

Preface

The *High-Dimensional Probability* proceedings continue a well-established tradition which began with the series of eight *International Conferences on Probability in Banach Spaces*, starting with Oberwolfach in 1975. An earlier conference on Gaussian processes with many of the same participants as the 1975 meeting was held in Strasbourg in 1973. The last Banach space meeting took place in Bowdoin, Maine, in 1991. It was decided in 1994 that, in order to reflect the widening audience and interests, the name of this series should be changed to the *International Conference on High-Dimensional Probability*.

The present volume is an outgrowth of the *Seventh High-Dimensional Probability Conference* (HDP VII) held at the superb *Institut d'Études Scientifiques de Cargèse* (IESC), France, May 26–30, 2014. The scope and the quality of the contributed papers show very well that high-dimensional probability (HDP) remains a vibrant and expanding area of mathematical research. Four of the participants of the first probability on Banach spaces meeting—Dick Dudley, Jim Kuelbs, Jørgen Hoffmann-Jørgensen, and Mike Marcus—have contributed papers to this volume.

HDP deals with a set of ideas and techniques whose origin can largely be traced back to the theory of Gaussian processes and, in particular, the study of their path properties. The original impetus was to characterize boundedness or continuity via geometric structures associated with random variables in high-dimensional or infinite-dimensional spaces. More precisely, these are geometric characteristics of the parameter space, equipped with the metric induced by the covariance structure of the process, described via metric entropy, majorizing measures and generic chaining.

This set of ideas and techniques turned out to be particularly fruitful in extending the classical limit theorems in probability, such as laws of large numbers, laws of iterated logarithm, and central limit theorems, to the context of Banach spaces and in the study of empirical processes.

Similar developments took place in other mathematical subfields such as convex geometry, asymptotic geometric analysis, additive combinatorics, and random matrices, to name but a few topics. Moreover, the methods of HDP, and especially its offshoot, the concentration of measure phenomenon, were found to have a number of important applications in these areas as well as in statistics, machine learning theory, and computer science. This breadth is very well illustrated by the contributions in the present volume.

Most of the papers in this volume were presented at HDP VII. The participants of this conference are grateful for the support of the Laboratoire Jean Alexandre Dieudonné of the Université de Nice Sophia-Antipolis, of the school of Mathematics at the Georgia Institute of Technology, of the CNRS, of the NSF (DMS Grant # 1441883), of the French Agence Nationale de la Recherche (ANR 2011 BS01 010 01 project Calibration), and of the IESC. The editors also thank Springer-Verlag for agreeing to publish the proceedings of HDP VII.

The papers in this volume aptly display the methods and breadth of HDP. They use a variety of techniques in their analysis that should be of interest to advanced students and researchers. This volume begins with a dedication to the memory of our close colleague and friend, Evarist Giné-Masdeu. It is followed by a collection of contributed papers that are organized into four general areas: inequalities and convexity, limit theorems, stochastic processes, and high-dimensional statistics. To give an idea of their scope, we briefly describe them by subject area in the order they appear in this volume.

Dedication to Evarist Giné-Masdeu

- *Evarist Giné-Masdeu July 31, 1944–March 15, 2015.* This article is made up of reminiscences of Evarist's life and work, from many of the people he touched and influenced.

Inequalities and Convexity

- *Stability of Cramer's Characterization of the Normal Laws in Information Distances*, by S.G. Bobkov, G.P. Chistyakov, and F. Götze. The authors establish the stability of Cramer's theorem, which states that if the convolution of two distributions is normal, both have to be normal. Stability is studied for probability measures that have a Gaussian convolution component with small variance. Quantitative estimates in terms of this variance are derived with respect to the total variation norm and the entropic distance. Part of the arguments used in the proof refine Sapogov-type theorems for random variables with finite second moment.
- *V.N. Sudakov's Work on Expected Suprema of Gaussian Processes*, by Richard M. Dudley. The paper is about two works of V.N. Sudakov on expected suprema of Gaussian processes. The first was a paper in the Japan-USSR Symposium on probability in 1973. In it he defined the expected supremum (without absolute values) of a Gaussian process with mean 0 and showed its usefulness. He gave an upper bound for it as a constant times a metric entropy integral, without proof. In 1976 he published the monograph, "Geometric Problems in the Theory

of Infinite-Dimensional Probability Distributions,” in Russian, translated into English in 1979. There he proved his inequality stated in 1973. In 1983, G. Pisier gave another proof. A persistent rumor says that R. Dudley first proved the inequality, but he disclaims this. He defined the metric entropy integral, as an equivalent sum in 1967 and then as an integral in 1973, but the expected supremum does not appear in these papers.

