *two* electrodes that generate a roughly uniform field perpendicular to the optic axis, and are used as beam deflectors (without focusing) for all kinds of charged particles. **Sextupoles** (also known as hexapoles) and **octupoles** are employed for correcting lens aberrations.

Figure 2.17a shows the general construction of a magnetic octupole lens, where iron-cored solenoid coils generate magnetic fields whose north and south poles alternate. Figure 2.17b shows the direction of the magnetic force on an electron along different radial directions. Along the dashed diagonal lines, this force is towards the optic axis (compressive) and is proportional to r^3 , as needed to cancel the effect of spherical aberration (r represents radial distance from the optic axis).

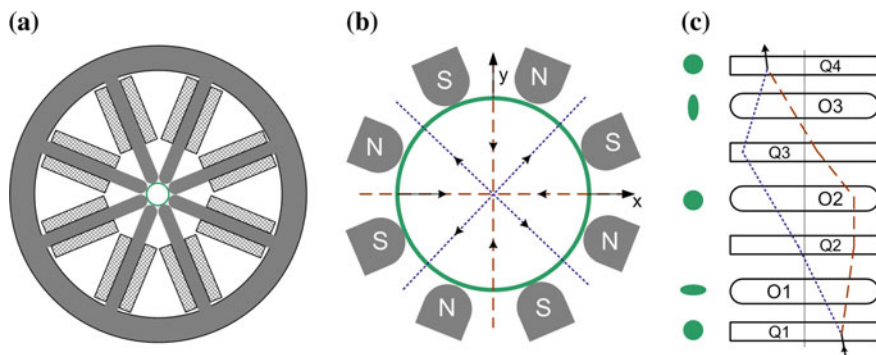


Fig. 2.17 **a** Construction of an octupole lens. Ferromagnetic material is shown in gray, solenoid-coil windings are cross-hatched. **b** Enlargement of the central region, showing the polarities of the magnetic poles and the direction of the magnetic force on an electron traveling in the vacuum contained within the circular beam tube. **c** Quadrupole-octupole spherical-aberration corrector, showing cross-sectional profiles of the beam at various planes and the path taken by the electrons traveling in two perpendicular (x - z and y - z) planes

Along the dotted lines the force is away from the axis and the aberration is increased.

In the octupole-quadrupole corrector (Fig. 2.17c) a magnetic quadrupole Q1 is used to stretch the beam into a thin ellipse oriented along one of the compressive directions (dashed ray) so that an octupole O1 can cancel spherical aberration along that direction, with little effect in other directions. A second quadrupole Q2 restores the beam into a circular shape, allowing a second octupole O2 to correct for fourfold astigmatism. A third quadrupole Q3 then stretches the beam into an ellipse elongated in a perpendicular direction, so that an appropriately oriented octupole O3 can correct for spherical aberration in that direction. A final quadrupole Q4 restores the beam to a circular cross-section, with spherical aberration corrected. This form of corrector is used in STEM instruments manufactured by the Nion Company.

Another form of spherical-aberration corrector uses magnetic sextupoles. A *short* sextupole would give radial forces proportional to r^2 (and therefore the same at coordinates $+r$ and $-r$) but two *long* sextupoles (oriented 60° apart) provide an appropriate form of correction. This design is manufactured by the CEOS and FEI companies and has been fitted to many commercial TEMs, while JEOL and Hitachi have developed their own sextupole designs. CEOS also manufactures a device containing magnetic and electrostatic multi-poles that can correct for chromatic as well as spherical aberration, a procedure that becomes necessary for obtaining atomic resolution at accelerating voltages below 50 kV.

As an alternative to reducing C_c , chromatic aberration can be controlled by reducing the energy spread ΔE of the electron beam, using an electron-beam **monochromator**, which is an electron spectrometer (see Sect. 6.9) followed by an energy-selecting slit. In the **Wien filter**, a magnetic field B_y and electrostatic field E_x are applied, perpendicular to each other and to the optic axis, such that the net force F on an electron is zero at some precise electron speed v given by:

$$F = (-e)E_x - (-e)B_y v = 0 \quad (2.23)$$

These electrons travel in a straight line and pass through a narrow slit; those moving at other speeds are deflected sideways and are absorbed by the slit blades. The Wien filter is usually placed just below the electron gun (where the electron speed v is small, allowing higher sensitivity) and is known as a gun monochromator; it can reduce ΔE to about 0.1 eV. Other spectrometer designs have been used for a monochromator, giving energy widths down to 0.01 eV (see Sect. 6.9).

2.8 Further Reading

As stated in Section 2.2, many aspects of electron optics are similar to those of geometrical optics, which is treated in many undergraduate textbooks. There is not much information available in the literature on the practical aspects of electron optics;

nowadays such expertise resides with commercial microscope manufacturers and is likely to be kept confidential. However, there are several books that deal with the electron-optic principles. For example:

Principles of Electron Optics by P.W. Hawkes and E. Kasper (Springer, 1996; ISBN: 978-0-12-333340-7) is a three-volume treatise dealing with electron-optical elements (including magnetic and electrostatic lenses, mirrors, and beam deflection systems) as well as electron scattering and image formation and interpretation, with an emphasis on underlying theory.

Handbook of Charged Particle Optics, ed. J. Orloff (CRC Press, 2nd edition, 2008; ISBN-13: 978-1420045543) contains chapters written by SEM, STEM, and FIB experts and includes aberration correction, which is also treated in *Aberration-Corrected Imaging in Transmission Electron Microscopy* by R. Erni (Imperial College Press, 2nd edition, 2015; ISBN-13: 978-1783265282).

An easy-to-read account of aberration correction using an electrostatic mirror is given by G.F. Rempfer, D.N. Delage, W.P. Skoczlas and O.H. Griffith in *Microscopy and Microanalysis* 3 (1997) 14-27: *Simultaneous correction of spherical and chromatic aberrations with an electron mirror: an optical achromat*.

Figure 2.13 is based on information given in *Foundations of Electron Optics* written (in German) by W. Glaser (Springer-Verlag, Vienna, 1952). For further insight into the operation of a magnetic lens, see *Transmission Electron Microscopy* by L. Reimer and H. Kohl (5th edition, Springer-Verlag, 2008).

Maxwell's rules for image formation are contained in the article: *On the general laws of optical instruments*, published in *Quarterly Journal of Pure and Applied Mathematics* 2 (1858) 233-246. Fermat's least-time principle provides an interesting alternative view of image formation by a glass lens, as discussed at http://tem-eels.ca/education/education_index.html.

<http://www.springer.com/978-3-319-39876-1>

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