- *Optimal Concentration of Information Content for Log-Concave Densities* by Matthieu Fradelizi, Mokshay Madiman, and Liyao Wang. The authors aim to generalize the fact that a standard Gaussian measure in \mathbb{R}^n is effectively concentrated in a thin shell around a sphere of radius \sqrt{n} . While one possible generalization of this—the notorious “thin-shell conjecture”—remains open, the authors demonstrate that another generalization is in fact true: any log-concave measure in high dimension is effectively concentrated in the annulus between two nested convex sets. While this fact was qualitatively demonstrated earlier by Bobkov and Madiman, the current contribution identifies sharp constants in the concentration inequalities and also provides a short and elegant proof.
- *Maximal Inequalities for Dependent Random Variables*, by J. Hoffmann-Jørgensen. Recall that a maximal inequality is an inequality estimating the maximum of partial sum of random variables or vectors in terms of the last sum. In the literature there exist plenty of maximal inequalities for sums of independent random variables. The present paper deals with dependent random variables satisfying some weak independence, for instance, maximal inequalities of the Rademacher-Menchoff type or of the Ottaviani-Levy type or maximal inequalities for negatively or positively correlated random variables or for random variables satisfying a Lipschitz mixing condition.
- *On the Order of the Central Moments of the Length of the Longest Common Subsequences in Random Words*, by Christian Houdré and Jinyong Ma. The authors study the order of the central moments of order r of the length of the longest common subsequences of two independent random words of size n whose letters are identically distributed and independently drawn from a finite alphabet. When all but one of the letters are drawn with small probabilities, which depend on the size of the alphabet, a lower bound of order $n^{r/2}$ is obtained. This complements a generic upper bound also of order $n^{r/2}$.
- *A Weighted Approximation Approach to the Study of the Empirical Wasserstein Distance*, by David M. Mason. The author shows that weighted approximation technology provides an effective set of tools to study the rate of convergence of the Wasserstein distance between the cumulative distribution function [c.d.f] and the empirical c.d.f. A crucial role is played by an exponential inequality for the weighted approximation to the uniform empirical process.
- *On the Product of Random Variables and Moments of Sums Under Dependence*, by Magda Peligrad. This paper establishes upper and lower bounds for the moments of products of dependent random vectors in terms of mixing coefficients. These bounds allow one to compare the maximum term, the characteristic function, the moment-generating function, and moments of sums of a dependent vector with the corresponding ones for an independent vector with the same

marginal distributions. The results show that moments of products and partial sums of a ϕ -mixing sequence are close in a certain sense to the corresponding ones of an independent sequence.

- *The Expected Norm of a Sum of Independent Random Matrices: An Elementary Approach*, by Joel A. Tropp. Random matrices have become a core tool in modern statistics, signal processing, numerical analysis, machine learning, and related areas. Tools from high-dimensional probability can be used to obtain powerful results that have wide applicability. Tropp's paper explains an important inequality for the spectral norm of a sum of independent random matrices. The result extends the classical inequality of Rosenthal, and the proof is based on elementary principles.
- *Fechner's Distribution and Connections to Skew Brownian Motion*, by Jon A. Wellner. Wellner's paper investigates two aspects of Fechner's two-piece normal distribution: (1) Connections with the mean-median-mode inequality and (strong) log-concavity (2) Connections with skew and oscillating Brownian motion processes.

Limit Theorems

- *Erdős-Rényi-Type Functional Limit Laws for Renewal Processes*, by Paul Deheuvels and Joseph G. Steinebach. The authors discuss functional versions of the celebrated Erdős-Rényi strong law of large numbers, originally stated as a local limit theorem for increments of partial sum processes. We work in the framework of renewal and first-passage-time processes through a duality argument which turns out to be deeply rooted in the theory of Orlicz spaces.
- *Limit Theorems for Quantile and Depth Regions for Stochastic Processes*, by James Kuelbs and Joel Zinn. Contours of multidimensional depth functions often characterize the distribution, so it has become of interest to consider structural properties and limit theorems for the sample contours. Kuelbs and Zinn continue this investigation in the context of Tukey-like depth for functional data. In particular, their results establish convergence of the Hausdorff distance for the empirical depth and quantile regions.
- *In Memory of Wenbo V. Li's Contributions*, by Q.M. Shao. Shao's notes are a tribute to Wenbo Li for his contributions to probability theory and related fields and to the probability community. He also discusses several of Wenbo's open questions.

Stochastic Processes

- *Orlicz Integrability of Additive Functionals of Harris Ergodic Markov Chains*, by Radosław Adamczak and Witold Bednorz. Adamczak and Bednorz consider integrability properties, expressed in terms of Orlicz functions, for "excursions" related to additive functionals of Harris Markov chains. Applying the obtained inequalities together with the regenerative decomposition of the functionals, we obtain limit theorems and exponential inequalities.

- *Bounds for Stochastic Processes on Product Index Spaces*, by Witold Bednorz. In many questions that concern stochastic processes, the index space of a given process has a natural product structure. In this paper, we formulate a general approach to bounding processes of this type. The idea is to use a so-called majorizing measure argument on one of the marginal index spaces and the entropy method on the other. We show that many known consequences of the Bernoulli theorem—complete characterization of sample boundedness for canonical processes of random signs—can be derived in this way. Moreover we establish some new consequences of the Bernoulli theorem, and finally we show the usefulness of our approach by obtaining short solutions to known problems in the theory of empirical processes.
- *Permanental Vectors and Self Decomposability*, by Nathalie Eisenbaum. Exponential variables and more generally gamma variables are self-decomposable. Does this property extend to the class of multivariate gamma distributions? We consider the subclass of the permanental vectors distributions and show that, obvious cases excepted, permanental vectors are never self-decomposable.
- *Permanental Random Variables, M -Matrices, and M -Permanents*, by Michael B. Marcus and Jay Rosen. Marcus and Rosen continue their study of permanental processes. These are stochastic processes that generalize processes that are squares of certain Gaussian processes. Their one-dimensional projections are gamma distributions, and they are determined by matrices, which, when symmetric, are covariance matrices of Gaussian processes. But this class of processes also includes those that are determined by matrices that are not symmetric. In their paper, they relate permanental processes determined by nonsymmetric matrices to those determined by related symmetric matrices.
- *Convergence in Law Implies Convergence in Total Variation for Polynomials in Independent Gaussian, Gamma or Beta Random Variables*, by Ivan Nourdin and Guillaume Poly. Nourdin and Poly consider a sequence of polynomials of bounded degree evaluated in independent Gaussian, gamma, or beta random variables. Whenever this sequence converges in law to a nonconstant distribution, they show that the limit distribution is automatically absolutely continuous (with respect to the Lebesgue measure) and that the convergence actually takes place in the total variation topology.

High-Dimensional Statistics

- *Perturbation of Linear Forms of Singular Vectors Under Gaussian Noise*, by Vladimir Koltchinskii and Dong Xia. The authors deal with the problem of estimation of linear forms of singular vectors of an $m \times n$ matrix A perturbed by a Gaussian noise. Concentration inequalities for linear forms of singular vectors of the perturbed matrix around properly rescaled linear forms of singular vectors of A are obtained. They imply, in particular, tight concentration bounds for the perturbed singular vectors in the ℓ_∞ -norm as well as a bias reduction method in the problem of estimation of linear forms.

- *Optimal Kernel Selection for Density Estimation*, by M. Lerasle, N. Magalhães, and P. Reynaud-Bouret. The authors provide new general kernel selection rules for least-squares density estimation thanks to penalized least-squares criteria. They derive optimal oracle inequalities using concentration tools and discuss the general problem of minimal penalty in this framework.

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